Fixed Production Capacity, Menu Cost and the Output-Inflation Relationship*

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Abstract

This paper analyzes the impact of inflation when firms face frictions in both price and quantity adjustments. A vast literature examines the consequences of price-adjustment costs assuming frictionless quantity adjustments. However, temporary quantity adjustments may very well be expensive, for example because continual adjustments of the optimal production plant are impossible. Moreover, recent findings suggest that frictions in quantity adjustment may remove the linkage between output and inflation. In this paper we show that this is not the case when inflation is anticipated. On the contrary, a predetermined production capacity may significantly amplify the consequences of price-adjustment costs.

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1 Introduction

It is by now well established that small fixed price-adjustment costs, the so-called menu costs, may cause nominal changes to have large real effects.[1] This result is derived assuming that quantities can be adjusted costlessly. Empirical evidence shows that menu costs are not trivial (Levy et al., 1997; Dutta et al., 1999), but there is also evidence that points to large fixed quantity-adjustment costs (Bresnahan and Ramey, 1994). For instance, a firm may have to commit to a given production plant which may make it very costly to adjust output in response to changes in demand. This would give rise to fixed costs that are not altered over the price cycle, contrary to the presumption in the menu cost literature.

An interesting question is whether the existence of non-trivial quantity-adjustment costs invalidates the menu-cost results. Andersen (1994 ch. 5, 1995) addresses this issue in a model with linear demand and cost functions. He shows that following a nominal disturbance, a fixed quantity-adjustment cost larger than the fixed price-adjustment cost is enough to keep output unchanged. Since Andersen considers "Knightian" uncertainty (i.e., a shock occurs although the agents are completely sure that it will not happen), the unchanged level of output is identical to what would be produced under complete certainty. Hence, Andersen's result indicates that output becomes independent of inflation when quantity-adjustment costs are sufficiently large.

In this paper, we follow another strand of the literature, see Sheshinski and Weiss (1977), Kuran (1986), Naish (1986), Danziger (1988), Konieczny (1990), and Bénabou and Konieczny (1994), by analyzing the case where there is no uncertainty, but a fully anticipated, constant rate of inflation. We consider the case where the firm has to commit to a given production plant which puts an upper limit on production and gives rise to fixed production costs. The underlying assumption is that it is too costly to adjust the production plant during a price cycle.[2] For

[1] See Akerlof and Yellen (1985), Mankiw (1985), Parkin (1986), Blanchard and Kiyotaki (1987) and Ball and Romer (1989, 1990).

[2] Similarly, in Danziger (2001) the existence of a fixed quantity-adjustment cost forces the firm to choose a constant permanent level of production. In Fluet and Phaneuf (1997) random demand shocks influence a firm's choice of technique.

tractability, we assume in addition that there is no discounting, a constant elasticity of demand, and linear production costs. Under these assumptions, we show that some of the results obtained in the menu-cost literature remain valid with a fixed capacity. Thus, the higher the rate of inflation, the higher is the initial real price and the lower is the terminal real price (Sheshinski and Weiss, 1977). Furthermore, the higher the rate of inflation, the lower is the average output (Kuran, 1986; Naish, 1986).[3]

The market alternates over time between a Keynesian regime, where production is determined by demand, and a Classical regime, where production is determined by supply. We show that the relative time spent in the Keynesian regime rises with the firm's monopoly power.

We also use the model to gauge the quantitative importance of frictions in quantity adjustment by comparing the size of the output loss caused by inflation with and without a fixed output capacity. For realistic values of the menu cost, the model suggests that the loss of output due to inflation is several times larger with a fixed output capacity than without. Thus, far from invalidating the previous finding of a negative output-inflation relationship, the introduction of a fixed output capacity amplifies the negative consequences of a price-adjustment cost.

