A Note on the Welfare Evaluation of Tax Reform with Non-Convex Preferences and Discrete Labor Supply

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September 2004
1 Introduction

The welfare evaluation of tax reform has traditionally been based on a standard convex labor-leisure framework. In this framework, marginal changes in work incentives create marginal adjustments in the number of hours worked. Empirical evidence shows that such continuous responses is an unrealistic description of actual labor supply behavior. Discrete labor supply adjustments seem to be the rule rather than the exception. In particular, most of the variation in labor supply occurs along the extensive margin (entry and exit) where people jump from zero hours to, say, 30 or 40 hours per week.

This note derives the welfare effect of tax reform in two different models of labor supply. The first model – called the extensive-intensive model – is a simple extension of the standard convex framework where continuous adjustments along the intensive margin is combined with discrete labor supply behavior along the extensive margin. The second model – the general discrete model – is a more fundamental departure from the standard framework. In this model, we assume that each individual has to choose from among a finite number of working hours (say, 0, 20, 30, and 40 hours per week), so that behavior is discrete along both margins of labor supply.

For empirical applications, there is another important difference between the two models. In the first model, we assume that individuals with the same observable characteristics have identical earnings if working. This may create a bias in the results if there is a lot of variation in earnings within groups, for example, because of differences in unobservable characteristics. In the second model, we assume that individuals with the same observable characteristics are distributed on an (exogenously given) discrete number of working hours. This model may therefore – at least in principle – provide more reliable results if there is a lot of intra-group variation in earnings.

In both models, we disregard income effects in labor supply. We know from previous work that the first model may be generalized to account for income effects without having any impact on the results (as long as the hours elasticities are interpreted as compensated elasticities). We expect that this result will also hold for the general discrete model.
2 The Extensive-Intensive Model

2.1 Labor Supply Behavior

The most common explanation for discrete behavior along the extensive margin is the presence of non-convexities in preferences or budget sets due to fixed work costs (Cogan, 1981) or concave work cost functions (Heim and Meyer, 2003). These work costs may be monetary costs (child care, transportation, clothing, etc.) or they could come in the form of time losses (e.g., commuting time and the time used preparing for and recovering from work). Moreover, emotional costs are perceivable resulting from the stress and the additional responsibilities associated with work. The various types of work costs may be fixed or they may depend in complex ways on working hours. But in general they tend to create economies of scale in the work decision, implying that very small working hours become non-optimal for the individual. Below we adopt a simple framework incorporating fixed work costs, denoted by $q$, which may capture some or all of the factors mentioned above.$^1$

For the fixed work costs, we adopt a stochastic formulation where each individual $i$ draws a fixed cost $q_i$ from a distribution $P_i(q)$ with density $p_i(q)$. As we shall see, this formulation implies that each individual in the population has a probability of labor market participation, which may be interpreted as an individual participation rate. The main advantage of the stochastic formulation is that it generates a smooth participation response at the individual level, where small changes in wages or taxes create small changes in the probability of participation. Hence we may capture the sensitivity of entry-exit behavior by setting elasticity parameters for each individual. Although the participation response is smooth in this way, it is also discrete in the sense that – conditional on entry – the individual never chooses very low hours of work.

Individuals choose labor supply behavior after the realization of their fixed cost of working. The labor earnings of individual $i$ if she enters the labor market is given by $w_i h_i$, where $w_i$ is the wage rate and $h_i$ is the hours of work. The tax system is described by a function $T(w_i h_i, z)$, where $z$ is an abstract parameter which we will use to capture policy reform. The tax function is a net payment to the public sector, embodying taxes as well as transfers. The tax/transfer system may be proportional, progressive, or regressive, and it may involve non-linearities. Attention is restricted, however, to the case of piecewise linearity such that marginal

$^1$ An additional source of non-convexity in the budget set which we incorporate in the analysis derives from the design of the tax-transfer system. This is mostly due to phase-out and discontinuities in the welfare system.
tax rates are locally constant.

