Tax Evasion, Information Reporting, and the Regressive Bias Prediction*

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Abstract

Models of rational tax evasion and optimal enforcement invariably predict a regressive bias in the effective tax system, which reduces redistribution in the economy. Using Danish administrative data, we show that a calibrated structural model of this type replicates moments and correlations of tax evasion and audit probabilities once we account for information reporting in the tax compliance game. When conditioning on information reporting, we find that both reduced-form evidence and simulations exhibit the predicted regressive bias. However, in the overall economy, this bias is negated by the tax agency’s use of information reports and revenue-maximizing disposition of audit resources.

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Keywords: Information Reporting, Regressive Bias, Tax Enforcement, Tax Evasion

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1 Introduction

In this article, we develop a structural model of tax evasion and enforcement in a population of taxpayers. Highly detailed Danish administrative data allows us to perform a meaningful calibration exercise to investigate the model’s ability to explain tax evasion and the tax agency’s enforcement strategy. We show that the model’s predictions closely match key empirical relationships in the data and, in particular, we provide the first empirical evidence of the regressively biased average effective tax rates predicted in the theoretical literature on tax evasion and optimal enforcement (Reinganum and Wilde, 1986; Cremer, Marchand, and Pestieau, 1990; Sanchez and Sobel, 1993; Erard and Feinstein, 1994).\footnote{As shown by Scotchmer (1992), the prediction of regressive bias is theoretically robust. Model variations in the literature, e.g., whether or not the agency can commit to audit strategies, consistently arrive at regressively biased effective average tax rates.}

The potential for tax evasion requires a distinction between the statutory tax system and the effective tax system. Tax evaders pay less taxes than they should and this implies a wedge between statutory and effective average tax rates. The regressive bias prediction states that this wedge is larger for high-income taxpayers than for low-income taxpayers. Thus, the tax system may be substantially less redistributive than intended by the tax code.

The intuition behind this prediction is the following: The tax compliance game played by the tax agency and taxpayers is a screening problem in which high-income taxpayers can increase their expected income by imitating low-income taxpayers. If not all taxpayers can be audited, the tax agency should optimally prioritize tax returns reporting low income. Rather than eliminating tax evasion altogether, budget-constrained optimal enforcement primarily discourages very low reports by high-income individuals. Due to the optimal regressivity in tax enforcement, evading taxes on the margin subjects a low-income taxpayer to a greater risk of getting caught than a high-income taxpayer, which tends to make high-income taxpayers evade more. In equilibrium, the decreasing relationship between the probability of audit and reported income and the increasing relationship between evaded taxes and true income lead to an increasing wedge between
the statutory average tax rate and the effective average tax rate as a function of true income, i.e., a regressive bias. Figure 1(a) illustrates how the wedge between the effective average tax rate, $\tau_{\text{eff}}$, and the average tax rate implied by the statutory tax system, $\tau$, is increasing in true income.

There is one important exception to the regressive bias result: when the tax agency uses *ex ante*-observable population variables, such as gender, age, occupation, or employer-reported salaries, to predict true incomes, there may be no bias or even progressive bias in the population as a whole. Scotchmer (1987) shows that when tax agencies facilitate prediction of taxpayers’ true income by dividing taxpayers into *audit groups*, upon which the agency conditions its enforcement strategy, effective average tax rates remain regressively biased within audit groups, but the direction of the bias between groups is ambiguous. The aggregate bias depends on the predictive power of the signals (i.e., the *ex ante*-known population variables) and the allocation of audit resources across audit groups. Consequently, the regressive bias prediction should be interpreted as a *within-audit-group* phenomenon. Figure 1(b) illustrates the aggregate relationship between effective average tax rates, $\tau_{\text{eff}}$, and true income, which is a composite of relationships within multiple audit groups, $\tau_{\text{eff}}^i$. Whereas the regressive bias prediction remains valid within audit groups, effective tax rates may be *progressively biased* across audit groups.

The mechanism driving the result is that some low-income taxpayers benefit from being high-income individuals within their audit group while some high-income taxpayers instead are low-income taxpayers within their audit group. This reclassification changes the risk of being audited and, hence, the *ex ante* effective tax rate. In addition, the tax agency can more efficiently target high-income individuals by modifying the distribution of audit resources between audit groups. If the observable signal of true income is stronger or audits are more abundant among high-income taxpayers, progressive bias between groups may dominate in the aggregate.

We combine insights from two main sources, Kleven, Knudsen, Kreiner, Pedersen, and Saez (2011) and Erard and Feinstein (1994). In the former, the authors collect a uniquely
detailed micro-data set based on a random sample of Danish taxpayers containing pre- and post-audit incomes and taxes, as well as reports on income, proxies for audit probabilities, etc. They show that third-party reported income is by far the best predictor of true income compared to other population variables. Since the Danish tax agency, SKAT, does in fact use these information reports extensively in its enforcement efforts, they are ideal for constructing audit groups. Based on this insight, we generalize Erard and Feinstein’s within-audit-group model to describe tax evasion and optimal enforcement both within and between audit groups. We calculate an internally consistent set of model parameters directly from data and calibrate the tax agency’s budget, the only unknown variable in this setup, to match the simulated level of tax evasion to data. We evaluate the model numerically and find that it convincingly replicates aggregate tax evasion behavior for both wage earners and the self-employed although these two groups differ markedly in terms of the propensity to evade taxes and the extent and distribution of third-party reported income. Whereas the simulation allows complex nonlinear relationships between the key variables, it does not allow for a more general formulation of behavioral heterogeneity, such as stemming from differences in gender, age, marital status, etc. However, we find that evidence from reduced-form regressions controlling for these sources of heterogeneity is consistent with the structural model’s comparative statics.

Both graphical evidence and reduced-form regressions exhibit clear evidence of a regressively biased effective average tax rates within audit groups. However, between audit groups, tax rates are progressively biased to such an extent that tax rates are actually progressively biased in total income. Thus, our findings support the regressive bias prediction, but we do not find that this leads to a substantial regressive bias in the redistribution between high- and low-income taxpayers, overall. In fact, the data exhibits the structure of Scotchmer (1987), in which unbiased or progressively biased tax enforcement in the aggregate economy is caused by the tax agency’s use of a priori information for the

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2Other recent articles demonstrate the importance of explicitly considering information reporting. Phillips (2010) demonstrates the predictive power of an indirect measure of third-party reported information in US data and Pomeranz (2013) demonstrates the general importance of information as a deterrent of VAT evasion in a sample of Chilean firms.
purpose of auditing a larger proportion of high-income taxpayers relative to low-income taxpayers.

Our results have important implications for policy. Due to the theoretical robustness of the regressive bias prediction, it has been argued (e.g., in Scotchmer, 1992) that governments should increase the progressivity of the income tax schedule to counter regressive bias inherent in optimal tax enforcement. However, our results imply that such a policy adjustment is undesirable. In the first place, adjusting tax rates cannot eliminate the inequity between taxpayers that evade taxes and taxpayers that do not. Secondly, there may be no regressive bias to correct in the aggregate. If such is the case, the policy priority is correcting the horizontal inequity between evaders and non-evaders rather than the distortion of redistribution between high- and low-income taxpayers – for this purpose allocating more resources to the tax agency or collecting more information *ex ante* are superior approaches.

We now proceed to the main body of the article. Section 2 develops our model of the tax compliance/evasion game. Section 3 outlines the context and features of the data. Section 4 describes the calibration of parameters and outlines the numerical strategy. Section 5 provides the evidence of regressively biased tax enforcement in the Danish economy. Section 6 concludes. Appendix A provides details of the equilibrium characterization and the numerical implementation of the model. Appendix B provides a description of black market activity in Denmark.

2 Theory: A Model of Income Tax Auditing Subject to Information Reporting

Several current theories are capable of analyzing behavior within audit groups, i.e., conditional on pre-defined groups based on *ex ante*-observable information. However, as we wish to simulate aggregate reporting behavior as well as the tax agency’s overall response, we need a model that can encompass a population of taxpayers, i.e., several audit groups and the optimal distribution of audit resources between them. To this end, we generalize
the model in Erard and Feinstein (1994) to incorporate a population that is heterogeneous in third-party income reports.

The model introduces noise in taxpayer reports by incorporating the stylized fact that some taxpayers report their incomes honestly, even when they have ample opportunity to evade taxes. As we demonstrate in Section 3, this is also the case in our data. As argued in Erard and Feinstein (1994), including inherently honest taxpayers increases the realism and usefulness of the model: it eliminates several potential equilibria and leaves them with a unique revenue-maximizing equilibrium prediction. Further, it eliminates the unrealistic feature of earlier models that the tax agency in equilibrium would know the true incomes of all taxpayers before the actual audit.

This model is well-suited for analyzing the behavior of rational tax evaders given that some taxpayers are in fact honest, but it does not address the reason why some report their incomes honestly and others do not. It provides a relatively simple framework for analyzing optimal enforcement in the face of this behavior and subject to the informational asymmetries inherent in the tax enforcement/compliance game.

The structure of the model is the following. The tax agency selects the audit regime subject to a budget constraint without being able to commit to an audit strategy. The audit schedule for a particular audit group (i.e., conditional on a particular third-party reported income level) is a function of taxpayers' reported residual incomes, i.e., income in excess of third-party reported income as we assume that third-party reported income is common knowledge. The tax agency allocates its resources across different strata of the population so as to equalize the shadow values of extending resources to auditing taxpayers with different amounts of third-party reported income. Whereas the distribution of true incomes, conditional on information reports, is known, actual true incomes of individual taxpayers are private information. Taxpayers choose income reports subject to their expectations about the audit regime. Lastly, the final tax returns and the audit schedule are realized, audits are conducted, and tax revenue and ex post utilities, as measured by income net of taxes and any penalty payments, are realized. In equilibrium, the probability that a particular taxpayer is audited depends both on the exogenous signal,
i.e., third-party reported income, and the endogenously determined reported income.

2.1 Individual Reporting Behavior

Individual taxpayers have true taxable incomes, $y$, and report taxable incomes, $\tilde{y}$. Part of true income, $z$, is reported by third parties and is common knowledge. Therefore, $y = z + u$, where $u$ is residual income, which can be positive or negative as it includes both, e.g., wages and deductions not reported by third parties, and is restricted to the domain $[u, \bar{u}]$. $u$ is \textit{ex ante} unknown and can only be ascertained by the tax agency by conducting a costly audit, which we assume reveals all of “true” residual income.\footnote{We assume that taxpayers do not incur a cost from filing taxes (time costs, hiring of a tax accountant, concealment costs etc.). Such costs have welfare consequences in the form of deadweight losses. Cremer and Gahvari (1994) show that a concealment technology that allows taxpayers to lower the probability of detection at a cost can affect the effective progressivity of the tax system. This may result in more or less progressivity depending on the exact specification of the concealment technology. However, their model assumes a constant audit probability, whereas our model implies a non-increasing audit probability on the domain of reports of dishonest taxpayers. In any case, whether or not such costs are important, our results in Section 5 indicate that they are not necessary to explain the correlation structure of effective average tax rates.} We denote the reported residual $x$, such that $x = \tilde{y} - z$.

Taxpayers are split into two types, honest and dishonest taxpayers. We define the densities of true income conditional on third-party reports $f_{u|z}^h$ and $f_{u|z}^d$ for honest and dishonest taxpayers, respectively, allowing the distributions to differ. The total density function is $f_{u|z} = f_{u|z}^h + f_{u|z}^d$ and $F_{u|z}$ is the conditional distribution function associated with $f_{u|z}$.

