Strategic Campaigns and Redistributive Politics

Christian Schultz*

July 2004, revised July 2006

Abstract

The paper investigates strategic, informative campaigning by two parties when politics concern redistribution. Voters are uncertain about whether parties favor special groups. Parties will target campaigns at groups where most votes are gained by informing about policies. In equilibrium, campaigning will be most intensive in groups where the uncertainty is largest and where voters are most mobile, most likely to vote, most receptive to campaigns and relatively uninformed initially. These groups will become more informed about policy. Parties will therefore gain more votes by treating these groups well. In the end these groups will gain from strategic campaigning. Conditions for positive and negative welfare effects are established.

JEL: D72, D82, H40

Keywords: Political Economy, Redistribution, Information, Campaigns

*Department of Economics, University of Copenhagen, Studiestraede 6, DK 1455 Copenhagen K, Denmark, CIE, EPRU and CES-ifo. E-mail: Christian.Schultz@econ.ku.dk, web: www.econ.ku.dk/CSchultz

†I benefitted from the comments of the editor, Leonardo Felli, two referees and discussions with Giuseppe Bertola, Douglas Hibbs, Niels Henrik Mörch von der Fehr, Peter Norman Sørensen and Jean Tirole, as well as seminar audiences in CORE, Goteborg, Florence, Oslo and Paris and EEA in Venice, while working on this paper.
1 Introduction

Political campaigns are an important part of electoral democracies. Candidates spend endless days on the campaign trail and parties spend huge amounts of campaign money. According to the Federal Election Commission, the parties and candidates are estimated to have spent roughly 3 billion US dollars in the 2000 US electoral cycle. Although, the US is probably the country where campaign money are most important, campaign expenditures are increasing in most developed democracies. Clearly, the campaigns have a purpose: influence voters so they are induced to vote for the campaigning party. While it is well documented that they actually do work, see e.g. Green and Krasno (1988) or Gerber (1998), the way may differ. Some voters may be attracted just by the fact that a party campaigns intensely and perhaps use nice campaigns. In the literature such campaigns are usually called persuasive, cf. Baron (1994). On the other hand, parties also spend considerable resources on informative campaigns, informing about their policies or perhaps about other parties’ policies. Such campaigns would seem beneficial from a social perspective as they allow voters to make more informed choices. But they also have implications for the parties’ strategic struggle, and the policies, which are adopted in the end. In this paper we will consider the effects of strategic, informative campaigning on redistributive policies when parties can target their campaigns to groups. The main questions we are interested in are: how will the parties spend their limited campaign resources, and which effects will this have on policies? Which groups will benefit? Are such campaigns to be considered good or bad?

The basic model of the paper is the model of redistributive politics originally introduced by Lindbeck and Weibull (1987) and extended by Dixit and Londregan (1996). Here, parties favor groups, whose vote-response is high when they are offered better policies\footnote{Dahlberg and Jonasson (2002) find that this prediction is supported in data from a Swedish temporary grant program}. In my model, voters are uncertain about whether parties are biased towards special groups. The voters’ information about politics differs. Some voters are uninformed and become in-
formed through political campaigns. Parties will target campaigns to those groups where campaigns are most effective. These are groups with many swing voters, with a high turnout (and a high increase in turnout in response to campaigning) and groups where the initial level of information is low. The campaigns will increase the information level in these groups and this makes the parties offer these groups better policies. In the end, these groups will benefit from the strategic campaigning. There is a complementarity between campaigns and spending - a campaign-multiplier - campaigns reinforce the effects of voter mobility.

Informative campaigns are often considered good since they allow voters to make informed choices. However, the results of this paper points to another effect. Informative campaigning creates a particular distribution of information in the electorate, which in turn affects real policies. A group’s information level is determined by the parties’ strategic incentives to gain most votes from the limited campaign money rather than considerations of fairness. This favors groups, who already gain from the strategic struggle among the parties, the mobile groups and the groups with high turnout. In this respect informative campaigning makes the distribution of policies over groups more unequal than if all groups had the same level of information and a limit on campaign spending (or finances) would make for a more equal distribution in society. Since the model is one of pure redistribution, all allocations are Pareto efficient. If, however, one takes a utilitarian perspective, and evaluates the sum of utilities in society, the fact that campaigns reinforce the skewness due to differences in voter mobility and participation rates is bad. However, there is a countervailing effect; campaigns are also targeted at groups where the level of information initially is low. These groups are initially disadvantaged, and therefore equality may be enhanced by campaigning. If reality one would often think of such groups being less educated, low income groups and campaigns may therefore benefit such weak groups. More receptive groups also gain. Whether these are less educated or more educated groups is not obvious. Presumably, well-educated are better at receiving information, but if the information level of well-educated is very high from news coverage, then there is not much to learn. Results reported
by Alvarez (1997, p 172) suggest that the latter effect is the most important. Although the model provides specific predictions about which characteristics make a group an attractive target for campaigning and therefore make the group gain, the total welfare effect depends on the strength of the different forces. A general welfare assessment is complicated by the fact that the model does not allow closed form solutions, but for the case where the ideological component of the parties’ preferences is almost egalitarian, it is shown that the welfare effect of campaigns are negative or positive depending on which of the above mentioned effects dominate.

The model predicts that only one party will campaign in a given group. The reason is that a party’s campaign informs voters about policies. A party will campaign in groups where it has good news to reveal: either that its policy is better than expected (positive campaigning) or that the other party’s policy is worse than expected (negative campaigning). The parties will therefore never campaign in the same groups. The model thus predicts that parties should have a tendency to focus on issues where their standing among voters is strong.

In reality, one often observes that parties campaign in the same groups. This does not necessarily contradict our results since we focus on informative campaigning. As is well known from for instance Baron (1994) parties will have incentives to direct persuasive campaigns to the same groups of voters.

A large literature considers the effects of lobby groups’ influence on politics through campaign donations. The effects of lobbying are by now well described and understood, see e.g. Baron (1994) and the book by Helpman and Grossman (2001). The lobbying literature usually assumes that campaigning is persuasive, so money directly buys votes. I show that campaign funds are also valuable for parties if campaigns are informative, and hence the incentive to induce lobby group payments remains. Although lobbies will not be considered in this paper, the model could serve as a building block in a model of lobbying.

We assume that the parties commit to their policy proposals before the election and that they inform truthfully about these. Clearly, this is a benchmark case. In the real world, politicians may lack a commitment device;
however, reputational forces will punish politicians who grossly disregard pre-electoral promises. Most of the world remembers Bush Sr.’s (in)famous "Read my lips" statement. Furthermore, we also want to investigate the effects of informative campaigning under the positive presumption, that they really are informative. Clearly, if they are cheap talk, they are either akin to persuasive campaigns or just a waste of money.

The organization of the paper is as follows: Section 2 shortly discusses some related literature. Section 3 introduces the model. Equilibrium is defined in section 4. Section 5 derives equilibrium policies and campaign choices. Section 6 discusses welfare, and section 7 offers some concluding remarks. Most proofs are in the Appendix.

2 Some related literature

A number of recent papers argue that parties offer favorable policies to well-informed parts of the electorate. Strömberg (2004a) considers the interplay between information and redistributive policies. In his model, information is provided by mass media rather than by parties. Voters are uncertain about the parties’ policies because they are unsure about the size of the different groups. Thus, mass media have a role informing voters about the policies of the parties. As in the present paper parties will target informed groups. Strömberg also shows that there will be a bias in the policy, since the mass media will have an own interest in informing particular groups that are attractive to advertisers. Strömberg (2004b) considers the impact of the spreading of radio for the allocation of funds under New Deal in the thirties in the US. The data show a clear pattern: more informed groups - with a high share of households owning a radio - receive larger funds as predicted by the theory. Although in a different context, Besley and Burgess (2002) report the same kind of result for India. Indian states with larger newspaper circulation tend to receive more public food distribution and calamity relief expenditure. Compared with the papers of Strömberg and Besley and Burgess, the important new element in the present paper is that the parties
choose the distribution of information strategically as part of the electoral struggle.

Informative campaigning is also considered by Austen-Smith (1987) in a model with a uni-dimensional policy space. He considers a model where there is uncertainty about parties’ policies. Risk adverse voters are therefore reluctant to vote for parties whose policies they only have stochastic information about. Informing the voters reduces the risk. Austen-Smith shows that in equilibrium informative campaigning makes ideological parties choose policies closer to the median voter’s preferred policy. While highly innovative, a problem with the analysis is that the source of the uncertainty is unexplained (in equilibrium, the positions of the parties are actually certain) and it is unexplained how campaigns contribute to reducing the uncertainty. In my model, voters are unsure about the ideological component of parties’ preferences and this gives uncertainty about the policies.

Prat (2002) is interested in whether a ban should be put on lobbies’ contributions to parties. He considers informative campaigning in a model where two office-seeking candidates may have different competencies. An interest group receives non-verifiable information about the competency of the candidates. The interest group offers money to the parties in return for a favorable policy position. The money is used on non-informative campaigning, but a separating equilibrium exists in which voters infer the candidates’ competency from the amount of campaign spending. Campaigns are thus both good and bad for voters, good since they provide information about the candidates’ competency, and bad since the parties distort policy in order to obtain money from the interest group. Therefore, whether a ban on campaigns will benefit voters is not clear. Potters, Sloof and Van Winden (1997) also discuss signalling through campaigning.

Coate (2004a) offers an interesting analysis of informative campaigns in a model of a uni-dimensional policy space. Two parties are policy-motivated; they each propose a candidate, which may be moderate or extreme (partisan). Party members prefer a partisan candidate, while moderate voters prefer a moderate candidate. While voters know a candidate’s party affiliation, they do not know his type. Informative campaigning can inform about the char-
acteristic of a candidate. Such campaigning benefits moderate candidates as it is possible to attract moderate voters from the other party. Parties receive contributions from partisan interest groups. Coate shows that a limit on contributions reduces expected campaign spending, but it also decrease the likelihood that parties will select moderate candidates. Coate (2004b) considers informative campaigning regarding candidates’ qualities or competencies. Politics is uni-dimensional and extreme lobby groups are informed about the qualities of candidates. A trade off arises, since qualified candidates will be able to raise money from lobby groups and can inform the electorate about their superior quality, which is a good thing. However, the candidates modify their policy in a more extreme direction in order to cater to the extreme lobby groups, which is a bad thing. Coate shows that under some conditions a limit on campaign contributions may then be Pareto improving. While the negative effects of informative campaigns in Coate’s papers derives from the power of extreme lobby groups, my argument is different. There are no lobby groups in the present model; the unequal distribution, which results from informative campaigning, derives from the unequal distribution of information the campaigns induce. Ashworth (2006) studies the effect of incumbency advantages in fund-raising in model similar to Coate’s. He finds that public matching funds improve welfare in districts where parties campaign in the absence of matching funds and reduces welfare in other districts.

