Cost Linkages Transmit Volatility Across Markets

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Abstract

We present and test a model where increasing marginal costs link foreign shocks to the domestic supply of an exporter. A positive shock in the foreign market spurs the exporter to increase exports. The additional output raises the marginal cost of production, which leads the exporter to reduce supply to the domestic market. Foreign shocks do not affect nonexporters’ domestic supplies through this marginal cost linkage. Therefore, exporters supply the domestic market at a structurally different volatility than nonexporters. Through this linkage, a rise in the variance of foreign shocks lowers the welfare for consumers of domestic varieties. Fifteen years of Danish firm-level census data confirm the existence of a cost linkage. Domestic sales volatility for exporters is on average 24 percent greater than for nonexporters. The difference remains after controlling for endogenous exporting status with instruments such as firm location and heteroskedastic propensity. This is evidence that firm level linkages transmit volatility from foreign to domestic markets.

Keywords: Exports, firm heterogeneity, volatility, increasing marginal costs.

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1. Introduction

Exporters are subject to demand volatilities in both domestic and foreign markets. If a firm’s production technology links its supply on the domestic market to its supply on the foreign markets, then a demand shock in one market leads to a reallocation of supply across all markets.\(^1\) These firm-level linkages between demand in one market and supply in another is a new channel through which export activity transmits shocks across markets.

This paper introduces firm-specific increasing marginal costs in a standard international trade model to link demand shocks in the foreign market to supply adjustments in the domestic market.\(^2\) A positive shock in the foreign market spurs the exporter to increase exports. This additional output raises the marginal cost of production, which leads the exporter to lower supply to the domestic market. Thus, increasing marginal costs link foreign shocks to an exporter’s domestic supply. Firms that do not export do not have this substitution channel. Therefore, exporters supply the domestic market at a structurally different volatility than nonexporters. If instead marginal costs are constant,\(^3\) the firm-level transmission channel does not exist and exporters and nonexporters supply the domestic market at the same volatility.

We provide evidence for a firm-level transmission channel consistent with increasing marginal costs using a census of Danish firm level data. We observe the full universe of Danish firms and examine about 18,000 firms operating between 1992 and 2006 ac-

\(^{1}\)For example, due to higher than expected demand on the domestic market, Apple Inc. delayed the introduction of its new iPad in several international markets (http://www.businessweek.com/news/2010-04-14/apple-delays-ipad-s-international-debut-after-misreading-demand.html).

\(^{2}\)For firm-level evidence on nonlinear production costs see Bresnahan and Ramey (1994) and Caballero et. al. (1997).

\(^{3}\)As is standard in the international trade literature e.g. Melitz (2003); Melitz and Ottaviano (2008).
counting for about 87 percent of gross economic activity in the manufacturing sector. After accounting for differences in size and endogenous selection into export status, we estimate the volatility of domestic sales to be 24 percent higher for exporters than for nonexporters. This is evidence that firm-level linkages transmit volatility across markets.

Identifying the channels by which shocks are transmitted across countries is an ongoing area of research. Empirical and simulation studies find country-level evidence that trade increases the transmission of shocks across countries (e.g. Kose and Yi (2006); Kose et. al. (2008)). A growing literature examines the determinants of volatility at a more disaggregated industry level. di Giovanni and Levchenko (2008) examine the risk content of trade, working through the impact of trade openness on individual sectors, changes in the comovement between sectors and changes in the pattern of specialization.

Our theory and results relate to a recent literature that examines increasing marginal costs and firm-level linkages between foreign and domestic sales in Melitz and Ottaviano (2008) type trade models. Spearot (2011) focuses on investment incentives rather than the feedback of demand shocks across markets studied here. Soderbery (2010) examines the impact of capacity constraints and shows that a tariff liberalization leads exporters to substitute sales across the domestic and foreign market. The substitution of sales across markets in his paper is similar to ours. The main difference is that our theory and empirics focus on the impact of cost linkages on volatility. An advantage of this approach is that our empirical test is based on observable trade status, while Soderbery’s test requires information about usually unobserved capacity constraints.

Concurrently in progress, Vannoorenbergh (2010) examines the impact of a exporting firm’s share of exports in total sales on its domestic sales volatility for a sample
of French exporters. He works within the CES framework of Melitz (2003) and introduces heterogeneity in the slope of the marginal cost curve. Our linear demand framework allows us to introduce firm heterogeneity through differences in initial marginal costs. This heterogeneity results in structurally different supply volatilities between exporting and nonexporting firms, regardless of whether there is heterogeneity in the slope of the marginal cost curve. Unfortunately, a marginal cost curve with a non-zero intercept cannot be nested in the CES approach to test the empirical relevance of both channels of heterogeneity. Therefore we conclude that our study complements Vannoorenberghe’s; both provide evidence for alternative sources of heterogeneity in two complementary trade models. Consequently, both sources of heterogeneity are important to explain firm level volatility and cost linkages.

This study makes several contributions. First, we examine the impact of the decision to export on a firm’s domestic sales volatility. This firm-level link is a new channel by which foreign volatility is transmitted to the domestic market. Second, our model shows that, through this firm-level link, foreign volatility reduces welfare for consumers of domestic varieties, but the net effect of exporting depends on the correlation of foreign and domestic shocks. Third, our empirics take advantage of recently developed treatment estimators that account for endogenous selection into export status but do not require exclusion restrictions (Klein and Vella, 2009). This resolves the usual identification problem where the export status of a firm is endogenous and it is difficult to find valid instruments that are excluded from the empirical model of interest. Fourth, we derive a measure of total supply volatility accounting for export status to provide evidence that the heterogeneity in sales volatility across Danish manufacturing firms is driven by heterogeneity in initial marginal costs.4

4As in Mills and Schumann (1985), our total supply volatility does not depend on the slope of the marginal cost curve but is a function of the initial marginal cost. This test does not rule out additional heterogeneity in the marginal cost slopes.
This evidence extends the results of Mills and Schumann (1985) and facilitates our understanding of the sources of firm heterogeneity.

The next section outlines a model of international trade with increasing marginal costs. Section 4 presents the data, explains the regression variables and tests the predictions. These results motivate some additional empirical questions as well as robustness checks. Several remarks conclude the paper.

2. Theoretical Model

In this section, we introduce a model of international trade that generates several testable predictions regarding the output volatilities of firms. The model reconsiders the constant marginal cost assumption that is standard in the literature. It incorporates increasing marginal costs in a Melitz and Ottaviano (2008) type trade model allowing for heterogeneity in the initial marginal cost.\footnote{We do not solve for general equilibrium, but our exogenous demand shocks can be interpreted in a general equilibrium framework. Specifically, Melitz and Ottaviano (2008) examine a linear demand and monopolistic competition framework, where each firm’s demand intercept is a function of market size, demand parameters and the number of competing firms. A change in foreign demand feeds back into the domestic demand intercept. From the viewpoint of a single firm, however, this general equilibrium feedback shock is indistinguishable from any exogenous shock to the demand intercept. Our theoretical, testable predictions focus on the volatility of a single firm in response to exogenous shocks (to the firm). To this end, a partial equilibrium analysis suffices.} Under conditions that match the patterns in the data, we derive the firm’s domestic supply and show its relationship to export status and firm size.

2.1. Setup

Firms produce horizontally differentiated varieties to potentially sell in both a domestic and a foreign market. Preferences in the domestic market can be represented
by
\[ U = q_0 + D \int_{i \in \Omega} q^d(i) - \frac{1}{2} \int_{i \in \Omega} (q^d(i))^2 \]

where \( q_0 \) is a numeraire good and \( \Omega \) is the set of firms selling a differentiated product. Symmetric preferences exist in the foreign market, where \( D \) and \( d \) are replaced with \( F \) and \( f \). The exogenous variables \( D \in (D, \overline{D}) \) and \( F \in (F, \overline{F}) \) are positive and bounded random variables. They represent aggregate market conditions faced by all firms in the two markets. Their distributions are known with means \( \mu_D, \mu_F \), variances \( \sigma^2_D, \sigma^2_F \) and correlation coefficient \( \rho \). Firm \( i \) sets firm-market-specific prices \( p^d(i), p^f(i) \) and quantities \( q^d(i), q^f(i) \) to maximize profits. From equation (2.1), it is straightforward to generate linear demand curves \( p^d(i) = D - q^d(i) \) and \( p^f(i) = F - q^f(i) \).

Firm \( i \)'s cost of production is denoted by \( c_i(Q(i)) \), where \( Q(i) = q^d(i) + q^f(i) \). The cost function \( c_i(Q(i)) \) can be represented by a quadratic expansion from initial output level \( Q_0 = 0 \),

\[ c_i(Q(i)) = c_i(0) + c'_i(0) Q(i) + \frac{1}{2} c''_i(0) Q(i)^2 \]

This study examines the simple case where \( c_i(0) = c(0), c'_i(\cdot) = \beta_i, \) and \( c''_i(\cdot) = \nu \).

Each firm has an idiosyncratic initial marginal cost of production determined by \( \beta_i \), where \( 0 < \beta_i < \overline{\beta} \) is positive and bounded. The firm’s marginal cost increases with output at a rate equal to \( \nu \geq 0 \). Like in Melitz and Ottaviano (2008), heterogeneity in (initial) marginal costs leads to heterogeneity in firm decisions. Firms know \( \beta_i \).

