Strategic Assortment Reduction by a Dominant Retailer

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ABSTRACT
In certain product categories, large discount retailers are known to offer shallower assortments than traditional retailers. In this paper, we investigate the competitive incentives for such assortment decisions and the implications for manufacturers’ distribution strategies. Our results show that if one retailer has the channel power to determine its assortment first, then it can strategically reduce its assortment by carrying only the popular variety while simultaneously inducing the rival retailer to carry both the specialty and popular varieties. The rival retailer then bears higher assortment costs, which leads to relaxed price competition for the commonly carried popular variety. We also show that when the manufacturer has relative channel power, it chooses alternatively to distribute both product varieties through both retailers. Our analysis suggests, therefore, that when a retailer becomes dominant in the distribution channel, it facilitates retail segmentation into discount shops, carrying limited product lines, and specialty shops carrying wider assortments. We also illustrate how retailer power leading to strategic assortment reduction can lead to lower consumer surplus.

Key words: channels of distribution; channel power; assortment; retailing; game theory
1. Introduction

Retailers and manufacturers today are increasingly subject to the decisions of a small circle of retailers that include mass merchandisers and wholesale clubs (Raju and Zhang 2005). These mass retailers and warehouse clubs are known to abandon certain specialty varieties and devote more shelf space to the more popular, high volume, brands.\(^1\) By carrying fewer SKU’s, they can cut their retailing costs by making it easier to track inventories (Drèze, Hoch, and Purk 1994).\(^2\) An obvious consequence of reducing assortments is to improve the bottom line as well as to offer more competitive prices. But when a dominant, discount retailer decides not to carry a manufacturer’s specialty products, it may have an effect on the distribution decisions by other channel members. If the dominant retailer is strategic, it evaluates the decisions of other channel members when making its assortment plans. This paper evaluates this aspect of the assortment decision.

Traditionally, it was the manufacturer who defined the breadth of its product line and distributed all varieties through complying retailers. The manufacturer’s decision criterion of whether to distribute a variety was whether there was sufficient demand to cover production and distribution costs. However, these notions of channel management must be reexamined in light of the well-documented shift in channel power toward major retailers (Kadiyali, Chintagunta and Vilcassim 2000). Indeed, these large retailers dictate to their vendors what should be made, in what colors, in what sizes and what they are made of (Bianco et al. 2003, Munson and Rosenblatt 1999). Our objective is to identify the consequence of this shift on manufacturer’s distribution decisions.

We define a retailer as being dominant in the sense that it has the ability to credibly commit not to carry one or more of a manufacturer’s varieties. Our results indicate that this modest gain in channel power is sufficient to upset the distribution outcome relative to the case when the manufacturer has full distributional control. For example, large discount retailers may have the reputation for carrying shallow assortments in order to dedicate valuable shelf space to

\(^1\) For example, Wal-Mart has only a limited selection in various product categories that include groceries (O’Keefe 2002) and baby goods (Desjardins 2005). Another successful mass merchant Target also has a limited assortment in different categories that range from consumer electronics (Master 2001) to automobile supplies (Discount Store News 1999).

\(^2\) According to a recent Fortune article, the strategy of Costco is to provide a limited selection of high quality products. By stocking fewer items it streamlines distribution and hastens inventory turns (Helyar and Harrington, 2003).
popular products. If this is communicated throughout the industry, manufacturers and competing retailers will react accordingly. Alternatively, a dominant retailer, as mentioned above, may dictate what products the manufacturers should make. In these cases, we show that the shift in channel power toward a dominant retailer may have profound implications on the way a manufacturer’s products are distributed, the profitability of a channel and its members, the degree of competition between retailers, and on consumer welfare.

In this paper, we use an analytical model of a manufacturer and two competing retailers to ask how distribution outcomes change when slightly shifting some authority in one of the channels. We find that a dominant retailer may have a profit incentive to refuse to distribute specialty varieties that extends beyond its own operational costs. In particular, if this retailer is strategic, it anticipates that competing retailers may choose to continue carrying a full line of products and, consequently, maintain high assortment costs. This passes on additional benefits to the dominant retailer in the form of relaxed price competition for the popular products. We show that such asymmetric distribution outcomes can arise by a modest shift in channel distribution authority to one of the retailers. We refer to this outcome as strategic assortment reduction and use our model to determine the conditions under which this arises.

Strategic assortment reduction occurs when a manufacturer prefers to distribute its full product line through a retailer but, when dominant, this retailer would rather choose to carry only the popular variety. As such, it reflects channel conflict, or diverging incentives between a manufacturer and an independent retailer regarding the assortment decision. These diverging incentives arise when the manufacturer is unable to set channel specific wholesale prices. Benefits that accrue to the channel through lower assortment costs in one channel are not fully captured by the manufacturer because its uniform wholesale price must reflect demand across the entire set of retailers.

It is interesting to point out that otherwise symmetric retailers may, in fact, carry different assortments. When one retailer wants to abandon a specialty variety, this lowers competition for it, thereby adding an incentive for another retailer to continue carrying it despite the additional assortment costs. This supports a folk wisdom in retailing that tells smaller, non-dominant retailers, who compete against low-cost discounters and the likes of Wal-Mart to find a niche

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3 Such inability is often imposed by legal or trade-specific constraints that require uniform list prices. We comment in section 4 on the implication for our results when these constraints may not hold.
rather than to try to duplicate its low pricing strategy. Rigby and Haas (2004), for example, suggest that there is a substantial segment of consumers who are willing to pay non-discount prices in return for a wider assortment. Our model indicates that a shift in channel authority plays a role in this retail segmentation story.

The growth of the discount retail format has raised concern about the availability of specialty products and the impact on consumer welfare. Given the size and dominance of these discounters, some critics argue that their narrow assortments make specialty varieties harder to find (e.g., Bianco et al. 2003). Our research formally investigates these issues and provides some support as well as some refutation of the critics’ concerns.

Specifically, we assess the conditions under which retail dominance leads the manufacturer to abandon a product following the dominant retailer’s decision to do so. A necessary condition for strategic assortment reduction is that the manufacturer does not want to eliminate a variety. Obviously, a product will not be produced if there is insufficient consumer demand. But, even if there is a profitable market for the product from the manufacturer’s point of view, a dominant retailer may wish not to carry it to avoid added assortment costs.

Our model also permits us to measure the consequence of strategic assortment reduction on consumer welfare. We show that when channel authority is shifted from the manufacturer to a dominant retailer it leads to unambiguously lower levels of consumer surplus. The intuition is that a retailer does not account for double marginalization losses in competing retail channels and is, therefore, quicker to abandon a specialty variety before the manufacturer would. And, because retailers carrying a specialty variety have less downstream competition, double-marginalization inefficiencies are enhanced. Thus, consumers, on the whole, suffer as a result of strategic assortment reduction. This result, therefore, legitimizes some of the critics’ concerns about the detriment of retail dominance on consumer well-being.

At a general level, our work falls in line with the growing body of research investigating the implications of shifting channel power in the retail sector. Dukes, Gal-Or, and Srinivasan (2006), Iyer and Villas-Boas (2003), and Raju and Zhang (2005), for example, examine the implications of changing channel power on the distribution of channel profits. The current work,

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4 According to Fortune, this is how “Central Market”, a new gourmet superstore recently opened by San Antonio-based H.E. Butt Grocery Co. whose H.E.B chain is the second largest private supermarket business in the country, competes with Wal-Mart. Central Market has a huge selection: some 30 types of apples, twenty kinds of homemade sausage, 2300 labels of wine and 400 types of beer (O’Keefe. 2002).
in contrast, examines the implication of this shift of channel power on distribution and product line decisions. This is similar in spirit to Geylani, Dukes and Srinivasan (2006), which investigates the implication of this shift on manufacturer’s joint promotions and advertising decisions.