Ortuno-Ortin and Schultz (2000) considers informative campaigning in a model of a uni-dimensional policy space where policy motivated parties receive public funding for campaigning. They show that informative campaigns contribute to making the parties’ policies converge since a more informed electorate responds more to policy moderation.

All of the above-mentioned papers with informative campaigning consider uni-dimensional policy spaces. Their aim is to consider different legislative regimes regarding campaign finance, in order to discuss the socially best regime. The present paper, on the other hand, focuses on the distributional aspects of the effects of strategic informative campaigning. It shows that there are distributional consequences of informative campaigning.

Konrad (2004) considers inverse campaigning in a model of redistributive
politics, where a party deliberately informs the whole electorate that a particular group is going to lose from its proposal or benefit from the opponent’s proposal. This may be sensible in situations where it is common knowledge that a particular fraction of the electorate will gain from the proposal, but where the identity of the winners and losers are unknown to (parts of) the electorate. An uninformed voter, who just knows that some fraction of voters will win, will revise the probability that she is a winner if she learns that particular other groups for sure will lose. In this way, inverse campaigning can indirectly be positive for a party. Since the present paper assumes that parties can inform a group directly about its policy through targeted campaigning, the effect Konrad focusses on is not present here.

3 Basics

We consider a society with \(n\) groups \(i = 1, ..., n\). Each group contains a continuum of citizens of size 1. Two parties \(A\) and \(B\) each proposes a distribution of per capita consumption before an election, \(c_A = (c_{A1}, ..., c_{An})\), and \(c_B = (c_{B1}, ..., c_{Bn})\), respectively. The proposals have to fulfil the budget constraint

\[
\sum_{i} c_{Ai} = \Omega.
\]

The parties are committed to these proposals; the winner of the election implements its proposal.

Voters become informed about policies in a variety of ways: through media, conversation, attending meetings, etc., and through parties’ campaigns. For the model, the important feature is that the parties can affect the share of informed voters by campaigning. Each party has an amount, \(M\), of campaign money, which can be used informing the different groups of voters about the policies.\(^2\)

\(^2\)There is ample evidence to this effect starting with a famous panel study by Lazarsfeld, Berelson and Gaudet (1944). Alvarez finds (p 172) that “there are substantial reductions in voter uncertainty of the positions of the candidates during the campaign that are directly related to the flow of information”. Ansolabehere and Iyengar (1995, p42) note that
We will assume that information is hard so a party cannot lie about policies. Evidently, this is a benchmark case. But as touched upon in the introduction, it is a benchmark case of interest. Political life has its reputations, a politician caught lying will be front page news, journalists and independent experts enjoy such prey\(^3\). Furthermore, as also argued in the Introduction, we are interested in evaluating informative campaigns on the presumption that they actually are informative. Voters, who remain uninformed about the policies, rely on their expectations. They are uncertain about the exact policies chosen by the parties but derive whatever inferences can be drawn from the fact that they remain uninformed.

We model the uncertainty as originating from an uncertainty about the parties’ preferences. The voters may be unsure about which faction of the party is the most influential, how the different factions strike deals within the party and they may also be unsure about leading politicians’ preferences. As an example, consider the newly elected Margaret Thatcher in UK. Everybody knew that she was a right-winger, but how stern and how tough she would be for instance in relation to trade unions was uncertain. Likewise, the newly

\(^3\)“exposure to advertising makes voters much more likely to refer to issues as reasons for supporting or opposing a candidate”. This is echoed by many authors: Popkin (1991) stresses information about candidate positions, Gelman and King (1993) stress the role of media and West (1993), stress the importance of TV-ads. Holbrook (1996) stresses that media’s coverage of the campaigns is mostly about campaign events (conventions, debates etc). So although a lot of campaigning is done through advertisements, direct mail, telephone calls and canvassing, the ways campaigns work are manifold. Holbrook (1996) answers his title "Do campaigns matter?", with a resounding “yes”, both for the electorates’ information and voting. Briand and Wattenberg (1996, p 172) find that “citizens recalling political advertising have the most accurate knowledge of the candidates’ issue positions”. Just, Crigler and Wallach (1990) report experiments showing that political advertising convey more accurate candidate issue positions than televised debates.

\(^3\)A recent Danish experience is the March 1998 general election. Polls showed a very close race between the ruling Socialdemocrats and the right wing opposition. Prime Minister Poul Nyrup Rasmussen made a very popular pledge not to change an early retirement scheme, which most economists found very unwise. The Socialdemocrats won the election and Prime Minister Nyrup reneged on his promise half a year later, changing the early retirement scheme. He immediately made front page news as a liar; the Socialdemocrats plummeted in the polls and never regained the lost. They lost the next election in 2001.
elected Tony Blair for sure had changed some priorities in Labour, but how much?

While voters are uncertain about the parties’ preferences, it is assumed that both parties know each other’s preferences. The motivation is that parties are lead by professional politicians, who spend time and effort to understand their competitors, they are assisted by full time employees, whose job it is exactly to gather such information and make the relevant assessments, and they are often in daily contact with leading politicians from other parties. For the leaders of the parties it is an important part of the strategic struggle to know the opponent. The incentive for a voter is much smaller (actually zero in our model, where there is a continuum of citizens in each group). In short, there is asymmetric information among voters and parties.

In principle, a party can choose to inform about its own policy only, about the other party’s policy only, or about both parties’ policies. However, rational voters, who observe a campaign message from a party, should ask themselves why the party withholds some information, if it indeed does. We will assume that the cost of reaching a voter is the same regardless of whether the information is about one of both of the parties’ policies. The idea is that the (significant part of the) cost of campaigning is associated with reaching the voters. A voter therefore knows that the party costlessly could provide more information, if it ”hides” information. Assume for instance, that party A informs about its own policy only. Had B’s policy been bad, party A would happily have informed about it. As there are only two types of party B in the model, there are only two possible equilibrium policies of party B, so a voter seeing A’s campaign will infer that B’s policy is good. Similarly, it does not help a party only to inform about the other party’s policy. Consequently, when a party decides to campaign in a group, it reveals information about both parties’ policies. Notice that the same unravelling argument applies even if there are more types of parties. A party would always want to reveal the most positive information, if it does not voters will exclude this case etc.

In the end a party therefore only has two options: inform about both parties’ policies or not inform at all, in this sense a party’s campaign is both positive (“my policy is good”) and negative (“his policy is bad”).


Some voters receive information from newspapers, television etc. regardless of whether the parties campaign or not. Let the fraction of voters in group \( i \), who are informed by other means than campaigns, be \( \Gamma_i \), where \( 0 \leq \Gamma_i < 1 \). This fraction may differ across groups, a highly educated group would presumably have a high \( \Gamma_i \).

Following the literature on informative advertising (see Butters (1977) and Grossman-Shapiro (1984)) it is assumed that the fraction of informed voters is increasing in the amount of campaigning. If party A spends \( m_{Ai} \) on campaigns in group \( i \), then the fraction

\[ \psi_i (m_{Ai}) = \gamma_i \psi (m_{Ai}) \]

of the uninformed voters in group \( i \) will see A’s campaign, where \( \gamma_i > 0, \psi > 0, \psi(0) = \infty, \psi'' < 0, \) and \( \gamma_i \psi (M) < 1 \). This in accordance with the “ceiling effect” discussed by Bartels (1988). He notes that there is evidence that campaign effects are not linearly correlated with exposure to campaign messages, but that the marginal effect declines. Some groups may be easier to inform than others, for a discussion of this see Zaller (1991). A group with a high \( \gamma_i \) is a group, which is easy to reach through campaigning; perhaps because it is well educated, watch much TV or reads a particular magazine. In principle one could also imagine that the \( \psi_i \) function depends on the party; it may be that a party is better to inform a particular group than the other party. We will however disregard such complications. We assume that the probability that an individual voter becomes informed by party A’s campaign is the same for all uninformed voters in group \( i \) and therefore equal to \( \psi_i (m_{Ai}) \). Similarly, a voter becomes informed by party B’s campaign with probability \( \psi_i (m_{Bi}) \).

For simplicity, we assume that the events that an uninformed voter becomes informed from A or from B are independent. The fraction of informed voters in group \( i \) therefore becomes

\[ \Psi_i = (\psi_i (m_{Ai}) + (1 - \psi_i (m_{Ai})) \psi_i (m_{Bi})) (1 - \Gamma_i) + \Gamma_i. \tag{2} \]

If consumption in group \( i \) is \( c_i \), a voter in the group derives utility

\[ u(c_i) = \ln c_i. \tag{3} \]
Voters may be ideologically attached to one of the parties. Voter \( v \) in group \( i \) has a ideological component in favor of party \( B \), equal to \( x_{iv} + y \), \( x_{iv} \) is a personal component, \( y \) a general component shared by all voters in the electorate. It represents a common shock to preferences in the electorate occurring just before the election, which may be positive or negative\(^4\). When voter \( v \) is informed about the parties’ policies, she prefers party \( A \) if

\[
x_{iv} + y \leq u(c_{Ai}) - u(c_{Bi}).
\]  

(4)

There is a distribution of ideological components in each district. In district \( i \), the ideological component is given by the distribution function \( \Phi_i \). To keep the analysis simple, we follow Persson and Tabellini (2000) and assume that \( \Phi_i \) is the uniform distribution, with support \([-\frac{1}{2\delta_i}, \frac{1}{2\delta_i}]\) and density \( \delta_i \). This restriction is inessential, as the results by Lindbeck and Weibull (1987) show. It will save us some concavity assumptions later. The fact that there is a zero mean is also inessential as long as the support is sufficiently large so that \( u(c_{Ai}) - u(c_{Bi}) \) (or the expected counterpart considered below) is contained in the support of \( \Phi_i \). As we show below, it is only the density which matters. The general ideological component is also uniformly distributed, with mean 0 and density \( \theta \). The parties and the voters are informed about the distribution of the ideological components, but the parties do not know the realization of the general component \( y \) before choosing policies and campaign expenditures.

A voter who is not informed about the parties’ policies will form expectations. Her expected utility from consumption if party \( A \) wins the election is denoted \( E_A u_i \). When voter \( v \) is uninformed, she will prefer party \( A \) if

\[
x_{iv} + y \leq E_A u_i - E_B u_i.
\]  

(5)

Informed and uninformed voters thus cast their vote differently (viz. equations (4) and (5))\(^5\).

\(^4\)In the recent (2002) German election, the Socialdemocratic Kansler, Schröder, gained support in polls just before the election following severe flooding in southern Germany. Allegedly, Schröder showed leadership. In the model this would be good realization of \( y \) for the Socialdemocrats.