Individual utility is specified in the following way

\[ u_i = w_i h_i - T(w_i h_i, z) - v_i(h_i) - q_i \cdot 1(h_i > 0), \]  

(1)

where \( 1(.) \) denotes the indicator function, and where disutility of labor \( v_i(h_i) \) is an increasing, convex function which is normalized such that \( v_i(0) = 0 \). Like several recent contributions analyzing the design of labor income taxation (Diamond, 1998; Saez, 2002; Immervoll et al., 2004), we adopt a quasi-linear specification of preferences hence excluding income effects on labor supply. The absence of income effects simplifies the analysis, and it is broadly in line with the empirical labor supply literature (e.g., Pencavel, 1986).

The maximization of (1) may be solved in two stages. First, we solve for the optimal hours of work conditional on labor force participation and, second, we consider the choice to enter the labor market at the optimal working hours. Given participation, \( h_i > 0 \), the optimum is characterized by the standard condition that the marginal rate of substitution between consumption and leisure equals the marginal net-of-tax wage. That is,

\[ (1 - m_i) w_i = v_i'(h_i), \]  

(2)

where \( m_i \equiv \partial T(\cdot) / \partial (w_i h_i) \) is the effective marginal tax rate on earnings. Since the \( T \)-function embodies transfers, the marginal tax rate include the marginal claw-back of benefits. This equation implicitly defines individual labor supply conditional on working as a function of the marginal consumer wage, \( h_i = h_i ((1 - m_i) w_i) \).

For the household to enter the labor market in the first place, the utility from participation must be greater than or equal to the utility from non-participation. This implies a participation constraint given by

\[ q_i \leq (1 - a_i) w_i h_i - v_i(h_i) \equiv \tilde{q}_i, \]  

(3)

where \( a_i \equiv [T(w_i h_i, z) - T(0, z)] / (w_i h_i) \) is the tax rate on labor force participation. It corresponds to an effective average tax rate, including the benefit reduction from entry in proportion to earnings (i.e., the average claw-back rate). Eq. (3) defines an upper bound \( \tilde{q}_i \) on the fixed cost of working. Individuals with a fixed cost below their threshold-value \( \tilde{q}_i \) decide to enter the labor market at \( h_i \) hours, while those with a fixed cost above the threshold choose to stay outside the labor force. By implication, the probability of participation for individual \( i \) is given
by \( P_i(q_i) = \int_0^{\bar{q}_i} p_i(q) \, dq \). Assuming \( N \) individuals in the population, we obtain aggregate labor supply simply by multiplying the participation rate and the hours of work for each individual and aggregating over all individuals, i.e.,

\[
L = \sum_{i=1}^{N} P_i(q_i) h_i \left( (1 - m_i) w_i \right),
\]

which completes the description of labor supply behavior. This expression emphasizes the joint role of the intensive and extensive margins in determining aggregate labor supply behavior, and it shows that the two margins are related to different tax/transfer parameters. While the choice of working hours depends on the effective marginal tax rate, the participation decision is determined by the cut-off fixed cost \( \bar{q}_i \) which is related to the effective average rate of taxation.

From eq. (4), the effect of tax reform on labor supply may be decomposed into its effect on hours of work for those who are working and its effect on labor force participation. Accordingly, we define for each individual

\[
\varepsilon_i \equiv h_i \left( (1 - m_i) w_i \right), \quad \eta_i \equiv p_i(q_i) \left( (1 - a_i) w_i h_i \right),
\]

where \( \varepsilon_i \) is the hours-of-work elasticity with respect to the marginal net-of-tax wage \( (1 - m_i) w_i \), while \( \eta_i \) is the participation elasticity with respect to the net-of-tax income gain from labor market entry \( (1 - a_i) w_i h_i \). Hence, each elasticity is defined with respect to the net-of-tax price which is relevant for the margin in question. The hours-of-work elasticity measures the sensitivity of working hours conditional on labor force participation, while the participation elasticity measures the sensitivity of the individual’s probability of working. Of course, if there is a large number of individuals of type \( i \), the elasticity \( \eta_i \) reflects the actual change in participation for these individuals.