More specifically, this paper contributes to the literature on retailing, variety, and assortment. There is a substantial literature in marketing that examines the trade-offs associated with the assortment decision. The early work of Baumol and Ide (1956) identifies that the benefit of wide assortments in attracting customers must be balanced with the additional stocking and inventory costs, and Nilsson and Høst (1987) offer operational tools to navigate these costs and benefits. This trade-off is present in our paper. However, we take a closer look at the impact of a store’s assortment on decisions upstream and across to competing stores.

More recently, Messinger and Narasimhan (1997) empirically verify a trend in consumers’ assessment of the assortment trade-off. Specifically, they show that growing opportunity costs of shopping (e.g. higher wages) have increased the value of assortment to grocery stores. Hoch, Bradlow and Wansink (1999) identify why and by how much a consumer cares about assortment at a given store. Briesch, Chitagunta and Fox (2005) examine the impact of assortment on consumers’ grocery store choice. Alternatively, Boatwright and Nunes (2001), Borle et al. (2005), and Broniarczyk, Hoyer and McAlister (1998) evaluate the impact of assortment reduction of a retailer on sales, customer retention, and consumers’ perception of the variety, respectively, at a given store. These works offer a deep examination of assortment on consumers’ store choices and perceptions. In contrast, we abstract from the consumer decision in order to evaluate the impact of a retailer’s assortment decision on channel management and distribution.

Another stream of research has assessed the impact of the evolving retail sector on upstream decisions and the variety of consumer products. For example, Marvel and Peck (2004), motivated by the apparent reduction in shirt and shoe size offerings, point out that retailers may have an incentive to carry limited varieties in order to segment and differentiate the market. Similarly, our paper points to strategic incentives to reduce assortment. In contrast, the strategic incentive of assortment reduction in our paper is to pass on costs to the competing retailer in order to gain a strategic advantage.
Allain and Waelbroeck (2006) and Inderst and Shaffer (2006) evaluate the impact of retail concentration on the upstream incentive to offer product variety. Allain and Waelbroeck (2006) argue that retail concentration may explain the documented reduction in new CD releases by limiting manufacturers ability to segment the market (e.g. with new and old releases). Inderst and Shaffer (2006) identify how a merger of non-competing retailers can commit to fewer products varieties in order to leverage market power over consumers and thereby increase its upstream buyer power. This encourages manufacturers’ to narrow their product lines. Overall economic welfare, therefore, declines, as a result of the retail merger. Neither of these papers, however, focuses on the strategic incentive of one retailer over another with respect to its assortment decision as is done in our paper.

Finally, this paper contributes to research on retail segmentation. Bhatnagar and Ratchford (2004) identify consumer heterogeneity with respect to breadth of assortment as a significant factor in determining consumer choice across retail formats. Our paper contributes to this reasoning by further suggesting that retail dominance also plays a natural role in segmenting discount stores from specialty stores. Zhu, Singh, & Dukes (2006), point out that the location of an entering discount retailer plays a role in altering consumer shopping patterns inducing traditional incumbent retailer to segment across income levels. Again, the current paper illustrates how resultant segmentation may arise simply from incentives within the channel.

The general model is presented and analyzed in the next section. This section also characterizes the equilibrium outcomes in both variants of the model: the manufacturer-dominant model and the retailer-dominant model. Section 3 evaluates the consequences of strategic assortment reduction on consumer welfare. Section 4 summarizes and concludes with managerial implications. An appendix contains the omitted details of the analysis and all proofs.

2. A Model of Assortment Choice

The objective of our analysis is to illustrate the consequence of retailer dominance on assortment outcomes. As such, our analytics can be seen as a controlled exercise to isolate the impact of changing the assortment decision maker. We consider a game-theoretic model consisting of three players: two retailers, $A$ and $B$, and a manufacturer, $M$.

The manufacturer has two products in its product line, call product 1 and 2, and distributes these products to two competing retailers. The game is played in three stages:
Stage 1.  \( M \) decides its distribution plan and then sets wholesale (supply) prices for the products it sells.

Stage 2.  Retailers \( A \) and \( B \) choose quantities, which are subsequently stocked and sold.

We consider two variants of this model. Each variant depicts one of the two channel dominant settings investigated. In the \( M \)-Dominant game, \( M \) fully determines the assortments for both the retailers in stage 1. This is contrasted with the \( A \)-Dominant game, which has an additional starting stage, referred to as Stage 0, in which retailer \( A \) announces which products it will (or won’t) carry in its assortment. Subsequently, in stage 1, \( M \) decides what to distribute through retailer \( B \). We employ this notion of dominance to illustrate the profound impact of a slight shift in channel distribution authority to one of the retailers. As we show, this additional channel power is sufficient to alter the equilibrium distribution, which may lower consumer welfare and, in some cases, be Pareto inferior for the channel.

The trade off in the assortment decision concerns assortment costs, which are defined by expenses associated with monitoring and handling additional SKU’s. These are incurred by the retailer and depend on number of products carried. Specifically, we suppose that each retailer faces the same marginal cost function, \( c(n) \), where \( n \) is the number of products it carries and \( c(2) > c(1) \geq 0 \). This specification reflects diseconomies of scale in the breadth of product assortment. Specifically, by carrying a lower number of SKU’s, retailers have lower inventory and handling costs. While these costs are borne directly by the retailer, they have implications for the manufacturer’s distribution strategy. This is due to the fact that assortment costs affect channel margins.

The products can be thought of as two varieties in the manufacturer’s product line. One variety is more popular than the other. For example, a compact disc (CD) manufacturer makes and distributes many varieties of music. There is a new CD from a popular band and then there is one from a lesser known band. Product 1 denotes the “popular” variety and product 2 the “specialty” variety. We impose the assumption that these products do not compete for each other. This simplifies the analysis and clarifies the strategic incentive in assortment reduction. We analyze the last two stages of the model as follows.

During stage 1, the manufacturer sets a wholesale price \( w_i \), for each product \( i \), provided it is to be sold. Retailers subsequently choose quantities in stage 2. Let \( q_i \), be the amount of
product $i$ bought by retailer $j = A, B$. Retailers then sell this quantity to the market. The retail price of each product is determined by the total quantity in the market:  

$$p_i(q_i^A, q_i^B) = a - b_i(q_i^A + q_i^B),$$  \hspace{1cm} (1)$$

where $a > 0$ and $b_i < b_2$. This specification captures our assumption that product 2 is a “specialty” variety. A larger coefficient $b$ indicates that the price needed to clear the shelves of this product declines more quickly with respect to quantity. Note that we assume the price of variety $i$ does not depend on the price of variety $j$, implying that the two varieties do not directly compete.  

Let $c^j = c(n^j), j = A, B$ denote the retailers’ marginal cost depending on the number $n^j$ of varieties carried. Retailer $j$, if she is to carry product $i$, chooses a quantity

$$q_i^j = \arg \max_{q>0}[a - b_i(q + q_i^{-j}) - c^j - w_i]q$$

where $q_i^{-j}$ represents the quantity of product $i$ carried by the rival retailer. If retailer $j$ does not carry product $i$ then $q_i^j = 0$. This yields the stage 2 quantities

$$q_i^j = \begin{cases} 0 & \text{if } j \text{ does not carry } i \\ \frac{a - w_i - 2c^j + c^{-j}}{3b_i} & \text{otherwise} \end{cases}$$ \hspace{1cm} (2)$$

and profit to retailer $j$ from product $i$$

$$\Pi_i^j = p_i(q_i^A, q_i^B) - c^j - w_i \left[q_i^j\right].$$ \hspace{1cm} (3)$$

Total profits for retailer $j$ are the product profits in (3) summed over products: $\Pi^j = \sum_{i=1,2} \Pi_i^j$. Given the quantities expressed in (2), $M$ chooses wholesale prices in stage 1 to maximize profits. Because of the competitive independence of products 1 and 2, profit maximization is equivalent to maximizing individual product profits, defined by

$$\Pi_i^M = w_i(q_i^A + q_i^B),$$ \hspace{1cm} (4)$$

5 In this sense, we model competition between the two retailers as “Cournot,” which lets us focus on the assortment decisions of channel members rather than defining details about consumer choice. This yields an identical outcome as a two-stage “Bertrand” game if one considers each retailer’s quantity decision (capacity) chosen first and followed by price setting (Kreps & Scheinkman, 1983).