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6Our utility function is a variant of Melitz and Ottaviano’s (2008). We make the simplifying assumptions that the degree of product differentiation is unity \( \left( \frac{d^2U}{dq^d(i)^2} = 1 \right) \) and that the consumer does not experience marginal disutility from total consumption over all varieties \( \int_{i \in \Omega} q^d(i) \), while adding uncertainty to the reservation prices \( D \) and \( F \).

7The linear cost function found in standard trade models, e.g. Melitz and Ottaviano (2008), assumes \( c_i(0) = c(0), c'_i(\cdot) = \beta_i, \) and \( c''_i(\cdot) = 0 \). Our model considers a nonzero quadratic term. Mills and Schumann (1985) provide evidence for heterogeneity in firm size and total supply volatility due to quadratic costs. They do not examine the impact of export activity. Our testable predictions do not rely on the heterogeneity of \( c''_i(\cdot) \) but rather of \( c'_i(\cdot) \), so we use a simple \( c''_i(\cdot) = \nu \).
before making any supply decisions.

To maximize profits, the firm must solve two related problems. First, fixed entry costs into the domestic and foreign markets require a firm to evaluate various entry scenarios. Second, after entry, in every period the firm observes the realizations of the demand shocks and determines the optimal allocation of output across its destination markets. To determine the firm’s optimal strategy, solve the problem backwards.

2.2. Optimal Supply

To keep the exposition concise, we focus on interior solutions where firms supply the markets for all possible realizations of the preference shocks in all time periods. In an expanded model, firms may supply to the foreign market without supplying to the domestic market. However, less than two percent of firms in our sample, accounting for less than 1% of total sales, serve the export market without also serving the domestic market. Therefore, we assume model parameters such that firms either supply only the domestic market (nonexporters) or supply both the domestic and the foreign market (exporters).

2.2.1. Nonexporters

Consider firms that sell only to the domestic market. These firms are denoted with subscript $n$. Firm $i$ selling only to the domestic market has output $Q_n(i) = q^d_n(i) \geq 0$. It supplies the optimal domestic quantity $q^d_n(i)$ such that the marginal revenue of

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8This restricts the distributions of the demand shocks and the cost parameters. If instead we assume CES utility, corner solutions are ruled out because consumers have an infinite willingness to pay to consume at least a small amount of any existing variety. An appendix that discusses all possible solutions and the necessary parameter restrictions is available upon request.
domestic sales equals the marginal cost:

$$D - 2q_n^d(i) = \beta_i + \nu q_n^d(i)$$

which results in the optimal supplied quantities $Q_n(i)$ and $q_n^d(i):^9$

$$Q_n(i) = q_n^d(i) = \frac{D - \beta_i}{2 + \nu}$$

The output of firm $i$ varies with $D$. Firms with lower initial marginal cost $\beta_i$ are larger and produce more output; this is in line with the predictions of the standard heterogeneous firm trade models, e.g. Melitz (2003).

2.2.2. Exporters

Let the subscript $e$ denote an exporting firm. Exporting firm $i$ sells to both markets (i.e. $q_f^e(i) > 0, q_d^e(i) > 0$). Firm $i$ supplies quantities that equate the marginal revenues of each market to the marginal cost.

$$D - 2q_e^d(i) = \beta_i + \nu (q_e^d(i) + q_e^f(i))$$

$$F - 2q_e^f(i) = \beta_i + \nu (q_e^d(i) + q_e^f(i))$$

'^9The second order conditions are consistent with profit maximization.
Solving this set of equations generates the optimal outputs (with $Q_e(i) = q_e^f(i) + q_e^d(i)$):

\begin{align*}
q_e^d(i) &= \frac{1}{4} \frac{(2 + \nu)D - \nu F - 2 \beta_i}{1 + \nu} \\
q_e^f(i) &= \frac{1}{4} \frac{(2 + \nu)F - \nu D - 2 \beta_i}{1 + \nu} \\
Q_e(i) &= \frac{1}{2} \frac{D + F - 2 \beta_i}{1 + \nu}
\end{align*}

As before, firms with lower $\beta_i$ are larger and produce more output. The key difference between this model and the standard constant marginal cost set-up ($\nu = 0$) is that the optimal domestic (foreign) supply of the exporter is a function of the foreign (domestic) shock. Therefore, exporters transmit foreign volatility to the domestic market.

2.3. Export Decision

To examine the effect of the decision to export, we find the profits of firms that do and do not export. The profits of a firm that sells to just the domestic market are:

$$\pi_n(D, \beta_i) = \frac{(D - \beta_i)^2}{2(2 + \nu)} - c(0)$$

while profits for a firm that sells to both the domestic and foreign markets are:

$$\pi_e(D, F, \beta_i) = \frac{\nu(D - F)^2}{8(1 + \nu)} + \frac{(D - \beta_i)^2}{4(1 + \nu)} + \frac{(F - \beta_i)^2}{4(1 + \nu)} - c(0)$$

The nonexporting and exporting profits are functions of the firm’s initial marginal cost and the realizations of demand in both markets. Firm $i$ compares the expectations of these profits when deciding whether to export. If it chooses to export, firm $i$ must pay an idiosyncratic fixed trade cost $K_i$. This could represent the firm’s prox-
imity to ports, or idiosyncratic foreign buyer networks. $K_i$ is drawn independently of $\beta_i$ and known before the firm makes its export decision. The expected value of exporting is the difference between the expected profits minus the cost of exporting $K_i$:

$$E[\pi_e(D,F,\beta_i)] - E[\pi_n(D,\beta_i)] = \frac{E[(2 + \nu) F - \nu D - 2\beta_i]^2}{8(2 + 3\nu + \nu^2)} - K_i$$

Equation (3) shows, sans $K_i$, firm $i$ will always make more profit by exporting. Consider a set of firms with $\beta_i = \beta$. There exists a $K^\beta \equiv \frac{E((2 + \nu) F - \nu D - 2\beta_i)^2}{8(2 + 3\nu + \nu^2)}$ such that only those firms with $K_i < K^\beta$ will find it profitable to export. Since $((2 + \nu) F - \nu D - 2\beta_i)$ is positive for exporters by equation (2b), $K^\beta$ decreases with $\beta$. The set of firms with $K_i < K^\beta$ contracts as $\beta$ increases.

**Lemma 2.1. Export Decisions** Firms with lower $\beta$ or lower entry costs $K$ are more likely to be exporters.

The idiosyncratic trade cost $K_i$ allows for the possibility that an exporting firm $i'$ has an (initial) marginal cost $\beta_{i'} = \beta_i$ equal to that of a nonexporting firm $i$. This is a departure from the Melitz (2003) and Melitz and Ottaviano (2008) models where exporters and nonexporters are partitioned via their marginal costs. This additional channel through which firms choose to export allows us to compare an exporter and a nonexporter with the same initial marginal cost $\beta_i$.

2.4. Domestic Supply

Now that the sets of exporters and nonexporters are defined, we compare the domestic supply quantities of an exporter $i'$ and an nonexporter $i$ with the same initial
marginal cost $\beta_\nu = \beta_i$. Subtract $q^d_e (i')$ from $q^d_n (i)$ to obtain

$$q^d_n (i) - q^d_e (i') = \frac{1}{4} \nu \left( \frac{(2 + \nu)F - \nu D + 2 \beta_i}{(2 + \nu)(1 + \nu)} \right) \geq 0.10$$

Nonexporting firm $i$ has a higher domestic supply than exporting firm $i'$ for all $\nu > 0$. Unlike firm $i$, exporting firm $i'$ must divide its output between two markets. We summarize this relation in the following lemma:

**Lemma 2.2. Domestic Supply** Consider exporter $i'$ and nonexporter $i$, where $\beta_\nu = \beta_i$:

1. If there are no cost linkages (i.e. $\nu = 0$), then nonexporter $i$ supplies the domestic market with the same quantity as exporter $i'$.

2. If there are cost linkages (i.e. $\nu > 0$), then nonexporter $i$ supplies the domestic market with a larger quantity than exporter $i'$.

Lemma 2.2 is testable. Controlling for heterogeneity in $\beta_i$, cost linkages lead to higher domestic supply for nonexporters than for exporters. Under the null, $\nu = 0$ and the exporters' domestic supply equals the nonexporters' domestic supply.

3. Volatility and Welfare

In the previous section, we show how increasing marginal costs modeled by $\nu > 0$ link foreign shocks to domestic supply, given by equation (2a). In this section, we show how the variances in $D$ and $F$ affect the volatilities of domestic supply in firms that do and do not export and we examine the impact of the linkages on welfare.

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10Since firm $i'$ exports, $q^e_e (i') > 0 \Rightarrow (2 + \nu)F - \nu D + 2 \beta_i > 0$. 

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3.1. Volatility and Linkages

We use the coefficient of variation \(cvo(\cdot)\) of supplied quantities as our measure of volatility. The coefficient of variation of the domestic supply of nonexporting firm \(i\) is

\[
cvo(q^d_n(i)) = \sqrt{\frac{VAR(q^d_n(i))}{E(q^d_n(i))}} = \frac{\sigma_D}{\mu_D - \beta_i} \quad (4)
\]

where \(E(q^d_n(i))\) is the expectation of \(q^d_n(i)\) over the possible realizations of \(D\). Equation (4) shows that large nonexporting firms with low initial marginal costs \(\beta_i\) supply the domestic market at lower volatilities than small firms with high initial marginal costs. The slope of the marginal cost curve, \(\nu\), does not impact the coefficient of variation of the nonexporting firm. Because the nonexporting firm’s domestic sales are also its total sales, the total output volatility is simply \(cvo(Q_n(i)) = cvo(q^d_n(i))\).