\(^5\)This is in accordance with the results of Bartels (1996). Using probit analysis on data
Voters may be more or less inclined to actually go to the voting booth and cast a vote. The inclination to vote may differ across groups. Let $\xi_i$ be the fraction of voters in group $i$, who actually cast a vote at the election, the voter turnout. For simplicity, we first assume that voter turnout is exogenous, the same among informed and uninformed voters, and independent of the ideology of particular voters. If informed voters were more inclined to vote, the vote response from informing voters would be larger than what is the case in the model at present. Later we will consider the case where voter turnout is responsive to campaigning.

We can now find the votes for party $A$ in district $i$

$$V_i = \xi_i \left[ \Psi_i \Phi_i \left( u(c_{Ai}) - u(c_{Bi}) - y \right) + (1 - \Psi_i) \left( E_A u_i - E_B u_i - y \right) \right].$$ (6)

Since the individual ideological component is uniformly distributed with density $\delta_i$ and mean 0, we have

$$V_i = \xi_i \left[ \frac{1}{2} + \delta_i \left( \Psi_i \left( u(c_{Ai}) - u(c_{Bi}) \right) + (1 - \Psi_i) \left( E_A u_i - E_B u_i \right) - y \right) \right].$$ (7)

The total number of votes cast for party $A$ is therefore

$$V_A = \sum_{i=1}^{n} V_i.$$ (8)

The probability party $A$ wins the election, $p_A$, equals the probability that $V_A \geq \frac{\sum \xi_i}{2}$. This depends on the realization of the general shock to preferences, $y$. Since $y$ is uniformly distributed with mean zero and density $\theta$, we have after a few manipulations

$$p_A = \frac{1}{2} + \frac{\theta}{\delta} \sum_{i=1}^{n} \xi_i \delta_i \left[ \Psi_i \left( u_i(c_{Ai}) - u(c_{Bi}) \right) + (1 - \Psi_i) \left( E_A u_i - E_B u_i \right) \right].$$ (9)

From US National Election Study surveys from six presidential elections, he concludes that individual voting differs significantly depending on whether the individual is informed or not. Aggregation does not eliminate this effect: according to his estimates, incumbents did almost five percent points better than in his hypothetical “fully informed” electorate. Shaw (1999) estimates campaign effects from an extensive data set on state-wide television advertising and candidate appearances from the 1988, 1992 and 1996 presidential campaigns in the US and concludes that campaigns influence electorates and electoral votes.
where \( \delta \equiv \sum_{i=1}^{n} \xi_i \delta_i \), i.e. sum of densities weighted by voter turnout. In each group \( \xi_i \delta_i \) is a measure of the "effective" mobility of votes, i.e. the increase in votes for a party if it improves the policy of the group so much that a voter in the group gains one util.

The parties are partly motivated by power and partly by ideology. If a party wins the election it receives rents \( R \). Furthermore, the party cares about a weighted average of voters’ utilities in the different groups. The uncertainty of uninformed voters stems from that they do not know how the parties weigh the utility of different groups. There are two types of each party. One type is neutral, type \( N \), and weighs all groups equally, while the other is biased towards particular groups and put larger weights on the utility of these groups, which we could think of as the party’s constituency. We will call this type, the biased type \( \Pi \). When party \( A \) is of type \( t \in \{N, \Pi\} \), its objective is to maximize

\[
U_A^t = p_A R + \sum_{i=1}^{n} \alpha_i^t \left(p_A u(c_{Ai}) + (1-p_A) u(c_{Bi})\right)
\]

(10)

The party cares for the rents of office and a weighted sum of the expected utility of different groups. We normalize so that the average weight \( \sum_{i=1}^{n} \alpha_i^t / n = 1 \). The neutral type of party \( A \) has \( \alpha_i^N = 1 \) for all \( i \), while the biased type weighs the groups differently as explained in detail below.

Party \( B \) of type \( t \) seeks to maximize

\[
U_B^t = (1-p_A) R + \sum_{i=1}^{n} \beta_i^t \left(p_A u(c_{Ai}) + (1-p_A) u(c_{Bi})\right)
\]

(11)

Again \( \sum_{i=1}^{n} \beta_i^t / n = 1 \), and the neutral type has \( \beta_i^N = 1 \) for all \( i \).

In order to facilitate the analysis we focus attention on a symmetric economy. Group \( i \) (\( i < n/2 \)) has the same characteristics as group \( n+1-i \), i.e. \( \gamma_i = \gamma_{n+1-i}, \xi_i = \xi_{n+1-i}, \Gamma_i = \Gamma_{n+1-i} \) and \( \delta_i = \delta_{n+1-i} \). The biased parties’

\[6\]

Of course, it would be more realistic to assume that there was infinitely many types of a party corresponding to varying degrees of bias. However, this would also complicate the analysis to follow; we follow the lead of a large literature and restrict attention to the two-type case.
preferences are symmetric, in the sense that \( \alpha_i^\Pi = \beta_{n+1-i}^\Pi \). The symmetry implies that when both parties are of the same type they are in fact in a symmetric situation. The biased type of a party attaches a weight larger than one to some groups, one can think as those as the party’s constituency. Party A’s constituency consists of groups \( i = 1, ..., n_a \leq n/2 \) and party B’s of groups \( i = n+1-n_a, ..., n \). The constituencies are therefore non-overlapping. The parties attach a weight less than one to the groups outside their constituencies.

We assume that either both parties are biased or they are both neutral. An interpretation is that some voters are unsure about how partisan politics really is. The probability that the parties are biased and attached to their constituencies is \( \pi \), where \( 0 < \pi < 1 \). Voters know the different \( \alpha_i \) and \( \beta_i \) and the probability \( \pi \), but they do not know parties’ types. Alternatively one could have assumed that it was independent events whether the parties were biased or not. Then there would be four possible configurations of types and this would make the formulas longer and some proofs more intricate without adding qualitatively to the paper. The important issue is that voters in the different groups are unsure about which party proposes the best policy for them, so that campaigning is potentially important. This feature is also secured in our formulation.

The timeline is as follows. First the parties’ types are realized and observed by both parties. Parties then decide on policies and the distribution of campaign expenditures simultaneously. Some voters receive information about the policies and some do not. The uninformed voters form expectations about the policies and voters vote. Finally, the winning party implements the policy it proposed.

4 Equilibrium

We solve the model for a Perfect Bayesian Equilibrium. In this equilibrium each party (of each type) maximizes its utility. A voter takes as given the equilibrium strategies of the parties and prefers party A, if she is informed
and (4) is fulfilled, or she is uninformed and (5) is fulfilled. Otherwise she prefers B. Let e.g. \( m'^{i}_{A} \) be the amount of campaign money party A spends informing voters in group \( i \), when parties are of type \( t \). A uninformed voter rationally derives whatever information is contained in the fact that she is uninformed. For instance, if the biased type of party B favors her group, she may think that since she was not informed by party B; party B is not favoring her group after all and is of the neutral type. Formally, she takes the campaign strategies as given and uses them to update her prior belief using Bayes’ rule. Her posterior belief that parties are biased is therefore

\[
\tilde{\pi}_i = \frac{(1 - \psi_i (m'^{n}_{A})) (1 - \psi_i (m'^{n}_{B})) \pi}{(1 - \psi_i (m'^{n}_{A})) (1 - \psi_i (m'^{n}_{B})) (1 - \pi) + (1 - \psi_i (m'^{n}_{A})) (1 - \psi_i (m'^{n}_{B})) \pi}. \tag{12}
\]

This is the probability of being uninformed when parties are biased, divided with the total probability of being uninformed. As \( \psi_i (M) < 1 \), we always have that \( 0 < \tilde{\pi}_i < 1 \). Let e.g. the policy of party A of type \( t \) be denoted \( c^t_A \). With posterior belief \( \tilde{\pi}_i \), the expected utility to a voter in group \( i \) if party A wins the election is

\[
E_A u_i = (1 - \tilde{\pi}_i) u(c^N_A) + \tilde{\pi}_i u(c^P_{Ai}). \tag{13}
\]

The expected utility if party B wins the election is

\[
E_B u_i = (1 - \tilde{\pi}_i) u(c^N_B) + \tilde{\pi}_i u(c^P_{Bi}). \tag{14}
\]

Since voters do not observe the distributions of the parties’ campaign expenditures, the parties take the uninformed voters’ beliefs as given. Therefore they also take the expected utilities in (14) and (13) as given.\(^7\)

In equilibrium parties also take each others’ strategies as given. Party A of type \( t \)'s problem is:

\[
\begin{align*}
\text{Given } t \text{ and } (\tilde{\pi}_i, c^t_{Bi}, m'^{i}_{Bi})_{i=1}^{n} & \max_{c_A, m_A} \ p_A R + \sum_{i=1}^{n} \alpha^t_i (p_A u(c_{Ai}) + (1 - p_A) u(c^t_{Bi})) \\
\text{sub } & \sum_{i} c_{Ai} = \Omega; \sum_{i} m_{Ai} = M; c_{Ai}, m_{Ai} \geq 0 .
\end{align*}
\tag{15}
\]

\(^7\)If voters were able to observe the campaign expenditures, a party would take into account that it could affect beliefs through the distribution of expenditures.
while $B$’s is

$$
\begin{align*}
\text{Given } t \text{ and } (\pi_i, c_{Ai}^t, m_{Ai}^t)_{i=1}^n \\
\max_{c_B, m_B} (1 - p_A) R + \sum_{i=1}^n \beta_i^t (p_A u(c_{Ai}^t) + (1 - p_A) u(c_{Bi}^t)) \\
\text{sub } \sum_i c_{Bi} = \Omega; \sum_i m_{Bi} = M; c_{Bi}, m_{Bi} \geq 0.
\end{align*}
$$

We can now define an equilibrium

**Definition.** An electoral equilibrium with strategic campaigns consists of policies and campaign expenditures in all groups for both parties $(c_{Ai}^t, m_{Ai}^t)_{i=1}^n$, $(c_{Bi}^t, m_{Bi}^t)_{i=1}^n$, $t \in \{N, \Pi\}$ and expected utilities for the voters of all groups $(E_Au_i, E_Bu_i)_{i=1}^n$ such that

1. Each party (of each type) chooses policies and campaign expenditures to maximize utility as in (15) and (16).

2. Each uninformed voter forms beliefs using the parties’ strategies and Bayes’ rule and evaluates expected utilities using these beliefs. For instance $E_Au_i$ is given by (13), where the beliefs are given by (12). The number of votes cast for party $A$ is then given by (8).