As is common in the theoretical literature, we assume that taxes are linear in income. However, we relax this assumption in the empirical implementation. Whereas honest taxpayers always report $x = u$, we assume that dishonest taxpayers are risk neutral and maximize expected utility given by expected income net of taxes and penalties

$$(1 - t) z + p(x|z) [(1 - t) u - \theta t (u - x)] + (1 - p(x|z)) [u - tx],$$

where $t$ is the tax rate, $\theta$ is the penalty rate on tax evasion, and $p(x|z)$ is the audit
probability for report \( x \) given the level of third-party reporting \( z \). The correct amount of taxes are paid with certainty on income reported by third parties, whereas taxes (and penalties) paid on residual income depends on both a taxpayer’s evasion behavior and whether or not the taxpayer is audited.

In optimum, the taxpayer’s choice must satisfy the first-order condition

\[
u = x + \frac{p(x|z) - \frac{1}{1+\theta}}{p'(x|z)}. \quad (1)
\]

It is clear from Equation (1) that for \( p(\cdot) = \frac{1}{1+\theta} \), \( x = u \) and evasion is discouraged completely. However, \( p \geq \frac{1}{1+\theta} \) is not compatible with equilibrium when the tax agency cannot commit to the audit regime: if evasion were completely discouraged, the tax agency would lower \( p \) for some \( x \) as a cost saving measure. Thus, in equilibrium \( p(\cdot) \in [0, \frac{1}{1+\theta}) \).

Furthermore, the incentive compatibility constraints on the tax agency’s optimization problem implies that audit functions are decreasing on the domain of income reports (see Erard and Feinstein (1994) for a detailed demonstration of this point).

Given that \( p'(x|z) \) is negative and \( p(x|z) < \frac{1}{1+\theta} \), increasing the audit probability will, ceteris paribus, lower tax evasion as the risk of getting caught is higher. Lowering \( p'(x|z) \) (increasing its absolute value) also reduces tax evasion by increasing the risk of audit from taxes evaded on the margin.\(^6\)

The equilibrium involves two types of behavior by tax evaders. Individuals with residual incomes below some threshold, \( u_{pool} \), pool at the lowest possible report, whereas, for individuals with residual incomes above this threshold, each \( u \) is associated with a unique \( x \), i.e., they separate in equilibrium. As seen in the Appendix, Figure A.1, the

\(^4\)We use a different specification for penalties in case of detected evasion compared to Erard and Feinstein (1994). We model penalties as proportional to evaded taxes rather than evaded income as this is also the structure of the actual Danish penalty system.

\(^5\)We have investigated the implications of assuming that tax evaders’ expected utilities take the CRRA form, using the parameter estimate of \( 1 - 0.741 \) from Andersen, Harrison, Lau, and Rutström (2008) for Danish citizens. We found that the structure of simulated data was very similar to the baseline simulation in this article, the main difference being that a given level of average tax evasion could be attained at a lower budget value. Therefore, we have chosen not to incorporate CRRA utilities as this change comes with a heavy cost, making the model well nigh impossible to solve for some parametrizations and levels of income, especially for the self-employed.

\(^6\)Taxpayers’ income returns must also satisfy the second-order condition, \( p''(x|z) (x - u) + 2p'(x|z) \leq 0 \).

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implication of this is a linear association between tax evasion and residual income for small \( u \) and a nonlinear association, determined by Equation (1), for large \( u \). For interior reports, the relationship is determined by the tradeoff between \( p(x|z) \) and \( p'(x|z) \) and is not necessarily increasing.

2.2 Optimal Audit Response

The tax agency chooses a continuum of audit schedules \( p(x|z) \) and a budget allocation \( B(z) \) for all \( z \). In this way, the informational aspect of using third-party reported incomes to predict true income is incorporated into the population-wide equilibrium.\(^7\) The audit schedule is chosen to maximize expected revenue (taxes plus fines)\(^8\)

\[
\int \left( \int_{x}^{u} \left[ p(x|z) \left( tE(y|x,z) + \theta t \left( E(y|x,z) - \tilde{y} \right) \right) + (1 - p(x|z)) t\tilde{y} \right] dF_{x|z} \right) dF_{z}
\]

subject to the budget constraint

\[
c \int \left( \int_{x}^{u} p(x|z) dF_{x|z} \right) dF_{z} \leq \int B(z) dF_{z} \equiv B, \tag{2}
\]

where \( F_{x|z} \) is the induced conditional distribution function for reported residual income, \( x \), given third-party reported income, \( z \); \( F_{z} \) is the marginal distribution function for \( z \); and \( B(z) \) is the proportion or density of the overall audit budget, \( B \), allocated to income reports with third-party reported income, \( z \). For each \((x, z)\), the tax agency must choose

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\(^7\)In principle, the tax agency could also condition audit schedules on other population variables such as gender, age, occupation, etc. However, as Kleven et al. (2011) show, these variables are less powerful as predictors. Conditioning on whether the taxpayer was audited in previous years would complicate matters as it would introduce a dynamic aspect to reporting decisions. However, as observations on past audits are not employed in SKAT’s actual audit scheme, this limitation is unlikely to affect the fit of our model. In addition, the statute of limitations for retrospective audits is limited to 14 months. In any case, the relatively few number of tax evaders in our data does not allow us to pursue this generalization.

\(^8\)Scotchmer (1992) shows that maximizing some measure of social welfare instead of expected revenue does not change the qualitative prediction that (within an audit group) there will be regressive bias, although it may change the distribution of resources across audit groups. The similarity of the observed and simulated distribution of resources, cf. Section 5, suggests that revenue maximization is not an inappropriate simplification in this context.
\[ \max_p \left\{ p[tE(y|x, z) + \theta t (E(y|x, z) - \bar{y})] + (1 - p)t\bar{y}] \right\} dF_{x|z} dF_z \\
- \lambda(z) c \left[ p \ dF_{x|z} - B(z) \right] dF_z, \]

where \( \lambda(z) \) is the Langrangian multiplier on the budget constraint. This implies a point-wise first-order condition

\[ tE(y|x, z) + \theta tE(y|x, z) - \theta t\bar{y} - t\bar{y} - \lambda(z) c \geq 0, \quad (3) \]

which is greater than, equal to, or less than zero as \( p = \frac{1}{1+\theta} \), \( p \in \left(0, \frac{1}{1+\theta}\right) \), or \( p = 0 \). We focus on the equilibrium in which the tax agency chooses a mixed strategy such that (3) holds with equality.\(^9\)

In Appendix A.1, we show that, for each \( z \), the equilibrium can be characterized by a second-order differential equation in \( p(x|z) \) supplemented with a boundary constraint that ensures optimal reporting by taxpayers pooling at the lowest possible report. The budget allocation across different \( z \) is pinned down by the requirement that the shadow value of increasing the budget, \( \lambda(z) \), must be the same for all \( z \), i.e., \( \lambda(z) = \lambda \), \( \forall z \), for an interior solution. The shadow value, \( \lambda \), is pinned down by the requirement that the tax agency’s overall budget, \( B \), may not be exceeded.

3 Data

SKAT’s tax collection efforts extensively employ information reports by third parties. During some year \( t \), incomes are earned and by the end of January in year \( t + 1 \), SKAT receives information reports from employers, banks, pension funds, and other entities – the so-called third-party income reports. In general, all income received as salary, private/public pensions, honorarium, unemployment benefits, etc., is subject to third-

\(^9\)The second-order condition is \( \frac{\partial E(y|x, z)}{\partial p(x|z)} \geq 0 \). In our simulations the solutions always satisfy this criterion.
party reporting as well as, e.g., mortgage interest payments and some capital income.\textsuperscript{10} Self-employment income is only rarely covered by information reporting, e.g., in cases where remuneration is paid by a public institution. Third parties do not have discretion as to whether or not to supply SKAT with this information. The informational requirement is entirely related to the type of income.

By mid-March, SKAT sends out pre-populated tax returns based on third-party information and other information that they possess about the taxpayers, such as their residence and workplace for calculating commuting allowances. Subsequently, taxpayers have until May 1 to correct their tax return; in case of no corrections, the pre-populated tax return counts as final. After the deadline, SKAT’s computerized system processes tax returns and attaches audit flags to returns that the system finds likely to contain errors.

Briefly, the audit flag system relies on third-party income reports and also a collection of auditing “best practices” that could be converted to algorithmic form, e.g., specific tax return compositions indicative of misreporting, cut-off rules based on expected incomes conditional on third-party reported income, etc. The flag system consists of a large number of flags, each of which is intended to signal tax evasion on particular line-items or combinations of line-items. The system does not as such assign a probability of audit or rank tax returns according to their likelihood of containing errors but assigns a recommended action, i.e., “audit” or “do not audit”. Although the flag system operates for both wage earners and the self-employed, in practice, it is only used for wage earners as the predictive power of the audit flags for self-employed has been judged too low. For the self-employed, further information is gathered on a case-by-case basis. SKAT uses correlates of true income such as bank deposits, consumption of housing, cars, and other durables to signal the likelihood of non-reported income. They may also seek information exchange with known tax shelters about foreign deposits or uncover such deposits indirectly by tracking purchases with foreign credit cards, but such information is much harder to gather. All in all, the workings of the audit regime is very different for the self-employed and much more resource intensive.

\textsuperscript{10}Dividends are reported by third parties, whereas capital gains were not reported in 2006/2007.
After the tax returns have been processed, tax examiners assess the flagged returns and decide whether or not to initiate an audit based on the information available, local knowledge, and auditing resources. If an audit discovers underreporting, the taxpayer may pay the taxes owed immediately or postpone the payment at an interest. If the tax examiner views the underreporting as deliberate, the tax agency may impose a fine according to a fining scheme depending on the assessed intentionality of the misreporting.

3.1 Experimental Design

The data originates from an experiment conducted by SKAT in the years 2006–2008, originally analyzed in Kleven et al. (2011), and is in many ways comparable to the US Taxpayer Compliance Measurement Program. The experiment involved a stratified random sample of 17,764 self-employed individuals and 25,020 wage earners and recipients of public transfers in Denmark. In the present study, we use a sample of non-treated wage earners and recipients of public transfers (referred to as “wage earners”) and a sample of non-treated self-employed for the fiscal year 2006. The sample of wage earners is a stratified random sample of 10,740 Danish taxpayers, and the sample of the self-employed is a random sample (non-stratified) of 8,890 taxpayers. The full populations of wage earners and self-employed, respectively, were approximately 4.2 million and 400,000 in 2006. For each taxpayer, SKAT conducted an unannounced audit after the deadline for changing the tax return (May 1, 2007). The tax audits were comprehensive in the sense that SKAT examined all items on the tax return, demanding documentation for all items on which SKAT did not possess information. Moreover, SKAT made a significant effort to have tax examiners perform homogeneous audits by, e.g., organizing training workshops and distributing detailed audit manuals. The audits took up 21 percent of the resources devoted to tax audits in 2007.

\footnote{In the original study in Kleven et al. (2011), some taxpayers were subject to treatments. These taxpayers received notifications prior to filing their final tax returns, indicating that they would be audited with either 50 or 100 percent probability.}

\footnote{Note the randomness of our sample as opposed to tax compliance data obtained from the regular audits that is heavily biased by over-sampling taxpayers who are likely to have misreported their income in either direction. The sampling strategy for wage earners involved a stratification on tax return complexity. For the self-employed no stratification scheme was employed.}
Of course, it is unlikely that tax examiners find all hidden income, such as that stemming from cash-only businesses and other black market activities. We focus our attention on the detectable part of tax evasion, and, in what follows, we will write true income when, in fact, we mean detectable income. In Section 5.4 we discuss the importance of non-detectable income for our results, using the best available data on the informal economy in Denmark.