6 This is made for simplicity. As we discuss in section 4, the presence of competition between varieties tends to reinforce the basic motive for strategic assortment reduction.
subject to (2) over \( w_i \geq 0 \) for \( i = 1,2 \). Total profits for the manufacturer are the product profits in (4) summed over products: \( \Pi^M = \sum_{i=1,2} \Pi_i^M \). Using \( M \)'s first order conditions for this maximization and the retailers’ optimal reactions in (2), a straightforward derivation leads to a general characterization, which, for brevity, is relegated to Lemma A.1 in the Appendix. This characterization allows us to simplify the analysis by considering four relevant distribution outcomes. These four outcomes are depicted in Figure 1.

The first outcome has the manufacturer distributing only the popular product, product 1. We call this outcome, single product with dual distribution \((S)\). Note that since the market for product 2’s market is smaller, distributing only product 2 through both retailers is always dominated by distributing only product 1. Forthwith, we ignore the single distribution of product 2. A second outcome, which we call full product line with dual distribution \((F)\) involves selling
the entire product line through both retailers. As an asymmetric case, it is possible to have product 1 distributed through retailer $A$ and both products through retailer $B$. This we term as *full product line with specialty distribution (Sp)*. Finally, it is possible for *full product line with exclusive distribution (Ex)* in which each retailer is given exclusive sale of one of the two products.

Obviously, the distributional arrangements depicted above do not represent all possible. We omit all strategically dominated arrangements such as distribution $S$ with product 2, as already mentioned above. Similarly, distributing both products exclusively through one retailer is dominated by $Ex$ whenever assortment costs are strictly positive. Finally, note that distributions $Sp$ and $Ex$ each have mirror counterparts with respect to retailers $A$ and $B$. However, we have assumed that retailers are symmetric, which implies that the outcomes depicted in Figure 1 (weakly) dominate them for $M$. This is not the case, however, when retailers have asymmetric assortments costs. We return to this case in section 2.3.

Without loss of generality, normalize retail cost by defining $c \equiv c(2) > c(1) = 0$. This normalization offers the interpretation that $c$ represents the increase in unit cost of carrying an additional product. This incorporates the notion that handling an additional product increases the marginal cost for all products because of the added sorting and tracking of multiple products in the store. Under this assumption, the relevant payoffs and quantities can be compactly expressed, which we do in the following proposition.

**PROPOSITION 1**

Let $c < a$ and $b_1 < b_2$. Payoffs in the four distribution outcomes given in Figure 1 are expressed as in Table 1.

Proposition 1 allows us to reduce the analysis of both the $M$ and $A$ Dominant games to a series of comparisons in payoffs. Our particular interest is in understanding the impact of retail dominance on the distribution outcome. To do this we identify regions of the parameters space in $a, c, b_1$ and $b_2$ such that, if held constant, the equilibrium distribution outcome changes when dominance changes – moving from the $M$-Dominant regime to the $A$-Dominant regime.
Table 1: Payoffs and Quantities in the Distribution Outcomes of Figure 1

<table>
<thead>
<tr>
<th>Outcome:</th>
<th>$S$</th>
<th>$F$</th>
<th>$Sp$</th>
<th>$Ex$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ Profit</td>
<td>$\Pi^M_S = \frac{a^2}{6b_1}$</td>
<td>$\Pi^M_F = \frac{(a-c)^2}{6b_1} + \frac{(a-c)^2}{6b_2}$</td>
<td>$\Pi^M_{Sp} = \frac{(a-x)^2}{6b_1} + \frac{(a-c)^2}{8b_2}$</td>
<td>$\Pi^M_{Ex} = \frac{a^2}{8b_1} + \frac{a^2}{8b_2}$</td>
</tr>
<tr>
<td>$A$ Profit</td>
<td>$\Pi^A_S = \frac{a^2}{36b_1}$</td>
<td>$\Pi^A_F = \frac{(a-c)^2}{36b_1} + \frac{(a-c)^2}{36b_2}$</td>
<td>$\Pi^A_{Sp} = \frac{(a-x)^2}{36b_1} + \frac{(a-c)^2}{16b_2}$</td>
<td>$\Pi^A_{Ex} = \frac{a^2}{16b_1}$</td>
</tr>
<tr>
<td>Quantity</td>
<td>$q_i^A = \frac{a}{6b_1}, i=1,2$</td>
<td>$q_i^A = \frac{a-c}{6b_1}, i=1,2$</td>
<td>$q_i^A = \frac{a-x}{6b_1}, i=1,2$</td>
<td>$q_i^A = \frac{a}{4b_1}$</td>
</tr>
<tr>
<td>$B$ Profit</td>
<td>$\Pi^B_S = \frac{a^2}{36b_1}$</td>
<td>$\Pi^B_F = \frac{(a-c)^2}{36b_1} + \frac{(a-c)^2}{36b_2}$</td>
<td>$\Pi^B_{Sp} = \frac{(a-x)^2}{36b_1} + \frac{(a-c)^2}{16b_2}$</td>
<td>$\Pi^B_{Ex} = \frac{a^2}{16b_1}$</td>
</tr>
<tr>
<td>Quantity</td>
<td>$q_i^B = \frac{a}{6b_1}, i=1,2$</td>
<td>$q_i^B = \frac{a-c}{6b_1}, i=1,2$</td>
<td>$q_i^B = \frac{a-x}{6b_1}, i=1,2$</td>
<td>$q_i^B = \frac{a}{4b_1}$</td>
</tr>
</tbody>
</table>

2.1 The $M$-Dominant Game

In the $M$-Dominant version of the game, $M$ announces which products it expects the two retailers to carry. This dominance can be implemented in the model by allowing $M$ to impose a penalty on a retailer who orders zero quantity of a product $M$ wants the retailer to take. In contrast, the $A$-Dominant version of the game has retailer $A$ declaring which of the manufacturer’s products it will and will not handle. The manufacturer then decides how to distribute its products under the constraint imposed by $A$’s declaration.

We first consider the $M$-Dominant regime and examine what distributional arrangements are optimal for $M$, given the parameters $a, c, b_1$ and $b_2$. This is done by a straightforward comparison of $M$’s profits in the first row of Table 1 over the parameter space. The orderings of these profits depend on the two ratios $c/a \in [0,1]$ and $b_1/b_2 \in [0,1]$. The first ratio represents the size of the assortment cost relative to the market size for the product category. The second ratio measures the distribution of the market across the two varieties. In particular, the larger the ratio, the more equally the market is distributed over the two varieties. In Proposition 2, we fully characterize the equilibria of the $M$-Dominant game.

**PROPOSITION 2**

Let $(c/a, b_1, b_2) \in [0,1]^2$ and denote $\zeta \equiv \frac{b_1-5}{3} \approx 0.64$. Then there exist functions $f(b_1/b_2), g(b_1/b_2), h(b_1/b_2)$ with $0 < g(b_1/b_2) < f(b_1/b_2) < 1$ for all $b_1/b_2 \in (0,1)$.
\[ g(b_1 / b_2) < h(b_1 / b_2) < f(b_1 / b_2) \] for all \( b_1 / b_2 \in (\frac{1}{3}, \zeta) \) and \( h(b_1 / b_2) < g(b_1 / b_2) \) for all \( b_1 / b_2 \in (\zeta, 1) \) which characterize the equilibria of the M-Dominant game as follows.