Applying a standard fixed point argument, we show in the Appendix

**Lemma 1** An equilibrium exists.

### 5 Campaigns and Policies

We now consider equilibrium choices of campaigns and policies. For party $A$ of type $t$ the first order conditions for an interior maximum are

$$
\frac{\partial p_A}{\partial c_A} \left( R + \sum_{i=1}^n \alpha_i^t (u(c_{Ai}^t) - u(c_{Bi}^t)) \right) + p_A \alpha_i^t u'(c_{Ai}^t) = \lambda \quad (17)
$$

$$
\frac{\partial p_A}{\partial m_{Ai}} \left( R + \sum_{i=1}^n \alpha_i^t (u(c_{Ai}^t) - u(c_{Bi}^t)) \right) = \mu. \quad (18)
$$

where $\lambda$ and $\mu$ are Lagrange multipliers. The amount of campaign spending is non-negative. If the non-negativity constraint binds in a group $i$, then the corresponding derivative, $\frac{\partial p_A}{\partial m_{Ai}}$, is negative and the optimal choice is $m_{Ai} = 0$. 

17
Using (9), (3) and the budget constraint we get

\[ c_t^{A_i} = \frac{\Psi_i \xi_i \alpha_i t \theta G_{tA}^{A} + p_A \alpha^t_i}{\sum_{j=1}^{n} \left( \Psi_j \xi_j \alpha_j t \theta G_{tA}^{A} + p_A \alpha^t_j \right)} \Omega. \]  \hspace{1cm} (19)

where

\[ G_{tA}^{A} = R + \sum_{i=1}^{n} \alpha^t_i \ln \left( \frac{c_{tA}^{A}}{c_{tB}^{A}} \right) \]  \hspace{1cm} (20)

is the gain from winning for party A of type t. Party B's policy is given by

\[ c_t^{B_i} = \frac{\Psi_j \xi_j \alpha_j t \theta G_{tB}^{B} + \left(1 - p_A\right) \beta^t_j}{\sum_{j=1}^{n} \left( \Psi_j \xi_j \alpha_j t \theta G_{tB}^{B} + \left(1 - p_A\right) \beta^t_j \right)} \Omega, \]  \hspace{1cm} (21)

where

\[ G_{tB}^{B} = R + \sum_{i=1}^{n} \beta^t_i \ln \left( \frac{c_{tB}^{B}}{c_{tA}^{B}} \right). \]  \hspace{1cm} (22)

Although we have not solved for an equilibrium yet, (19) and (21) convey a nice insight. As in the models of Lindbeck and Weibull (1987) and Dixit and Londregan (1996), the parties favor a district, where the density of voters at the cutpoint, \( \delta_i \), is high and where the inclination to vote, \( \xi_i \), is high. Shifting resources to such a district will give many votes. However, for this effect to work, voters have to be informed about the parties’ policies; \( \Psi_i \) has to be high. The term \( \Psi_i \xi_i \alpha_i t \theta \) equals the expected increase in votes from offering the group a policy which is one util better, so \( \Psi_i \xi_i \alpha_i t \theta G_{tA}^{A} \) is the increase in expected gain to the party from offering such a better policy. Groups with many informed voters will gain; this is just as in Strömberg (2002b). 8

8As touched upon in the Introduction, Dahlberg and Johansson (2002) find support for the Lindbeck-Weibull (1987) model in Swedish data. Milligan and Smart (2003) consider a federal program for interregional development transfers distributed by two government agencies covering different parts of Canada. For the Atlantic regions, the pattern of spending accords with the predictions of Lindbeck-Weibull, spending is larger in electoral districts with close races and where plurality in the previous vote was small. Cadot et al. (2002) find that the regional allocation of spending on roads in France also shows such a pattern. Wright (1974) reports that US federal spending in the period 1933-40 were positively related to a state’s “political productivity”, which is a measure depending on the number of electoral votes per capita, variability in the incumbents vote share in past elections, and closeness of presidential elections.
When both parties are neutral they both weigh all groups equally. When they are biased, they are also in a symmetric situation. A symmetric equilibrium therefore exists in which, \( c_{A_i}^{\Pi} = c_{B_n+1-i}^{\Pi} \), \( c_{A_i}^{N} = c_{B_i}^{N} \) (for all \( i \)), they win with probability \( \frac{1}{2} \) each, and where the gain from winning is the same for the parties, equal to \( R \) when they are neutral and \( G = G_A^{\Pi} = G_B^{\Pi} \) when they are partisan. In such an equilibrium the policies of the neutral types are given by

\[
c_{A_i}^{N} = c_{B_i}^{N} = \frac{\Psi_i \xi_i \delta \gamma R + \frac{1}{2} \alpha_i}{\sum_{j=1}^{n} (\Psi_j \xi_j \delta \gamma R + \frac{1}{2} \alpha_j)} \Omega,
\]

and the policies of the biased types are given by

\[
c_{A_i}^{*} = \frac{\Psi_i \xi_i \delta \gamma G + \frac{1}{2} \alpha_i}{\sum_{j=1}^{n} (\Psi_j \xi_j \delta \gamma G + \frac{1}{2} \beta_j)} \Omega \quad \text{and} \quad c_{B_i}^{*} = \frac{\Psi_i \xi_i \delta \gamma G + \frac{1}{2} \beta_i}{\sum_{j=1}^{n} (\Psi_j \xi_j \delta \gamma G + \frac{1}{2} \beta_j)} \Omega,
\]

In the Appendix it is verified, that this constitutes an equilibrium. We will focus on this equilibrium. Here we have that

\[
\frac{c_{A_i}^{\Pi}}{c_{B_i}^{\Pi}} = \frac{\Psi_i \xi_i \delta \gamma G + \frac{1}{2} \alpha_i}{\Psi_i \xi_i \delta \gamma G + \frac{1}{2} \beta_i} \Omega
\]

From which it is clear that

\[
\alpha_i^{\Pi} > \beta_i^{\Pi} \iff c_{A_i}^{\Pi} > c_{B_i}^{\Pi}
\]

If party \( A \) is more concerned with group \( i \) than party \( B \) is, it offers the group a better policy than \( B \) does. Party \( A \) is interested in revealing that its policy is better for the voters of group \( i \). In fact, we have

**Lemma 2** At most one party campaigns in a group. This is the party whose policy is relatively better than expected for the group’s voters. If parties are biased, then \( A \) campaigns in groups where \( \alpha_i^{\Pi} > \beta_i^{\Pi} \) and \( B \) campaigns in groups where \( \alpha_i^{\Pi} < \beta_i^{\Pi} \). If parties are neutral, \( A \) campaigns where \( \alpha_i^{\Pi} < \beta_i^{\Pi} \) and \( B \) where \( \alpha_i^{\Pi} > \beta_i^{\Pi} \). No campaigning occurs in groups where \( \alpha_i^{\Pi} = \beta_i^{\Pi} \).

All proofs, which are not in the text, are in the Appendix.
There will be campaigning in all groups where the uninformed voters are uncertain about the policies, and no campaigning where there are not. Parties will campaign where they have positive news to reveal. A biased type, campaigns in the groups it favors, and a neutral type campaigns in groups, who its biased type would disfavor.

We now show that there is an equilibrium, where the amount of campaigning, and therefore the information level of a group is independent of types of the parties. In this equilibrium, the posterior belief of the uninformed voters in all groups therefore equals the prior.

Lemma 3 There exists an equilibrium, where the posterior belief in all groups equals the prior. In this equilibrium, the identity of the party who campaigns in a group depends on the type of the parties, but the amount of campaigning in a group is the same independent of the types.

The key to the Lemma is that the parties are in a symmetric situation. If parties are biased and $\alpha_i^I > \beta_i^I$, then party A has the incentive to campaign in group $i$ as A’s policy compares more favorably with B’s policy than expected. The incentive depends on the size of the positive “utility surprise” in group $i$ compared with other groups $j$, where A campaigns. If, on the other hand, parties are neutral, party B campaigns in group $i$ (and the other groups biased A would campaign in). B’s incentive is also governed by the size of the surprise in group $i$ compared with the other groups, and this comparison is the same as when biased A campaigns. The only difference is the sign. Therefore the two parties have equally strong incentives to campaign in group $i$, and the amount of campaign spending is independent of types. Because of this there is no new information in being uninformed. Hence, an uninformed voter does not revise her prior.

In the sequel we will focus on the equilibrium of Lemma 3, where the posterior equals the prior. In this equilibrium, the parties do no influence the beliefs of the uninformed voters. This feature distinguishes it from other potential equilibria in the model where the Bayesian updating using the parties’ strategies would lead to different beliefs. The fact that beliefs stay at the prior also makes for simplicity.
Let $m_i$ denote the *total* level of campaigning in group $i$. We now have

**Proposition 1** The level of campaigning and information is larger in groups, which are mobile, receptive, have high voter turnout, where the uncertainty about the parties’ preferences is large, and where there are many initially uninformed voters.

The parties campaign most intensively where many votes can be gained. This is in groups where the uncertainty is large and the policy surprise therefore large, in groups where voters are receptive, so that the informational effect of a given amount of campaigning is large, in mobile groups and in groups with high voter turnout where a given surprise changes many voters’ vote.\(^9\)

Groups where the initial information level is low are also attractive to campaign in. The party who offers the group a relatively good policy will want to inform about it and campaigns are effective, since there are so many uninformed voters. The incentives to campaign in such a group are therefore strong.

In the model $\Gamma_i$ and $\gamma_i$ are independent parameters. If, however, it is the case that very well informed groups are hard to inform further, so that a high $\Gamma_i$ goes hand in hand with a low $\gamma_i$, then groups with low $\Gamma_i$ will, of course, experience even more campaigning.

In order to assess which groups gain from campaigning, we use as a point of reference the case where there are no campaigns. Then $\Psi_i = \Gamma_i$ for all groups $i$. Let e.g. $c_{Ai}$ denote the policy party $A$ offers group $i$ if parties are of type $t$.

From equations (23) and (24), we know that the consumption level offered to a group is increasing in its information level. Combining this with Proposition 1 directly gives

---

\(^{9}\)These results are in line with the empirical results of Nagler and Leighley (1992), who investigate campaign expenditures in different states in 1972. They find these to be higher in states with close races and more voters.
Theorem 1 Groups which are mobile, receptive, initially relatively uninformed, where the uncertainty about the parties’ preferences is large, or have high voter turnout gain from strategic campaigning.

The groups where campaigns attract many votes will be targeted by the parties and this makes these groups well-informed, which in turn makes the parties offer them favorable policies.