For each taxpayer, we have income and tax records as reported by third parties, the final return as potentially changed by the taxpayer, and the post-audit return. In addition, the data contains information on the generated audit flags that would normally constitute a basis for selecting taxpayers for audits as well as a “compliance rating” reflecting the auditor’s assessment of the degree to which discovered misreporting reflected deliberate fraud or accidental under/over-reporting.

### 3.2 Tax Compliance in Denmark

Table 1 presents descriptive statistics on major income components for the two samples of wage earners and self-employed, respectively. The table shows sample means with standard errors of means in parentheses – all numbers for wage earners are calculated accounting for the stratification scheme. Column (1) presents pre-audit figures measured at the deadline, May 1, and column (5) shows figures reported by third-parties. Self-reported figures (the difference between (1) and (5)) are shown in column (6). Negative figures mean that taxpayers on average adjust the number downwards to less than what third-parties have reported. Columns (2)–(4) describe how the figures in (1) were adjusted by the tax examiners during the audits. Columns (3) and (4) split the audit adjustments into positive (meaning underreporting) and negative (meaning overreporting) adjustments while column (2) holds the average net adjustment, i.e., the sum of (3) and (4).

[Table 1 about here]

Panel A of Table 1 shows figures on total income and total taxes for wage earners. Total income is defined as the sum of labor market income, transfers, capital income,
stock income, self-employment income, and foreign income less deductions. Pre-audit total income is on average a little less than 200,000 DKK with a significantly positive net adjustment from SKAT of almost 1,700 DKK. The positive net adjustment reflects an asymmetry in the reporting behavior with underreporting being more than ten times as high as the overreporting on average. Third-party reported total income is slightly higher than pre-audit total income mainly due to deductions not included in the third-party reports, implying a negative residual (i.e., self-reported) total income.

In Panel B we show descriptive statistics for the sample of self-employed taxpayers. As with wage earners, we find a pronounced asymmetry in net audit adjustments corresponding to much higher underreporting compared to overreporting for the self-employed. The main difference compared to wage earners is spelled out in the average level of self-reported income. Income sources of self-employed are to a much lesser extent covered by the system of third-party reporting, resulting in an almost even split between income reported by third parties and self-reported income. This provides SKAT with a much greater challenge in discovering unreported income.

We get a further idea as to where the opportunities to evade taxes are prevalent by looking at taxpayers’ behavior and conditioning on the informational environment. In Table 2 we separate taxpayers according to whether or not their entire income was reported to the tax agency by a third party. Panel A shows the shares of under-/overreporting and correct reports for each sample (wage earners and self-employed, respectively). All figures in the table are calculated accounting for stratification whenever applicable. The overall population weighted share of compliers, given by wage earners not underreporting, amounts to approximately 94 percent for wage earners. For the self-employed, approximately 65 percent comply. To address taxpayers with ample opportunity to evade taxes, Panel B shows shares of particular groups conditional on whether or not their entire income is reported by a third-party (standard errors in parentheses). For example, less than 2 percent of wage earners with all income reported by third parties underreport

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13 In this article, we focus on aggregate measures of income and taxes. Descriptive statistics on the components of income as well as a description of the Danish tax system is available in Pinje and Boserup (2011).
taxes. For wage earners with some income not reported by third parties, this share is much higher, yet a substantial share of over 80 percent (depending on the definition of compliance) are found to comply with the tax laws despite having ample opportunity to evade.

[Table 2 about here]

Only few self-employed taxpayers (3.4 percent) have their entire income reported by third parties, underlining the tax agency’s challenge in securing tax revenue from these taxpayers. Further, almost 35 percent are found to underreport their taxes. The share of self-employed who do not underreport their taxes is again high (95 percent) for those with all income reported by third-parties and much lower (64 percent) for those with some income not covered by the system of third-party reporting, albeit still a substantial share comply with given tax laws. Strikingly, wage earners and self-employed who have all their income reported by third parties do not differ significantly in reporting behavior, whereas this is not the case when some income is not reported by third parties.

### 3.3 Effective Tax Rates

The measure of effective average tax rates takes into account the extent of tax evasion, the risk of detection, and the penalties paid in case of detection, all of which affect expected payments to the tax agency. As such, we must restrict our analysis of this phenomenon to the individuals for whom we have information about the enforcement regime, i.e., wage earners. The *ex ante* effective average tax rate can be calculated from data as

\[
\tau^{\text{eff}} = P \cdot \left( \frac{T + \Theta(T - \tilde{T}, \tilde{T})}{Y} \right) + (1 - P) \cdot \tilde{T},
\]

where \( P \) is the probability of getting caught, \( T \) and \( \tilde{T} \) are taxes on true and reported income, respectively, \( Y \) is true income, and \( \Theta(\cdot, \cdot) \) is a nonlinear function describing the penalty for underreporting taxes as a function of underreported taxes and the assessed
intentionality of evasion, $I$.\textsuperscript{14,15} With a probability $P$, evasion is detected and the taxpayer pays the full taxes due plus a penalty that is proportional to the amount of taxes evaded. With a probability $1 - P$, evasion goes undetected and the taxpayer only pays taxes on reported income. As tax evasion diminishes or as the risk of detection or the penalty increase, the effective average tax rate will increase, \textit{ceteris paribus}.

We denote by $\tau$ the nominal average tax rate, defined in the usual way, $\tau = T/Y$. As a matter of convenience, we define the tax rate bias as $\tau - \tau_{\text{eff}}$. This allows us to compare how much statutory and effective tax rates differ when both vary across individuals in the sample. For honest taxpayers, the tax rate bias is zero by definition.

$Y$, $T$, and $\tilde{T}$ are observed in the data as post-audit total income and taxes, and pre-audit taxes. We use SKAT’s audit flag system as a proxy for the probability of getting caught for wage earners. Not all taxpayers with flags are audited equally thoroughly, so we assume that the probability of audit is proportional to the number of flags assigned to a tax return.\textsuperscript{16} Specifically, we calculate our proxy for the probability of detection simply as the ratio of flags assigned to a tax return to the maximal number of flags assigned to any tax return. With this approach, the audit rate among wage earners is 3.3 percent. This is slightly lower than the total population audit rate of 4.2 percent reported by Kleven et al. (2011). As this rate includes audits of the self-employed, who, presumably, are audited relatively more intensively, the average audit rate suggested by our proxy seems reasonable.

We specify the penalty function, $\Theta(T - \tilde{T}, I)$, using the actual rules for calculating penalties for tax evasion and the compliance rating system applied by the tax examiners during the audits.\textsuperscript{17}

\textsuperscript{14}The model variant of Equation (4) is Equation (A.4), which we use to calculate effective average tax rates in model simulations.

\textsuperscript{15}$I$ is the so-called compliance rating assigned by tax examiners. Details in the Appendix, Section A.3.3.

\textsuperscript{16}Or equivalently, we may assume that each part of the tax return, to which an audit flag corresponds, is audited with probability 1, in which case $P$ reflects the share of a tax return that is audited.

\textsuperscript{17}See Appendix A.3.3 for details.
4 Calibration

We construct a set of parameters for the purpose of simulating the model that are internally consistent, i.e., they all derive from the same data set. For all income distributions and parameters estimated for wage earners, we account for the stratification scheme applied in the data collection process. We provide full details on the calibration and numerical solution algorithm in the Appendix. Here we provide a brief overview.

We use the taxpayer data to construct the income distributions needed in the model, estimating income distributions separately for the two groups, wage earners and the self-employed. As income measure we use total income defined as the sum of labor market income, transfers, capital income, stock income, self-employment income, and foreign income less deductions.

Our data contains too few tax evaders to estimate separate bivariate income distributions for tax evaders and honest taxpayers. Therefore, we restrict these distributions to being identical except for the most salient difference, a large number of wage earners with zero residual incomes, \( u \), reporting zero residual income, \( x \), corresponding to large mass points in the conditional residual income distributions of honest taxpayers, \( f_{u|x}^h \). We allow our estimates of these mass points to vary in \( z \) as it is much more common for low-income taxpayers to have no residual income compared to high-income taxpayers.

The self-employed are characterized by having a small proportion of their incomes reported by third parties. Therefore, they have mostly residual income and the mass points in the residual income distributions are essentially zero. For the same reason, the residual income distributions exhibit much greater variance compared to wage earners.\(^{18}\)

As we have restricted the conditional residual income distributions for honest taxpayers and tax evaders to be identical for \( u \neq 0 \), we can parametrize the difference in the densities of each group by a single parameter, \( Q \). As mentioned above, we do not allow this parameter to vary in \( z \), but we allow it to differ between the groups of wage earners and the self-employed to account, to some degree, for differences in the propensity for

\(^{18}\)In the Appendix, Figure A.3, we provide graphical illustrations of the difference between wage earners and the self-employed in terms of the bivariate income distributions.
evasion, e.g., due to self-selection into these employment categories.

With this restriction on conditional densities, we can write $f_{u|z}^h = Qf_{u|z}(u)$, if $u \neq 0$, $f_{u|z}^h = M(z)$, if $u = 0$, and $f_{u|z}^d = (1 - Q)f_{u|z}(u)$, where $M(z) \geq 0$ is the mass point of the residual income distribution for honest taxpayers at $z$. Thus, for some $z$, the share of honest taxpayers is $Q$ when $u \neq 0$, whereas it is $M(z)$ when $u = 0$.

Under the assumptions of our model, we can identify $Q$ in our data as the share of taxpayers with non-zero residual income in the sample who do not underreport income on their tax returns. In the model, any tax evader with $u > u$ always evades some amount, so, for our purposes, those who do not evade are revealed to be honest. The calculation of the parameter values for wage earners and the self-employed is shown in Table 2, i.e., 0.855 and 0.640, respectively. Not surprisingly, $Q$ is estimated to be much smaller among the self-employed, reflecting a larger propensity for tax evasion.

The model has a fixed penalty factor, $\theta$, as opposed to the actual penalty function described in Appendix A.3.3. We approximate an appropriate value of $\theta$ by calculating the average penalty rate for the sample of tax evaders, accounting for the relative shares of wage earners and self-employed in the population. We take a simple approach and use the OLS slope coefficient between calculated penalties, $\Theta(\cdot, \cdot)$, and underreported taxes as our value of $\theta$. The resulting penalty rate on underreported taxes is 1.15, which is levied on taxes evaded.

We estimate a marginal tax function, $t(z)$, using local mean smoothing of marginal tax rates on the entire sample of wage earners and self-employed accounting for the relative shares of wage earners and self-employed in the population. We allow the approximated tax rates to vary in $z$ to partially account for the progressiveness of Danish income taxes. Because our data set contains all line items, we can calculate each taxpayer’s marginal tax

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19 As a technical matter, we also need to account for the few taxpayers who overreport income on their tax returns. See Appendix, Section A.3.2, for a discussion of this point.

20 While it is possible to solve the model with a more general (e.g., non-linear) tax rate function, we cannot, in this model setup, implement the actual Danish marginal tax rate function as there is no unique mapping from $(z,u)$ to the marginal tax rate. Incorporating the complexity of taxation in Denmark would require a model of tax enforcement specific to individual line items and incorporating the equilibrium distribution of audit resources across these line items, as well as across audit groups. This is possible in theory, but we cannot evaluate such a model empirically using this data set as the number of tax evaders, distributed on line items, is too small.
rate on all income components, such as earnings, capital income, stock income, etc. To obtain an average marginal effective tax rate, conditional on \( z \), we then weight marginal taxes of different components according to their relative size on a taxpayer's tax return. The average of the estimates of \( t(z) \) is 0.442, but the function includes values as low as 0.124 for taxpayers with little earnings in excess of the personal allowance and as high as 0.521 for taxpayers facing the top income tax rate.