### 2.2 Tax Reform and Welfare

To study the relationship between tax policy and efficiency, we derive the excess burden of taxation. With quasi-linear utility, this is defined as \( D = U_0 - U - R \), where \( U \) and \( U_0 \) are aggregate utilitarian welfare with and without taxation, respectively, while \( R \) denotes aggregate government revenue. The (expected) aggregate utilitarian welfare with taxation may be derived from eqs (1)-(3), which gives

\[
U = \sum_{i=1}^{N} \left[ \int_0^{\bar{q}_i} \left[ w_i h_i - T(w_i h_i, z) - v_i(h_i) - q_i p_i(q) dq - \int_{\bar{q}_i}^{\infty} T(0, z) p_i(q) \, dq \right] \right],
\]

5
where the first term in square brackets represents individual welfare when working, while the second term represents welfare when not working. The aggregate government revenue may be written as

\[ R = \sum_{i=1}^{N} \left[ \int_{0}^{\bar{q}_i} T(w_i h_i, z) p_i(q) dq + \int_{\bar{q}_i}^{\infty} T(0, z) p_i(q) dq \right], \tag{7} \]

where the first term is the revenue from the labor income tax, while the last term reflects expenditures from transfers to those out of work. To derive the effect of a tax reform, we consider a marginal change in the \( z \)-parameter. By inserting eqs. (6) and (7) in the definition of the excess burden and differentiating with respect to \( z \), we obtain

\[ \frac{dD}{dz} = -\sum_{i=1}^{N} \left[ m_i w_i \frac{dh_i}{dz} P_i(\bar{q}_i) + a_i w_i h_i \frac{dP_i(\bar{q}_i)}{dz} \right], \tag{8} \]

where we have used the first-order conditions (2) and (3). This expression reflects that the marginal deadweight burden is given by the effect on government revenue from behavioral responses (e.g., Auerbach, 1985; Immervoll et al., 2004). The expression shows how the behavioral revenue effect is related to the two different margins of labor supply response. The first term captures the revenue effect from the change in the optimal hours of work for those who are working. The second term gives the effect on revenue brought about by the tax-induced change in labor force participation. This second effect on efficiency is related to the tax rate on labor force participation \( a_i \), whereas the efficiency effect from changed working hours depends on the tax burden on the last dollar earned \( m_i \).\(^2\)

Using eqs (2)-(5), the marginal deadweight loss in proportion to aggregate labor income (GDP) may be written as

\[ \frac{dD}{dz} \sum_{i=1}^{N} w_i h_i P_i(\bar{q}_i) = \sum_{i=1}^{N} \left[ \frac{m_i w_i}{1-m_i} \frac{\partial m_i}{\partial z} \cdot \varepsilon_i + \frac{a_i}{1-a_i} \frac{\partial a_i}{\partial z} \cdot \eta_i \right] s_i, \tag{9} \]

where \( s_i \equiv w_i h_i P_i(\bar{q}_i) / \left( \sum_{i=1}^{N} w_i h_i P_i(\bar{q}_i) \right) \) is the (expected) wage share of individual \( i \), and where \( \partial a_i / \partial z \equiv \left[ \frac{\partial T(w_i h_i, z)}{\partial z} - \frac{\partial T(0, z)}{\partial z} \right] / (w_i h_i) \) is the impact on the effective average tax rate from the reform. The first term in this expression looks familiar, since it reflects a classic Harberger-style formula for the marginal deadweight burden of taxation. It shows that the

\(^2\)From the above expression, we may confirm that the welfare effects from extensive responses would disappear within a convex framework, as pointed out in Section 4.1. In such framework, new entrants into the labor market work infinitesimal hours, \( h_i \approx 0 \). As a consequence, we have \( T(w_i h_i, z) - T(0, z) = a_i w_i h_i \approx 0 \) for these individuals. This would imply that the revenue effect created by participation responses (the second term in eq. 8) equals zero to a first order.
welfare loss depends on the level of the marginal tax rate, the increase in the marginal tax rate, and the elasticity of hours of work. The second component in the expression reflects the deadweight loss due to changed labor supply behavior along the extensive margin. This effect is related to the level of the average tax rate, the change in the average tax rate because of the reform, as well as the sensitivity of entry-exit behavior as measured by the labor force participation elasticity.