(i) If \( 0 < b_1 / b_2 < \frac{1}{3} \) then the unique equilibrium outcome is

\[ F \text{ if and only if } 0 < c / a < g(b_1 / b_2); \]
\[ Sp \text{ if and only if } g(b_1 / b_2) < c / a < f(b_1 / b_2); \]
\[ S \text{ if and only if } f(b_1 / b_2) < c / a < 1. \]

(ii) If \( \frac{1}{3} < b_1 / b_2 < \zeta \) then the unique equilibrium outcome is

\[ F \text{ if and only if } 0 < c / a < g(b_1 / b_2); \]
\[ Sp \text{ if and only if } g(b_1 / b_2) < c / a < h(b_1 / b_2); \]
\[ Ex \text{ if and only if } h(b_1 / b_2) < c / a < 1. \]

(iii) If \( \zeta < b_1 / b_2 < 1 \) then the unique equilibrium outcome is

\[ F \text{ if and only if } 0 < c / a < g(\zeta); \]
\[ Ex \text{ if and only if } g(\zeta) < c / a < 1. \]

The interpretation of this proposition is facilitated by examining the graph in Figure 2, which depicts the equilibrium outcomes in all regions of the parameter space \((b_1 / b_2, c / a)\). The function \( f \) is defined by \( M \)'s indifference across distribution outcomes \( S \) and \( Sp \). Similarly, the functions \( g \) and \( h \) are defined by \( M \)'s indifference between outcomes \( Sp \) and \( F \) and between outcomes \( Ex \) and \( Sp \), respectively.

Part (i) of Proposition 2 corresponds to the case when product 1 is significantly more popular than product 2 – low values of \( b_1 / b_2 \). When assortment costs are relatively high, \( M \) chooses to only distribute the popular product through both retailers (outcome \( S \)). As assortment costs decrease somewhat, it becomes optimal for \( M \) to introduce the specialty variety through one of the retailers (outcome \( Sp \)). And finally, when assortment costs decrease sufficiently, full distribution through both retailers is optimal for the manufacturer (outcome \( F \)).

Parts (ii) and (iii) of Proposition 2 correspond to the cases where the relative popularity of product 2 is increased. In particular, with the exception of an intermediate region, if the ratio \( b_1 / b_2 \) exceeds \( 1/3 \), the manufacturer views the two products similarly and outcomes \( Ex \) and \( F \) dominate the parameter space. That is, when products are more similar in market size, \( M \)'s trade-
off is simply the benefit of additional revenue from broader market coverage versus the channel losses from assortment costs. Therefore, it is optimal for $M$ to establish exclusive territories for each product (outcome $Ex$) when assortment costs are large and open the market for both products (outcome $F$) as when assortment costs decrease.

![Figure 2: Equilibrium Outcomes in M-Dominant Game](image)

## 2.2 The $A$-Dominant Game: Strategic Assortment Reduction

We now turn to an analysis of the $A$-Dominant game. In this game, retailer $A$ has the channel power with regard only to its assortment. What to distribute through retailer $B$ remains in $M$’s control. Obviously, if it is optimal for $M$ to distribute only one product – always product 1 – through retailer $A$, then $A$ will never refuse to carry it and, as a result, $A$’s channel power would be of no consequence. Therefore, the relevant region for the analysis of the $A$-Dominant game is the region in which it is optimal for $M$ to choose $F$ in the $M$-Dominant game. To make matters precise we define the following notation. Let $\Lambda = \{Sp, Ex, S\}$ be the set of possible outcomes in which retailer $A$ does not carry product 2 and $\Theta \subset [0,1]^2$ be the parameter space such that $\Pi^M_k > \Pi^M_x$ for all $x \in \Lambda$. Specifically,
\[ \Theta = \{(b_1/b_2,c/a) \in [0,1]^2 \mid c/a < \min\{g(b_1/b_2), g(\zeta)\}\}, \]

which corresponds to the lower region of the graph in Figure 2.

Refusing to carry product 2 is optimal for \( A \) only if she prefers another outcome \( x \in \Lambda \) over \( F \). This is a necessary, but not sufficient, condition for strategic assortment reduction. In particular, if retailer \( A \) is fully strategic, which is assumed, then it evaluates the consequence of refusing to carry product 2 on \( M \)'s distribution strategy in its other retail channel. Suppose that \( A \) rejects product 2 in period 0. Then \( M \) has three options with respect to retailer \( B \): distribute both varieties, product 1 only, or product 2 only. This corresponds to the outcomes in \( \Lambda \). We can now state the exact conditions for a strategic assortment reduction in equilibrium as follows: \( x \) is a strategic assortment reduction equilibrium outcome if and only the following two conditions hold:

(i) \( \Pi_x^A > \Pi_f^A; \) and (ii) \( \Pi_x^M > \Pi_y^M \) for all \( y \neq x; y \in \Lambda \).

Condition (i) says that retailer \( A \) has, in fact, an incentive to change the outcome from \( F \), while condition (ii) requires that \( M \)'s subsequent distribution strategy is optimal given \( A \) does not carry product 2.

From (ii), we can immediately rule out \( S \) as a possible equilibrium outcome in this game. Recall that \( M \) prefers the outcome \( Sp \) to \( S \) in regions below the curve \( f \) in Figure 2. And since \( \Theta \) lies entirely below the curve \( f \), assortment costs are always sufficiently low so that \( M \) prefers to distribute both varieties through \( B \) over distributing only product 1.

With \( S \) ruled out as an optimal choice for \( M \), we can use the results of Propositions 1 and 2 to determine her optimal choice among \( Ex \) and \( Sp \) for parameters in in \( \Theta \). In particular, because the curve defined by \( h \) determines \( M \)'s indifference between these two outcomes, we conclude that \( M \) prefers \( Ex \) for regions of \( \Theta \) above \( h \) and \( Sp \) for regions below \( h \).

Finally, we turn to condition (i) to determine retailer \( A \)'s optimal decision whether to carry product 2 given \( M \)'s reaction described above. First observe that a comparison of profit expressions in Proposition 1 yields \( F \) as retailer \( A \)'s preferred outcome over \( Sp \) if and only if

\[ c/a < k(b_1/b_2) \equiv \frac{\gamma - 1}{\gamma + \frac{1}{2}}, \quad \text{where} \quad \gamma \equiv \sqrt{1 + \frac{b_2}{b_1}}. \quad (5) \]

The indifference curve defined by \( k \) lies everywhere below \( g \) which implies that \( F \) is the equilibrium outcome of the \( A \)-Dominant game for all \( (b_1/b_2,c/a) \) with \( c/a < k(b_1/b_2) \). In the
regions of $\Theta$ above $k$ and below the curves $g$ and $h$, retailer $A$ prefers $Sp$ over $F$ and the manufacturer earns the highest profit with $Sp$ given that she cannot implement $F$. Thus, $Sp$ must be the equilibrium outcome in this region. See Figure 3.

Finally we examine $A$’s preferences in the remaining sector of $\Theta$ above the curve $h$, which is the region $M$ establishes exclusive territories ($Ex$) in reaction to $A$ not carrying product 2. It is readily checked by comparing profits $\Pi_0^A$ and $\Pi_\varphi^A$ that $A$ prefers $Ex$ over $F$ if and only if

$$c / a > l(b_1 / b_2) \equiv 1 - \frac{3 \sqrt{b_1 / b_2}}{2 \sqrt{1 + b_1 / b_2}}. \quad (6)$$

Furthermore, the indifference curve defined by $l$ lies everywhere below $h$, which implies that condition (i) holds for $Ex$ in this region. The equilibrium of the $A$-Dominant game is now fully characterized.