To simulate the model, we find individual solutions, \( (p, \frac{\partial p}{\partial x}) \), to Equation (A.2) in the Appendix. For each \( z \), solutions are found numerically using methods of ordinary differential equations. The optimal budget allocation, which in our simulations is always interior, is found by equating shadow prices of increasing the budget density across levels of \( z \). Without loss of generality, we normalize the per-audit cost, \( c \), to 1 such that overall budget parameters \( B \) can be interpreted as the share of the population subject to audit within the groups of wage earners and the self-employed. Since the mean level of evasion is inversely proportional to total tax revenue, it is monotonically declining in \( B \). To calibrate \( B \), we use the estimated income distribution to simulate a population of taxpayers: we vary \( B \) until the average level of evasion for tax evaders matches the level observed in the data, approximately DKK 8,312 for wage earners and DKK 25,991 for the self-employed. The resulting budget values are \( B = 0.0412 \) and \( B = 0.4565 \), respectively.

In Appendix A, we perform a robustness check to ensure that the close correspondence between the aggregates of the data and the baseline simulation are not substantially changed by small perturbations of the key parameters of the model.

5 Results

Our results are organized into two parts. In the first part, we present a graphical comparison of the relationships between key economic variables in observed data and a large simulated population (approx. 200,000 observations). This part provides the grounds based on which we conclude that the model is able to replicate the aggregate relationships in the data, e.g., between tax evasion and residual income, and that data and model
simulations suggest the same impact of regressively biased tax enforcement. Whereas the simulation allows complex nonlinear relationships between the key variables, it does not allow for a more general formulation of behavioral heterogeneity, such as stemming from differences in gender, age, marital status, etc. Therefore, in the second part, we present regression analyses between the key economic variables in observed data, controlling for these sources of heterogeneity. In addition, these regressions allow us to control for the between-audit-group and the within-audit-group variation in key economic variables jointly, identifying each type of correlation separately.\footnote{Note that these regressions are a convenient way of representing the relationships between key variables, accounting for heterogeneity, covariation between income variables, and outliers. While the regressions are consistent with the regressive/progressive structure of effective average tax rates being caused by regressive tax enforcement and the distribution of information reports and audit resources, they do not, as such, provide evidence about the causal relationship between these variables.}

5.1 Tax Evasion and Enforcement

In Figure 2 we compare the observed distribution of flags across third-party reported and residual income with the optimal distribution obtained in the simulations. In Panel (a) we show, for each individual, third-party reported income (in thousands of DKK) and the ratio of flags to the maximally observed number of flags assigned to any return. In addition, we show the local-average ratios and 95 percent confidence bounds using local mean smoothing. In Panel (b) we show individual and average observations of audit probabilities from simulated data. Generally, the audit intensity is increasing in third-party reported income, reflecting the fact that higher-income taxpayers find it relatively easier to evade taxes since the conditional variance of true residual income is larger. As we do not know how the number of flags assigned to tax returns translates into the likelihood of being audited, it is not surprising that there is a level difference between the two graphs. This reflects the fact that our minimal assumptions proxy for the empirical audit probability suggests an audit rate among wage earners of 3.3 percent, whereas the audit rate required to calibrate the model is 4.1 percent. Nonetheless, the graphs have very similar profiles. Both are increasing in third-party reported income and the audit intensity is especially high in the right tail of the distribution. We calculate a goodness-
of-fit measure of the simulations as the correlation between local means for observed and simulated data using 200 evenly spaced points on the domain of the local means curves. The similarity in the graphs is borne out in a correlation coefficient between local means of 0.803.

[Figure 2 about here]

Panels (c) and (d) show the empirical and simulated covariation of reported residual incomes \((x)\), denoted in thousands of DKK, and the probability of audit. As in Panel (a), there is a level difference between the two graphs. However, under our minimal assumption that the number of flags is positively correlated with the actual likelihood of audit, Panel (c) suggests that the actual likelihood of an audit is distributed across the distribution of reported residual income in a manner broadly consistent with a revenue-maximizing tax agency.\(^{22}\) This conclusion is reinforced by the relatively high correlation (0.797) between local means of the share of flags and simulated audit probabilities in the distribution of reported residual incomes.

The simulations accurately reproduce the covariance structure of tax evasion with respect to the composition of the tax return in terms of third-party reported income and residual income. Figure 3 shows empirical and simulated covariation of tax evasion \((u - x)\) and residual income \((u)\), denoted in thousands of DKK for both wage earners and the self-employed. For each panel, we show individual data points and local means with 95 percent confidence intervals across the domain of residual incomes using local mean smoothing. As shown in Panel (b) and (d), the local means of simulated tax evasion are highly correlated (correlation coefficients 0.951 and 0.966, respectively) with the local means of observed tax evasion for both wage earners and the self-employed. Moreover, except for a slight clustering of wage earners with small negative residual incomes, but relatively large degrees of evasion in the observed data, the distribution of individual data

\(^{22}\)Note that the increasing average probability of audit for \(x > 0\) is perfectly consistent with audit probability functions being strictly decreasing, conditional on \(z\). In the simulations, the average audit probability is increasing for \(x > 0\) because the equilibrium audit intensity and the variance of \(u|z\) are increasing in \(z\). Therefore, the higher is a taxpayer’s \(z\), there more likely it is, on average, that he is audited which, in equilibrium, lessens the degree to which he evades taxes, making it more likely that he reports a positive residual income.
points also closely resembles that observed in the data. Although the self-employed evade more taxes on average, in neither data nor simulations do the self-employed appear to be more prone to evasion on the margin. Rather, the self-employed evade more taxes because they tend to have larger incomes and because less of that income is revealed by third parties. Finally, in equilibrium, tax evasion for the self-employed is curtailed to a large extent by intensive auditing.

In conjunction, Figures 2 and 3 suggest that the direct evidence of tax evasion and the indirect evidence on the Danish tax agency’s enforcement strategy is consistent with our theory of rational tax evaders and a revenue-maximizing tax agency. Moreover, since, as shown in Figure 2, the budget intensity increases with third-party reported income, we should expect Scotchmer’s conjecture of progressive/regressive bias conditional on residual/third-party reported income to be borne out in both data and simulations.

5.2 Effective Tax Rate Bias

We calculate the bias of effective average tax rates as described in Section 3.3, \( \tau - \tau_{\text{eff}} \), for data using the actual tax and penalty systems while for simulations using our approximations of a constant penalty rate, \( \theta \), and a set of constant marginal tax rates, \( t_z \), that vary with third-party reported income.

In Figure 4 we display, for individual and simulated data, our calculation of effective tax rate bias and either third-party reported income or residual income. Panel (a) shows observations from the data set and Panel (b) shows simulated data in the case of third-party reported income. In each panel we also show local means calculated using local mean smoothing. Both data and simulations exhibit effective average tax rates that are progressively biased with the bias decreasing towards 0 as third-party reported income increases. Moreover, the estimated local means are highly correlated (correlation coefficient 0.974).

Panels (c) and (d) show the corresponding figures of the data points and local means of effective tax rates and residual income. Panels (c) and (d) share the same overall
shape, namely, effective tax rates relatively unbiased (flat) in negative residual income but strongly biased and increasing in positive residual income. In both panels, the tax bias seems to decrease slightly at very high positive residual incomes. For the simulations, this reflects, similar to Figures 2(a) and 2(c), that high residual income is more common among taxpayers that also have large third-party reported incomes and that are audited relatively intensely. The structure of the data also seems consistent with this explanation. Again, local means of effective tax bias in data and simulations in the distribution of residual income are highly correlated. The correlation, however, is somewhat smaller than for the progressive bias, reflecting the fact that regressive bias is generated partly by the allocation of audit probabilities within audit groups which, in the data, we observe imperfectly.

[Figure 4 about here]

5.3 Regressions

Another way to assess the correlation structure in the data is to run reduced-form regressions, as we have done in Table 3, controlling for within-audit-group and between-audit-group variation and, in all regressions, using additional controls for age, age-squared, and indicator variables for taxcenter, industry/employment status, marriage to self-employed taxpayer, homeownership, state-church membership, joint tax return, and gender.23

First, Panel A shows estimates from running a median regression on the sample of tax evaders of tax evasion on true residual income, \( u \), and third-party reported income, \( z \), as well as a median regression of evasion on true total income, \( y \). Panel D shows equivalent regressions for the self-employed. We allow slope coefficients to differ depending on whether \( u \) is positive or negative as we can see from Figure 3 that evasion behavior

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23 A previous version of this article presents Table 3 without including these additional controls. The regression coefficients are in all cases fairly close for the two specifications, the additional controls mainly being significant in the audit-probability regression. This is because the additional controls, e.g., gender and homeownership, are good predictors of whether a taxpayer reports his or her income correctly but not of the magnitude of evasion. In the previous version, we used a pseudo \( R^2 \) as defined in Koenker and Machado (1999), which constitutes a local measure of goodness of fit for the median. For our purposes, we need a goodness-of-fit measure for the entire distribution, so we use the logical equivalent of the usual OLS \( R^2 \), defined in Table 3.
is markedly different for positive and negative true residual incomes. As the sample is relatively small and includes a number of outliers, we use median regressions in order that these data points do not drive the results. Whereas evasion does not appear to be increasing in total income $y$, it is, in fact, strongly increasing (0.379 [0.011] for wage earners 0.032 [0.001] for the self-employed) in positive residual income, which is also reflected in the difference in pseudo $R^2$. As we can see from Figure 3, this is because positive residual income is much easier to disguise – many tax evaders for whom $u > 0$ simply evade the entire amount of their residual income. For the tax agency, these taxpayers are indistinguishable from the many honest taxpayers reporting around $u = 0$, so this type of evasion is costly to uncover. For wage earners this is an especially attractive strategy due to the large mass of honest taxpayers reporting $x = 0$. For the self-employed there is virtually no excess mass of honest taxpayers reported $x = 0$, but it is still the case that the conditional distributions of residual income given third-party reported income is centered around $u = 0$ which makes such a reporting strategy attractive. As we also noted above, Table 3 suggests that the observed average marginal propensity to evade taxes is smaller for the self-employed than for wage earners. In our model, this is explained by the much higher audit rate for self-employed compared to wage earners. Because the self-employed on average have higher incomes and are subject to less third-party reporting, a self-employed taxpayer would tend to evade more than a wage earner for the same audit risk. Despite the high audit rate for self-employed, they nevertheless evade substantially more than wage earners.

[Table 3 about here]

Next, Panel B shows marginal effects, multiplied by a factor 100 for readability, from a Tobit regression of audit flag intensity (our empirical counterpart to the audit probability) on third-party reported income, $z$, and reported residual income, $x$, allowing slopes to

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24 In the model, this difference in evasion behavior is due to i) the incentive compatibility constraint dictates higher audit probabilities at lower reported residual incomes, which discourages evasion on negative residual income relative to positive residual income, and ii) the large mass point of honest tax reports at $x = 0$ combined with the incentive compatibility constraint necessitates low audit probabilities on the positive side, which encourages evasion on positive residual income relative to negative residual income.
differ depending on whether \( x \) is positive or negative, and a Tobit regression of audit flag intensity on total reported income, \( \tilde{y} \).\(^{25}\) As the audit intensity is a function of not only reported residual income and third-party reported income, but also the distribution of true residual income at a given level of third-party reported income, we also include a function of \( u \).\(^{26}\) We find that the correlations exhibited by SKAT’s audit flags are consistent with Scotchmer’s conjecture that a population-wide description of an optimal enforcement regime should entail decreasing audit probabilities within audit groups but increasing probability of audit between groups, exhibited by the negative coefficient on \( x \cdot D_{x \leq 0} \) and a positive coefficient on \( z \).