A priori one might have wondered whether the standard convex framework could be saved by a reinterpretation of the labor supply elasticity. Following this interpretation, one would introduce extensive responses into the framework simply by using estimates of the total labor supply elasticity including both margins of response. The above analysis demonstrates that, in general, this approach is not correct, since labor force participation is related to a different tax wedge than are working hours. The analysis also shows that the size of the error made by the conventional model depends on the degree to which the observed variation in aggregate labor supply is concentrated on the extensive margin. This point was also emphasized in Kleven and Kreiner (2004) in the context of the marginal cost of public funds.

Having said that, it should be noted that there is one special case for which a reinterpretation of the conventional model is valid. This is the case of a linear Negative Income Tax (NIT), which grants a lump sum transfer \( B \) to all individuals in the economy (participants and non-participants) and then imposes a constant marginal tax rate on labor income, \( m_i = m \forall i \). Thus, the tax burden on labor market entry for individual \( i \) becomes \( T(w_i h_i, z) - T(0, z) = mw_i h_i \), which implies a participation tax rate \( a_i = m \). Moreover, if the tax reform is simply a change of tax/transfer parameters within the framework of the NIT, we would have \( \frac{\partial a_i}{\partial z} = \frac{\partial m}{\partial z} \). Inserting in eq. (9), we get

\[
\frac{dD/dz}{\sum_{i=1}^{N} w_i h_i P_i (\bar{q}_i)} = \frac{m}{1 - m} \frac{\partial m}{\partial z} \cdot (\varepsilon + \eta),
\]

(10)

where \( \varepsilon \equiv \sum_{i=1}^{N} \varepsilon_i s_i \) and \( \eta \equiv \sum_{i=1}^{N} \eta_i s_i \) are weighted averages of individual elasticities. This corresponds to a standard Harberger-type formula, with the intensive and extensive elasticities being lumped together into a total labor supply elasticity \( \varepsilon + \eta \).

\footnote{It does not cause problems that the intensive and extensive elasticities are defined with respect to different prices. With an NIT, the elasticity \( \eta \) as defined in eq. (5) becomes exactly identical to the participation elasticity with respect to the wage rate. Therefore, the elasticity \( \varepsilon + \eta \) in the above expression may be interpreted as the wage elasticity of aggregate labor supply.}
3 A General Discrete Model

3.1 Labor Supply Behavior

The model in the previous section had only discrete labor supply responses along the extensive margin. Conditional on working, marginal changes in the incentives to work created marginal changes in the hours worked. In this section, we set-up a model where all labor supply adjustments are discrete.

We consider a population of I individuals. Each individual can choose between N + 1 states. The states differ with respect to the number of hours worked, denoted by \( h(0) < h(1) < \ldots < h(N) \), where \( h(0) = 0 \) corresponds to being out of the labor force. Individual \( i \) is characterized by a wage rate \( w_i \) which gives rise to the earnings \( w_i h(0) < w_i h(1) < \ldots < w_i h(N) \) across the different states. Working is associated with different types of costs as discussed in the previous section. Some costs are fixed while others depend on the number of hours worked (e.g., the loss of leisure time when working). The work costs at the different states are characterized by a vector \( q_i = (q_i(0), \ldots, q_i(N)) \) where \( q_i(0) = 0 \). We assume that the vector of work costs is drawn from a multivariate distribution with the joint density \( p_i(q_i) \) and the cumulative distribution function \( P_i(q_i) \). This formulation implies that each individual is assigned a probability of being in each state.

Utility for individual \( i \) at \( h(j) \) hours is described by

\[
 u_i(j) = c_i(j) - q_i(j),
\]

(11)

where \( c_i(j) \) is consumption in state \( j \) while \( q_i(j) \) is the cost of working measured in units of consumption. Consumption is given by \( c_i(j) = w_i h(j) - T(w_i h(j)) \), where \( T(\cdot) \) is a tax function which – in contrast to the previous model – could be non-differentiable.