**PROPOSITION 3** Let $g$ and $h$ be the functions determined in Proposition 1,

$$\zeta \equiv \frac{4 \sqrt{5} - 5}{5} \approx 0.64, \text{ and } (b_1 / b_2, c / a) \text{ be in } \Theta. \text{ Then the equilibria of the } A \text{-Dominant game is described as follows.}$$

(i) If $0 < b_1 / b_2 < \zeta$ then the unique equilibrium outcome is

- $F$ if and only if $0 < c / a < k(b_1 / b_2)$;
- $Sp$ if and only if $k(b_1 / b_2) < c / a < g(b_1 / b_2)$.

(ii) If $\zeta < b_1 / b_2 < 1$ then the unique equilibrium outcome is

- $F$ if and only if $0 < c / a < k(b_1 / b_2)$;
- $Sp$ if and only if $k(b_1 / b_2) < c / a < h(b_1 / b_2)$;
- $Ex$ if and only if $\max \{h(b_1 / b_2), k(b_1 / b_2)\} < c / a < g(\zeta)$.

Proposition 3 establishes that $A$’s dominance in determining assortment does, in fact, alter the distribution of $M$’s products. Specifically, when $Ex$ or $Sp$ is the equilibrium of the $A$-Dominant game, $M$ would have chosen $F$ in the $M$-Dominant game. Proposition 3 also states that $A$’s channel dominance need not always alter the distribution. In fact, for low assortment costs, both $M$ and $A$ prefer the outcome be $F$.

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7 This is shown in the proof of Proposition 3.
Figure 3 summarizes the equilibrium outcomes in the *A-Dominant* game and is helpful for interpreting the previous result. The regions above curve $k$ indicate when *strategic assortment reduction* is an equilibrium outcome. The intuition behind $M$ and $A$’s diverging preferences can be seen by understanding when the channel member switches from $F$ to $Sp$ as assortment costs increase. As indicated in Figure 3, $A$’s indifference curve $k$ lies everywhere below $M$’s indifference curve $g$. For any fixed market size distribution, $b_1 / b_2$, retailer $A$ prefers $Sp$ over $F$ for lower assortment costs $c/a$ than does $M$. The reason is that $A$ accrues benefits from higher assortment costs in the $Sp$ outcome (from Proposition 1, $\partial \Pi_{Sp}^A / \partial c > 0$). When $A$’s rival $B$ has higher costs, its retail price for product 1 is higher, and $A$ benefits from the *relaxed price competition*, as a result.

The manufacturer, $M$, on the other hand, does not fully benefit from this relaxed retail price competition because it is unable to price discriminate, by assumption. Thus it cannot unilaterally raise its wholesale price to capture the additional channel surpluses with $A$. In fact, $M$ is forced to reduce its uniform wholesale price (in $Sp$, $w_1 = \frac{1}{2}(a - \frac{c}{2})$) as assortment costs increase, which is another benefit for $A$ in the $Sp$ outcome. These benefits imply that $A$ will abandon product 2 before it is optimal for $M$ to do so. Therefore, $Sp$ is a strategic assortment reduction equilibrium in the region indicated by Proposition 3.

This result can be seen in a broader context of retail segmentation. In the strategic assortment reduction outcome $Sp$, the dominant retailer, $A$, lowers its costs while inducing higher costs on its rival. In addition, $A$ carries higher volumes than $B$: $q_1^A - q_1^B = c / b_1 > 0$. Our analysis suggests, therefore, that when a retailer becomes dominant in the distribution channel, it may facilitate retail segmentation into discount shops, carrying large volumes of limited product lines, and specialty shops, which carry wider assortments and “harder-to-find” varieties.

Part (ii) of Proposition 3 says that exclusive distribution of products 1 and 2 ($Ex$) can also be a strategic assortment reduction outcome in equilibrium. This is due, however, to the fact that in regions above the indifference curve $h$, $M$ prefers $Ex$ over $Sp$. As product 2 becomes relatively more popular (higher $b_1 / b_2$), $M$ favors exclusive distribution and does not incur the channel losses associated with higher assortment costs. Unlike in the $Sp$ outcome discussed above, $Ex$ affords the manufacturer individualized wholesale pricing. Thus, the added channel surplus that comes with low retail costs can be absorbed with wholesale price, $w_2$. 

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In contrast, $A$ would prefer that $M$ distribute product 1, in addition to product 2, through retailer $B$ and increase his assortment costs. When retailer $A$ refuses to distribute product 2, she would actually prefer that $M$ implement $Sp$ rather than $Ex$. Consequently, in the $A$-Dominant game, retailer $A$ forces an outcome that is second-best for itself. Nevertheless, $A$’s refusal to carry product 2 is strategically optimal since $F$ is a worse outcome than $Ex$. The above discussion implies that retail dominance in assortment choice can lead to Pareto inferior outcomes for the entire channel.

2.3 Asymmetric Assortment Costs

We previously assumed that the only distinction between the two retailers was that $A$ acquired the ability to move first and announce its refusal to carry one of the manufacturer’s products. It is natural to suppose that this ability may come with other advantages, such as being more efficient. Suppose, for instance, that retailer $A$ has lower assortment costs than retailer $B$. From the overall channel perspective, in this case, there are obviously advantages to using $A$ to distribute both varieties. Recall the specialty distribution ($Sp$) from Figure 1. If this is optimal for $M$, then
clearly $M$ would choose $A$ over $B$ for distributing the entire product line. This corresponds to $SpA$ in Figure 4. The question we ask here is: would $A$ abandon to carry the specialty variety even though it is more efficient in assortment than its rival $B$? As we show, the answer is yes.

Formally, let $0 = c^j(1) < c^j(2) \equiv c^j$ for $j = A, B$ with $c^A < c^B$, which reflects the efficiency advantage of retailer $A$ over $B$. We evaluate the consequence of a shift in channel power from the manufacturer to retailer $A$ by considering the $M$-Dominant and $A$-Dominant games in a similar vein as before. In order to draw on the previous analysis, define $c = \frac{c^A + c^B}{2}$ to be the average assortment cost and consider points $(b_1 / b_2, c / a) \in [0,1]^2$ which lead to $Sp$ in the $M$-Dominant game of section 2.1. Specifically, restrict attention to the region defined by

$$\Theta_{Sp} = \{(b_1 / b_2, c / a) \in [0,1]^2 \mid g(b_1 / b_2) < c / a < \min\{f(b_1 / b_2), h(b_1 / b_2)\}\},$$

which is the region denoted by $Sp$ in Figure 2. By construction, all parameter constellations residing in $\Theta_{Sp}$ lead to some form of specialty distribution being optimal for the manufacturer. The fact that $c^A < c^B$ implies $SpA$ is the optimal specialty distribution for $M$ and is, thus, the equilibrium outcome in the $M$-Dominant game. In the case of asymmetric costs, strategic assortment reduction is said to occur whenever $SpA$ is the equilibrium of the $M$-Dominant game, but $SpB$ is the equilibrium of the $A$-Dominant game.