The correlation of audit probability and income within and between audit groups translates into a significant positive correlation between audit probability and total reported income, \( \tilde{y} \). Thus, despite the decreasing relationship within audit groups, third-party information reporting allows SKAT to audit taxpayers with high incomes more frequently.\(^{27}\)

Lastly, in Panel C of Table 3 we run a median regression for the effective tax rate bias (in percentage points) either on true residual income, \( u \), with slopes allowed to differ on the positive and negative domain of \( u \), and third-party reported income, \( z \), or on true total income, \( y \). Since the data on tax evasion and audit flags seem consistent with the mechanisms driving the theoretical prediction of regressively biased effective average tax rates within audit groups, it is not surprising that we find a regressive bias within audit groups (i.e., a positive coefficient on \( u \cdot D_{u \geq 0} \)) and progressive bias between audit groups (i.e., a negative coefficient on \( z \)). These effects combine to make tax rates progressively

\(^{25}\) Left and right censoring of the audit flag intensity in the Tobit regressions is at 0 and 1, respectively.

\(^{26}\) In fact, as we can see in Figure 2, in the aggregate for both observed and simulated data there is a slight positive correlation between the audit intensity and reported residual income for large/positive values of reported residual income. In the model there is a negative relationship between audit intensity and reported residual income within audit groups. The increasing relationship in the Figure 2(b) is caused by intense auditing of individuals with high levels of third-party reported income who also, on average, report relatively high residual incomes. The empirical distribution of audit intensity on third-party reported income in Figure 2(a) is consistent with this allocation of resources.

\(^{27}\) The intention to audit high-income taxpayers with higher probability is not a specific feature of Danish tax enforcement. Internal Revenue Service (2012) shows how, in 2011, 1.0 percent of taxpayers with incomes less than $200,000 were audited, 3.9 percent of taxpayers with income in the range of $200,000-1,000,000 were audited, and 12.5 percent of taxpayers with incomes over $1,000,000 were audited.
biased in total income, $y$. As shown above, the progressive bias between audit groups derives from the fact the SKAT intensively audits taxpayers with high $z$.

Overall, Table 3 suggests a correlation structure of effective tax rates as depicted in the stylized Figure 1(b). The data supports the theoretical prediction that effective tax rates are regressive within audit groups. Between audit groups, there is a progressive bias such that average tax rates are actually progressively biased in total total income.

Table 3 highlights the necessity of controlling for the information available to tax agencies in empirical work focused on the comparative statics of tax evasion and audit probabilities. Whereas both data and our simulations produce a relatively straightforward positive correlation between residual income and tax evasion and a relatively straightforward negative correlation between reported residual income and the audit probability, analyses without information controls may grossly underestimate the strength of these relationships.\textsuperscript{28}

5.4 Non-Detectable Income

A potential problem for the robustness and validity of our results concerns non-detected tax evasion. As we discuss in Section 3, some unreported income is almost certainly missing from our measures of tax evasion, despite SKAT’s diligent effort in making audits comprehensive. In particular, black market income is likely hard to detect. In Appendix B.1, we briefly present the best available evidence on the distribution of black market income in the Danish population based on survey data collected by the Rockwool Foundation Research Unit. This evidence suggests that black market income may be of a non-negligible magnitude averaging approximately 3,143 DKK in the population. In comparison, the population weighted average underreported income is 3,619 DKK, cf. Table 1, Panel C.

If black market income is completely non-detectable, the presence of such income will affect neither the taxpayers’ nor the tax agency’s optimization criteria.\textsuperscript{29} Consequently,

\textsuperscript{28}For example, this may explain why Clotfelter (1983), who includes rough controls for audit groups based on intervals of adjusted gross income, finds a strongly positive effect of income on tax evasion, whereas Feinstein (1991), who does not include such controls, finds no significant effect.

\textsuperscript{29}For taxpayers, this hinges on the assumption of risk neutrality. For example, if taxpayers are risk
the equilibrium of the model is unaffected, and the calibration exercise in this section remains valid because we fit the model to average tax evasion not including black market income.

Of course, even if black market income is completely non-detectable, it implies a measurement error in true residual income and translates into an underestimation of the effective average tax rate bias. Given that black market income is negatively correlated with reported income (cf. Appendix B.1) and third-party reported income is a very large part of reported income (approximately 95 percent in the population), we can deduce that black market income is also negatively correlated with third-party reported income. Therefore, accounting for black market income implies a level shift in the effective tax rate bias as a function of third-party reported income and, in addition, that this level shift is largest for taxpayers with little third-party reported income. As a result, including black market income implies a stronger progressive bias.

With respect to the regressive bias, the effect of including black market income depends on how black market income and detectable residual income are correlated. For the group of “wage earners”, we know that low-income earners and recipients of public transfers more frequently provide black market labor. These individuals have little to no detectable residual income, and, as black market income constitutes positive residual income and at the same time increases the effective tax rate bias, including black market income will tend to strengthen the positive relationship between tax rate bias and residual income (i.e., the regressive bias) for positive residual incomes depicted in Figure 4(c).

All in all, black market activities strengthen the distortions of the statutory tax system already generated by tax evasion and enforcement with respect to the formal economy.\textsuperscript{30}

\textsuperscript{30}If, instead, it is assumed that black market income is partially detectable, the structure of the model is changed. For example, we can think of situation wherein the share of evaded income that is detected, conditional on being audited, is subject to a constant probability of detection or a variable probability of detection for which the detection probability is modulated by the thoroughness with which a return is audited. However, as long the detection probability is constant or depends only on the factors that enter into $p(x|z)$, we expect that the equilibrium solution to the model will be very similar. From the point of view of a taxpayer, what counts is the compound probability of having evasion detected, so the solution of the altered differential equation describing the equilibrium will change so as to make the effective audit probability, the product of the audit and detection probabilities, conform to the same restrictions imposed on the audit probabilities in our model. In this case, the main difference in the model will be higher required budget values for the realized levels of evasion.
6 Concluding Remarks

This article highlights the importance of information in tax enforcement and advances the literature in the direction of developing a full-fledged structural model of tax evasion and enforcement. We find evidence in favor of the regressive bias prediction and Scotchmer’s (1987) conjecture that it is crucial to distinguish regressive bias within an audit group from aggregate or between-group variation. Using detailed administrative data, we find evidence suggesting that, whereas effective tax rates are regressively biased within audit groups as theory suggests, this relationship is negated by a progressive bias between audit groups induced by the distribution of audit resources and third-party information. The outcome is that tax rates are progressively biased in total income.

Based on data on the distribution of black market income in Denmark, we argue that our results are also robust to the lack of non-detectable income in our data. In fact, as discussed in Section 5.4, the data on the distribution of black market income suggests that our finding of regressive and progressive tax rate bias within and between audit groups, respectively, are lower bounds on the actual distortions of the statutory tax system.

The degree to which optimal tax enforcement distorts progressive redistribution is determined by how well tax agencies’ can predict true incomes. Whereas, in Denmark, the regressiveness within audit groups is counteracted by a large information collection effort, the model implies that, in an enforcement regime with much less third-party reported information, optimal enforcement may well induce a substantial regressive bias in the redistribution among taxpayers.

Models of tax evasion and enforcement that do not consider third-party reported information suggest that regressive bias can be countered simply by adjusting marginal tax rates. Once we allow for population heterogeneity of behavior and income composition, this is no longer feasible. Our results suggest an obvious policy to ameliorate these distortions: increasing the share of income reported by third parties will reduce both the extent of evasion and the regressive bias in tax enforcement.

In addition to providing empirical evidence on the regressivity of tax enforcement, this article contributes to the empirical literature on tax evasion by helping explain the
behavior of tax evaders. We find that the reporting behavior of tax evaders closely resembles the behavior predicted by our model. We focus on the implications, rather than the explanations, of honest reporting by some taxpayers as an exogenous feature of the information available to tax agencies. In other words, we do not explain why some taxpayers are honest and some are not. In another vein in the literature, behavioral/sociological explanations for honest tax reporting are explored; for example, guilt and shame (e.g., Grasmick and Bursick, 1990), fairness (e.g., Spicer and Becker, 1980), and trust in government (e.g., Slemrod, 2003; Torgler, 2003). Therefore, our study is a necessary complement to the extant literature on tax evasion. All in all, there seems to be a role for both expected-utility maximization and behavioral/social extensions in explaining the reporting behavior of taxpayers.
Table 1. Tax Compliance in Denmark, Income Year 2006.

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</tr>
<tr>
<td><strong>C. Wage earners and self-employed</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Income</td>
<td>202,310</td>
<td>3,367</td>
<td>3,619</td>
<td>-252</td>
<td>192,324</td>
<td>9,987</td>
</tr>
<tr>
<td></td>
<td>(1,883)</td>
<td>(469)</td>
<td>(467)</td>
<td>(24)</td>
<td>(1,774)</td>
<td>(715)</td>
</tr>
<tr>
<td>Total Tax</td>
<td>68,439</td>
<td>1,331</td>
<td>1,416</td>
<td>-86</td>
<td>(231)</td>
<td>(9)</td>
</tr>
</tbody>
</table>

Notes: Panel A shows descriptive statistics for a stratified random sample of 10,740 taxpayers denoted as wage earners (incl. unemployed, pensioners, etc.). Due to the stratification strategy employed by SKAT, the sample contains 74.6 percent “heavy” taxpayers (i.e., with high-complexity tax returns) and 25.4 percent “light” taxpayers, whereas the population has 32.6 percent heavy taxpayers and 67.4 percent light taxpayers. In Panel B the sample consists of 8,890 randomly selected self-employed taxpayers. No stratification was employed. Panel C provides descriptive statistics for wage earners and self-employed combined using population weights.

Total income is defined as the sum of labor market income, transfers, capital income, stock income, self-employment income, and foreign income less deductions. In the table, deductions are given as a negative amount. Reported income is the sum of third-party reported income and self-reported income. Standard errors of means in parentheses. All estimates for wage earners are population weighted. All amounts in DKK (1 USD ≈ 6 DKK in 2006).
Table 2. Reporting Behavior of Danish Wage Earners and the Self-Employed, 2006 Incomes.