The optimal working hours for individual \( i \), denoted by \( h_i \), is obtained by finding the state \( j \) which provides the highest utility level,

\[
 h_i = \{h(j) \text{ such that } u_i(j) \geq u_i(k) \ \forall k\}.
\]

(12)

For each individual, let us denote by \( Q_i(j) \) the set of vectors \( q_i \) such that working hours \( h(j) \) are optimal for the individual. That is,

\[
 Q_i(j) = \{q_i \text{ such that } u_i(j) \geq u_i(k) \ \forall k\}.
\]

(13)
The measure of $Q_i(j)$ w.r.t. the distribution $P_i(q_i)$ is denoted by $P_i(j)$ which is the probability that individual $i$ chooses $h(j)$ hours of work. This implies that the expected labor supply of individual $i$ may be written as

$$L_i = \sum_{j=0}^{N} P_i(j) h(j).$$

The number of hours at each state is exogenous and all the variation in labor supply occurs therefore through changes in the probabilities $P_i(j)$. These probabilities are endogenous and depend on the consumption possibilities at the different states. Notice that the probability distribution $P_i(j)$ satisfies the following properties

$$\sum_{j=0}^{N} P_i(j) = 1, \quad (14)$$

$$\sum_{j=0}^{N} \frac{\partial P_i(k)}{\partial c_i(j)} = 0, \quad (15)$$

where we have used eq. (13). The first relationship is a simple identity. The second relationship states that the individual probability distribution is unaffected by a lump sum transfer or, equivalently, that there are no income effects on labor supply. Since the probability distribution is then independent of the overall consumption level, the probability of being in a given state depends only on the differences in consumption levels between the current state and the other states.

Without further restrictions, the problem becomes intractable because an individual may jump between any two states following a (small) tax reform. We therefore follow Saez (2002) by assuming that intensive responses from a marginal reform will take place only across adjacent hours of work, i.e., from $h(j)$ to $h(j-1)$ or $h(j+1)$. Thus, we assume that a marginal change in the consumption level at $h(j)$ hours relative to the consumption level at $h(k)$ hours will not induce the individual to change behavior unless the two states are adjacent, i.e.

$$\frac{\partial P_i(j)}{\partial (c_i(j) - c_i(k))} = 0 \text{ for } k = 1, \ldots, j-2, j+2, \ldots, N \text{ and } j = 1, \ldots, N-1, \quad (16)$$

$$\frac{\partial P_i(N)}{\partial (c_i(N) - c_i(k))} = 0 \text{ unless } k = 0, N-1. \quad (17)$$

Notice that this assumption does not rule out the possibility that an individual may jump from some given state to state 0. We do not want to exclude this behavior because adjustments in
labor supply along the extensive margin often occur from a significant number of hours to zero hours.

A sufficient condition for the behavioral assumption in (16)-(17) to be optimal would be if the average tax rate $T(w_i h(j)) / (w_i h(j))$ is non-decreasing in earnings $w_i h(j)$, and the work-cost per hour $q(j) / h(j)$ is U-shaped in the number of hours (corresponding to a standard average cost curve). In this case, it may be that an individual working $h(j)$ hours can obtain the same utility level at $h(j-1)$, $h(j+1)$ or at $h(0)$ hours, but never at, say, $h(j-2)$ hours. Since marginal changes in the consumption levels can only induce a movement between two states if their initial utility levels are identical, it may only be optimal for the individual to jump to one of the neighbor states or to the non-working state following a small tax reform.

For those states which individual $i$ may move between, it must be the case that

\[
\frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j-1))} = -\frac{\partial P_i(j-1)}{\partial (c_i(j) - c_i(j-1))}, \tag{18}
\]

\[
\frac{\partial P_i(j)}{\partial (c_i(j) - c_i(0))} = -\frac{\partial P_i(0)}{\partial (c_i(j) - c_i(0))}. \tag{19}
\]

These two identities imply that if the probability of being in a given state increases because that state has become economically more attractive relative to another state, then the probability increase is matched by a reduction in the probability of being in the other state.

In order to measure the labor supply responsiveness to economic incentives, we define an intensive elasticity, $\varepsilon_{ij}$, and an extensive elasticity, $\eta_{ij}$, for each individual at each state: 

\[
\varepsilon_{ij} = \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j-1))} \frac{c_i(j) - c_i(j-1)}{P_i(j)} \frac{h(j) - h(j-1)}{h(j)}, \tag{20}
\]

\[
\eta_{ij} = \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(0))} \frac{c_i(j) - c_i(0)}{P_i(j)}. \tag{21}
\]

The intensive elasticity measures the expected percentage change in working hours following a one percent increase in the consumption difference between two neighbor states. Since the number of hours is exogenous at each state, the intensive elasticity also measures the expected percentage change in working hours following a one percent increase in the (average) hourly net-wage. The extensive elasticity measures the percentage change in the probability of labor market participation at state $j$ following a one percent increase in the consumption difference between working in state $j$ and staying out of the labor market.