For the exporting firm, the coefficients of variation are:\(^\text{11}\)

\[
cvo(q^d_e(i)) = \sqrt{\frac{(2 + \nu)^2 \sigma_D^2 + \nu^2 \sigma_F^2 - 2(2 + \nu)\nu \rho \sigma_D \sigma_F}{(2 + \nu) \mu_D - \nu \mu_F - 2\beta_i}} \quad (5a)
\]

\[
cvo(Q_e(i)) = \sqrt{\frac{\sigma_D^2 + \sigma_F^2 + 2 \rho \sigma_D \sigma_F}{\mu_D + \mu_F - 2\beta_i}} \quad (5b)
\]

Equation (5) shows that large exporters with low initial marginal costs \(\beta_i\) supply the domestic market at a lower volatility than small exporters.

The total supply volatility of the exporter (5b) extends the supply volatility of the nonexporter (4) to account for the variance on the foreign market and the correlation between the domestic and foreign shock. As in Mills and Schumann (1985), the

\(^{\text{11}}\)To compute the volatilities for the exporter we assume that domestic and export quantities are always positive. However, in our data, a portion of exporting firms have zero exports in some years. To examine robustness with respect to this assumption, the estimates based on specification (9) examine the volatility differences between these permanent and marginal exporters. A robustness test that drops all exporters that don’t supply the foreign market when they supply the domestic market confirms our conclusions.
expressions for total supply volatility, \((5b)\) and \((4)\) do not depend on the slope of the marginal costs, \(\nu\). That is, even if cost linkages are heterogenous across firms, only heterogeneity in \(\beta_i\) drives the variation in the coefficient of variation of total supply. We exploit this fact in the empirics to provide evidence for heterogeneity in \(\beta_i\).

With our measures of volatility in hand, we can now compare the domestic supply volatilities of an exporter \(i'\) and a nonexporter \(i\) with \(\beta_{i'} = \beta_i\). A comparison of equations \((4)\) and \((5a)\) reveals that they are structurally distinct; the predicted volatility of exporters is different from that of nonexporters. We can express the difference as a ratio:

\[
\frac{cvo(q^d_e (i'))}{cvo(q^d_n (i))} = \frac{E(q^d_n (i))}{E(q^d_e (i'))} \sqrt{\frac{1}{16 (1 + \nu)^2} \left( \frac{(2 + \nu)^2 + \nu^2 \left( \frac{\sigma_F}{\sigma_D} \right)^2 - 2\rho \nu (2 + \nu) \left( \frac{\sigma_F}{\sigma_D} \right)}{(2 + \nu)^2 + \nu^2 \left( \frac{\sigma_F}{\sigma_D} \right)^2} \right)}
\]

The ratio of domestic volatilities depends on the values of \(\sigma_D, \sigma_F, \rho\) and \(\nu\). We present the following proposition:

**Proposition 3.1.** Consider exporter \(i'\) and nonexporter \(i\), where \(\beta_{i'} = \beta_i\).

1. If there are no cost linkages \((\nu = 0)\), then \(cvo(q^d_e (i')) = cvo(q^d_n (i))\).
2. If there are cost linkages \((\nu > 0)\) and \(\rho\) is below some sufficient cutoff \(\rho^* > 0\), then \(cvo(q^d_e (i')) > cvo(q^d_n (i))\).

**Proof** Proposition 3.1:

1. Let \(\nu = 0\). The expression under the radical is equal to 1. By lemma 2.2, \(\frac{E(q^d_n (i))}{E(q^d_e (i'))} = 1\). Thus, \(\frac{cvo(q^d_e (i'))}{cvo(q^d_n (i))} = 1\).
2. Let \(\nu > 0\) and consider \(\rho^* \equiv \left( \frac{\nu(\nu^2 + 8\nu + 8)}{2(2+\nu)^3} \frac{\sigma_D}{\sigma_F} + \frac{1}{2} \frac{\nu}{2+\nu} \frac{\sigma_F}{\sigma_D} \right) > 0\). For all \(\rho < \rho^*\), the expression under the radical is greater than 1. By lemma 2.2, \(\frac{E(q^d_n (i))}{E(q^d_e (i'))} > 1\). Thus, \(\frac{cvo(q^d_e (i'))}{cvo(q^d_n (i))} > 1\).
Part 1 of Proposition 3.1 leads to a clear and falsifiable hypothesis: If there are no cost linkages, then the domestic supply volatility of an exporter equals the supply volatility of a nonexporter. If the data show that the supply volatilities of exporters are structurally different from those of nonexporters, then we can reject this no-cost-linkage hypothesis.

Whether cost linkages result in a structurally higher or lower volatility on the domestic market is an empirical question; for \( \rho \) close to 1 and \( \sigma_F > \sigma_D \), it may be that \( \frac{\text{cvo}(q^d(i))}{\text{cvo}(q^d(i'))} < 1 \).\(^{12}\) However, if the data show \( \frac{\text{cvo}(q^d(i))}{\text{cvo}(q^d(i'))} > 1 \), then that is evidence for the presence of cost linkages as well as an imperfect demand correlation that results in a structurally greater volatility as given by part 2 of proposition 3.1.\(^{13}\) Proposition 3.1 is the main prediction we take to the data in the empirical section.

### 3.2. The Net Welfare Effects of Foreign Volatility

Finally, we examine the consequences of cost linkages and foreign volatility on the welfare of consumers of domestic varieties. If cost linkages exist, then consumers of domestic varieties will be exposed to foreign volatility if they purchase goods made by exporters. The welfare effect of this volatility link is determined by the share of exporters in the economy and the variance of the foreign shocks.

Let \( s \in [0, 1] \) be the share of exporters among the active firms selling to the domestic market. Let \( N \) denote the total mass of active domestic firms, which will remain constant in this discussion.

We measure the welfare effect of volatility as the difference between the expected util-

\(^{12}\)We don’t derive the exact conditions for this case, because it is empirically irrelevant in our data.  
\(^{13}\)Note that the conditions derived in proposition 3.1 are the same even with firm-specific heterogeneity in \( \nu \) or firm specific variances and correlation coefficients. This implies that the test is valid even with firm-specific heterogeneity in \( \nu \) or firm specific variances and correlation coefficients.
ity $E [U (D, F)]$ and the utility the economy would realize if there were no volatility on the domestic and foreign markets $U (\mu_D, \mu_F)$. Holding fixed the number of firms $N$ and the share of exporters $s$, the impact of domestic and foreign volatility on welfare is:

$$W (\sigma_D^2, \sigma_F^2, \rho, s, N) = E [U (D, F) - U (\mu_D, \mu_F) | \sigma_D^2, \sigma_F^2, \rho, s, N].$$

From the definition of utility in equation (2.1) and some algebraic manipulation,\(^{14}\) we find

$$W (\sigma_D^2, \sigma_F^2, \rho, s, N) = Nw_1 \sigma_D^2 - Nw_2 \sigma_F^2 - Nw_3 \rho \sigma_D \sigma_F$$

where

$$w_1 = \frac{48 + 128 \nu + 112 \nu^2 + 32 \nu^3 + (8 \nu^2 + 16 \nu^3 + 7 \nu^4) s}{32 (1 + \nu)^2 (2 + \nu)^2},$$

$$w_2 = \frac{s \nu^2}{32 (1 + \nu)},$$

$$w_3 = \frac{s \nu (2 + 3 \nu)}{16 (1 + \nu)^2}.$$ 

Note that $w_j, j \in \{1, 2, 3\}$ are all nonnegative. The welfare effect of foreign volatility $W (\sigma_D^2, \sigma_F^2, s)$ is:

$$\frac{dW}{d\sigma_F} = -2Nw_1 \sigma_F - Nw_3 \rho \sigma_D \sigma_F \leq 0$$

As foreign volatility increases, the consumers in the domestic market will suffer. Note that $\frac{dW}{d\sigma_F} |_{\nu=0} = 0$ : if there are no foreign linkages, foreign volatility does not affect domestic consumers’ utility. This is because the foreign shocks are passed through to the domestic supply via the firms’ cost. With constant marginal costs this channel does not exist, because the foreign shock would not impact the domestic supply of

\(^{14}\)The algebra is more fleshed out in Appendix A-2.
the exporters.\textsuperscript{15}

Taking the cross partial of the welfare effect of volatility with respect to foreign demand variance and the share of exporters gives us

\[
\frac{d^2 W}{d\sigma_F ds} = -\frac{2N\nu^2}{32(1+\nu)}\sigma_F - \frac{\nu(2+3\nu)N}{16(1+\nu)^2}\rho\sigma_D.
\]

The share of exporters magnifies the welfare effect of foreign volatility. However, that does not mean more exporters are bad for domestic consumers. The total welfare effect of increasing exporters is

\[
\frac{dW}{ds} = \frac{8\nu^2 + 16\nu^3 + 7\nu^4}{32(1+\nu)^2(2+\nu)^2}N\sigma_D^2 - \frac{\nu^2}{32(1+\nu)}\sigma_F^2 - \frac{\nu(2+3\nu)}{16(1+\nu)^2}\rho\sigma_D\sigma_F
\]

which depends on the volatility of domestic consumer’s preference $\sigma_D$ versus the volatility of foreign shocks transmitted to the domestic market $\sigma_F$. If domestic consumer’s tastes vary greatly compared to foreign volatility, having more exporters actually benefits domestic consumers because exporters are more elastic in responding to changes in consumer tastes. This gain may or may not outweigh the negative effects of transmitting foreign volatility to the domestic market.