In the $A$-Dominant game, the question of whether $A$ will abandon product 2 or not is determined by the order relation between profits

$$\Pi_{SpA}^A = \frac{1}{36a}(a - \frac{5c^A}{2})^2 + \frac{1}{16b_2}(a - c^A)^2 \quad \text{and} \quad \Pi_{SpB}^A = \frac{1}{36a}(a + \frac{5c^B}{2})^2,$$

which can be derived applying Lemma A.1 in the appendix. When comparing these profits, note that the relative assortment cost difference increases $A$’s benefit to $SpB$

$$\left(\frac{\partial \Pi_{SpB}^A}{\partial c^B} > 0 \right) \left(\frac{\partial \Pi_{SpA}^A}{\partial c^A} > 0 \right).$$

Furthermore, this benefit is stronger for a relatively popular product 1 (small ratio $b_1 / b_2$). This latter condition is similar to the manufacturer’s necessary condition for choosing a specialty distribution in the $M$-Dominant game. In particular, $(b_1 / b_2, c / a) \in \Theta_{Sp}$ ensures that $b_1 / b_2 \leq \zeta \approx 0.64$. This reasoning suggests that $A$ will, in fact, always abandon the specialty product whenever $M$ finds it optimal to use $A$ for a specialty distribution. This is formally established in Proposition 4.
PROPOSITION 4 Let $c^A < c^B$, $(b_1 / b_2, c^j / a)$ be in $\Theta_{Sp}$ for $j = A, B$. Then the equilibrium of the M-Dominant game is SpA and the equilibrium of the A-Dominant game is SpB.

This result implies that despite being more efficient in handling assortment, $A$ prefers an inefficient rival to carry the burden of assortment. In addition, retailer $A$ abandons the specialty variety in the A-Dominant game whenever and despite the fact that SpA is optimal for the manufacturer.

This last result has direct implications for the channel. In particular, $A$’s dominance will always lower channel efficiencies. Moreover, because this leads to higher retailer costs, consumers pay higher prices and consume less in SpB than in SpA. Therefore, $A$’s dominance lowers consumer welfare. We investigate the consumer welfare issue in more detail in the next section.

3. Strategic Assortment Reduction and Consumer Welfare

In this section we ask whether strategic assortment reduction, as a consequence of retail dominance in the channel, increases or decreases the welfare of consumers. Consumers benefit when more products are available through more channels, but suffer the portion of assortment costs passed through retail prices. In the asymmetric cost case of section 2.3, this trade-off was
clear because strategic assortment reduction simply resulted in higher retailing costs without changing the number of distribution channels. However, as we saw in section 2.2, strategic assortment reduction reduces the number of distribution channels of product 2 while simultaneously lowering costs. Thus, understanding the consumer welfare trade-off in the symmetric case requires additional analysis, to which the remainder of this section is devoted.

To investigate the impact of strategic assortment reduction on consumer welfare, we determine the outcome that maximizes consumer surplus, which is computed as follows. For a given outcome \( x \in \{F, Sp, Ex\} \), denote total output for product \( i \) as \( q_i(x) = q_{i1} + q_{i2} \), \( i = 1, 2 \) where \( q_{ij} \) are the individual quantities listed in Table 1 from Proposition 1 under the corresponding outcome. Then consumer surplus under outcome \( x \) is

\[
CS_x = \sum_{i=1,2} \left\{ \int_0^{q_i(x)} \left[ p_i(q) - p_i(q_i(x)) \right] dq \right\} = \sum_{i=1,2} \frac{b_i}{2} \left[ q_i(x) \right]^2 ,
\]

which is the area between the demand curve (1) and the price. As the right-hand side expression in (8) indicates, consumer surplus depends crucially on the output \( q_i \) of the two varieties. Obviously, the level of output of each product, in turn, depends on the number of retail outlets through which the products are distributed. This suggests that full distribution, \( F \), has the most quantity, followed by the specialty distribution, \( Sp \) and finally exclusive territories, \( Ex \). However, as retailers take on a second product, assortment costs raise retail costs and lower output. The following proposition confirms that the former effect dominates in the region \( \Theta \).

**PROPOSITION 5** For \( (b_1 / b_2, c / a) \) in \( \Theta \), consumer surplus is maximized with the outcome \( F \) and minimized with the outcome \( Ex \). Specifically, \( CS_F > CS_{Sp} > CS_{Ex} \).

The implication of this proposition is that any strategic assortment reduction induced by \( A \)’s dominance lowers consumer surplus. In fact, under the conditions leading to outcome \( Ex \) in equilibrium (Proposition 3, (ii)) of the \( A-Dominant \) game, consumer surplus is minimized.

This result is related to inefficiencies associated with traditional monopoly power, which each retailer has to some degree. Specifically, retailer \( A \) abandons product 2 at lower levels of assortment costs \( c \) than is socially optimal because these costs affect the retailer’s profit on all inframarginal units sold. Consequently, when \( A \) implements \( Sp \) it adversely affects consumers.
The adverse affect that comes with this strategic assortment reduction is partially counter-veiled by the fact that A’s costs are lower in Sp leading to higher output (lower prices) for product 1 than in F. In fact, from Proposition 1, we note that total output of the popular variety increases going from F to Sp.

\[
q_1(Sp) = \frac{a - \frac{c}{b_2}}{3b_1} > \frac{a - c}{3b_1} = q_1(F).
\]

This implies directly that consumers of the popular variety, product 1, benefit from strategic assortment reduction. Proposition 4, however, tells us that this benefit is overshadowed by the loss in consumer surplus due to lower output in the specialty variety, product 2.

When \( b_1 / b_2 \) becomes larger (crossing over indifference curve \( h \) in Figure 3), this adverse effect on consumer surplus is doubled. In this case, the manufacturer, when evaluating the distribution through retailer B, makes the same assessment as made by A above and abandons one of the products to lower assortment costs. And, because the profitability of each product is relatively similar when \( b_1 / b_2 \) is large, the channel, and in particular, the manufacturer, is better off distributing only product 2 to retailer B.

4. Conclusion & Managerial Implications

This paper has sought to identify whether a retailer has strategic incentives that deviate from a manufacturer’s with respect to assortment. We have illustrated theoretically that a retailer may be quicker to abandon a manufacturer’s specialty variety when there are increasing costs associated with the number of SKU’s. In a competitive retail setting, a first-moving retailer can remove a specialty variety from its assortment while anticipating that a rival retailer will want to carry it in spite of the additional costs. This fact has several implications for managers that are brought out in our analysis.

First, the ability of one retailer to commit to its assortment decision before other channel members may have great impact on the distributional outcomes. In particular, by refusing to carry a manufacturer’s specialty variety, the dominant retailer can lower its own costs while simultaneously inducing the competing retailer to carry the manufacturer’s full product line. Thus, the competing retailer bears the higher assortment costs thereby giving the dominant retailer a cost advantage. For manufacturers, this indicates added channel coordination problems as a result of reduced competition for the specialty product.
Second, a dominant retailer may have a profit incentive to refuse to distribute specialty varieties that extend beyond its own operational costs. Particularly, by not distributing the specialty product, the dominant retailer can pass on the assortment costs to the competing retailer. Because of these higher costs, the rival retailer’s consumer price is higher. The dominant retailer benefits from the relaxed price competition for the commonly carried popular product, as a result.

Third, under conditions supporting a strategic assortment reduction, consumer welfare is unambiguously reduced. Consumers of the popular product may get lower prices, but lower competition for the specialty product leads to monopoly inefficiencies as well as more severe losses to double-marginalization. Thus, the growth of the discount retail format is not always in the consumer’s best interest.

Fourth, strategic assortment reduction outcomes reflect retail segmentation in the form of a discounter – the dominant, low-cost retailer – and a specialty shop – the high-cost, wide assortment retailer. Recognizing this segmentation may help manufacturers tailor other elements of the marketing mix as it applies to specific channels. For instance, manufacturers with wide product lines may want to focus training for sales staff at specialty shops in order to direct consumers to the product with best suited features. Sales staff at the dominant retailer, on the other hand, need not receive such training.

In our analysis, we made several simplifying assumptions in order to dig out the impact of retail dominance on assortment outcomes. For instance, we have not considered competitive effects between varieties. By introducing this dimension of competition, the dominant retailer would have an extra incentive to loose the specialty variety to insulate its popular variety from this additional competitive pressure. This reasoning implies that inter-variety competition tends to reinforce the motivation for strategic assortment reduction.