Notice that the movement from $h(1)$ to $h(0)$ hours can be interpreted as either an extensive
or an intensive response. Adopting the convention that it is an extensive response, we have that 
ε_{i1} = 0 by definition.

### 3.2 Tax Reform and Welfare

Like the first model, the excess burden of taxation is defined as \( D = U_0 - U - R \), where \( U \) and \( U_0 \) are aggregate utilitarian welfare with and without taxation, respectively, while \( R \) denotes aggregate government revenue. Using eq. (11), the (expected) aggregate utilitarian welfare with taxation may be written as

\[
U = \sum_{i=0}^{I} \sum_{j=0}^{N} P_i (j) u_i (j) = \sum_{i=0}^{I} \sum_{j=0}^{N} P_i (j) [w_i h (j) - T (w_i h (j)) - q_i (j)], \tag{22}
\]

while the aggregate government revenue becomes

\[
R = \sum_{i=0}^{I} \sum_{j=0}^{N} P_i (j) T (w_i h (j)). \tag{23}
\]

We consider a small tax reform which creates marginal changes in the tax function at the different states, \( \{dT (w_i h (j))\}_{i,j} \). The change in the excess burden becomes \( dD = -dU - dR \). From eq. (22), the change in aggregate utilitarian welfare may be written as

\[
dU = \sum_{i=0}^{I} \sum_{j=0}^{N} [dP_i (j) u_i (j) + P_i (j) du_i (j)].
\]

The first term in the bracket vanishes because of the envelope theorem. A marginal change in the tax system may only give rise to a jump from the current state to another state if the initial utility levels are the same at the two states. Hence, if the term \( dP_i (j) u_i (j) \) has some given positive value at one state then there will be another state with a corresponding negative value. Therefore,

\[
dU = - \sum_{i=0}^{I} \sum_{j=0}^{N} P_i (j) dT_i (j), \tag{24}
\]

where we have used eq. (22) and the definition \( T_i (j) \equiv T (w_i h (j)) \).

From eq. (23), we obtain the change in government revenue following a small reform

\[
dR = \sum_{i=0}^{I} \sum_{j=0}^{N} [dP_i (j) T_i (j) + P_i (j) dT_i (j)]. \tag{25}
\]
We may now obtain the change in excess burden by combining eqs (24) and (25). This gives

\[ dD = -dU - dR = -\sum_{i=0}^{I} \sum_{j=0}^{N} dP_i (j) T_i (j). \]

This expression reflects the general insight that changes in deadweight burden are given by the effects on government revenue from behavioral responses. In this model, the behavioral responses are reflected in the changes in the individual probability distributions. Each agent is, ceteris paribus, more likely to be at a state which is favored by the tax reform relative to the other states.

By deriving the changes in the probability distribution from the tax reform, the above relationship may be rewritten to

\[ dD = \sum_{i=0}^{I} \sum_{j=0}^{N} \left[ \sum_{k \neq j} \frac{\partial P_i (j)}{\partial (c_i (j) - c_i (k))} \cdot d (T_i (j) - T_i (k)) \right] T_i (j), \]

where we have used the definition of consumption and the fact that the probability of being at a given state only depends on the differences in consumption levels.