4. Empirical Evidence

Proposition 3.1 gives a test for cost linkages in firm level data. This section explains the data, defines the estimation specifications and employs several estimators to test Proposition 3.1. Other testable predictions of the model and various robustness checks are also discussed.

\textsuperscript{15}Appendix A-2 shows that these results are robust to a more general utility function when we allow for the product differentiation term to differ (i.e. $\frac{d^2 U}{d^2 q(i)} = \gamma \neq 1$)
4.1. Data

We employ Statistics Denmark’s firm-level Account Statistics dataset for the years 1992 to 2006. This dataset is compiled from annual censuses of the economic activity of all Danish firms. For each year $t$ we have annual firm-level output as total sales and annual firm-level export sales in Danish Kroner (DKK) for each firm $i$. Domestic output is measured as the difference between total sales and export sales. Each firm’s product is classified according to 320 six digit industries defined by the classification of economic activities in the European Community (NACE6).

Cleaning the data preserves about 87 percent of the gross total output, but drops a large amount of observations. The combined raw dataset contains 67871 firms registered with the Danish government. The sample drops 36854 firms that report zero employees over their entire lifespan. The aggregate sales of these firms are less than 1 percent of the sum of the total sales over all firms. This suggests that the vast majority of these are likely “firms” created by individuals for tax write-off purposes. Clerical entry error drops 1655 firms as they violate annual reporting requirements and 88 firms because they show negative sales. Also dropped were 1670 firms who switched 4 digit industries but kept the same name. The theory assumes that firms that supply the export market also supply the domestic market. Only 858 of our firms export for a total of 1463 $i - t$ couples without supplying the domestic market. This is less than 1 percent of our observations. The sample drops these export-only firms. Finally, to compute a measure of volatility requires a minimum number of observations. Dropping firms that lack the necessary observations or variation, the final dataset contains 18398 firms. Columns (1) and (2) of Table 1 provide the summary statistics for these firms separating exporters from nonexporters. Given the export orientation and location of Denmark it is not a surprise that a relatively large share of about 40 percent of firms try the export market at least once. It is standard
in firm-level data sets that exporters are large. The same is true here. Whether we measure size by employment or total output, the mean exporting firm is about twice as large as the average nonexporter. The bottom line is that the identification approach is hard on the number of observations, but preserves a significant proportion of the economic activity.

Our main Proposition 3.1 concerns the relative domestic quantities of exporters and nonexporters, but we have only domestic sales data for the entire sample period. We do have data on domestic quantities at the 8 digit Combined Nomenclature product level for years 1997-2006 for 5051 firms in our sample. However, that data is voluntarily self-reported by the firms, so the accuracy is debatable. A sales analogue of Proposition 3.1 requires distributional assumptions on $D$ and $F$ and results in an intractable prediction. Therefore, we use domestic sales as a proxy for domestic quantity in our empirics. As a test of how well sales proxies for quantity, we regress log domestic sales on log domestic quantities for the core product of the 5051 firms available. We find, when controlling for firm fixed-effects, that a one percent increase in domestic quantities results in a $0.55 \pm 0.01$ percent increase in domestic sales.\footnote{Without fixed effects the elasticity is close to unity.} We also have export quantities for our exporters. When controlling for firm fixed-effects, a one percent increase in export quantities results in a $0.81 \pm 0.001$ percent increase in export sales.\footnote{While the variation in sales seems to capture the variation in quantities well, it is also worth pointing out that we can derive an analog to proposition 3.1 in terms of sales. The intuition is the same. Exporters and nonexporters have the same sales on the domestic market if there are no cost linkages. Therefore the volatilities in terms of sales are the same under the null. Whether exporters have a higher or lower volatility than nonexporters is an empirical question, just as with quantities. We derive the proposition in terms of quantities because the relationships are more intuitive and the intuition readily leads into the welfare derivation.} While the correspondence is not 1-1, firm sales predict quantities well.

Our tests require an empirical analogue to the theoretical coefficient of variation.
Let \( x_{it} \) be the quantity of interest, where \( x \) denotes either Domestic Sales or Total Sales. To remove time trends, specify

\[
ln(x_{it}) = \alpha_i + \zeta_i(T_t) + \epsilon_{it}.
\]

where \( T_t \) is a linear time trend. Because log differences approximate percentage change, \( \epsilon_{it} \) is approximately equal to the percentage deviation of \( x_{it} \) from its time varying mean \(^{18}\)

\[
\hat{\epsilon}_{it} = \ln(x_{it}) - \ln(exp(\hat{\alpha}_iT_t^{\hat{\zeta_i}})) \approx \frac{x_{it} - E(x_{it})}{E(x_{it})},
\]

and

\[
\sqrt{\sum_t [\hat{\epsilon}_{it}^2]} \approx \sqrt{\left[ \sum_t \left( \frac{x_{it} - E(x_{it})}{E(x_{it})} \right)^2 \right]} = \sqrt{\text{Variance}_i \left( \frac{x_{it}}{E(x_{it})} \right)}.
\]

where \( \text{Variance}_i(x_{it}) \) is across-time variance of \( x_{it} \). The most right hand side expression captures the coefficient of variation of \( x_{it} \) with respect to a time trending mean. For each firm define the measure of volatility as

\[
\text{rmse}(x) \equiv \sqrt{\sum_t [\hat{\epsilon}_{it}^2]}.
\]

This statistic captures the average percentage deviation of the random variable of interest around its time trending mean.

All of our tests require a measure of firm size. The theory and data suggest an input and an output based definition of size. The input based measure is the average workforce by firm, \((Employment_i)\). The output based measure computes the average

\(^{18}\)Table 2 shows our root mean square errors are close to zero, allowing us to make this approximation
output share within the industry. Let $j$ denote a NACE6 industry. Then the output share of firm $i$ that belongs to industry $j$ is \[ \text{Output Share}_{it} = \frac{\text{Total Sales}_{it}}{\sum_{i \in j} \text{Total Sales}_{i}}. \] Let $\text{Output Share}_t$ denote the simple average of $\text{Output Share}_{it}$ over time.

In our endogeneity robustness checks, we use other firm variables such as capital per worker, materials per worker, and energy per worker. These are gathered from the same firm censuses as firms are obliged to report their annual depreciated value of capital as well as their purchases of energy and materials. In addition, most firms report the municipality or zip code in which the firm is incorporated.

The last variable suggested by the model is the firm’s trade status. Figure 1 separates the average size distribution by export activity. The log of average work force, our proxy for firm size, is plotted on the horizontal axis. The height of each bar corresponds to the total count of firms within a given range of size. Any bar may be separated into three parts. The light grey shaded portion counts any firm that exports in all periods when it is also active on the domestic market. These Permanent exporters comprise 3360 firms. The dark grey shaded portion counts any firm that exports in some, but not all of the periods when it is active on the domestic market. These Marginal exporters comprise 4034 firms. The group of All Exporters combines the group of Marginal and Permanent exporters. The black portion counts the number of firms that never export; a total of 11004 firms. The permanent exporters with a mean total sales value of 140mio DKK tend to be larger than marginal exporters with a mean total sales value of about 30mio DKK. The same ranking holds for the domestic sales where the marginal exporters command mean sales of about 23mio while the permanent exporters’ average sales ring in at 63mio. For completeness, columns (3) and (4) of Table 1 split the group of exporters in column (2) into marginal and permanent exporters. The statistics are consistent with the notion of recent trade theory that small exporters are closer to
the productivity cut off which makes exports profitable. Small exporters are on the export margin as they have not grown their productivity and size to make exporting a permanently profitable position. Let $S$ be either of three export statuses such that $S \in \{\text{Permanent, Marginal, All Exporters}\}$. Let $s_i$ be firm $i$’s observed export status. Choose an export status $S$ and let the indicator be defined as

$$\mathbb{I}[S]_i \equiv \begin{cases} 1 & \text{if } s_i = S \\ 0 & \text{if } s_i \neq S. \end{cases}$$

This means that $\mathbb{I}[S]_i$ defines three separate export indicators, one for each possible export status: marginal, permanent and the combined all exporters.

Table 2 summarizes the regression variables in logs as they are applied in the specifications by export status. Overall the summary statistics suggest significant variation in the variables of interest across all firms and within the different samples split by export status. The regressions employ cross-firm variation across and within various export statuses to test the predictions from the theory.

4.2. Empirical Model and Identification

The base specification

$$\ln(\text{rmse}(x)_i) = \gamma_j + \zeta_1 \mathbb{I}[S]_i + \zeta_2 \ln(\text{Size}_i) + \text{Controls}_i + u_i$$

relates the log volatility of the firm-level variable $x_i$ to log firm size and the export activity indicator $\mathbb{I}[S]_i$ defined in (4.1). The NACE6 industry specific intercept $\gamma_j$ accounts for cross-industry-specific volatility that may be systematically correlated with the export potential of the industry. The identification assumption is that, within industries, firms are subject to the same fundamentals. This implies they are
subject to the same long run (annual) shock.