2 The Model

We consider a monopolistic firm that produces a non-storable good and has a time-invariant demand function $z_t^{-\alpha}$, where z_t is the real price of the good at time t and $\alpha > 1$. The firm faces a fixed price-adjustment cost $c > 0$, implying that the price will not be adjusted continuously. The firm chooses a fixed production capacity, $\tilde{Y}$, which is an upper bound on the size of the production. Due to the high adjustment

[3]The negative relationship between average output and inflation in Kuran (1986) and Naish (1986) depends on the absence of discounting and the specific functional assumptions. Danziger (1988) shows that with discounting, small inflation rates lift average output above the static monopoly output. Konieczny (1990) and Benabou and Konieczny (1994) consider general profit and demand functions, and show that the relationship between average output and inflation depends on the shapes of these functions. Furthermore, Benabou and Konieczny provide a complete characterization for small inflation rates and small menu costs, and they give examples where the relationship between average output and inflation is positive.

cost involved, it is never profitable for the firm to change its capacity.[4]

The choice of the production capacity gives rise to a fixed cost $k\tilde{Y}$ at each point in time, where $k > 0$ is the real cost per unit of capacity. However, we also assume that the firm produces only what it can sell, that is, the actual output is $\min\left(z_t^{-\alpha}, \tilde{Y}\right)$. To simplify matters we assume that there is no variable cost of production. Later, in Section 4, we relax this assumption.

The presence of a price-adjustment cost together with a fixed capacity give rise to rationing of either the firm or the consumers. Thus, the firm may be in a Keynesian regime where output is determined by demand, $\tilde{Y} > z_t^{-\alpha}$, or in a Classical regime where output is determined by supply, $\tilde{Y} < z_t^{-\alpha}$. Let $\tilde{z}$ be the real price at which demand exactly equals the production capacity, that is $\tilde{z} \equiv \tilde{Y}^{-1/\alpha}$. The real profit of the firm at time t is $\Pi\left(z_t, \tilde{Y}\right) \equiv z_t \min\left(z_t^{-\alpha}, \tilde{Y}\right) - k\tilde{Y}$.

< Figure 1 >

The upper curve in Figure 1 illustrates the real profit as function of the real price if the production capacity could be adjusted costlessly. In this case the real profit is $z_t^{1-\alpha} - kz_t^{-\alpha}$, which is maximized for the real price $\hat{z} \equiv \frac{\alpha k}{\alpha-1}$ and the quantity $\hat{Y} \equiv \left(\frac{\alpha k}{\alpha-1}\right)^{-\alpha}$. The lower curve shows the real profit when the firm fixes its production capacity at $\tilde{Y}$. Profits are identical only at the real price $\tilde{z}$, which yields the maximum of $\Pi\left(z_t, \tilde{Y}\right)$ for the capacity level $\tilde{Y}$, and at the real price k, which yields a real profit of zero in both cases. Otherwise real profits are always lower with the fixed production capacity. The real profit becomes zero for a firm with a fixed capacity when $z_t = \left(k\tilde{Y}\right)^{\frac{1}{1-\alpha}}$, whereas the real profit with a variable production capacity is positive as long as $z_t > k$.

[4] A sufficient condition is that the cost of adjusting the capacity is at least as high as the price-adjustment cost. For the case of a fixed cost of quantity adjustment, Danziger (2001) argues that this can be seen by assuming "that the firm incurs only a single adjustment cost if the price and quantity are adjusted simultaneously. It is then optimal for the firm to choose the same quantity at each adjustment, that is, the firm adjusts the price but never the quantity. Since it would have been costless to also adjust the quantity, it follows that with a separate cost of quantity adjustment at least equal to the cost of price adjustment, the firm chooses to adjust only its price and never its quantity." The present case of a cost of adjusting the capacity is identical to the case of a fixed cost of quantity adjustment if there is no variable cost of production. A positive variable cost of production makes an adjustment of the capacity even less attractive.

The general price level increases at the constant rate $\mu > 0$. Due to the price-adjustment cost, the firm keeps its nominal price unchanged for a fixed period of time denoted by T, and then increases it to a new level. The initial real price at the beginning of a period with a constant nominal price is denoted by S. After $\tau \geq 0$ of the period has elapsed, the real price has been reduced to $z_\tau = Se^{-\mu\tau}$, and as τ tends to T, the real price converges to the terminal real price $s \equiv Se^{-\mu T}$. The length of time from the beginning of the period until the demand equals the firm's output capacity is denoted by $\tilde{T}$; that is, $\tilde{z} = Se^{-\mu\tilde{T}} \Leftrightarrow \tilde{T} = \frac{1}{\mu}\ln\frac{S}{\tilde{z}}$.[5]