We define \( m_{ij} \) and \( a_{ij} \) as the marginal tax rate and the participation tax rate, respectively, of individual \( i \) at state \( j \):

\[ m_{ij} = \frac{T_i (j) - T_i (j - 1)}{w_i h (j) - w_i h (j - 1)}, \quad a_{ij} = \frac{T_i (j) - T_i (0)}{w_i h (j)}, \]

(28)

where the changes in the tax rates are defined as

\[ dm_{ij} = \frac{d (T_i (j) - T_i (j - 1))}{w_i h (j) - w_i h (j - 1)}, \quad da_{ij} = \frac{d (T_i (j) - T_i (0))}{w_i h (j)}. \]

(29)

Using these definitions, the behavioral assumptions in (16) and (17), the properties of the probability distribution in (18) and (19), and the labor supply elasticities in (20) and (21), we may rewrite eq. (27) to (see appendix)

\[ \frac{dD}{W} = \sum_{i=0}^{I} \sum_{j=1}^{N} \left[ \frac{m_{ij}}{1 - m_{ij}} \cdot dm_{ij} \cdot \varepsilon_{ij} + \frac{a_{ij}}{1 - a_{ij}} \cdot da_{ij} \cdot \eta_{ij} \right] s_{ij}, \]

(30)

where \( W \equiv \sum_{i=0}^{I} \sum_{j=1}^{N} P_i (j) w_i h (j) \) is the (expected) aggregate wage income while \( s_{ij} \equiv [P_i (j) w_i h (j)] / W \) are wage shares, defined as the earnings of individual \( i \) at state \( j \) relative to aggregate wage income.
Notice the similarity between the above result and the result (9) of the previous model. The main difference between these two results is that the first model assumes that an individual with a given set of characteristics always works a certain number of hours while the second model assumes that the individual may work a discrete number of hours according to some probability distribution. This implies that the last model may provide more reliable empirical results if there is a lot of variation in hours within groups of individuals with the same observable characteristics.

4 Derivation of eq. (30)

From eq. (27), we have that
\[
\frac{dD}{dD_i} = \sum_{j=0}^{N} \left[ \sum_{k \neq j} \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(k))} \cdot d(T_i(j) - T_i(k)) \right] T_i(j).
\]

This expression may be simplified using the assumption in (16)-(17). In order to do so, we need to account for the fact that the states 0, 1 and \( N \) are different from the other states. In states \( j = 2, 3, ..., N - 1 \), the labor supply response will consist of three terms: intensive responses from \( j \) to \( (j - 1) \) and from \( j \) to \( (j + 1) \) as well as an extensive response from \( j \) to 0. By contrast, for state 0 there will be \( N \) extensive responses (0 to 1, 2, ..., \( N \)). Next, state 1 is special because the intensive response 1 to 0 is also the extensive response. In other words, for state 1 the response will have only two terms (from 1 to 0 and from 1 to 2). Likewise, for state \( N \) the response will have only two terms, because individuals cannot increase hours here. These considerations imply
\[
\frac{dD}{dD_i} = \sum_{k=1}^{N} \frac{\partial P_i(0)}{\partial (c_i(0) - c_i(k))} \cdot d(T_i(0) - T_i(k)) \cdot T_i(0)
+ \left[ \frac{\partial P_i(1)}{\partial (c_i(1) - c_i(0))} \cdot d(T_i(1) - T_i(0)) + \frac{\partial P_i(1)}{\partial (c_i(1) - c_i(2))} \cdot d(T_i(1) - T_i(2)) \right] T_i(1)
+ \sum_{j=2}^{N-1} \left[ \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j-1))} \cdot d(T_i(j) - T_i(j-1)) 
+ \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j+1))} \cdot d(T_i(j) - T_i(j+1)) + \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(0))} \cdot d(T_i(j) - T_i(0)) \right] T_i(j)
+ \left[ \frac{\partial P_i(N)}{\partial (c_i(N) - c_i(N-1))} \cdot d(T_i(N) - T_i(N-1)) 
+ \frac{\partial P_i(N)}{\partial (c_i(N) - c_i(0))} \cdot d(T_i(N) - T_i(0)) \right] T_i(N).
\]
Using the properties in (18) and (19), the above expression may be rewritten to

\[ dD_i = \sum_{k=1}^{N} \frac{\partial P_i(k)}{\partial (c_i(k) - c_i(0))} \cdot d(T_i(0) - T_i(k)) \cdot T_i(0) \]
\[ + \left[ \frac{\partial P_i(1)}{\partial (c_i(1) - c_i(0))} \cdot d(T_i(1) - T_i(0)) + \frac{\partial P_i(2)}{\partial (c_i(2) - c_i(1))} \cdot d(T_i(1) - T_i(2)) \right] T_i(1) \]
\[ + \sum_{j=2}^{N-1} \left[ \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j-1))} \cdot d(T_i(j) - T_i(j-1)) \right] T_i(j) \]
\[ + \left[ \frac{\partial P_i(j+1)}{\partial (c_i(j+1) - c_i(j))} \cdot d(T_i(j+1) - T_i(j)) \right] T_i(j) \]
\[ + \left[ \frac{\partial P_i(N)}{\partial (c_i(N) - c_i(0))} \cdot d(T_i(N) - T_i(0)) \right] T_i(N). \]