<table>
<thead>
<tr>
<th>Observations</th>
<th>Wage earners</th>
<th>Self-employed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entire income reported by third-parties?</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>A.</td>
<td>Share</td>
<td>Share</td>
</tr>
<tr>
<td># underreported</td>
<td>0.010</td>
<td>0.049</td>
</tr>
<tr>
<td># correct</td>
<td>0.653</td>
<td>0.269</td>
</tr>
<tr>
<td># overreported</td>
<td>0.003</td>
<td>0.016</td>
</tr>
<tr>
<td>Total reports</td>
<td>0.665</td>
<td>0.335</td>
</tr>
</tbody>
</table>

B. | Share of sub-sample | Share of sub-sample | Share of sub-sample | Share of sub-sample |
| Correct reports | 0.979 | 0.809 | 0.943 | 0.590 |
|                  | (0.002) | (0.011) | (0.055) | (0.006) |
| Not underreporting | 0.984 | 0.855 | 0.950 | 0.640 |
|                  | (0.002) | (0.010) | (0.055) | (0.007) |
| “Honest” taxpayers* | 0.988 | 0.901 | 0.957 | 0.690 |
|                  | (0.002) | (0.008) | (0.055) | (0.007) |

Notes: Standard errors of fractions in parentheses. The sample of wage earners is a stratified random sample. Fractions and standard errors are calculated subject to the stratification scheme. “Wage earners” also include recipients of benefits. The sample of self-employed is a non-stratified random sample. *Calculated imposing the assumption that unintentional underreporting is as frequent as (unintentional) overreporting — i.e., symmetry in reporting errors. For example, for the self-employed (right-most column), the (unstratified) calculation is simply \((0.570 + 2 \cdot 0.048)/0.966 \approx 0.690\). For wage earners, we provide a population weighted estimate.
### Table 3. Evasion Behavior, Tax Enforcement, and Tax Bias – Regressions on Data Sample.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>A. Evasion</th>
<th>B. Audit flag intensity</th>
<th>C. Tax bias (in pct. points)</th>
<th>D. Evasion</th>
</tr>
</thead>
<tbody>
<tr>
<td>u \cdot D_{u \leq 0}</td>
<td>-0.000</td>
<td>-0.029 **</td>
<td>-0.004</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.010)</td>
<td>(0.004)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>u \cdot D_{u &gt; 0}</td>
<td>0.379 ***</td>
<td>-0.000</td>
<td>0.030 ***</td>
<td>0.032 ***</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>z</td>
<td>0.001</td>
<td>0.001 *</td>
<td>-0.004 ***</td>
<td>0.008 ***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>y</td>
<td>-0.003 **</td>
<td>-</td>
<td>-0.002 **</td>
<td>0.017 ***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>x \cdot D_{x \leq 0}</td>
<td>-</td>
<td>-0.037 ***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x \cdot D_{x &gt; 0}</td>
<td>-</td>
<td>0.002</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\tilde{y}</td>
<td>-</td>
<td>0.002 ***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Estimation method**
- Median regression
- Tobit regression
- Median regression
- Median regression

<table>
<thead>
<tr>
<th>Additional controls included</th>
<th>Median regression</th>
<th>Tobit regression</th>
<th>Median regression</th>
<th>Median regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample</th>
<th>Median regression</th>
<th>Tobit regression</th>
<th>Median regression</th>
<th>Median regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaders</td>
<td>905</td>
<td>10,584</td>
<td>900</td>
<td>2,980</td>
</tr>
<tr>
<td>Observations</td>
<td>905</td>
<td>10,584</td>
<td>900</td>
<td>2,980</td>
</tr>
<tr>
<td>Ols. left-censored</td>
<td>-</td>
<td>8,555</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ols. right-censored</td>
<td>-</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pseudo R²</td>
<td>0.74</td>
<td>0.30</td>
<td>0.49</td>
<td>0.26</td>
</tr>
</tbody>
</table>

**Notes:** *p < 0.05, **p < 0.01, ***p < 0.001. All samples are restricted to only contain taxpayers with strictly positive true total income.

**Variable definitions:** Additional controls include age, age-squared, and indicator variables for taxcenter, industry/employment status, marriage to self-employed taxpayer, homeownership, state-church membership, joint tax return, and gender. Monetary variables are in '000 DKK (u, z, y, \tilde{y}, x, and evasion). Audit flag intensity is defined as no. of flags on the tax return divided by maximum no. of flags on any tax return (13) so that audit flag intensity is distributed on the unit interval. Tax bias is the percentage point difference between the true average tax rate and the effective average tax rate (reflecting both evasion behavior, audit probabilities, and penalty rates).

**Wage earners:** Numbers in parentheses are stratified standard errors, bootstrapped in the case of median regressions. The sample contains wage earners and benefit recipients with detected tax evasion. In the Tobit regressions, we present marginal effects conditional on the audit probability being in (0, 1) (multiplied by a factor of 100 for readability). Due to missing values in the compliance-rating variable, there are 5 fewer observations in the tax-bias regression.

**Self-employed:** Numbers in parentheses are bootstrapped standard errors. The sample contains self-employed taxpayers with detected tax evasion.

**Goodness-of-fit measures:** In median regressions, pseudo $R^2 = 1 - \sum (y_i - \tilde{y}_i)^2 / \sum (y_i - \text{median}(y))^2$, whereas in Tobit regressions the McFadden pseudo $R^2$ is used.
Figure 1. Correlation Structure of Effective Average Tax Rates.

Notes: $\tau$ is the statutory average tax rate (here, constant at $\tau = t$), $\tau_{i}^{\text{eff}}$ is the effective average tax rate within audit group $i$, and $\tau^{\text{eff}}$ is the aggregate effective average tax rate.
Figure 2. Observed and Simulated Audit Probabilities for Wage Earners.

Notes: Panels (a) and (c) show, for the subsample of only tax evading wage earners and recipients of benefits (905 obs.), the number of flags per taxpayer as a share of the maximally observed number of flags across the distribution of third-party reported income, \( z \), and residual income, \( u \), respectively. Panels (b) and (d) show the simulated audit probability (\( \sim 200,000 \) obs.) across the distribution of third-party reported income, \( z \), and residual income, \( u \), respectively. In all panels, the dotted lines give the local average of the observations together with 95 percent confidence bands using local mean smoothing with the Epanechnikov kernel function and a rule-of-thumb bandwidth. The correlations noted in Panels (b) and (d) are calculated between local means using 200 evenly spaced points on the domain of the local means curves. The local mean smoothing in Panels (a) and (c) does not account for the stratification scheme. Income is defined as the sum of all income less deductions and is measured in ’000 DKK. 1 USD \( \approx 6 \) DKK (in 2006). In Panels (b) and (d), the budget is allocated such that approximately 4.1 percent of all wage earners and recipients of benefits are audited.
Figure 3. Observed and Simulated Tax Evasion Across the Distribution of True Residual Income for Wage Earners and the Self-Employed.

Notes: Panels (a) and (c) show observed tax evasion across the distribution of true, i.e., post-audit, residual income, $u$, for wage earners (905 obs.) and self-employed (2,980 obs.), respectively. Panels (b) and (d) show simulated tax evasion across the distribution of true, i.e., post-audit, residual income, $u$, for wage earners and self-employed (~200,000 obs. for each group), respectively. In both panels, the dotted lines give the local average of the observations together with 95 percent confidence bands using local mean smoothing with the Epanechnikov kernel function and a rule-of-thumb bandwidth. The correlations noted in Panels (b) and (d) are calculated between local means using 200 evenly spaced points on the domain of the local means curves. The local mean smoothing in Panels (a) and (c) does not account for the stratification scheme. The simulated data in Panels (b) and (d) is not stratified. Income is defined as the sum of all income less deductions and is measured in ’000 DKK. 1 USD $\approx$ 6 DKK (in 2006). In Panel (b), the budget is allocated such that approximately 4.1 percent of all wage earners and recipients of benefits are audited. In Panel (d), the fraction of self-employed taxpayers audited is approximately 45.7 percent. Note, however, that the self-employed make up a much smaller group (approx. 400,000) compared to wage earners (approx. 4.2 million).
(a) Observed tax bias for tax evaders across the distribution of third-party reported income.

(b) Simulated tax bias for tax evaders across the distribution of third-party reported income.

(c) Observed tax bias for tax evaders across the distribution of true residual income.

(d) Simulated tax bias for tax evaders across the distribution of true residual income.

**Figure 4.** Observed and Simulated Tax Rate Bias for Wage Earners.

**Notes:** The effective tax rate bias, $\tau - \tau_{\text{eff}}$, is the difference between the average statutory tax rate and the average effective tax rate as implied by the tax system, tax enforcement, and tax evasion behavior. Panels (a) and (c) show the observed tax bias as a function of third-party reported income, $z$, and residual income, $u$, respectively, for the subsample of tax evading wage earners and recipients of benefits (900 obs.). Tax rate bias is calculated as in Equation (4). Panels (b) and (d) show the simulated tax bias as a function of third-party reported income, $z$, and residual income, $u$, respectively, for tax evading wage earners and recipients of benefits (~200,000 obs.). Tax rate bias is calculated as described in Equation (A.4). In all panels, the dotted lines give the local average of the observations together with 95 percent confidence bands using local mean smoothing with the Epanechnikov kernel function and a rule-of-thumb bandwidth. The correlations noted in Panels (b) and (d) are calculated between local means using 200 evenly spaced points on the domain of the local means curves. The local mean smoothing in Panels (a) and (c) do not account for the stratification scheme. Income is defined as the sum of all income less deductions and is measured in '000 DKK. 1 USD $\approx$ 6 DKK (in 2006). In Panels (b) and (d), the budget is allocated such that approximately 4.1 percent of all wage earners and recipients of benefits are audited.
Appendix

A.1 Characterizing the equilibrium

As mentioned, our model is a generalization of the model in Erard and Feinstein (1994). Specifically, our model simplifies to theirs if i) $z$ is zero for all individuals, such that $F_{u|z} = F_u = F_y$, and ii) the ratio of honest to dishonest taxpayers, $\frac{f^h_u(u)}{f^d_u(u)}$, is constant on $[\underline{u}, \overline{u}]$. In this case, the problem becomes that of a partial optimization for a fixed $B(z)$ within an audit group. In this simpler version of the model, Erard and Feinstein (1994) show that the equilibrium audit and evasion functions have a number of useful properties. Due to the incentive constraints on reporting for high-income taxpayers, the audit function $p(x|z)$ is decreasing and continuous in reported income. The reporting function, $x(u|z)$ is strictly increasing in an upper region of the income domain and constant in a lower region as some taxpayers pool at the lowest possible report. As the audit and reporting functions are continuous and differentiable on the interior of the reporting domain, it is possible to solve for the equilibrium using methods of differential equations. In addition, as pooling occurs only at the lowest report, where the differential equation is undefined, sufficient conditions for equilibrium can be obtained by checking that the solution to the differential equation also satisfies the tax agency’s first-order condition for the lowest report, equivalent to (A.3) below. In the same way, we can leverage these properties to solve for the population-wide equilibrium as a range of within-audit-group equilibria coupled with the optimal budget distribution, $B(z)$.

The unique revenue-maximizing equilibrium of the model is described by the collection of functions, $u(x|z)$ and $p(x|z)$, and the budget distribution, $B(z)$. Once $p(x|z)$ is determined, $u(x|z)$ is implicitly defined as the solution to the taxpayers’ first-order condition, and the tax agency chooses $p(x|z)$ such that (3) holds with equality. The two equations are connected by the tax agency’s conditional expectation of taxpayers’ true income given the reported income and third-party reports, $E(y|x, z)$, which is

$$E(y|x, z) = z + \frac{f^h_{u|z}(x)x + f^d_{u|z}(u(x|z))}{f^h_{u|z}(x) + f^d_{u|z}(u(x|z))} \frac{\partial u(x|z)}{\partial x} u(x), \quad (A.1)$$
where the derivative $\frac{\partial u(x,z)}{\partial x}$ is derived from (1) by differentiating implicitly to get $\frac{\partial u}{\partial x} = 2 + \frac{p''(x)(x-u)}{p'(x)}$.

We can then derive a second-order differential equation in $p(x|z)$, (A.2), which determines the optimal equilibrium responses $p(x|z)$ and $x(u|z)$ in audit group $z$ using the expressions for $E(y|x,z)$, $u(x|z)$, $\frac{\partial u}{\partial x}$, and the tax agency’s first-order condition to get

$$p''(x) = \left( \frac{f^h_u(x)}{f^d_u(u(x))} \left[ \frac{p(x)-\frac{1}{\theta+t}}{p'(x)} - \frac{\lambda c}{\theta+t} \right] - 2 \right) \cdot p'(x) \left( \frac{1}{1+\theta} - p(x) \right)^{-1},$$

suppressing $z$ for convenience.