Lindbeck and Weibull (1987) showed that mobile groups would gain, since they offered a solid vote response to better policies. As is clear from Theorem 1, this effect is reinforced by informative campaigning. Groups with a solid vote response are interesting targets for campaigning. The campaigning makes these groups better informed and therefore even more attractive for the parties to cater to. There is a complementarity between campaigning and spending, which gives rise to a campaign-multiplier.

The strategic distortion identified by Lindbeck and Weibull is thus aggravated by informative campaigning. From an egalitarian perspective, this feature of informative campaigning is not good and a reduction in campaign funds would improve on the situation. A social planner, maximizing the sum of utility in society, would find this aspect of campaigning as welfare decreasing. Clearly, as the model is one of pure redistribution all allocations are Pareto Optimal.

The evaluation of campaigns may change when we consider asymmetries in initial information. Groups, who are initially relatively uninformed, will experience more campaigning and their level of information is thus relatively much increased, which make the parties offer them better policies. In this respect campaigning may lead to a more even distribution in equilibrium, which is beneficial from a welfare perspective. In the next section, where we consider welfare more in detail in a tractable example, it is shown that this may indeed be the case.

An important objective of real world campaigning, which is not included in the model at present, is that campaigns, beside being informative, are often directed at making people vote.\textsuperscript{10} It is straightforward to include such an

\textsuperscript{10}Rekkas (2002) estimate a random coefficients discrete choice model using Canadian
effect in the model. Suppose that turnout depends positively on campaigns and that the effect differs across groups so that

$$\xi_i = \kappa_i \xi(m),$$

where $\xi'(m) > 0$ and $\xi''(m) < 0$ and $\kappa_i > 0$ is a parameter representing the group’s responsiveness. A group with a high turnout response to campaigning has a high $\kappa_i$. For the model to be well behaved it is necessary that the total response to more campaigning in a group is concave. A sufficient condition ensuring this in all groups is that

$$\frac{\partial^2 (\psi(m) \xi(m))}{\partial m^2} < 0.$$  \hfill (27)

**Theorem 2** Suppose condition (27) is fulfilled. Groups where turnout is more responsive to campaigning benefit from strategic campaigning.

If campaigns not only inform but also induce people to vote, groups with a responsive turnout win. This conforms the general principle behind all results of the model. The more responsive a group is in a broad sense, the more it wins.

### 6 Welfare

An assessment of the welfare consequences of campaigning is difficult, since the model does not allow closed form solutions\(^{11}\). In this section we investigate the special case where biased types of parties have an almost egalitarian elections data. Her empirical results confirm the importance of campaign expenditure for the outcome of elections. Interestingly, the own- and cross-expenditure vote share elasticities show that political campaign spending not only redistributes voters across parties but also decreases the size of the abstaining group of the electorate. Matsusaka and Palda (1999) also find that campaign spending increases voter turnout. On a similar note, Gerber and Green (2000) report that in a field experiment, voter turnout was increased substantially by personal canvassing.

\(^{11}\)From a technical point of view, the problem is that the expressions for the policies of the biased types (24) are not on closed form, as $G$ depends on the policies, cf (20). Similarly, there are no closed form expressions for campaign spending.
ideology, so that $\alpha_i^{II} = 1 + \varepsilon$ for all groups $i = 1, \ldots, n$, in $A$’s constituency and $\alpha_i^{II} = 1 - \frac{n_i}{n} \varepsilon$ in the rest. Similarly $\beta_i^{II} = 1 + \varepsilon$ for groups in $B$’s constituency and $\beta_i^{II} = 1 - \frac{n_i}{n} \varepsilon$ in the rest. We look at the limit as $\varepsilon \to 0$, here the parties have no particular preference for any group, so differences in policies across groups are due to the parties’ strategic concerns. We assume that $\psi(m_i) = \sqrt{m_i}$ and $\sqrt{M} \leq \max \frac{M}{\gamma_i}$, so

$$\Psi_i(m_i) = \gamma_i \sqrt{m_i} (1 - \Gamma_i) + \Gamma_i.$$  \hfill (28)

So far we have not discussed where the campaign resources stem from. In the real world they stem from private donations and public funding of political parties. We will here - for simplicity - consider the case where the campaigns are publicly financed so that when each party receives campaign funds equal to $M$, the resources left for distribution among the groups are $\Omega - 2M$.

The level of consumption offered by party $A$ of type II in district $i$ is given by

$$c_i^{II} = \frac{\psi_i \xi \delta \nu \rho G + \frac{1}{2} \alpha_i^{II}}{\sum_{j=1}^{n} (\psi_j \xi \delta \nu \rho G + \frac{1}{2} \alpha_i^{II})} (\Omega - 2M) \to \frac{\psi_i \xi \delta \nu \rho R + \frac{1}{2}}{\sum_{j=1}^{n} \psi_j \xi \delta \nu \rho R + \frac{a}{2}} (\Omega - 2M) \quad \text{for } \varepsilon \to 0$$

In fact, as $\varepsilon \to 0 : c_i^{II} \to c_i^{NI} = c_i^{NI}$ and $c_i^{II} \to c_i^{NI} = c_i^{NI}$ as can be seen from (23) and (24). In the limit as $\varepsilon \to 0$, both parties offer the same policies to a particular group. The gain from winning therefore approaches $R$ as $\varepsilon \to 0$. Let $t_i \equiv (\gamma_i \sqrt{m_i} (1 - \Gamma_i) + \Gamma_i) \xi \delta \nu \rho R + \frac{1}{2}$, then

$$c_i = \frac{t_i}{\sum_{j=1}^{n} t_j} (\Omega - 2M) \quad \text{ (29)}$$

Total welfare equals

$$W = \sum_{i=1}^{n} \ln c_i$$

Letting $t'_i \equiv \frac{dt_i}{dM}$ the marginal welfare effect of increasing $M$ can be written

$$\frac{dW}{dM} = \sum_{i=1}^{n} t'_i - n \frac{\sum_{j=1}^{n} t'_i}{\sum_{j=1}^{n} t_j} - \frac{2}{\Omega - 2M} \quad \text{ (30)}$$
The amount of available resources, $\Omega$, and campaign resources, $M$, are important for the results. If $\Omega$ is low and $M$ high, the welfare effects of increasing $M$ are bound to be negative. In the following we will assume that $\Omega$ is large and $M$ relatively small so that we can disregard the term $-2\frac{2}{\Omega-2M}$. Evidently, when we find a negative welfare effect even disregarding this term, the total effect is more negative. When we find a positive effect disregarding this term, the total effect is positive provided $\Omega$ is sufficiently large and $M$ sufficiently small. Disregarding $-2\frac{2}{\Omega-2M}$ (30) gives that the welfare effect of increasing the campaign resources is positive if and only if

$$\sum_{i=1}^{n} \frac{t'_i}{t_i} - n\sum_{j=1}^{n} \frac{t'_j}{t_j} > 0$$

(31)

Using this condition we find

**Proposition 2** When the biased parties are almost egalitarian
- if groups only differ in voter turnout and/or in mobility ($\xi_i \delta_i \neq \xi_j \delta_j$),
- or if groups only differ in receptiveness ($\gamma_i \neq \gamma_j$),
then an increase in campaign resources $M$ decreases welfare.
- If groups only differ in initial level of information ($\Gamma_i \neq \Gamma_j$) and $M$ is sufficiently small,
then an increase in campaign resources $M$ increases welfare.
- If groups only differ in initial level of information ($\Gamma_i \neq \Gamma_j$) and $M$ is sufficiently large,
then it depends on parameter values whether an increase in campaign resources $M$ increases or decreases welfare.

Campaigns decrease welfare, when the important difference between the groups stems from difference in turn out ($\xi$), density of the swing voters ($\delta$), or receptiveness for campaigns. In these cases, campaigns reinforce the skewed distribution already present in the original Lindbeck and Weibull model - the parties’ strategic concerns induce them to exploit the campaign-multiplier, and this decreases overall welfare. This is so even though the biased types are almost egalitarian in their ideology.
Campaigns increase welfare when some groups have a low fraction of initially informed voters and campaign resources are sufficiently small. The low information level makes these groups an interesting target for campaigns, and in equilibrium the groups gain and total welfare is increased. When campaign resources become abundant a welfare assessment is more complicated and it depends on parameter values whether and increase in $M$ increases or decreases welfare. Welfare effects can be negative for high $M$ since the initially uninformed groups are so attractive to campaign in that their equilibrium share of consumption exceeds that of the initially well informed groups. Increasing $M$ further will benefit the initially uninformed further and make the distribution even more skewed.

When $M$ becomes large enough, the condition $\sqrt{M} \leq \max \frac{1}{\gamma_i}$ is violated. Consider a slight change in $\psi$, so that

$$\psi(m) = \begin{cases} \sqrt{m} & \text{if } m \leq \frac{1}{\gamma_i} \\ 1 & \text{if } m > \frac{1}{\gamma_i} \end{cases}$$

Suppose groups only differ in $\Gamma_i$ but are almost alike, so that half of each party’s constituency has $\Gamma_i = \Gamma_I$ and the other half $\Gamma_i = \Gamma_I + \eta$, where $\eta > 0$ is small ($\eta \to 0$). In the Appendix we show that increasing $M$ decreases welfare for all $R, \xi, \delta, \Gamma, \gamma$, if $M$ is sufficiently large, i.e. when

$$\gamma \sqrt{\frac{M}{n}} > \frac{1}{2\sqrt{2}} \approx .35$$

One may ask, which policies would be Pareto improving? If the positive welfare effects of campaigns dominate, it obviously helps to give public funding to parties’ campaigns. If the negative effects dominate, this is not a good idea and a cap on campaign resources would improve welfare. Ideally, one would want to increase the information level in groups which are disadvantaged in the distributional game, but it is not clear who should implement this policy; the parties have a different agenda. If, however, it was somehow done, the parties would react. If information were increased in a particular group, the parties would partly offset this by campaigning less in the group.

One could imagine general laws stipulating that public campaigns should be directed at increasing the information level in all groups. Regardless of
whether it would in fact be implementable, it may give negative results. For example one finds in a numeric example with four groups where \( n_a = 2 \) and where \( R = 1, \theta = 1, \xi_1 = .6, \xi_2 = .5 \) and \( \delta_1 = \delta_2 = \gamma_1 = \gamma_2 = 1 \), that increasing \( \Gamma_1 = \Gamma_2 \) from .5 to .7 decreases welfare.

7 Concluding remarks

Strategic informative campaigning benefits groups, who are responsive to campaigning. The reasons for this responsiveness may be that there are many swing voters; many voters, who listen to campaigns; many voters, who actually cast a ballot and whose turnout increases much in response to campaigning. Groups where uncertainty about the parties’ preferences and thus policies are particularly large also offer a solid vote response to campaigns. So do groups with relatively many initially uninformed voters. Furthermore, the parties will never direct informative campaigns at the same groups. Parties want to reveal the positive message that their policy is better than expected compared with the rival’s, and, in the nature of things, only one party has something positive to reveal to a particular group.