Two approaches are available to examine non linearity within the group of exporters. First, split export activity by marginal and permanent export status according to (4.1) and specify

$$ln(rmse(x)_i) = \gamma_j + \zeta_1 \mathbb{I}[\text{Permanent}]_i + \zeta_2 \mathbb{I}[\text{Marginal}]_i + \zeta_2 ln(\text{Size}_i) + u_i$$

This specification emphasizes difference in volatility by export status. The coefficient $\zeta_1$ identifies the difference in volatility between permanent and nonexporters. Coefficient $\zeta_2$ identifies the difference in volatility between marginal and nonexporters. The difference $\zeta_2 - \zeta_1$ identifies a structural difference between permanent and marginal exporters.

Second, volatility differences between exporters and nonexporters may be heterogeneous in size. To examine this hypothesis interact the export indicator with firm size to obtain

$$ln(rmse(x)_i) = \gamma_j + \zeta_1 \mathbb{I}[\text{All Exporters}]_i + \zeta_2 \mathbb{I}[\text{All Exporters}]_i \times ln(\text{Size}_i) + \zeta_3 ln(\text{Size}_i) + u_i$$

The coefficient $\zeta_2$ of this specification identifies size-driven heterogeneity the relative volatility of exporters compared to nonexporters.

4.3. Baseline Estimation Results: Impact of Export Activity on Domestic Volatility

Proposition 3.1 suggests a test for market linkages based on specification (8). Let the dependent variable be the domestic sales volatility of firm $i$. If marginal costs
are constant \((\nu = 0)\), then the theory suggests the null hypothesis \(H_0: \zeta_1 = 0\). If instead marginal costs increase \((\nu > 0)\), then the theory predicts \(H_a: \zeta_1 \neq 0\).

Table 3 reports the Ordinary Least Squares (OLS) estimates that test \(H_0\). Column 1 reports the estimates for specification (8). Exporters supply the domestic market at a volatility that is, on average, 24 percent higher than that of nonexporters. First, this result rejects the null hypothesis and provides evidence for firm-level linkages across markets. Second, the structural impact of exporting is both statistically and economically significant. It provides evidence that exporters channel volatility from the foreign to the domestic market.\(^{19}\)

Consistent with the theory and the existing literature, larger firms supply the domestic market at lower volatility. Given the trade status of the firm, doubling employment lowers the domestic sales volatility by about 3.4 percent.

Column 2 splits the impact of export activity on volatility by marginal and permanent export status according to specification (9). Accounting for differences in size, marginal exporters operate with higher volatility than permanent exporters. Both types operate with higher volatility than nonexporters.\(^{20}\)

To examine whether the previous result is simply an artifact of heterogeneity in size, Column 3 interacts the export activity indicator with average employment. The coefficient on the interaction between export status and average size is not statistically significant. This implies that the increased volatility of marginal exporters is not

\(^{19}\)In a robustness check consistent with Mills and Schumman (1985), we add in industry output share as an added explanatory variable. When controlling employment, output share had no significant effect on domestic volatility.

\(^{20}\)A Wald test rejects the null hypothesis that the average volatility for the permanent exporters is the same as the volatility of the marginal exporters with a p value of less than 1 percent. We also estimated a specification including output share. Output Share does not have a significant impact on volatility in this specification.
driven by heterogeneity in size, but is inherent to the export status.\textsuperscript{21}

4.4. \textit{Endogeneity of Export Status}

We examine two sources of endogeneity. First, firms may self select into export status. To control for this we apply a treatment estimator developed by Klein and Vella (2009). This instrumental variable estimator does not require exclusion restrictions (variables outside the model that predict export status). Instead, if there is heteroskedasticity in the selection specification, then this heteroskedasticity can be exploited to identify the consistent impact of treatment in a Two Stage Least Squares Estimation. They apply this approach estimating nonparametric probability models. To keep this discussion compact, we apply the approach via a heteroskedastic probit estimator as discussed by Millimet and Tchernis (2010):

1. Estimate the heteroskedastic probit $Pr(Exporting_i) = \phi(X_i\zeta/S(X_i\gamma))$, where $X_i$ is the vector of regressors in the main estimating equation (8). $S()$ is the form of heteroskedasticity.\textsuperscript{22}

2. Find the predicted probabilities of exporting $\hat{Pr}(Exporting_i)$ for each firm $i$.

3. Apply a Two Stage Least Squares estimator using $\hat{Pr}(Exporting_i)$ as an instrument for $I[All\ Exporters]_i$.

A second potential source of endogeneity are shocks transmitted through the input channels. Without accounting for differences in the factor demands, this may result in omitted variable bias. We add as controls in equation (8) materials per unit of

\textsuperscript{21}We also estimated a specification with the permanent and marginal export indicators including an interaction of the indicators with size. Again the interactions were not significant and a specification test (F-test) did not provide any evidence that the interactions should be included in the specification.

\textsuperscript{22}We tried several specifications for $S$. The one that consistently converged with the highest $R^2$ was an exponential of a linear combination of log employment, log capital/materials/energy per worker, and a employment–capital-per-worker interaction term.
labor, energy per unit of labor and capital per unit of labor to examine this source of endogeneity.

Table 4 reports five specifications. Column 1 reports the base specification for comparison. Column 2 reports the base specification augmented with input variables. Column 3 reports the estimates of the heteroskedastic probit model applied to generate the instruments for export status in the Two Stage Least Squares estimates in columns 4 and 5. Across the specifications, augmenting the specification with input information and/or to instrument for export status does not significantly change the conclusions.

We also added zip code dummies as explanatory variables in the heteroskedastic probit model. The idea is that firms located in zip codes closer to ports and borders have lower $K_i$ and are more likely to trade, but are still subject to the same domestic shocks. This drops the number of observations as some firms do not report their locations. We still found that exporters supply the domestic market at a volatility that is $19 - 24$ percent greater than that of nonexporters.\footnote{Other specifications generated similar results. We tried NACE6 domestic sales share or total output share instead of employment as measures for size. Instead of using per worker measures of energy, capital, and materials, we tried per kroner of total sales and per kroner of domestic sales. We report per worker as this is consistent with Bernard and Jensen (1995).} Firm size still has a negative and significant impact on the domestic supply volatility. The results are available upon request.

We employ several other robustness checks in our test of Proposition 3.1, including additional/alternative controls and alternative domestic sales data. These are summarized in A-3.
4.5. Additional Results

4.5.1. Total Output Volatility

To examine the impact of export activity on total volatility, we estimate the specifications derived in section 4.2, but with the volatility of total sales instead of domestic sales as the dependent variable. With a few notable exceptions, the implications of export status on total firm volatility are the same as for domestic volatility. Table 5 reports the results.

Across all columns, Table 5 shows that larger firms have lower total output volatility. This is evidence for heterogeneity working through the marginal cost intercepts. Equations (5b) and (2c) show that the cost linkages $\nu$ do not enter the total supply volatility. Therefore, after accounting for export status, the variation in the total supply volatility across firms must be driven by heterogeneity in the marginal cost intercept.\(^{24}\)

Across all columns, Table 5 shows that export activity raises a firm’s total volatility. Within the structure of the model, this result suggests that the variance of foreign shocks is high compared to that of domestic shocks. To see this, we divide equation (5b) by (4) and, with some manipulation, obtain

$$cvo(Q_e(i)) > cvo(Q_n(i)) \iff \sqrt{1 + \frac{\sigma_F^2}{\sigma_D^2} + 2\rho \frac{\sigma_F}{\sigma_D}} > 1 + \frac{\mu_F - \beta_i}{\mu_D - \beta_i} \quad (11)$$

Exporters have higher total output volatilities if the variance of foreign shocks is much greater than the variance of domestic shocks. To see this in (11), suppose $\mu_D = \mu_F$ and $\rho > 0$. Exporters would then have higher total volatility if and only

\(^{24}\)This is consistent with Mills and Schumann (1985) who also apply firm size as a proxy for the productivity parameter $\beta$. 

26
if the variance of foreign shocks is more than three times that of domestic shocks. If markets are symmetric and imperfectly correlated ($\mu_D = \mu_F$, $\sigma^2_F = \sigma^2_D$, $\rho < 1$), the total output volatility of the exporter is lower than that of the nonexporter. On first inspection this may seem counterintuitive, as the exporter faces more sources of volatility. However, the exporter is able to diversify its sources of volatility across two markets instead of only one. This diversification mitigates or negates the extra output volatility introduced by the foreign market.

Column (3) shows that exporting raises the total sales volatility for marginal exporters by about 16 percent. On the other hand, permanent exporters that are well established on the export market show a volatility that is about 12 percent lower than the volatility of nonexporters. This implies that permanent exporters use foreign markets to stabilize their total output.

Column (4) shows that the difference in volatility between marginal and permanent exporters is partially explained by heterogeneity in size. The interaction term of export status and average employment suggests that larger exporters have lower volatility than small exporters. Evaluating the first difference, the expected increase in total sales volatility by exporting is $0.215 - 0.075 \ln(\text{Employment}_i)$. The expected impact of exporting is positive for all firms with fewer than one quadrillion employees (so every firm in the sample).

4.5.2. Domestic Supply

We can test lemma 2.2 as additional support for our model. Our model predicts that after accounting for productivity differences, exporters supply the domestic market at a structurally lower quantity than nonexporters if $\nu > 0$.