However, as some taxpayers pool at the lowest report, to obtain sufficient conditions for equilibrium, we must check the tax agency’s first-order condition at $x = u$ separately as

$$E(u|x = u, z) = \frac{f^h_{u|z}(x) + \int_{u_{\text{pool}}}^u f^d_{u|z}(u) \, du}{f^h_{u|z}(x) + \int_{u_{\text{pool}}}^u f^d_{u|z}(u) \, du} = \frac{\lambda(z) c}{t + \theta t} + u,$$

where $u_{\text{pool}}$ is the residual income at which taxpayers (in this audit group) begin to pool at the lowest possible report.

Thus, given the equilibrium $\lambda(z)$, we can characterize the unique within-group equilibrium from Equations (A.3) and (A.2). By Equation (2), each $\lambda(z)$ corresponds to a required budget allocation, $B(z)$. Finally, the budget allocation across different $z$ is pinned down by the requirement that the shadow value of increasing the budget, $\lambda(z)$, must be the same for all $z$, i.e., $\lambda(z) = \lambda, \forall z$, for an interior solution. The shadow value, $\lambda$, is pinned down by the requirement that the tax agency’s overall budget, $B$, may not be exceeded.

Figure A.1 depicts the equilibrium functions for audit probabilities, income evasion, the effective average tax rate, and the induced density of income reports in a particularly simple version of the model, where $z = 0$ for all taxpayers. In addition, we set $B$ at 10 percent, $\log(u) \sim N(3.42, 0.3^2)$ truncated on $[20, 44]$, $Q = 0.4$, and $t = 0.5$.

Panel (a) shows the audit schedule, $p(x)$: it starts in $u$, is downward sloping, and

\footnote{Notice that $f_{x|z}(x(u)) = f_{u|z}(u(x)) \frac{\partial u(x,z)}{\partial x} = f_{u|z}(u(x)) \frac{\partial u(x,z)}{\partial x}$ as the SOC implies that $\frac{\partial u}{\partial x} \geq 0$ in interior optimum.}
Figure A.1. Equilibrium Responses and Tax Bias.

Notes: All panels display an example of equilibrium functions. The specification closely resembles an example employed in Erard and Feinstein (1994) in a model without third-party reporting. Equivalently, this could be an example of the solution for a particular $z$ in our model including third-party reporting. This example is produced assuming $B = 10$ percent, $\log(u) \sim \mathcal{N}(3.42, 0.3^2)$ truncated on $[20, 44]$, $Q = 0.4$, and $t = 0.5$. Income is measured in '000 $.
terminates in $p(\bar{x}) = 0$. This form balances the need to audit in order to raise revenue with the cost of doing so. The negative slope reflects the need to discourage high-income taxpayers from reporting too low incomes.

Panel (b) shows the amount of evasion as a function of true income. The linear increase in the first part of the graph reflects pooling of dishonest taxpayers: for a given audit schedule, there will be some level of residual income, $u_{\text{pool}}$ in $[u, \bar{u}]$, for which the most profitable report is $u$. Consequently, all taxpayers with residual incomes $u < u_{\text{pool}}$ also report $x = u$. Therefore, there will be a point mass in the induced distribution of reports, $f_x(x)$. After this pooling point, evasion falls rapidly in income until evasion again becomes increasing in income as the probability of detection becomes sufficiently low.

Panel (c) shows the effect of the optimal audit schedule on the ex ante effective tax rate, $\tau_{\text{eff}}$, which is calculated as the ratio of expected payments (taxes and penalties) to true income

$$\tau_{\text{eff}} = \frac{p(x) \cdot (ty + \theta t(y - \tilde{y})) + (1 - p(x)) \cdot t\tilde{y}}{y}. \quad (A.4)$$

The declining profile of $p(x)$ together with the high propensity to evade taxes of high-income taxpayers result in a negative relationship between the effective tax rate and income. Therefore, high-income taxpayers pay significantly less than the statutory tax rate, which, in the case of Figure 1(c), is $t = 0.5$, and we get regressively biased effective average tax rates.

Panel (d) shows the induced distribution of incomes and reports. The top graph is the original income distribution, which in this case is lognormal. The lower graph shows the distribution of induced reports, i.e., the equilibrium response of all taxpayers to the audit schedule. The right part of the graph is just a scaling of the original income distribution by $Q$ while the left part is a weighted average of reports by honest and dishonest taxpayers. The whole graph is somewhat lower than the original income distribution as there is a mass point of dishonest taxpayers reporting at $u$, the mass point being equal to the area between the graphs.
A.2 Numerical Implementation

We approximate the equilibrium solution by discretizing $z$ into an evenly spaced grid-point vector of dimension 200.\textsuperscript{32} Equilibrium functions for other values of $z$ are approximated by interpolation. For each grid point, we solve the second-order ordinary differential equation (ODE) in (A.2) for many values of $\bar{\tau}$, where $\bar{\tau} \equiv \tau(\bar{\eta})$. The ODE algorithm is initialized using $p(\bar{\tau}) = 0$ and $p'(\bar{\tau}) = \left(\frac{1}{1+\theta}\right)/\left((\bar{\eta} - \bar{\tau})\right)$, cf. (1). For each value of $\bar{\tau}$ and $z$, we need a corresponding value of $\lambda(z)$, the shadow value of increasing the budget size. However, $\lambda(z)$ and $\bar{\tau}$ are not separately identified. Therefore, we must take a heuristic approach, solving for each $\bar{\tau}$ the ODE for many values of $\lambda$ until one is found that satisfies the equilibrium conditions everywhere, in particular at $x = \bar{\tau}$. In practice, we do not merely guess repeatedly at $\lambda(z)$, but employ a search algorithm to find the $\lambda(z)$ that satisfies (A.3); this provides a candidate $\lambda(z)$ corresponding to a particular $\bar{\tau}$ that satisfies the FOC everywhere with a small error tolerance.

Figure A.2(a) shows a set of solutions of $p(x|z)$ for a particular $z$, each solution corresponding to a different value of $\bar{\tau}$. As shown in Panel (b), higher values of $\bar{\tau}$ correspond to lower values of $\lambda(z)$ (and higher values of $B(z)$) and imply that a larger proportion of taxpayers are audited. When this algorithm has executed for all grid points of $z$, we can determine the optimal budget allocation using the fact that in an interior equilibrium $\lambda(z)$ must be equalized across different levels of $z$.

Equation (A.2) can be solved by standard numerical methods. We employ a Runge-Kutta-type algorithm developed in Shampine (2009), which outperforms most standard ODE algorithms in terms of precision and robustness. However, two main numerical issues must be resolved.

First, due to point mass in $f_{u|z}^h$ at $u = 0$, $E(u|x, z)$ is discontinuous at $x = 0$, which induces what is known as a “singularity” in the differential equation. We take a standard approach to this problem and approximate solutions for which $\bar{\tau} > 0$ by substituting the logical function $1_{(x=0)}$ with a smooth, differentiable approximation. The resulting

\textsuperscript{32}The model for the self-employed is substantially more computationally intensive so there we use only 100 grid points. Of course, this implies that interpolations will be less precise, but this does not appear to be important. Likewise, solutions using only 50 grid points are graphically indistinguishable in terms of Figure 3.
Figure A.2. Solutions Examples for Wage Earners.

Notes: $\pi$ is defined as the lowest value of $x$ that solves $p(x) = 0$, i.e., the highest report of dishonest taxpayers. The domain of $\pi$ in Panel (b) is limited to highlight the graph around $\pi = 0$. Reported residual, $x$, is measured in '000 DKK (1 USD ≈ 6 DKK in 2006).

A function displays a relatively smooth transition from 0 to 1 in a small band around $x = 0$. An alternative approach is to split the ODE algorithm in two, corresponding to the domains $[u, 0)$ and $[0, \pi]$, and identifying the discontinuous jump in $p'(x)$ from the equations characterizing the equilibrium and the measure of point mass at $x = 0$. However, as the size of this discontinuity cannot be identified analytically, this introduces an element of imprecision in the algorithm which, in our experience, may negatively affect the robustness of the algorithm.

Second, the ODE algorithm may fail to converge if we allow the conditional density function to take values extremely close to 0 as the ratio $f_{u|x}(y) f_{d|x}(u(x))$ may diverge toward infinity. Estimating the density $f_{u|x}$ as a bivariate kernel density is numerically inconvenient as it tends to result in “troughs” of zero density in the interior of the domain of some conditional distributions. Instead, we estimate $f_{u|x}$ as a lognormal mixture distribution.

A.3 Calibration

A.3.1 Income Distributions

We use the taxpayer data to construct the income distributions needed in the model. In principle, the densities of honest and dishonest taxpayers can be estimated separately, but with the size of our data set this would introduce a large element of uncertainty in
estimates of dishonest taxpayers. This is important because richer wage earners are much more likely to have non-zero residual income than poorer wage earners. However, for the self-employed there are very few individuals without some residual income, and we can estimate income distributions without accounting for a mass point.

In practice, to fit the simultaneous distribution of $z$ and $u$, we exclude any honest taxpayers in $u = 0$ and fit a mixed lognormal distribution using 6 component distributions. Our results do not appear to alter significantly if, instead, a kernel estimation is used, although this approach can drastically increase the duration of computations. By using a sufficient number of component distributions the difference between this distribution and a bivariate kernel distribution becomes negligible. The distribution of the mass point of wage earners at $u = 0$ across $z$ is estimated separately as local means around each grid point of $z$.

Of course, the lognormal distribution is not defined on domains that include negative values. In estimating the mixture distribution, we create a simple additive mapping of the observations to a set of “virtual residual incomes” that are entirely positive and use the mapping to obtain the actual bivariate income distribution. The resulting distribution is indeed very close to that obtained by using a bivariate kernel density algorithm.

Lastly, we truncate the domain of the potential tax evaders’ conditional true income distributions where the densities are very close to zero to keep the fraction in equation (A.2) from diverging to infinity. Specifically, we truncate the unrestricted conditional densities at the 0.25 percent and 99.75 percent fractiles. The resulting supports of the conditional distributions vary in $z$ as illustrated in Figure A.3(a) for wage earners (dark) and the self-employed (light). Briefly, the variance of $u|z$ is generally increasing in $z$. However, the taxpayers with very low or negative $z$ have relatively complicated income compositions. Therefore, they have a high variance of $u|z$ and, for wage earners, a relatively small mass point at $u = 0$. In Figure A.3(b), we compare examples of the conditional residual income densities for wage earners and the self-employed for the same value of third-party reported income, in this case set to wage earners’ median value of third-party reported income, approx. 233,000 DKK. Recall that the conditional densities
(a) The support of $u$ across audit groups for wage earners (dark) and the self-employed (light).

(b) $f(u|z)$ at median values of $z$ for wage earners (solid) and the self-employed (dashed).

Figure A.3. Bivariate Income Distributions, $z$ and $u$

Notes: The estimated conditional densities of $u|z$ for wage earners and benefit recipients are truncated at the 0.25 and 99.75 percent fractiles of the unrestricted conditional distributions. The conditional densities are shown for a value of $z = 233,288$ DKK, corresponding to the median value for wage earners. The conditional density for wage earners is shown without the mass point located at $u = 0$ which accounts for approximately 58 percent of wage earners at this value of $z$. Residual income, $u$, and third-party reported income, $z$, are measured in ’000 DKK (1 USD ≈ 6 DKK in 2006).

of wage earners include a mass point at $u = 0$, here approx. 0.58, which is not shown in this figure. Thus, although it can be readily seen that the conditional densities of wage earners are more concentrated around $u = 0$ compared to the conditional densities of the self-employed, this difference is, in fact, much more pronounced.