The manufacturer may also have strategic variables other than wholesale prices at its disposal, which we have not modeled. Advertising and other promotional activity by the manufacturer are bound to be altered as a result of the shift in channel power and this may change the distributional incentives discussed in our model. In addition, relaxing the restriction that wholesale prices are uniform and allowing more complicated wholesale pricing contracts

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8 Gorman (2001) provides a recent illustration from the cookware category, in which manufacturers offer specialty retailers training that instructs floor personnel how to position and sell new products.
will clearly improve the ability to extract retailer surpluses and align assortment incentives within the channel.

Appendix

Lemma A.1 (Equilibrium of the Subgame Starting at Stage 1)

(i) When product i is carried by both retailers, the manufacturer’s and retailers’ product-specific profits are, respectively,

\[
\Pi_i^M = \frac{1}{6b_i} \left( a - \frac{c^A + c^B}{2} \right)^2; \quad \Pi_i^j = \frac{1}{36b_i} \left( a - \frac{7c^j - 5c^{-j}}{2} \right)^2,
\]

with the distribution given by

\[
q_i^j = \frac{1}{6b_i} \left( a - \frac{7c^j - 5c^{-j}}{2} \right) \quad q_i^{-j} = \frac{1}{6b_i} \left( a - \frac{7c^{-j} - 5c^j}{2} \right)
\]

for \(j = 1,2; -j \neq j\).

(ii) When product i is carried by only retailer j then product-specific profits and quantity are, respectively,

\[
\Pi_i^{u} = \frac{(a - c^j)^2}{8b_i}; \quad \Pi_i^{j} = \frac{(a - c^j)^2}{16b_i}, \quad q_i = \frac{a - c^j}{4b_i}.
\]

Proof of Lemma A.1

Retailers’ optimal reactions to wholesale prices \(w_i, i = 1,2\) are given in equation (2). For \(i = 1,2\), the manufacturer maximizes \(\Pi_i^M = w_i(q_i^A + q_i^B)\), subject to the reactions in (2) over \(w_i \geq 0\). Substitute these values back in to (2) to obtain equilibrium quantities. Equilibrium profits for product i are computed by substituting the optimal \(w_i\)’s and \(q_i^j\) into (3) and (4). Q.E.D.

Proof of Proposition 1

The expressions given Table 1 follow from Lemma A.1 as follows. For outcome \(F\), substitute \(c^A = c^B = c\) into the expressions in part (i) for products \(i = 1,2\) and compute profits \(\Pi_i^j = \sum_i \Pi_i^j\) for \(j = M, A,\) and \(B\). For \(Sp\), substitute \(c^A = 0\) and \(c^B = c\), with product 1 carried by both retailers using expressions of part (i) of Lemma A.1 and product 2 carried by retailer \(B\).
only using expressions of part (ii) of the lemma. Profits are $\Pi^j = \sum_i \Pi^j_i$ for $j = M$ and $B$ and $\Pi^A = \Pi^A_1$. For $Ex$, substitute $c^A = c^B = 0$, with product 1 carried by $A$ and product 2 carried by $B$ using the expressions in part (ii) of the above lemma. Profits are $\Pi^M = \Pi^M_1 + \Pi^M_2$, $\Pi^A = \Pi^A_1$, and $\Pi^B = \Pi^B_2$. For $S$, substitute $c^A = c^B = 0$, with product 1 carried by $A$ and $B$ using expressions of part (ii) of the lemma. Profits are $\Pi^j = \Pi^j_1$, for $j = M, A$ and $B$. Q.E.D.

**Proof of Proposition 2**

In the $M$-Dominant game, the manufacturer implements its preferred distributional strategy based on outcome leading to the most profits. Comparing profit levels across these four outcomes requires pair-wise comparisons using the profit expressions in Proposition 1. Specifically, five (5) such comparisons are sufficient to determine the equilibrium in all regions of the parameter space $[0,1]^2$. Direct comparisons of profits leads to the following:

\[
\Pi^M_F > \Pi^M_E \iff \frac{c}{a} > 1 - \sqrt{3}/2 . \tag{A.1}
\]

\[
\Pi^M_S > \Pi^M_E \iff \frac{b_1}{b_2} < 1/3 . \tag{A.2}
\]

\[
\Pi^M_F > \Pi^M_S \iff \frac{c}{a} > (\delta - 1)/ (\delta - \frac{1}{2}) \equiv g(\frac{b_1}{b_2}) . \tag{A.3}
\]

where $\delta = \sqrt{1 + b_1/(4b_2)}$. The function $g$ is strictly increasing in $b_1/b_2$ on $[0,1]$ and represents $M$’s indifference curve for $Sp$ and $F$. The last two comparisons that are need are the following.

\[
\Pi^M_S > \Pi^M_{Sp} \iff \frac{b_1}{b_2} < \frac{(4-c/a)(c/a)}{3(1-c/a)} \equiv \hat{f}(c/a) . \tag{A.4}
\]

\[
\Pi^M_{Sp} > \Pi^M_{Ex} \iff \frac{b_1}{b_2} > \frac{1}{1-(1-c/a)^2} \equiv \tilde{h}(c/a) . \tag{A.5}
\]

Because $\hat{f}$ is strictly increasing and $\tilde{h}$ is strictly decreasing in $c/a$ on $[0,1]$, we define for $b_1/b_2 \in [0,1]$, $f(\frac{b_1}{b_2}) \equiv \hat{f}^{-1}(b_1/b_2)$ and $h(b_1/b_2) \equiv \tilde{h}^{-1}(b_1/b_2)$, which represent $M$’s indifference curve for outcomes $S$ versus $Sp$ and $Ex$ versus $Sp$, respectively. Conditions (A.4) and (A.5) can be rewritten in the canonical form

\[
\Pi^M_S > \Pi^M_{Sp} \iff \frac{c}{a} > f(\frac{b_1}{b_2}) . \tag{A.4'}
\]

\[
\Pi^M_{Sp} > \Pi^M_{Ex} \iff \frac{c}{a} < h(\frac{b_1}{b_2}) . \tag{A.5'}
\]
The function $g$ is strictly increasing in $b_1/b_2$ on $[0,1]$ and represents $M$’s indifference curve for $Sp$ and $F$. It is verified (numerically) that $f(b_1/b_2) > g(b_1/b_2)$ for all $b_1/b_2 > 0$, as claimed in the condition of the proposition. Furthermore, since $f$ and $g$ are increasing and intersect $h$ at exactly one point (specifically, at $1/3$ and at $\zeta$, respectively) we can write the following:

$$f(b_1/b_2) \leq h(b_1/b_2) \iff b_1/b_2 \leq 1/3;$$ (A.6)

$$g(b_1/b_2) \leq h(b_1/b_2) \iff b_1/b_2 \leq \zeta \equiv \sqrt{3}/2.$$ (A.7)

Note that $1/3 < \zeta$. To show (i), let $b_1/b_2 \in [0,1/3)$ then (A.2) implies $Ex$ is dominated by $S$ and thus can never be an equilibrium for any $c/a$. If $0 < c/a < g(b_1/b_2)$ then (A.3) and (A.4’) imply $\Pi^M_F > \Pi^M_{Sp} > \Pi^M_S$, yielding $F$ as the equilibrium. If $g(b_1/b_2) < c/a < f(b_1/b_2)$, then (A.3) and (A.4’) imply $\Pi^M_S > \Pi^M_{Sp} > \Pi^M_F$, yielding $Sp$ as the equilibrium. Finally if $f(b_1/b_2) < c/a < 1$, then (A.3) and (A.4’) $\Pi^M_{Sp} > \Pi^M_S > \Pi^M_F$. To show (ii), let $b_1/b_2 \in (1/3, \zeta)$. (A.2) implies $S$ is dominated by $Ex$. Conditions (A.3) (A.5’) and (A.6) imply the ordering required for the equilibrium description in the proposition. To show (iii), let $b_1/b_2 \in (\zeta,1]$. (A.2) implies $S$ is dominated by $Ex$ and conditions (A.3), (A.6) and (A.7) imply that $Ex$ dominates $Sp$. Therefore, $F$ and $Ex$ are the only outcomes possible in equilibrium. Finally, (A.1) implies the ordering required for the equilibrium description in the proposition. Q.E.D.