Table 7 shows that across most specifications, we find support for $\nu > 0$. When
controlling for employment, exporters exhibit \((1 - \exp(-.116)) = 11\) percent lower sales in the Danish market than their non-exporting peers in the same NACE6 industry. This difference rises to 24 percent when controlling for energy, materials, and capital per worker. When accounting for the endogeneity of export status with our IV strategy, this difference drops to 2 percent and is statistically insignificant. So, we cannot completely reject the assumption of \(\nu = 0\) by testing just lemma 2.2.

### 4.5.3. Input Demand Volatility

Next we examine how export status and size impact the firm’s factor demand volatility. If factor markets are flexible and firms can easily adjust their inputs, we expect the changes in output to go hand in hand with changes in the factor demands.\(^{25}\) This implies that export status and size predict the factor demand volatility. On the other hand, firms may always respond to shocks by adjusting prices, the speed of the machinery or the number of working hours. In this case we expect no systematic relationship between export status, size and input demand volatility. Columns (1), (3) and (5) of Table 6 report estimates of specification (8) where the dependent variable is the volatility of labor, energy and capital. All three specifications show that exporting raises the input demand volatility and larger firms hire inputs at lower volatility. This suggests that exporters prefer flexible factor markets that allow them to easily adjust their inputs.

Column (2), (4) and (6) of Table 6 separate the export activity impact for marginal and permanent exporters. The results show that the impact of exporting on factor demand volatility is driven by the firms on the export margin. On average, a perma-

\(^{25}\)If the factor demand volatilities confirm the results from the total sales volatility discussed in section 4.5.1, then this is additional evidence that the variation we identify is driven by changes in outputs. Firms don’t need to hire and fire to change prices, but they do change their inputs to adjust output.
A permanent exporter operates with a significantly lower or equal volatility as a nonexporter. This result is consistent with the total output volatility results in the previous section. High output volatility feeds back to the firm’s inputs. Permanent exporters are able to stabilize their outputs, which helps stabilizes their inputs.

5. Conclusion

This paper introduces increasing marginal costs into a model of trade with stochastic linear demands. Increasing marginal costs result in firm-level market linkages; a shock in one market results in a reallocation of output across all markets.

Our results support the existence of cost linkages. Exporters supply the domestic market at a volatility 24 percent greater than that of nonexporters. This suggests that linkages across markets impact the stability of local markets. The linkages transmit foreign volatilities through to the domestic supply of exporters. The added domestic volatility reduces the welfare of consumers who purchase goods from exporters. This welfare reducing transmission of shocks from the foreign to the domestic market has not been previously examined in the literature.

The existing literature examines the impact of trade on the transmission of shocks across countries and examines the risk content of trade at a more aggregate level. This paper provides evidence for a specific transmission channel working through firm level incentives. This is important to understand by which channels shocks are disseminated through the trade network.

Finally, we find evidence that permanent exporters substitute quantities across markets to lower their total output and input volatilities. This is important for export promotion officials that identify and give recommendations to firms on how to succeed in export markets.
6. Acknowledgements

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7. Figures

Figure 1: Firm Distribution by Export Status

Permanent exporters are defined as the firms that supply the export and domestic markets simultaneously in all periods. Marginal exporters are defined as the firms that supply the export market in some periods when they supply the domestic market.
8. Tables

Table 1: Summary of Danish Firms

<table>
<thead>
<tr>
<th></th>
<th>All Firms</th>
<th>All Exporters</th>
<th>Marginal Exporters</th>
<th>Permanent Exporters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Sales</td>
<td>38</td>
<td>79</td>
<td>30</td>
<td>140</td>
</tr>
<tr>
<td>Mean Domestic Sales</td>
<td>23</td>
<td>41</td>
<td>23</td>
<td>63</td>
</tr>
<tr>
<td>Mean Employment</td>
<td>27.5</td>
<td>54</td>
<td>25</td>
<td>88</td>
</tr>
<tr>
<td>Mean Exports</td>
<td>15</td>
<td>38</td>
<td>7.1</td>
<td>75</td>
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<tr>
<td>No. of Firms</td>
<td>18398</td>
<td>7394</td>
<td>4034</td>
<td>3360</td>
</tr>
</tbody>
</table>

Employment in persons. All else in Millions of DKK.
Table 2: Summary Statistics of Regression Variables

<table>
<thead>
<tr>
<th></th>
<th>All Firms</th>
<th>All Exporters</th>
<th>Marginal Exporters</th>
<th>Permanent Exporters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev</td>
<td>Mean</td>
<td>Std. Dev</td>
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<tr>
<td>Employment</td>
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<td>1.33</td>
<td>2.66</td>
<td>1.43</td>
</tr>
<tr>
<td>Output Share</td>
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<td>1.78</td>
<td>-4.66</td>
<td>1.79</td>
</tr>
<tr>
<td>Energy per worker</td>
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<td>0.79</td>
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<tr>
<td>Materials per worker</td>
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<td>0.77</td>
<td>6.05</td>
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<tr>
<td>Capital per worker</td>
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<td>0.98</td>
<td>5.32</td>
<td>0.97</td>
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<td>Volatility of Domestic Sales</td>
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<td>Total Sales</td>
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<td>-0.98</td>
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<td>7394</td>
<td>4034</td>
<td>3360</td>
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Consistent with the specifications all variables are in logs.
Table 3: Impact of Export Status on Domestic Supply Volatility

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<td>Export Indicator:</td>
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<tr>
<td>All Exporters</td>
<td>.238</td>
<td>.262</td>
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<tr>
<td></td>
<td>(.017)***</td>
<td>(.029)***</td>
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<tr>
<td>Marginal Exporters</td>
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<td>.256</td>
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<tr>
<td></td>
<td>(.018)***</td>
<td></td>
<td></td>
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<tr>
<td>Permanent Exporters</td>
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<td>.198</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.024)***</td>
<td></td>
<td></td>
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<tr>
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<td>Employment</td>
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<td>-.030</td>
<td>-.027</td>
</tr>
<tr>
<td></td>
<td>(.007)***</td>
<td>(.007)***</td>
<td>(.01)***</td>
</tr>
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<td>Fixed Effects</td>
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<td>NACE6</td>
<td>NACE6</td>
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<tr>
<td>Interaction:</td>
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<tr>
<td>All Exporters × Employment</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.013)***</td>
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<td>.084</td>
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<tr>
<td>$F$ statistic</td>
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<td>70.698</td>
<td>66.012</td>
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</table>

Dependent variable: log of the volatility of total sales by firm, $ln(rmse(x)_i)$. Robust standard errors are reported in parenthesis (NACE6 clustered standard errors do not change results). Estimator: Ordinary Least Squares. Employment variable is in Log.
Table 4: OLS and Heteroskedastic Probit Estimates Accounting for Endogeneity

<table>
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<tr>
<th>Estimator</th>
<th>OLS (1)</th>
<th>OLS (2)</th>
<th>Heterosked. (3)</th>
<th>2SLS (4)</th>
<th>2SLS (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Exporters</td>
<td>.231 (0.022)***</td>
<td>.220 (0.022)***</td>
<td>.247 (0.114)**</td>
<td>.242 (0.108)**</td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>-.035 (0.009)***</td>
<td>-.031 (0.009)***</td>
<td>.936 (0.292)***</td>
<td>-.035 (0.018)*</td>
<td>-.035 (0.019)**</td>
</tr>
<tr>
<td>Energy per worker</td>
<td>.042 (0.024)*</td>
<td>.008 (0.057)</td>
<td>.042 (0.024)*</td>
<td>.046 (0.018)**</td>
<td></td>
</tr>
<tr>
<td>Materials per worker</td>
<td>.002 (0.032)</td>
<td>.647 (0.216)***</td>
<td>-.001 (0.034)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital per worker</td>
<td>.011 (0.009)</td>
<td>-.029 (0.032)</td>
<td>.011 (0.009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Const.</td>
<td>-1.513 (0.019)***</td>
<td>-1.680 (0.152)***</td>
<td>-2.352 (0.151)***</td>
<td>-2.313 (0.096)***</td>
<td></td>
</tr>
<tr>
<td>Controls</td>
<td>NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
</tr>
<tr>
<td>Obs.</td>
<td>17164</td>
<td>17164</td>
<td>17164</td>
<td>17164</td>
<td>17164</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.089</td>
<td>.09</td>
<td>.09</td>
<td>.09</td>
<td>.09</td>
</tr>
<tr>
<td>$F$ statistic</td>
<td>61.056</td>
<td>26.788</td>
<td>.44</td>
<td>.44</td>
<td></td>
</tr>
</tbody>
</table>

Dependent variable: log of domestic sales volatility in columns 1,2,4,5; Export at least once in column 3. Column 1 reports the base specification. Column 2 reports the base specification including the inputs as additional controls. Column 3 reports the 1st stage probit estimation of exporting using the Klein and Vella (2009) method. Column 4 reports the resultant IV results using the coefficients from column 3 to predict the probability of exporting. Column 5 repeats Column 4, but removes materials per worker and capital per worker as explanatory variables since they are insignificant in 4. Errors are robust (NACE6 clustered standard errors do not change results). These regressions have fewer observations than in Table 3, because some firms do not report energy, materials, and/or capital. All variables except ”All Exporters” are in Log.
Table 5: Impact of Export Status on Total Supply Volatility