A.3.2 Honesty

We can write conditional densities $f^h_{u|z} = Q f_{u|z}(u) + 1_{(u=0)} M(z)$ and $f^d_{u|z} = (1-Q) f_{u|z}(u)$, where $1_{(\cdot)}$ is the indicator function. Thus, for $u \neq 0$ the share of honest taxpayers is $Q$, whereas for $u = 0$ it is $M(z)$, where $M(z) \geq 0$ is the mass point at $u = 0$ for some level of third-party reporting, $z$. To determine an appropriate value of the parameter, $Q$, we must account for the fact that, in reality, some taxpayers seem to make reporting mistakes. For example, in the data some reports are adjusted downward by the auditor, which means that, in the absence of an audit, the taxpayer would have payed more than intended by the statutory tax system.

We approach the problem in the following way. First, we assume that no taxpayer will try to evade taxes on income that is reported by a third party (this assumption is borne
out in the data for wage earners as shown in Table 2). Secondly, in keeping with the
model, we disregard the fact that some taxpayers make reporting mistakes. A revenue-
maximizing tax agency is indifferent about the motivation for underreporting and about
overreporting.\footnote{We do not consider the, rather implausible, scenario that the tax agency might refrain from auditing
certain groups because this would reveal overreporting by some taxpayers thus lowering collected revenue.} As a consequence, taxpayers reporting too large taxable incomes are
treated as if they are exactly compliant and taxpayers that underreport taxable incomes
by mistake are treated as tax evaders. Then we separate taxpayers by whether they
underreported taxes (non-compliant taxpayers, \( x < u \)) or reported correctly/overreported
taxes (compliant taxpayers, \( x \geq u \)). Compliant taxpayers are decomposed into those
with zero residual income and non-zero residual income. We define the parameter \( Q \) as
the ratio of compliant taxpayers with non-zero residual income to the total number of
taxpayers with non-zero residual income in the sample. The idea is that having some
income not subject to third-party reporting provides taxpayers with ample opportunity
for evasion. By not seizing the opportunity, they reveal themselves as being honest in
the present context. Table 2 shows this decomposition. First, note that among wage
earners whose entire income is reported by third parties, the compliance rate is 97.9
percent. Among those wage earners that have some of their income not reported by
third parties, the compliance rate is 80.9 percent. The number of honest taxpayers is the
sum of those reporting correctly and those overreporting by mistake, which corresponds
to \( Q = 85.5 \) percent. The residual consists of both dishonest wage earners and wage
earners underreporting by mistake whom we cannot distinguish. To partially control
for self-selection into occupations according to a taxpayer’s proclivity to evade taxes, we
calculate \( Q \) separately for the self-employed as shown in Table 2. The resulting value,
\( Q = 64.0 \) percent, is indeed substantially lower and suggests that this distinction is
important.

A.3.3 Penalty

In Denmark, evasion penalties are calculated as a factor on taxes evaded; that factor,
however, varies for the amount evaded and the intentionality of evasion as assessed by
the auditor. In the case of intentional tax evasion, the fine is calculated as 1 times evaded
taxes under 30,000 DKK and 2 times the evaded taxes exceeding 30,000 DKK. In the case
of gross negligence, the rates are instead 0.5 times evaded taxes not exceeding 30,000 DKK
and 1 times evaded taxes exceeding 30,000 DKK. Fortunately, the compliance ratings
in the data are exactly intended to measure the degree of intentionality of uncovered
tax evasion. Compliance ratings take on values in \{0, 1, 2, \ldots, 6\} indicating decreasing
degrees of intentionality of misreporting. According to this classification, compliance
ratings of 0, 1, or 2 signify deliberate tax evasion, whereas 3, \ldots, 6 signify gross negligence
(approaching 3) or innocent mistakes (approaching 6). Using these classifications, we can
accurately calculate the penalty rate applicable for each individual tax evader. Assuming,
e.g., that innocent mistakes (rated 6) are not penalized or that the threshold in compliance
ratings between intentional evasion and gross negligence is between 1 and 2 or between 3
and 4, turns out not to affect the results we present in Section 5.

The model has a fixed penalty factor, \(\theta\), as opposed to the more complicated penalty
function, \(\Theta(\cdot, \cdot)\) described above. We approximate an appropriate value of \(\theta\) by calculat-
ing the average penalty rate for the sample of tax evaders accounting for stratification
between light and heavy taxpayers within the group of wage earners and for the relative
shares of wage earners and self-employed in the population. We take a simple approach
and use the OLS slope coefficient between calculated penalties, \(\Theta(\cdot, \cdot)\), and underreported
taxes as our value of \(\theta\). The resulting penalty rate on underreported taxes is 1.15.

### A.4 Sensitivity Analysis

To show that our conclusions are robust to changes in parameters, we present in Figure
A.4 the simulated results of parameter changes for wage earners. We do this by changing
the key parameters \(t_z\), \(\theta\), and \(Q\), and for each permutation letting \(B\) be calibrated to
match simulated and observed average tax evasion among evaders. This we do for 27
permutations of the key parameters, i.e., all combinations of \(-10\%, 0\%, +10\%\) changes to
the set of parameters, but otherwise using the same setup as in the baseline simulation.\(^{34}\)

\(^{34}\)For \(t_z\) the changes are implemented as across-the-board increases/decreases in the marginal tax rate.
Focusing on the mechanism driving the regressive bias within audit groups, Panels (a) and (b) of Figure A.4 show local averages of audit probabilities as a function of reported income and local averages of tax evasion as a function of true residual income, respectively. For audit probabilities, the changes are relatively minor, the main effects being a level shift in the maximal audit probability corresponding to changes in $\theta$. For tax evasion, the local averages are all qualitatively similar, although the impact of parameter changes are larger among taxpayers with larger residual incomes.

![Diagram of Audit Probability vs. Reported Residual Income](image)

![Diagram of Tax Evasion vs. True Residual Income](image)

![Diagram of Effective Tax Rate Bias vs. Third-Party Reported Income](image)

![Diagram of Effective Tax Rate Bias vs. True Residual Income](image)

**Figure A.4. Robustness Checks for Simulations of Wage Earners.**

*Notes:* This figure checks the robustness of our simulation results graphically by plotting variations in estimated local means on the basis of simulations with parameter permutations. We simulate the changes for wage earners in the four key relationships of the model, (a) audit probability as a function of reported residual income, (b) tax evasion as a function of true residual income, (c) regressive tax bias within audit groups as a function of true residual income, and (d) progressive tax bias between audit groups as a function of third-party reported income. The local averages depicted in the four panels are calculated in a similar manner to Figures 2–4, using local mean smoothing with the Epanechnikov kernel function and a rule-of-thumb bandwidth. We simulate the model for 10 percent parameters variations around the baseline estimates of $t_z$, $\theta$, and $Q$, corresponding to 27 separate simulations. Thus, Panel (a) depicts variation in local means around the baseline simulation depicted in Figure 2(d) and similarly Panels (b)–(d) correspond to variations around the baseline simulated local means depicted in Figures 3(b), 4(b), and 4(d), respectively. All amounts in '000 DKK (1 USD $\approx$ 6 DKK in 2006).
The structure of tax rate bias within and between audit groups in the simulations is also highly robust. The progressive bias between audit groups, shown in Panel (d), is virtually unchanged as it is generated mainly by the distribution of audit resources in the population, which is more or less unchanged by the parameter changes. The impact of parameter changes on the regressive bias within audit groups, shown in Panel (c), is more substantial as it compounds the effects of parameter changes shown in Panels (a) and (b). However, in all cases the qualitative relation between effective tax rate bias is very similar to the baseline simulation.

Varying the model parameters also affects the correlations of local averages in data and simulations, although not to a large extent. For the relationship between residual income and tax evasion, for example, the correlation coefficient lies between 0.917 and 0.954 compared to the baseline of 0.951. The most variable correlation is the relationship between third-party reported income and the probability of audit which lies between 0.615 and 0.933 compared to the baseline of 0.803. However, this largely does not affect the correspondence of the progressive bias relationships between data and simulations – the correlation of local averages for third-party reported income and effective tax rate bias lies between 0.966 and 0.976 compared to the baseline correlation of 0.970. The regressive bias relationship varies more as it is affected by changes in both tax evasion and the audit probabilities within audit groups and lies between 0.797 and 0.910 compared to the baseline correlation of 0.903.

B.1 Black Market Activities

A potentially important avenue for tax evasion is black market income. This type of income is much harder to discover by tax auditors and, thus, less likely to be included in our data despite the intensive auditing of tax returns for the experiment. To quantify the extent of black market activities in the population at large, we utilize survey data collected by the Danish Rockwool Foundation Research Unit. They have since 1985 collected survey data for random samples of the population in an attempt to quantify the incidence of and return to black market activity. Although the surveys in principle collect
identifiable information, such as social security numbers, the data set is anonymized. As such, no one has ever been prosecuted for having black market income due to participation in these surveys. Unfortunately, there were no surveys carried out in 2006 so we use two surveys from 2005 and two from 2007 instead (all amounts in 2006-prices). The surveys contain many variables, but we focus on measures of the incidence of and return to the supply of black market services (i.e., remuneration for black market labor). In Figure B.1, we show the incidence of black market work and the return to this activity across the distribution of reported income. Panel (a) shows the share of taxpayers having performed black market work for 20th fractiles of the reported income distribution in the sample. The figure indicates that black market work is more common among low-income taxpayers, whereas middle and top earners figure less prominently.\textsuperscript{35} Panel (b) shows the average black market income across the distribution of reported income for the entire sample, unconditional on whether or not taxpayers have participated in black market work, using local mean smoothing. Again, mainly low-income taxpayers have black market labor income. This is consistent with a comprehensive study by the Danish Economic Council in 2011 (DØRS, 2011) using the same data as here but for all available years, which concluded that black market earnings were negatively correlated with total reported income. In addition, DØRS (2011) finds that the self-employed and low-income wage earners most frequently supply labor on the black market, and black market wages are substantially higher for the self-employed, indicating that this group of taxpayers on average earns larger black market incomes than wage earners.

\textsuperscript{35}In the survey samples from 2005 and 2007 that we use, there are very few top earners. But in DØRS (2011), which uses a larger sample spanning more waves of the survey, there is a clear picture that top earners supply black market work less frequently than the middle income earners.
(a) Share of taxpayers selling black market services by 20th fractiles of the reported income distribution.

(b) Mean black market income across the reported income distribution.

Figure B.1. Size and Distribution of the Black Market Economy in Denmark.

Notes: The data on black market activity stems from a survey collection effort undertaken by the Rockwool Foundation Research Unit, and income data stems from linked administrative data. The Rockwool Foundation Research Unit’s surveys on black market income has been collected since 1985. For each wave in the collection effort, surveys are dispatched to individuals, both wage earners and the self-employed, with the understanding that their answers are kept anonymous. As such, no one has ever been prosecuted for acknowledging black market income unreported on their tax return in the surveys. Unfortunately, the surveys were not collected in 2006. Instead, we have obtained data for four surveys collected in 2005 and 2007 (in 2006 prices), for a total sample of surveyed individuals of 3,806. Only 10 individuals did not respond so the sample of responsive individuals is 3,796. Of these individuals, 560 responded that they had sold services on the black market during the last 12 months with an average income of 19,439 DKK. In the total sample of responsive individuals, this corresponds to an average black market income of 3,143 DKK. Panel (a) shows the share of taxpayers in the sample selling black market services by 20th fractiles of the distribution of total reported income. Panel (b) shows the average return to black market activity in the sample using local mean smoothing with the Epanechnikov kernel function and a rule-of-thumb bandwidth. All amounts are in '000 DKK (1 USD ≈ 6 DKK in 2006).

Source: Rockwool Foundation Research Unit and own calculations.
References