**Proof of Proposition 3**

As argued in the text, $A$ can only implement $M$’s second-best outcome ($F$ being the first), which is either $Sp$ or $Ex$. From (A.5’) in the proof of Proposition 2, we already know that $M$’s second-best is $Sp$ for $(b_1/b_2, c/a)$ below $h$ and $Ex$ above. (i.e., $Sp$ and $Ex$ are the corresponding equilibrium of the subgame starting in period 1.) $A$, in period 0, will not abandon product 2 if (5) holds. Further, note that $k$, as defined in (5) satisfies the following. For any $b_1/b_2 \in (0,1)$,

$$k(b_1/b_2) < g(b_1/b_2), \quad (A.8)$$

$$k(b_1/b_2) \leq h(b_1/b_2), \quad \text{for } b_1/b_2 \leq \eta, \quad (A.9)$$

where $\eta \approx 0.99$. To verify (A.8), observe that both $k$ and $g$ are both continuous and increasing and then it can be shown that $k(l) < g(l)$ and $k(x) = g(x)$ has no solution in $(0,1)$. The
condition (A.9) can be verified by first noting that both $k$ and $h$ are continuous with $k$ strictly increasing and $h$ strictly decreasing on $(0,1)$. This implies that they cross at most once, which occurs at $\eta$. The inequalities in (A.9) then follow from the fact that $k(0) = 0 < h(0) = 2 - \sqrt{3}$. It follows from these two conditions that for any $b_1 / b_2 \in (0,1)$, $F$ is the equilibrium if
\[
c / a < k(b_1 / b_2).
\]

Let $b_1 / b_2 \in (0,\eta)$ and $c / a \in (k(b_1 / b_2), \min\{g(b_1 / b_2), h(b_1 / b_2)\}$, retailer $A$ implements $Sp$ when abandoning product 2. Because $Sp$ is more profitable than $F$ for retailer $A$, it is the equilibrium outcome. Finally, let $b_1 / b_2 \in (\zeta, \eta)$ and $c / a \in (\max\{h(b_1 / b_2), k(b_1 / b_2)\}, g(\zeta))$. Then retailer $A$ implements $Ex$ when abandoning product 2. $A$ finds this more profitable than $F$ if condition (6) holds. Observe that $l'(b_1 / b_2) < 0$ for all $b_1 / b_2$ and that $l(\zeta) \approx 0.04174 < \frac{\sqrt{3} - 1}{4} = h(1)$. Thus,
\[
l(b_1 / b_2) < l(\zeta) < h(1) < h(b_1 / b_2)
\]
for all $b_1 / b_2 \in (\zeta,1]$, where the last inequality follows from the fact that $h$ is decreasing. Hence, $Ex$ is the equilibrium outcome in the region $b_1 / b_2 \in (\zeta,\eta)$ and
\[
c / a \in (\max\{h(b_1 / b_2), k(b_1 / b_2)\}, g(\zeta)).
\]

Q.E.D.

**Proof of Proposition 4**

First consider the $M$-Dominant game. Clearly, $\Pi_{SpA}^M > \Pi_{SpB}^M$ for $c^A < c^B$, where profit expressions can be directly deduced from Lemma A.1. Also, the fact that $c^A < c$ implies that $SpA$ dominates $F$, $Ex$, and $S$ for parameter constellations $(b_1 / b_2, c / a) \in \Theta_{Sp}$. Hence, $SpA$ is optimal for $M$ and therefore is the equilibrium outcome for the $M$-Dominant game.

In the $A$-Dominant game, if retailer $A$ abandons product 2 in Stage 0, then $(b_1 / b_2, c^A / a) \in \Theta_{Sp}$ and Proposition 2 imply that $M$’s optimal strategy is to distribute both products through retailer $B$. That is, $SpB$ is the equilibrium of the subgame starting in Stage 1 given that retailer $A$ does not carry product 2. Finally, $SpB$ is the equilibrium of the overall game if and only if $\Pi_{SpA}^A < \Pi_{SpB}^A$. A direct comparisons of profit expressions in (7) implies that the following condition is sufficient:
\[
\frac{b_1}{b_2} < \frac{8}{3} \frac{c^A}{a} \left(\frac{2-c^A / a}{1-c^A / a}\right)^2 \equiv p(c^A / a).
\]

(A.10)
First note that it can be directly verified that \( p \) is strictly increasing in \( c^a / a \). Therefore, \( p(c^a / a) > p[g(b_1 / b_2)] \) since \( c^a / a > g(b_1 / b_2) \) by the assumption \((b_1 / b_2, c^a / a) \in \Theta_{Sp}\). Finally, the condition (A.10) follows by noting that

\[
p[g(b_1 / b_2)] = \frac{8}{3} g(b_1 / b_2) (4\delta)(\delta - \frac{1}{2}) \]

\[
= \frac{32}{3} \left[ (1 + \frac{b_1}{4b_2}) - \sqrt{1 + \frac{b_1}{4b_2}} \right] > \frac{b_1}{b_2}.
\]

Q.E.D.

**Proof of Proposition 5**

Using the expressions for output quantities from Table 1 in equation (8), we arrive at the following:

\[
CS_F = \frac{(a-c)^2}{9} \left( \frac{1}{b_1} + \frac{1}{b_2} \right)
\]

\[
CS_{Sp} = \frac{(a-c/2)^2}{18b_1} + \frac{(a-c)^2}{32b_2}
\]

\[
CS_{Ex} = \frac{a^2}{32} \left( \frac{1}{b_1} + \frac{1}{b_2} \right).
\]

We first show that \( CS_F > CS_{Sp} \) everywhere in \( \Theta \). Generally,

\[
CS_F > CS_{Sp} \quad \iff \quad c / a < \frac{\sigma - 1}{\sigma - \frac{1}{2}} \quad (A.11)
\]

where \( \sigma = \sqrt{2 + \frac{23b_1}{16b_2}} \). The right-hand side of (A.11) is increasing and greater than \( \frac{2(\sqrt{2} - 1)}{4 \sqrt{2} - 1} \) for all \( b_1 / b_2 \). Since \( \frac{2(\sqrt{2} - 1)}{4 \sqrt{2} - 1} > 1 - \frac{\sqrt{3}}{2} = g(\zeta) \), it follows that \( CS_F > CS_{Sp} \) everywhere in \( \Theta \).

Comparing consumer surplus in \( Sp \) and in \( Ex \), we have the general condition

\[
CS_{Sp} > CS_{Ex} \quad \iff \quad \frac{b_1}{b_2} < \frac{\frac{4}{9}(2 - c / a)^2 - 1}{1 - (1-c/a)^2} = \hat{m}(c/a).
\]

(A.12)
Observe that \( \hat{m} \) is decreasing and \( \hat{m}(c/a) = 1 \Rightarrow c/a = \frac{17 - \sqrt{55}}{13} > g(\zeta) \). Therefore,
\[
(b_1/b_2, c/a) \in \Theta \Rightarrow c/a < g(\zeta) \Rightarrow \hat{m}(c/a) \geq 1 > b_1/b_2.
\]
Hence, (A.12) holds everywhere in \( \Theta \).

Q.E.D.
References