<table>
<thead>
<tr>
<th>Export Indicator:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Exporters</td>
<td>.066</td>
<td>.074</td>
<td>.215</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.017)**</td>
<td>(.017)**</td>
<td>(.028)**</td>
<td></td>
</tr>
<tr>
<td>Marginal Exporters</td>
<td></td>
<td></td>
<td></td>
<td>.155</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(.018)**</td>
</tr>
<tr>
<td>Permanent Exporters</td>
<td></td>
<td></td>
<td>-.116</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(.023)**</td>
</tr>
<tr>
<td>Controls:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Employment</td>
<td>-.075</td>
<td>-.041</td>
<td>-.033</td>
<td>-.002</td>
</tr>
<tr>
<td></td>
<td>(.006)**</td>
<td>(.013)**</td>
<td>(.013)**</td>
<td>(.015)</td>
</tr>
<tr>
<td>Log Output Share (log)</td>
<td>-.036</td>
<td>-.023</td>
<td>-.030</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.012)**</td>
<td>(.012)*</td>
<td>(.012)**</td>
<td></td>
</tr>
<tr>
<td>Fixed Effects</td>
<td>NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
</tr>
<tr>
<td>Interaction:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Exporters × Employment</td>
<td></td>
<td></td>
<td></td>
<td>-.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(.012)**</td>
</tr>
<tr>
<td>Obs.</td>
<td>18398</td>
<td>18398</td>
<td>18398</td>
<td>18398</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.054</td>
<td>.054</td>
<td>.062</td>
<td>.056</td>
</tr>
<tr>
<td>$F$ statistic</td>
<td>73.376</td>
<td>53.03</td>
<td>93.285</td>
<td>55.805</td>
</tr>
</tbody>
</table>

The dependent variable is the natural log of the volatility of total sales by firm. Robust standard errors are reported in parenthesis. Estimator: Ordinary Least Squares.
Table 6: Impact of Export Activity on Input Demand Volatility

<table>
<thead>
<tr>
<th></th>
<th>Employment</th>
<th>Energy</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>All Exporters</td>
<td>.042</td>
<td>.076</td>
<td>.063</td>
</tr>
<tr>
<td></td>
<td>(.016)***</td>
<td>(.016)***</td>
<td>(.021)***</td>
</tr>
<tr>
<td>Permanent Exporters</td>
<td>-.069</td>
<td>-.037</td>
<td>-.094</td>
</tr>
<tr>
<td></td>
<td>(.023)***</td>
<td>(.023)***</td>
<td>(.029)***</td>
</tr>
<tr>
<td>Marginal Exporters</td>
<td>.084</td>
<td>.119</td>
<td>.122</td>
</tr>
<tr>
<td></td>
<td>(.017)***</td>
<td>(.016)***</td>
<td>(.022)***</td>
</tr>
</tbody>
</table>

Controls:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Employment</td>
<td>-.212</td>
<td>-.198</td>
<td>-.100</td>
<td>-.085</td>
<td>-.152</td>
</tr>
<tr>
<td></td>
<td>(.006)***</td>
<td>(.007)***</td>
<td>(.007)***</td>
<td>(.007)***</td>
<td>(.008)***</td>
</tr>
<tr>
<td>Fixed Effects</td>
<td>NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
</tr>
</tbody>
</table>

Obs. 14055 14055 14055 14055 14055 14055

$R^2$ .161 .164 .124 .128 .097 .102

$F$ statistic 711.051 510.383 115.169 107.068 197.397 163.436

Dependent variables are the logs of the volatilities of each firm’s employment level, volatility of expenditure on energy and volatility of the value of the capital stock. Robust standard errors are in parenthesis. Estimator: Ordinary Least Squares.
Table 7: OLS and Heteroskedastic Probit Estimates of the Effect of Exporting on Domestic Supply

<table>
<thead>
<tr>
<th></th>
<th>OLS (1)</th>
<th>OLS Heterosk. Probit (2)</th>
<th>2SLS (3)</th>
<th>2SLS (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exported at least once</td>
<td>-.116</td>
<td>-.270</td>
<td>-.023</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.033)**</td>
<td>(.029)**</td>
<td>(.075)</td>
<td></td>
</tr>
<tr>
<td>Employment (log)</td>
<td>.848</td>
<td>.893</td>
<td>.908</td>
<td>.855</td>
</tr>
<tr>
<td></td>
<td>(.010)**</td>
<td>(.008)**</td>
<td>(.297)**</td>
<td>(.014)**</td>
</tr>
<tr>
<td>Energy per worker (log)</td>
<td>.123</td>
<td>.013</td>
<td>.121</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.021)**</td>
<td>(.056)</td>
<td>(.022)**</td>
<td></td>
</tr>
<tr>
<td>Materials per worker (log)</td>
<td>.395</td>
<td>.633</td>
<td>.369</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.023)**</td>
<td>(.221)**</td>
<td>(.023)**</td>
<td></td>
</tr>
<tr>
<td>Capital per worker (log)</td>
<td>.039</td>
<td>-.017</td>
<td>.040</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.006)**</td>
<td>(.031)</td>
<td>(.006)**</td>
<td></td>
</tr>
<tr>
<td>Const.</td>
<td>13.825</td>
<td>11.006</td>
<td>11.431</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.023)**</td>
<td>(.099)**</td>
<td>(.097)**</td>
<td></td>
</tr>
<tr>
<td>Controls</td>
<td>NACE6</td>
<td>1NACE6</td>
<td>NACE6</td>
<td>NACE6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&amp; ZIP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>16935</td>
<td>16935</td>
<td>16935</td>
<td>16935</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.802</td>
<td>.86</td>
<td>.855</td>
<td></td>
</tr>
<tr>
<td>$F$ statistic</td>
<td>3612.51</td>
<td>2811.289</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dependent variable: log of domestic sales. Column 1 reports the base specification. Column 2 reports the base specification including the inputs as additional controls. Column 3 reports the 1st stage probit estimation of exporting using the Klein and Vella (2009) method. Column 4 reports the resultant IV results using the coefficients from column 3 to predict the probability of exporting. Errors are robust and clustered around NACE6 industries. These regressions have fewer observations than in Table 3, because some firms do not report energy, materials, and/or capital.
Appendix Not For Publication

A-1. Alternative Theories

A-1.1. Perfect Competition on the Export Market

Suppose firms face perfect competition on the foreign market so that the foreign demand is characterized by $p^f(i) = F$. Domestic demand and production cost remain at $p^d(i) = D - q^d(i)$ and $c_i(Q(i)) = c(0) + \beta_i Q(i) + \frac{1}{2}Q(i)^2$. The resulting optimal quantities are

$$q^d(i) = \begin{cases} 
\frac{D - \beta_i}{3} & \text{if } F < \frac{1}{3}D - \frac{2}{3}\beta_i \\
\frac{D - F}{2} & \text{if } \frac{1}{3}D - \frac{2}{3}\beta_i < F < D \\
0 & \text{if } F > D 
\end{cases} \quad (A-1)$$

$$q^f(i) = \begin{cases} 
0 & \text{if } F < \frac{1}{3}D - \frac{2}{3}\beta_i \\
F - \beta_i - \frac{D - F}{2} & \text{if } \frac{1}{3}D - \frac{2}{3}\beta_i < F < D \\
F - \beta_i & \text{if } F > D 
\end{cases} \quad (A-2)$$

$$Q(i) = \begin{cases} 
\frac{D - \beta_i}{3} & \text{if } F < \frac{1}{3}D - \frac{2}{3}\beta_i \\
F - \beta_i & \text{if } \frac{1}{3}D - \frac{2}{3}\beta_i < F < D \\
F - \beta_i & \text{if } F > D 
\end{cases} \quad (A-3)$$

In this setup, an exporting firm’s domestic quantities, $\frac{D - F}{2}$, do not depend on $\beta_i$. The resulting domestic volatilities should not depend on size. This is not consistent with the regression results. Specification (10) interacts the export indicator with size. If size does not matter for exporters, then the impact of the interaction term should exactly offset the direct impact of size. The coefficient on the interaction term is negative and not significant. This implies that size does matter even in the group of exporting firms.
A-1.2. Heterogeneity in Demand

Suppose heterogeneity in demand such that the demand equations are

\[ p^d(i) = \lambda_i D - q^d(i) \]  \hspace{1cm} (A-4)

\[ p^f(i) = \lambda_i F - q^f(i) \]  \hspace{1cm} (A-5)

where \( \lambda_i > 1 \) is a firm-specific popularity (quality) term that add multiplicatively to the reservation price. Firms with high \( \lambda_i \) have higher demands for given demand shocks \( D \) and \( F \). Let the costs be linear \( (c''(\cdot) = 0) \)

Given these adjustments to the theory, the profit-maximizing export and domestic quantities are

\[ q^f(i) = \frac{\lambda_i F - \beta_i}{2} \hspace{0.5cm} \text{if} \hspace{0.5cm} F > \frac{\beta_i}{\lambda_i}, \text{else} \hspace{0.5cm} 0 \]  \hspace{1cm} (A-6)

\[ q^d(i) = \frac{\lambda_i D - \beta_i}{2} \hspace{0.5cm} \text{if} \hspace{0.5cm} D > \frac{\beta_i}{\lambda_i}, \text{else} \hspace{0.5cm} 0 \]  \hspace{1cm} (A-7)