Groups, who are the subject of intensive campaigning, end up being very well-informed. Parties wish to offer attractive policies to such groups as many votes can be gained this way. In equilibrium, such groups therefore gain from the strategic campaigning. From a welfare point of view, this may be good or bad. If some groups are disadvantaged because of few initially informed voters, campaigns may improve welfare. However, if the main differences between groups are in turnout, swing voters and receptiveness to campaigns, campaigns reinforce a skewed distribution and decrease welfare.

In the particular setting of our model, the uncertainty is about how much the parties favor different groups. Groups where this uncertainty is large gain. However, the uncertainty may concern other issues such as the size of the group or the distribution of ideological slant in the group. In these cases, it will still be true that groups where the uncertainty is particularly large will benefit from strategic campaigning. These are testable predictions.
of the model: informative campaigns are intense in groups where there is large uncertainty about policies; they are intense in groups, which contain many mobile voters who are inclined to vote, and in groups which are initially relatively uninformed and who are receptive, and parties will never use informative campaigns in the same groups. Furthermore, groups which are subject to intense campaigning gain in the distributional conflict.

The model does not include organized lobbies. Since strategic campaigning will benefit particular groups, these groups will have a stronger incentive to form lobby groups and support parties with campaign finance. It will be a subject of further research to investigate the interplay between lobbying and strategic campaigning.

Informative campaigns are often considered good since they allow voters to make more informed choices. One may value this feature per se, and one may also conceive of kinds of imperfect information, where more informed choices certainly are preferable\(^\text{12}\). In the present case, where politics concern redistribution among heterogeneous groups, campaigns may enhance welfare, but they may also easily end up aggravating the skewness resulting from the strategic behavior of the parties and then they do not enhance welfare.

8 Appendix

Proof of Lemma 1

Since \( u \) is strictly concave and \( \Phi_i \) is the uniform distribution, it follows from (6), (13), (14), (11) and (10) that the objective functions of the parties are strictly concave and continuous in the parties choice variables and continuous in the vector of beliefs \( \bar{\pi} \equiv (\bar{\pi}_i)_{i=1}^n \). Furthermore, the choice sets of the parties are strictly convex and compact. Hence the vector of optimal choices of the parties given by (15) and (16),

\[
\sigma \equiv ((a_N^A, m_N^A), (c_B^N, m_B^N), (c_B^H, m_B^H))
\]

is a continuous function of \((\bar{\pi}, \sigma)\). Call this function \( s(\cdot) \). We also have that the vector \( \bar{\pi} \) is a continuous function of \( \sigma \), which takes values in a convex compact set, call this function

\(^{12}\)See for instance Schultz (2002) for the case where politics is uni-dimensional.
Now consider the compound mapping $\sigma \to \sigma'$ defined by

$$\sigma' = s(p(\sigma), \sigma)$$

This is a continuous function mapping a compact, convex set into itself, hence it has a fixed point. This fixed point is an equilibrium. ■

**Proof of the existence of a symmetrical equilibrium.**

We need to check that $c'_{Ai} = c'_{Bi+1-i}$, $p_A = \frac{1}{2}$ and $G_A^\Pi = G_B^\Pi$. From (24) we have that

$$c'_{Ai} = \frac{\Psi_i \xi_i \delta_i \theta G + \frac{1}{2} \alpha'_i}{\sum_{j=1}^{\alpha_i} \left( \Psi_j \xi_j \delta_j \theta G + \frac{1}{2} \alpha'_j \right)} \Omega = \frac{\Psi_{n+1-i} \xi_{n+1-i} \delta_{n+1-i} \theta G + \frac{1}{2} \beta'_i}{\sum_{j=1}^{\beta_i} \left( \Psi_j \xi_j \delta_j \theta G + \frac{1}{2} \beta'_j \right)} \Omega = c'_{Bi+1-i}$$

and using (20) and (22) we get

$$G_A^\Pi = \left( R + \sum_{i=1}^{n} \alpha_i^\Pi \ln \left( \frac{c_{Ai}^\Pi}{c_{Bi}^\Pi} \right) \right) = \left( R + \sum_{i=1}^{n} \beta_i^\Pi \ln \left( \frac{c_{Bi+1-i}^\Pi}{c_{Ai+1-i}^\Pi} \right) \right) = G_B^\Pi = G$$

Inserting into (9) it is easily checked that indeed $p_A = \frac{1}{2}$ when $c_{Ai}^\Pi = c_{Bi+1-i}^\Pi$ and $c_{Ai}^N = c_{Bi}^N$. Hence the symmetric equilibrium exists. ■

**Proof of Lemma 2**

If $t = \Pi$, the difference between the actual and the expected utility difference in group $i$ is

$$u(c_{Ai}^\Pi) - u(c_{Bi}^\Pi) - (E_A u_i - E_B u_i) = (1 - \pi_i) (u(c_{Ai}^\Pi) - u_i(c_{Bi}^\Pi)),$$

and if $t = N$ it is

$$u_i(c_{Ai}^N) - u(c_{Bi}^N) - (E_A u_i - E_B u_i) = -\pi_i (u(c_{Ai}^\Pi) - u(c_{Bi}^\Pi)).$$

Using (9) and (18), the first order condition for an interior choice of campaign expenditures for party $A$ of type $\Pi$ in group $i$, $m_{Ai}^\Pi$, can be written

$$\frac{\partial \Psi_i \xi_i \delta_i \theta}{\partial m_{Ai}^\Pi} G_A^\Pi (u(c_{Ai}^\Pi) - u(c_{Bi}^\Pi) - (E_B u_i - E_A u_i)) = \mu.$$ 

29
If the left hand side is negative, the non-negativity constraint binds and $m_{A_i}^\Pi = 0$.

Using (19), (21) and (32), we can rewrite (34)

$$\frac{\partial \Psi_i}{\partial m_{A_i}} \xi_i \delta_i \frac{\theta}{\delta} (1 - \pi_i) \ln \left( \frac{c_{A_i}^\Pi}{c_{B_i}^\Pi} \right) G_A^\Pi = \mu$$

(35)

Hence, a biased party $A$, only campaigns in groups where where $c_{A_i}^\Pi > c_{B_i}^\Pi$.

Recall that $\frac{\partial \Psi_i}{\partial m_{A_i}} = (1 - \psi_i (m_{B_i})) \frac{\partial \psi_i}{\partial m_{A_i}}$, and $(1 - \psi_i (m_{B_i})) > 0$. Furthermore, $\psi(0) = \infty$, so party $A$ campaigns in all groups where $c_{A_i}^\Pi > c_{B_i}^\Pi$. From (26) these are groups where $\alpha_{i}^\Pi > \beta_{i}^\Pi$.

Party $B$'s first order condition, when it is biased is

$$\frac{\partial \Psi_i}{\partial m_{B_i}} \delta_i \xi_i \frac{\theta}{\delta} (1 - \pi_i) \ln \left( \frac{c_{B_i}^\Pi}{c_{A_i}^\Pi} \right) G_B^\Pi = \mu$$

(36)

Party $B$ campaigns where $c_{B_i}^\Pi > c_{A_i}^\Pi$.

When the parties are neutral, they propose the same policies, and the first order condition in groups where $A$ campaigns is

$$- \frac{\partial \Psi_i}{\partial m_{A_i}} \xi_i \delta_i \frac{\theta}{\delta} \pi_i \ln \left( \frac{c_{A_i}^\Pi}{c_{B_i}^\Pi} \right) R = \mu$$

(37)

The left hand side is positive in groups where $c_{B_i}^\Pi > c_{A_i}^\Pi$, i.e. where $A$'s policy would have been worse than $B$'s if the parties were biased.

Finally, party $B$'s first order condition, when parties are neutral is

$$- \frac{\partial \Psi_i}{\partial m_{B_i}} \xi_i \delta_i \frac{\theta}{\delta} \pi_i \ln \left( \frac{c_{B_i}^\Pi}{c_{A_i}^\Pi} \right) R = \mu$$

(38)

We see that $B$ then campaigns in groups where $c_{B_i}^\Pi < c_{A_i}^\Pi$.

**Proof of Lemma 3**

Consider a group where $\alpha_{i}^\Pi > \beta_{i}^\Pi$. Here party $A$ campaigns if $t = \Pi$ and $B$ campaigns if $t = N$. Using (12) the posterior belief for an uninformed voter in the group is

$$\bar{\pi}_i = \frac{(1 - \psi_i (m_{A_i}^\Pi)) \pi}{(1 - \psi_i (m_{A_i}^\Pi)) \pi + (1 - \psi_i (m_{B_i}^\Pi)) (1 - \pi)}$$

(39)
In a group where \( \alpha_i^\Pi < \beta_i^\Pi \) B campaigns when parties are biased, and A when they are not, and the posterior belief is therefore

\[
\bar{\pi}_i = \frac{(1 - \psi_i (m_{Bi}^\Pi)) \pi}{(1 - \psi_i (m_{Ai}^\Pi)) (1 - \pi) + (1 - \psi_i (m_{Bi}^\Pi)) \pi}
\]  

(40)

Suppose that \( \bar{\pi}_i = \pi \) for all \( i \). Consider a group \( i \) in \( B' \)'s constituency, where \( \beta_i^\Pi > 1 > \alpha_i^\Pi \). If party \( B \) is biased, party \( B \) campaigns in this group and the campaign expenditure is determined by the (set of) first order conditions (36). If on the other hand \( t = N \), then party \( A \) campaigns in the group, and the campaign expenditure is determined by the set of first order conditions (37). When \( \bar{\pi}_i = \pi \) for all \( i \), then these two set of equations are equivalent and the solutions \( (m_{Bi}^\Pi)_{i|\beta_i>1} \) and \( (m_{Ai}^N)_{i|\beta_i>1} \) are the same. Hence, \( m_{Bi}^\Pi = m_{Ai}^N \). Using (40) we see that in this case it is indeed true that \( \bar{\pi}_i = \pi \) is a solution to the Bayesian updating of the voters. Clearly, the same argument can be applied for all groups. In groups where \( \alpha_i^\Pi = \beta_i^\Pi \), there is no campaigning so trivially, \( \bar{\pi}_i = \pi \).