The firm's average real profit over a period with a constant nominal price is given by

$$V \equiv \frac{1}{T}\left[\int_0^T \Pi\left(Se^{-\mu\tau}, \tilde{Y}\right) d\tau - c\right],$$

which, after substituting for $\Pi(\cdot)$, gives

$$V = \frac{1}{T}\left[\int_0^{\tilde{T}} \left(Se^{-\mu\tau}\right)^{1-\alpha} d\tau + \int_{\tilde{T}}^{T} Se^{-\mu\tau}\tilde{Y} d\tau - c\right] - k\tilde{Y},$$

where the first integral is the revenue in the first part of the period where the firm is rationed by consumer demand, and the second integral is the revenue in the second part of the period where the firm rations its customers. Integrating and substituting for T and $\tilde{T}$ yield

$$V = \frac{1}{\ln(S/s)}\left(\frac{\alpha\tilde{Y}^{1-1/\alpha} - S^{1-\alpha}}{\alpha - 1} - s\tilde{Y} - \mu c\right) - k\tilde{Y}. \tag{1}$$

The firm chooses S, s, and $\tilde{Y}$ in order to maximize V. The first-order conditions are

$$\frac{\partial V}{\partial S} = \frac{1}{S\ln(S/s)}\left(-V + S^{1-\alpha} - k\tilde{Y}\right) = 0, \tag{2}$$

$$\frac{\partial V}{\partial s} = \frac{1}{s\ln(S/s)}\left[V - (s-k)\tilde{Y}\right] = 0, \tag{3}$$

$$\frac{\partial V}{\partial \tilde{Y}} = \frac{\tilde{Y}^{-1/\alpha} - s}{\ln(S/s)} - k = 0. \tag{4}$$

[5] Here we presume that $\tilde{z} \in (s, S)$, or equivalently, that $\tilde{T} \in (0, T)$. Theorem 1 shows that the solution satisfies this condition.

The first two conditions are standard and state that the real profits in the beginning and end of a period with a constant nominal price equal the average real profit over the period. If the real profit in the beginning of a period would exceed (be less than) the average real profit, the firm could increase the average real profit by increasing (decreasing) the initial real price. Similarly, if the real profit in the end of a period would exceed (be less than) the average real profit, the firm could increase the average real profit by decreasing (increasing) the terminal real price. The third condition is due to the fixed production capacity and may be rewritten as

$$\frac{\tilde{z} - s}{\ln(\tilde{z}/s)}(T - \tilde{T}) = kT,$$

where the left-hand side is the marginal gain whereas the right-hand side is the marginal loss of raising output capacity. A higher output capacity raises revenue, but only in the part of the period where the firm is not constrained by consumer demand, that is, only in the Classical regime. On the other hand, the cost of the increased capacity has to be paid throughout the period, that is, both in the Classical and Keynesian regimes.

3 The Impact of Inflation

If there were no fixed cost of price-adjustment, the nominal price would be adjusted continuously at the rate of inflation. The real price and the output would always be at their profit-maximizing levels, $\hat{z}$ and $\hat{Y}$. The following theorem characterizes the firm's optimal strategy with a price-adjustment cost and a fixed production capacity, and compares the strategy to the benchmark case without a price-adjustment cost:

Theorem 1 (i) $\frac{\tilde{T}}{T} = \frac{1}{\alpha}$.
(ii) $s < \hat{z} < \tilde{z} < S$ *and* $\tilde{Y} < \hat{Y}$.
(iii) $\frac{dS}{d\mu} > 0$, $\frac{ds}{d\mu} < 0$, $\frac{d\tilde{z}}{d\mu} > 0$ *and* $\frac{d\tilde{Y}}{d\mu} < 0$.

Proof. See the appendix.

Thus, the fraction of time spend in the Keynesian regime equals the inverse of the absolute value of the elasticity of demand, which is the degree of monopoly power as measured by the Lerner index. A higher degree of monopoly power makes it profitable to raise the price. Hence, both the initial and terminal real price increase, which for a given capacity (and therefore a given $\tilde{z}$) increase the relative time spent in the Keynesian regime. Furthermore, the higher real price in the Classical regime increases the marginal revenue from expanding the output capacity, which by itself tends to increase the relative time in the Keynesian regime. Although the longer relative time in the Keynesian regime tends to reduce the gains from expanding the capacity, and therefore pulls in the opposite direction, the overall result is that a higher degree of monopoly power increases the relative time in the Keynesian regime.

It is quite intuitive that the real price exceeds its profit-maximizing level in the beginning of a period with a constant nominal price, i.e., that $\hat{z} < S$, and that the real price is less than its profit-maximizing level in the end of a period, i.e., that $s < \hat{z}$. Furthermore, the higher the inflation rate, the higher is the initial real price and the lower is the terminal real price. These results are identical to what is found in models with a price-adjustment cost, but no constraint on the output capacity (see Sheshinski and Weiss, 1977).