These quantity functions are the same whether the firm is an exporter or not. The corresponding coefficients of variation are

\[ cvo(q^f_e) = \frac{\sigma_f}{\mu_F - \frac{\beta_i}{\lambda_i}} \]  \hspace{1cm} (A-8)

\[ cvo(q^d_e) = cvo(q^d_n) = \frac{\sigma_d}{\mu_D - \frac{\beta_i}{\lambda_i}} \]  \hspace{1cm} (A-9)

In this set-up, larger firms (low \( \beta_i \) or high \( \lambda_i \) still have lower coefficients of variation. However, since the coefficient of variation of domestic supply is identical for exporters and nonexporters, this model predicts that, once controlling for size, exporters and nonexporters have identical domestic volatilities. Our estimation results reject this prediction.
A second theory would have additive demand heterogeneity. In this case, the demand equations would be

\[ p^d(i) = \lambda_i + D - q^d(i) \]  
\[ p^f(i) = \lambda_i + F - q^f(i) \]

where \( \lambda_i \) again is a firm-specific popularity (quality) term. This time, it adds additively to the reservation price. Given these adjustments, the maximizing export and domestic quantities are

\[ q^f(i) = \frac{\lambda_i + F - \beta_i}{2} \text{ if } F > \beta_i - \lambda_i, \text{ else } 0 \]  
\[ q^d(i) = \frac{\lambda_i + D - \beta_i}{2} \text{ if } D > \beta_i - \lambda_i, \text{ else } 0 \]

These quantity functions are the same whether the firm is an exporter or not. The corresponding coefficients of variation are

\[ cvo(q^f_e) = \frac{\sigma_f}{\mu_F - \beta_i + \lambda_i} \]  
\[ cvo(q^d) = cvo(q^d_n) = \frac{\sigma_d}{\mu_D - \beta_i + \lambda_i} \]

In this additive scenario, an increase in \( \lambda \) affects the coefficient of variation in exactly the same way as a decrease in \( \beta_i \). Again, the \( cvo \) function is the same for exporters and nonexporters: this theory does not generate the structural difference between exporters and nonexporters. In short, our data rejects both theories of demand heterogeneity and linear cost.
A-2. Welfare

In this section, we estimate the welfare effect of volatility given a more general utility function. Let the utility in the domestic market be

\[ U(D, F) = q_0 + D \int_{i \in \Omega} q^d(i) - \frac{\gamma}{2} \int_{i \in \Omega} (q^d(i))^2 \]

where \( \gamma \) is the Melitz & Ottaviano (2008) degree of differentiation, which this study has assumed to equal 1. Exporter and nonexporter domestic quantities under this case is

\[ q^d_n(i) = \frac{D - \beta_i}{2\gamma + \nu} \]
\[ q^d_e(i) = \frac{(2\gamma + \nu) D - 2\gamma\beta_i - \nu F}{(4\gamma^2 + 4\gamma\nu)} \]

Let \( s_i = 1 \) if firm \( i \) is an exporter and 0 otherwise. The domestic quantity supplied of any firm \( i \) can then be expressed as a function \( q^d(\beta_i, s_i, D, F) \) of its initial marginal cost \( \beta_i \), its exporting status \( s_i \), and the reservation prices \( D \) and \( F \) :

\[ q^d(\beta_i, s_i, D, F) = \frac{D - \beta_i}{2\gamma + \nu} (1 - s_i) + \frac{(2\gamma + \nu) D - 2\gamma\beta_i - \nu F}{(4\gamma^2 + 4\gamma\nu)} s_i. \]

We measure the welfare effect of volatility \( W(\sigma_D^2, \sigma_F^2, \rho, \{s_i\}) \) as the difference between the expected utility given volatility parameters \( \sigma_D^2, \sigma_F^2, \rho, \{s_i\}_{i \in \Omega} \) and utility at the expected values (i.e. the utility if there were no volatility in \( D \) and \( F \)):

\[ W(\sigma_D^2, \sigma_F^2, \rho, \{s_i\}) = E(U(D, F) | \sigma_D^2, \sigma_F^2, \rho, \{s_i\}) - U(\mu_D, \mu_F) | \{s_i\} \]

We can substitute \( q^d(i) = q^d(\beta_i, s_i, D, F) \) into the expression for \( W(\sigma_D^2, \sigma_F^2, \rho, \{s_i\}) \)
and manipulate it algebraically to get:

\[
W \left( \sigma_D^2, \sigma_F^2, \rho, \{s_i\} \right) = E \left( \int_{i \in \Omega} \omega_{1i} D^2 - \omega_{2i} F^2 - \omega_{3i} DF \right)
\]

\[
\omega_{1i} = \frac{48 \gamma^4 + 128 \gamma^3 \nu + 8 \gamma^2 (14 + s_i) \nu^2 + 16 \gamma (2 + s_i) \nu^3 + (8 - s_i) s_i \nu^4}{32 \gamma (\gamma + \nu)^2 (2 \gamma + \nu)^2}
\]

\[
\omega_{2i} = \frac{s_i^2 \nu^2}{32 g (g + \nu)}
\]

\[
\omega_{3i} = \frac{s_i \nu (4 \gamma^2 + 8 \gamma \nu + (4 - s_i) \nu^2)}{16 \gamma (\nu + \gamma)^2 (2 \gamma + \nu)}.
\]

which equals

\[
W \left( \sigma_D^2, \sigma_F^2, \rho, \{s_i\} \right) = \sigma_D^2 \int_{i \in \Omega} \omega_{1i} - \sigma_F^2 \int_{i \in \Omega} \omega_{2i} - \rho \sigma_D \sigma_F \int_{i \in \Omega} \omega_{3i}.
\]

Assume there are a mass \(N\) firms. Then the fraction of exporters \(s = \frac{1}{N} \int_{i \in \Omega} s_i\). Now the coefficients can be rewritten as

\[
N \omega_1 = N \int_{i \in \Omega} \omega_{1i} = \frac{48 \gamma^4 + 128 \gamma^3 \nu + 112 \gamma^2 \nu^2 + 32 \gamma \nu^3 + (8 \gamma^2 \nu^2 + 16 \gamma \nu^3 + 7 \nu^4) s \, N}{32 \gamma (\gamma + \nu)^2 (2 \gamma + \nu)^2}
\]

\[
N \omega_2 = N \int_{i \in \Omega} \omega_{2i} = \frac{s \nu^2}{32 \gamma (\gamma + \nu)} \, N
\]

\[
N \omega_3 = N \int_{i \in \Omega} \omega_{3i} = \frac{s \nu (2 \gamma + 3 \nu)}{16 \gamma (\gamma + \nu)^2} \, N
\]

Note that \(\omega_1, \omega_2\) and \(\omega_3\) are all nonnegative. Now we can define \(W\) in terms of the number of firms \(N\) and the overall fraction of exporters \(s\):

\[
W \left( \sigma_D^2, \sigma_F^2, \rho, \{s_i\} \right) = W \left( \sigma_D^2, \sigma_F^2, \rho, s, N \right) = W \left( \sigma_D^2, \sigma_F^2, \rho, \{s_i\} \right) :
\]

\[
W \left( \sigma_D^2, \sigma_F^2, \rho, s, N \right) = N \omega_1 \sigma_D^2 - N \omega_2 \sigma_F^2 - N \omega_3 \rho \sigma_D \sigma_F
\]
And take the derivative with respect to $\sigma_F$:

$$\frac{dW}{d\sigma_F} = -2N\omega_2\sigma_F - N\omega_3\rho\sigma_D \leq 0$$

which is still negative and equals 0 at $\nu = 0$. Note that at $\gamma = 1$, the $\omega$ terms reduce to those of (6).

A-3. Additional Robustness Checks

Because not all firms are in the markets for all periods, variation in firm level variances may be driven by the number of observations available to compute firm level volatility. To account for this potential variation we augment our specifications with indicators for the number of observations used to calculate total and domestic volatility. These controls do not significantly change our results.

Mills and Schumann (1985) regress total sales volatility on various measures of firms size and industry specific volatility. We repeat this exercise and arrive at similar conclusions.

We also augmented our main specifications with zip code indicators. We still found that exporters supply the domestic market at a volatility that is 24 percent greater than that of nonexporters. The identifying variation in these specifications is within zip codes and within industries and allows for a stricter version of our identification assumption: As long as firms that are located in the same zip code and supply the same industry are subject to the same level of aggregate demand volatility, a structural difference in the domestic supply volatility between exporters and nonexporters on the domestic market is evidence for a firm-level cost linkage. Firm size still has a negative and significant impact on the domestic supply volatility.
Applying an alternative data source,\textsuperscript{26} experimenting with alternative measures of output and industry controls we find no significant difference in our results.

Firms usually report the 6 digit product in which they had the highest sales that year. This may change from year to year. When we control for a firm’s 6-digit product category, we use the firm’s modal 6 digit product category. In a robustness check, we also use the firm’s initial product category. The results differ in the third or fourth digit. The results from all the robustness checks are available upon request.

\textsuperscript{26}Sales from Denmark’s Value Added Tax (VAT) data banks for the years 1994-2006. Total sales data from the VAT closely track those from Account Statistics. A regression of VAT sales on Account statistics output for the years 1994-2006 finds a marginal effect of 0.947 with a Std. Err of 0.00038.