**Proof of Proposition 1**

We will focus on groups in which party \( A \) campaigns when the parties are biased. By symmetry the same results are true in groups where \( B \) campaigns when parties are biased. Lemma 3 implies that we can write the first order condition (35) of party \( A \) of type \( \Pi \) in a group where it campaigns as

\[
\gamma_i \frac{\partial \psi}{\partial m_{Ai}} \frac{(1 - \Gamma_i) \theta \xi_i \delta_i (1 - \pi)}{\delta} \ln \left( \frac{\Psi_i \xi_i \delta_i \theta G + \alpha_i^\Pi}{\Psi_i \xi_i \delta_i \theta G + \beta_i^\Pi} \right) = \mu
\]  

(41)

and there are similar expressions for the other first order conditions. Notice that \( \pi \) is the same across groups, independent of the other variables.

Let

\[
X_{Ai} = \frac{\partial^2 \psi}{\partial (m_{Ai})^2} \ln \left( \frac{\Psi_i \xi_i \delta_i \theta G + \alpha_i^\Pi}{\Psi_i \xi_i \delta_i \theta G + \beta_i^\Pi} \right)
\]

and

\[
+ \left( \frac{\partial \psi}{\partial m_{Ai}} \right)^2 \ln \left( \frac{\Psi_i \xi_i \delta_i \theta G}{\Psi_i \xi_i \delta_i \theta G + \alpha_i^\Pi} \right) - \frac{\xi_i \delta_i \theta R}{\Psi_i \xi_i \delta_i \theta G + \alpha_i^\Pi}
\]

then \( X_{Bi} < 0 \) for \( \alpha_i^\Pi > \beta_i^\Pi \). Applying the implicit function theorem on (41),
we then get that for \( \alpha_i^\Pi > \beta_i^\Pi \),

\[
\frac{d m_i^\Pi}{d \alpha_i^\Pi} = - \frac{\partial \psi}{\partial m_{Ai}} \frac{1}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi} > 0 \quad \text{and} \quad \frac{d m_i^\Pi}{d \beta_i^\Pi} = - \frac{\partial \psi}{\partial m_{Ai}} \frac{1}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \beta_i^\Pi} < 0
\]

Hence, if we compare two groups \( i \) and \( j \) which differ only in that \( \alpha_i^\Pi - \beta_j^\Pi > 0 \), then the solutions to the set of first order conditions (41) must fulfill \( m_{Ai}^\Pi > m_{Aj}^\Pi \), i.e. biased type of \( A \) campaigns more in group \( i \) than group \( j \). If parties are neutral party \( B \) campaigns in these groups and we know from Lemma 3 that \( m_{Bi}^N = m_{Ai}^\Pi \) and \( m_{Bj}^N = m_{Ai}^\Pi \). So when parties are neutral we also have \( m_{Bi}^N > m_{Bj}^N \).

Similarly,

\[
\frac{d m_i^\Pi}{d \delta_i} = - \frac{\partial \psi}{\partial m_{Ai}} \left( \ln \left( \frac{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \beta_i^\Pi} \right) + \frac{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi} - \frac{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \beta_i^\Pi} \right) \tag{42}
\]

The sign of this expression equals the sign of the numerator. For \( \alpha_i^\Pi \) and \( \beta_i^\Pi \) close to one, the numerator approximately equals

\[
\left( \Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi - 1 \right) + \frac{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi} - \frac{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \beta_i^\Pi}
\]

which equals

\[
- \frac{\partial \psi}{\partial m_{Ai}} \frac{1}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi} \Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \beta_i^\Pi \left( \alpha_i^\Pi - \beta_i^\Pi \right) \alpha_i^\Pi
\]

which is positive for \( \alpha_i^\Pi > \beta_i^\Pi \). Furthermore it is straightforward to check that the numerator of (42) is increasing in \( \alpha_i^\Pi \). We conclude that for \( \alpha_i^\Pi > \beta_i^\Pi \), \( \frac{d m_i^\Pi}{d \beta_i^\Pi} > 0 \) and accordingly that if groups \( i \) and \( j \) only differ in that \( \delta_i > \delta_j \), then \( m_{Ai}^\Pi > m_{Ai}^\Pi \). Since \( \gamma_i, \xi_i \) and \( \delta_i \) enter symmetrically in the first order conditions, Proposition 1 is also true for these variables.

Finally, consider two groups where the only difference consists in \( \Gamma_i > \Gamma_j \). Totally differentiating

\[
\frac{d m_i^\Pi}{d \Gamma_i} = - \frac{\gamma_i \frac{\partial \psi}{\partial m_{Ai}} \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi}{d \Gamma_i} \left( \frac{\partial \ln \left( \frac{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \beta_i^\Pi} \right)}{d \Gamma_i} \right) \right) - \ln \left( \frac{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \alpha_i^\Pi}{\Psi_i \frac{\xi_i \delta_i}{\delta} \theta G + \beta_i^\Pi} \right)
\]

\[13\]We use the approximation \( \ln x \approx x - 1 \) for \( x \) close to one.
As

$$\frac{\partial \ln \left( \frac{\Psi_i \xi_i \delta G + \alpha_i^\Pi}{\Psi_i \xi_i \delta G + \beta_i^\Pi} \right)}{\partial \Gamma_i} = \frac{1}{\Psi_i \xi_i \delta G + \beta_i^\Pi} \frac{\xi_i \delta_i \theta R \left( \Psi_i \xi_i \delta G + \beta_i^\Pi \right) - \xi_i \delta_i \theta R \left( \Psi_i \xi_i \delta G + \alpha_i^\Pi \right)}{\left( \Psi_i \xi_i \delta G + \beta_i^\Pi \right)^2} < 0$$

we get

$$\frac{dm_i^\Pi}{d\Gamma_i} < 0$$

and it follows that $m_i^\Pi_{A_i} < m_i^\Pi_{A_j}$. ■

**Proof of Theorem 2**

With $\xi_i = \kappa_i \xi (m)$, $\frac{\partial (\Psi_i (m_B_i) \kappa_i (m_B_i))}{\partial m_B}$ the first order condition (41) becomes

$$\gamma_i \kappa_i \frac{\partial^2 \left( \psi (m) \xi (m) \right)}{\partial^2 m} (1 - \Gamma_i) \theta \delta_i (1 - \pi) \frac{\ln \left( \frac{\Psi_i \xi_i \delta G + \alpha_i^\Pi}{\Psi_i \xi_i \delta G + \beta_i^\Pi} \right)}{\delta} = \mu$$

It straightforward check that under condition (27) the steps of Proposition 1 as well as Theorem 1 can be retraced. ■

**Proof of Proposition 2**

In order to shorten notation, let $z_i \equiv \xi_i \delta_i \frac{\theta}{\delta}$.

Inserting for $t_i$ we get

$$\frac{t_i'}{t_i} = \frac{\gamma_i \sqrt{m_i} (1 - \Gamma_i) z_i R}{\left( \gamma_i \sqrt{m_i} (1 - \Gamma_i) + \Gamma_i \right) z_i R + \frac{1}{2} dM} \frac{dm_i}{dM}$$

Letting

$$a_i \equiv \frac{\Gamma_i + \frac{1}{2Rz_i}}{\gamma_i (1 - \Gamma_i)}$$

we get

$$\frac{t_i'}{t_i} = \frac{1}{m_i + a_i \sqrt{m_i} dM} \frac{dm_i}{dM} \quad (43)$$

In groups where the biased type of party $A$ campaigns ($i = 1, ..., n_a$) (or in groups where the biased type of $B$ campaigns ($i = n + 1 - n_a, ..., n$)), the first order condition (35) for choice of $m_i$ becomes (using (25) and (28)).

$$\gamma_i \frac{1}{2 \sqrt{m_i}} (1 - \Gamma_i) z_i (1 - \pi) \ln \left( \frac{\Psi_i z_i G (\varepsilon) + \frac{1}{2} (1 + \varepsilon)}{\Psi_i z_i G (\varepsilon) + \frac{1}{2} (1 - \frac{n_a}{n} \varepsilon)} \right) G (\varepsilon) = \mu (\varepsilon)$$

33
As $\varepsilon \to 0$, we can use the approximation $\ln(x) \approx x - 1$, so that
\[
\ln \left( \frac{\Psi_i \gamma_i G(\varepsilon) + \frac{1}{2} (1 + \varepsilon)}{\Psi_i \gamma_i G(\varepsilon) + \frac{1}{2} (1 - \frac{na}{n} \varepsilon)} \right) \approx \frac{1}{2} \frac{\frac{na}{n} \varepsilon}{(1 - \frac{na}{n} \varepsilon)}
\]
and letting $a_i(\varepsilon) = \frac{1 - \frac{na}{n}}{2\gamma_i(1 - \frac{na}{n})\gamma_i}$ we can rewrite the first order condition
\[
m_i + a_i(\varepsilon) \sqrt{m_i} = \frac{(1 - \pi) \frac{na}{n} \varepsilon}{4\mu(\varepsilon)}
\]
In the limit as $\varepsilon \to 0$ the choice of campaign expenditures fulfill
\[
m_i + a_i \sqrt{m_i} = m_j + a_j \sqrt{m_j} = k
\]
The implicit function theorem and (45) gives
\[
\frac{dm_i}{dM} = \frac{1}{1 + a_i \frac{1}{2\sqrt{m_i}}} \frac{dk}{dM}
\]
Using the fact that $\sum_{i=1}^{na} \frac{dm_i}{dM} = 1$ this gives for $i = 1, \ldots, na$ and by symmetry also for $i = n + 1 - na, \ldots, n$
\[
\frac{dm_i}{dM} = \frac{1}{\sum_{j=1}^{na} \frac{1}{1 + a_j \frac{1}{2\sqrt{m_j}}}}
\]
If there exists $i$ such that $na < i < n + 1 - na$, then for those $i : \frac{dm_i}{dM} = 0$

Using the first order condition (45), we can rewrite (43)
\[
\frac{t'_i}{t_i} = \frac{1}{k} \frac{dm_i}{dM}
\]
We can therefore rewrite condition (31), as
\[
\frac{dW}{dM} > 0 \iff \sum_{i=1}^{n} \frac{1}{k} \frac{dm_i}{dM} - n \frac{\sum_{i=1}^{n} \frac{1}{k} \frac{dm_i}{dM} t_i}{\sum_{j=1}^{n} t_j} > 0
\]
using the fact that $\sum_{i=1}^{na} \frac{dm_i}{dM} = 1$ and $\sum_{i=n+1-na}^{n} \frac{dm_i}{dM} = 1$ this gives
\[
\frac{dW}{dM} > 0 \iff \frac{2}{n} > \sum_{i=1}^{n} \frac{dm_i}{dM} \frac{t_i}{\sum_{j=1}^{n} t_j}
\]
or
\[ \frac{dW}{dM} > 0 \iff \frac{2}{n} > \frac{\sum_{i=1}^{n} \partial m_i c_i}{\partial M \Omega}, \quad (48) \]