After changing the index from \( k \) to \( j \) in the first summation and collecting all the terms involving extensive responses, we have

\[ dD_i = \sum_{j=1}^{N} \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(0))} \cdot d(T_i(j) - T_i(0)) \cdot [T_i(j) - T_i(0)] \]
\[ + \frac{\partial P_i(2)}{\partial (c_i(2) - c_i(1))} \cdot d(T_i(1) - T_i(2)) \cdot T_i(1) \]
\[ + \sum_{j=2}^{N-1} \left[ \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j-1))} \cdot d(T_i(j) - T_i(j-1)) \right] T_i(j) \]
\[ + \left[ \frac{\partial P_i(j+1)}{\partial (c_i(j+1) - c_i(j))} \cdot d(T_i(j+1) - T_i(j)) \right] T_i(j) \]
\[ + \frac{\partial P_i(N)}{\partial (c_i(N) - c_i(0))} \cdot d(T_i(N) - T_i(0)) \cdot T_i(N). \]

Next, we rewrite the terms in the second summation:

\[ dD_i = \sum_{j=1}^{N} \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(0))} \cdot d(T_i(j) - T_i(0)) \cdot [T_i(j) - T_i(0)] \]
\[ + \frac{\partial P_i(2)}{\partial (c_i(2) - c_i(1))} \cdot d(T_i(1) - T_i(2)) \cdot T_i(1) \]
\[ + \sum_{j=2}^{N-1} \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j-1))} \cdot d(T_i(j) - T_i(j-1)) \cdot T_i(j) \]
\[ + \sum_{j=3}^{N} \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j-1))} \cdot d(T_i(j-1) - T_i(j)) \cdot T_i(j-1) \]
\[ + \frac{\partial P_i(N)}{\partial (c_i(N) - c_i(0))} \cdot d(T_i(N) - T_i(N-1)) \cdot T_i(N). \]
After collecting the terms involving intensive responses, we obtain

\[
dD_i = \sum_{j=1}^{N} \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(0))} \cdot d(T_i(j) - T_i(0)) \cdot [T_i(j) - T_i(0)]
\]

\[
+ \sum_{j=2}^{N} \frac{\partial P_i(j)}{\partial (c_i(j) - c_i(j - 1))} \cdot d(T_i(j) - T_i(j - 1)) \cdot [T_i(j) - T_i(j - 1)].
\]

We now substitute the labor supply elasticities in (20) and (21) into the above expression. This gives

\[
dD_i = \sum_{j=1}^{N} \eta_{ij} \cdot \frac{d(T_i(j) - T_i(0))}{w_j h(j)} \cdot \frac{T_i(j) - T_i(0)}{c_i(j) - c_i(0)} \cdot P_i(j) w_j h(j) +
\]

\[
\sum_{j=2}^{N} \varepsilon_{ij} \cdot \frac{d(T_i(j) - T_i(j - 1))}{w_j h(j) - w_j h(j - 1)} \cdot \frac{T_i(j) - T_i(j - 1)}{c_i(j) - c_i(j - 1)} \cdot P_i(j) w_j h(j).
\]

Insertion of the consumption levels and the tax rate definitions in (28) and (29) gives

\[
dD_i = \sum_{j=1}^{N} \left[ \frac{a_{ij}}{1 - a_{ij}} \cdot da_{ij} \cdot \eta_{ij} + \frac{m_{ij}}{1 - m_{ij}} \cdot dm_{ij} \cdot \varepsilon_{ij} \right] P_i(j) w_i h(j),
\]

where we have used the fact that \( \varepsilon_{i0} = 0 \). Finally, eq. (30) may be derived by aggregating the above expression over all individuals.

References