The fixed output capacity, $\tilde{Y}$, and therefore also the actual output during the entire period, are below the profit-maximizing output level if there were no price-adjustment cost, $\hat{Y}$. Moreover, the output capacity decreases with the rate of inflation.[6] When the capacity is fixed, the increase in the initial real price reduces profits to a large extent in the beginning of a period with a constant nominal price, because the firm cannot accommodate the reduction in demand in the beginning of a period by reducing its production capacity and thereby its costs. In isolation, this effect tends to make the firm reduce the output capacity. However, the lowering of the terminal real price has a large detrimental effect on the profits in the end of a

[6] In the case of deflation, the output capacity is also less than $\hat{Y}$ and decreases with deflation. To grasp the intuition for this, note that with deflation in Figure 1 the firm starts with a low real price and ends with a high real price, i.e., moves in the opposite direction than in the case of inflation. However, since there is no discounting, it does not matter for the firm whether it starts with the high or low real price. Accordingly, if the general price level would decrease at the rate of $-\mu < 0$, the initial (terminal) real price would equal the terminal (initial) real price for the case where the general price level increases at the rate of μ, and the output capacity would be the same as in the case where the general price level increases at the rate of μ.

period, since the firm is unable to satisfy the extra demand. This effect tends to make the firm increase the capacity. Theorem 1 shows that in the case of constant-elasticity demand and constant real unit cost, the first effect dominates and it is optimal for the firm to reduce the fixed output capacity. Consequently, the capacity falls with inflation.

Following previous studies, our main interest is the relationship between inflation and the average output defined as

$$\bar{Y} \equiv \frac{1}{T}\left[\int_0^{\tilde{T}} \left(Se^{-\mu\tau}\right)^{-\alpha} d\tau + \tilde{Y}\left(T - \tilde{T}\right)\right],$$

where the first term in the bracket is the output sold in the first part of a period where the firm sells less than its capacity, whereas the second term is the output sold in the second part of a period where the firm sells its entire production capacity.

Integrating and substituting $\tilde{T} = \frac{1}{\mu}\ln\frac{S}{\tilde{z}}$ yield

$$\bar{Y} = \tilde{Y}\frac{1-\left(\frac{S}{\tilde{z}}\right)^{-\alpha}}{\alpha\mu T} + \tilde{Y}\left(1 - \frac{\ln\frac{S}{\tilde{z}}}{\mu T}\right). \tag{5}$$

Further substituting $S/\tilde{z} = (S/s)^{1/\alpha}$, which is derived from conditions (2) and (3), and $\mu T = \ln(S/s)$ then yield

$$\bar{Y} \equiv \frac{\tilde{Y}}{\alpha}\left(\alpha - 1 + \frac{1-\frac{s}{S}}{\ln\frac{S}{s}}\right).$$

We can now establish

Theorem 2 $\frac{d\bar{Y}/\bar{Y}}{d\mu/\mu} < \frac{d\tilde{Y}/\tilde{Y}}{d\mu/\mu} < 0.$

Proof. We only need to show that the last term inside the parenthesis in the expression for $\bar{Y}$ is decreasing in μ. Since we know from Theorem 1 that S/s is increasing in μ, this is equivalent to showing that this last term is decreasing in S/s. However, this is true since its derivative with respect to S/s has the same sign as $1 - \frac{S}{s} + \ln\frac{S}{s}$, and the latter decreases in $\frac{S}{s}$ and approaches 0 for $\frac{S}{s} \to 1$. $\square$

Hence, not only does the average output decrease with the rate of inflation, as does the production capacity, the negative effect of the rate of inflation on the average output is proportionally larger than on the output capacity. To understand this, note from result (i) in Theorem 1 that the firm spends the same relative time in the Keynesian and Classical regimes for all inflation rates. Since output in the Classical regime falls with the same amount as the production capacity, Theorem 2 implies that the rise in inflation reduces the average output in the Keynesian regime proportionally more than the production capacity. This occurs because the higher inflation makes the firm increase the initial price (cf. Theorem 1), thereby reducing the demand at all points in time in the Keynesian regime.

In conclusion, the presence of a quantity-adjustment cost does not alter the result that with constant-elasticity demand and constant real unit cost, inflation reduces output (see Kuran, 1986; Naish, 1986).