Let e.g. \( \Delta_i^m \) denote the difference between \( \frac{dm_i}{dM} \) and the mean, then
\[ \Delta_i^m = \frac{dm_i}{dM} - \frac{2}{n} \quad \text{and} \quad \Delta_i^c = \frac{c_i}{\Omega} - \frac{1}{n} \]

and \( \sum_{i=1}^{n} \Delta_i^m = \sum_{i=1}^{n} \Delta_i^c = 0 \). Then we can rewrite (48)
\[ \frac{dW}{dM} > 0 \iff \frac{2}{n} > \frac{\sum_{i=1}^{n} \left( \Delta_i^m + \frac{2}{n} \right) \left( \Delta_i^c + \frac{1}{n} \right)}{\sum_{i=1}^{n} \Delta_i^m \Delta_i^c} \]

or
\[ \frac{dW}{dM} > 0 \iff \frac{2}{n} > \frac{\sum_{i=1}^{n} \left( \Delta_i^m \Delta_i^c + \frac{2}{n} \Delta_i^c + \frac{1}{n} \right)}{\sum_{i=1}^{n} \Delta_i^m \Delta_i^c} \]

which, gives
\[ \frac{dW}{dM} > 0 \iff 0 > \sum_{i=1}^{n} \Delta_i^m \Delta_i^c \]

From (45) \( a_i > a_j \Rightarrow m_i < m_j \). Using this, the definition of \( a_i \) and (47) we get that
\[ \Gamma_i > \Gamma_j \text{ or } \gamma_i < \gamma_j \text{ or } z_i < z_j \Rightarrow a_i > a_j \Rightarrow m_i < m_j \quad \text{and} \quad \frac{dm_i}{dM} < \frac{dm_j}{dM} \quad \text{and} \quad \Delta_i^m < \Delta_j^m \quad \text{(49)} \]

Recall that
\[ t_i \equiv (\gamma_i \sqrt{m_i} (1 - \Gamma_i) + \Gamma_i) z_i R + \frac{1}{2}, \quad (50) \]

Together with (49) and (29), this gives us
\[ \gamma_i < \gamma_j \text{ or } z_i < z_j \Rightarrow t_i < t_j \text{ and } \Delta_i^c < \Delta_j^c \quad \text{(51)} \]

**Suppose groups differ only wrt \( \gamma_i \).** Wlog renumber the groups so that \( \gamma_1 \leq ... \leq \gamma_n \), where at least one inequality is strict. Then by (49) and (51) we have that
\[ \Delta_i^m \leq \Delta_{i+1}^m \quad \text{and} \quad \Delta_i^c \leq \Delta_{i+1}^c \]

for all \( i < n \), and there exists at least one \( i \) such that \( \Delta_i^m < \Delta_{i+1}^m \) and one \( i \) such that \( \Delta_i^c < \Delta_{i+1}^c \). Since \( \sum_{i=1}^{n} \Delta_i^m = 0 \) and \( \sum_{i=1}^{n} \Delta_i^c = 0 \) there is an \( i \),
\( i_m \), such that for \( i < i_m, \Delta_i^m < 0 \) and for \( i \geq i_m, \Delta_i^m > 0 \). Similarly, there is \( i_c \), such that for \( i < i_c, \Delta_i^c < 0 \) and for \( i \geq i_c, \Delta_i^c > 0 \). Wlog assume that \( i_c \leq i_m \).

As \( \sum_{i=1}^{i_m} \Delta_i^m = 0 \) we have that \( \sum_{i=i_m}^{i-1} \Delta_i^m = -\sum_{i=i_m}^{n} \Delta_i^m \). Now

\[
\sum_{i=1}^{n} \Delta_i^m \Delta_i^c = \sum_{i=1}^{i_m-1} \Delta_i^m \Delta_i^c + \sum_{i=i_m}^{n} \Delta_i^m \Delta_i^c > \sum_{i=1}^{i_m-1} \Delta_i^m \Delta_{i-1}^c + \sum_{i=i_m}^{n} \Delta_i^m \Delta_i^c
\]

where the inequality follows from the fact that \( \Delta_{i_m-1}^c \geq 0, \Delta_{i_m-1}^c \geq \Delta_{j}^c, i = 1, \ldots, i_m-2, \sum_{i=1}^{i_m-1} \Delta_i^m < 0 \) and similarly that \( 0 < \Delta_i^c \leq \Delta_i^c, i = i_m+1, \ldots, n \) and \( \sum_{i=i_m}^{n} \Delta_i^m > 0 \).

Moreover,

\[
\sum_{i=1}^{i_m-1} \Delta_i^m \Delta_{i-1}^c + \sum_{i=i_m}^{n} \Delta_i^m \Delta_i^c = (\Delta_i^m - \Delta_{i-1}^c) \sum_{i=i_m}^{n} \Delta_i^m \geq 0
\]

which proves that

\[
\sum_{i=1}^{n} \Delta_i^m \Delta_i^c > 0
\]

so

\[
\frac{dW}{dM} < 0
\]

If groups differ wrt \( z_i \) only, an identical proof shows that also in this case \( \frac{dW}{dM} < 0 \).

**Consider the case where groups differ wrt \( \Gamma_i \) only.** Wlog assume that \( \Gamma_1 \leq \ldots \leq \Gamma_n, \) then \( \Delta_i^m \geq \Delta_{i+1}^m \) for all \( i < n \). As is clear from (50) the relation between \( \Delta_i^c \) and \( \Delta_{i+1}^c \) is not obvious. However, if \( M \) is sufficiently small, then \( \sqrt{m_i} \) is small and we have that \( t_i < t_{i+1} \) and therefore \( \Delta_i^c \leq \Delta_{i+1}^c \) for all \( i < n \). This implies that

\[
\sum_{i=1}^{n} \Delta_i^m \Delta_i^c < 0 \quad \text{so} \quad \frac{dW}{dM} > 0
\]

if \( M \) is sufficiently small.

For larger \( M \) a general result is not available (since we do not have a closed form solution for \( m_{i_i} \), it is difficult to assess which term dominates in \( t_i \)). An increase in \( M \) decreases welfare if \( t_i \geq t_{i+1} \) so that \( \Delta_i^c \geq \Delta_{i+1}^c \) for all \( i < n \) (with at least one strict inequality).
Consider an example where \( n_a = n/2 \) (\( n \) even). Let \( \Gamma_i = \Gamma_I \) for \( i = 1, \ldots, n/4 \) and \( \Gamma_i = \Gamma_{II} = \Gamma_1 + \eta \) for \( i = n/4 + 1, \ldots, n/2 \), where \( \eta > 0 \) is small. In the limit as \( \eta \to 0 \), the districts are alike. Let \( m_I \) denote campaign expenditures in groups \( i = 1, \ldots, n/4 \) and \( m_{II} \) campaign expenditures in groups \( i = n/4 + 1, \ldots, n/2 \). Then \( \Delta_I^m > \Delta_{II}^m \) and an increase in \( M \) decreases welfare if \( \Delta_I^c > \Delta_{II}^c \), i.e. if \( t_I > t_{II} \). We have that \( t_I > t_{II} \) if

\[
\gamma \sqrt{m_I} (1 - \Gamma_I) + \Gamma_I - (\gamma \sqrt{m_{II}} (1 - \Gamma_{II}) + \Gamma_{II}) > 0 \tag{52}
\]

Using the implicit function theorem on the first order condition (45) and evaluating in \( \eta = 0 \), we find that

\[
\frac{dm_I}{d\eta} = \frac{1}{\gamma} \frac{1}{1 - \Gamma_I} \frac{2M}{n} + \gamma \frac{1}{\sqrt{\frac{2M}{n}}} \frac{1 - \Gamma_I + \frac{\Gamma_I}{1 - \Gamma_I}}{1 - \Gamma_I} + 2
\]

Furthermore, differentiating the left hand side of (52) wrt \( \eta \) and evaluating at \( \eta = 0 \) gives

\[
\frac{d(LHS)}{d\eta} = \gamma \frac{1}{\sqrt{\frac{2M}{n}}} (1 - \Gamma_I) \frac{dm_I}{d\eta} + \gamma \sqrt{\frac{2M}{n}} - 1
\]

For small \( \eta \), we have that \( t_I > t_{II} \) if \( \frac{d(LHS)}{d\eta} > 0 \). Inserting for \( \frac{dm_I}{d\eta} \) we get

\[
\frac{d(LHS)}{d\eta} = \frac{1 + \frac{\Gamma_I + \frac{\Gamma_I}{\sqrt{\frac{2M}{n}}}}{1 - \Gamma_I}}{2 + \frac{1}{\gamma} \sqrt{\frac{2M}{n}}} + \gamma \sqrt{\frac{2M}{n}} - 1 \tag{53}
\]

By assumption, the maximal value of \( M \) is \( \frac{1}{\gamma} \). Evaluating \( \frac{d(LHS)}{d\eta} \) at \( M = \frac{1}{\gamma} \) gives

\[
\frac{d(LHS)}{d\eta} = \sqrt{\frac{2n}{n}} + \frac{1 + \frac{\Gamma_I + \frac{\Gamma_I}{\sqrt{\frac{2M}{n}}}}{1 - \Gamma_I}}{2 + \sqrt{\frac{2M}{n}}} - 1
\]

which is positive if

\[
\left( 2 - \sqrt{\frac{n}{2}} \right) \frac{\Gamma_I + \frac{\Gamma_I}{\sqrt{\frac{2M}{n}}}}{1 - \Gamma_I} > 1
\]

This necessarily takes that \( n < 8 \).
If \( n \geq 8 \), equation (53) can only be fulfilled if \( M > \frac{1}{\gamma} \), implying that \( \gamma \sqrt{M} > 1 \), which does not make economic sense. If, however,

\[
\psi(m) = \begin{cases} 
\sqrt{m} & \text{if } m \leq \frac{1}{\gamma^2} \\
1 & \text{if } m > \frac{1}{\gamma^2}
\end{cases}
\]

then (53) is positive if

\[
\left(\gamma \sqrt{\frac{2M}{n}} - \frac{1}{2}\right) \left(1 + \frac{\Gamma_I + \frac{1}{2\sqrt{n}}}{1 - \Gamma_I}\right) > \left(\frac{1}{2} - \gamma \sqrt{\frac{2M}{n}}\right) \gamma \sqrt{\frac{2M}{n}}
\]

which is fulfilled for all relevant \( \Gamma_I, R \) and \( z \) if \( M \) is so large that

\[
\gamma \sqrt{\frac{M}{n}} > \frac{1}{2\sqrt{2}} \approx .35
\]

References


[34] Rekkas, Marie (2002), “The Impact of Campaign Spending on Votes in Multiparty Elections”, mimeo, Simon Fraser University