4 Simulations with a Generalized Model

Until now we have assumed that all production costs are fixed. This implies, for example, that the firm cannot reduce its costs by lowering the production below its capacity. However, it is often the case that firms can save on raw materials and other inputs by lowering production. Therefore, when we in this section calibrate the quantitative importance of the capacity constraint on the output-inflation relationship, we use a generalized framework which permits variable costs of production. Moreover, the framework enables us to compare the results to the standard model with no constrains on quantity adjustments as that model arises as a special case of the generalized model (when there are only variable costs of production).

Like before, it is assumed that the firm chooses a given production capacity $\tilde{Y}$, which determines an upper limit to the size of the production. The capacity gives rise to a fixed cost equal to $k_F \geq 0$ per unit of capacity. In addition, the firm faces a variable cost of production equal to $k_V \geq 0$ per unit of production. The main difference between these two types of costs is that the fixed cost has to be paid independent of the size of production, while the firm is able to reduce the variable cost by producing less than the capacity. It is assumed that $k_F + k_V > 0$.

The firm's average profit over a period with a constant nominal price is now given by

$$
\begin{aligned}
V &= \frac{1}{T}\int_0^{\tilde{T}}\left[\left(Se^{-\mu\tau}\right)^{1-\alpha} - k_F\tilde{Y} - k_V\left(Se^{-\mu\tau}\right)^{-\alpha}\right]d\tau \\
&\quad + \frac{1}{T}\int_{\tilde{T}}^{T}\left[Se^{-\mu\tau} - k_F - k_V\right]\tilde{Y}d\tau - \frac{c}{T}.
\end{aligned}
$$

Integrating and substituting for T and $\tilde{T}$ yield

$$
V = \frac{1}{\ln(S/s)}\left[\frac{\alpha\tilde{Y}^{1-1/\alpha} - S^{1-\alpha}}{\alpha-1} + \beta k\frac{\tilde{Y}\left(\alpha\ln\frac{S}{\tilde{z}} - 1\right) + S^{-\alpha}}{\alpha} - s\tilde{Y} - \mu c\right] - k\tilde{Y},
$$

where $k \equiv k_F + k_V$ is total unit cost of production and $\beta \equiv k_V/k = k_V/\left(k_F + k_V\right)$ is the share of variable cost in total cost. This expression for the firm's average profit is identical to (1) if $\beta = 0$. The first-order conditions now become

$$
\begin{aligned}
\frac{\partial V}{\partial S} &= \frac{1}{S\ln(S/s)}\left[-V + S^{1-\alpha} - k\tilde{Y} + \beta k\left(\tilde{Y} - S^{-\alpha}\right)\right] = 0, && (6)\\
\frac{\partial V}{\partial s} &= \frac{1}{s\ln(S/s)}\left[V - (s-k)\tilde{Y}\right] = 0, && (7)\\
\frac{\partial V}{\partial \tilde{Y}} &= \frac{\tilde{Y}^{-1/\alpha} - s + \beta k\left(\ln S + \frac{1}{\alpha}\ln\tilde{Y}\right)}{\ln(S/s)} - k = 0. && (8)
\end{aligned}
$$

Using the solution to these conditions it is again possible to derive the average quantity from equation (5).

Insert $\tilde{Y} = \tilde{z}^{-\alpha}$ in condition (8) to obtain that

$$
\tilde{z} - s + \beta k\ln\frac{S}{\tilde{z}} = k\ln\frac{S}{s},
$$

according to which $\tilde{z} \to s$ as $\beta \to 1$. By setting $\tilde{z}$ equal to s in the two other first-order conditions it is clear that the solution converges to the result of the standard model without a fixed capacity. Intuitively, when $\beta \to 1$ the cost of capacity become negligible and the firm therefore chooses a sufficiently high capacity level that it never rations its customers. This implies that the firm is always in the Keynesian regime and the solution replicates that of the standard model which only features variable costs of production.

We are now able to simulate the output-inflation relationship using conditions (6)-(8) and equation (5). In doing so, we measure the loss of output as a proportion of the frictionless output level, i.e. $1 - \bar{Y}/\hat{Y}$. Moreover, by manipulating the expressions it is possible to show that the output loss is uniquely determined from knowledge of (the absolute value of) the demand elasticity, α, and of $\mu\psi$, where $\psi \equiv c/(\hat{z}\hat{Y})$ is the menu cost as a proportion of the firm's frictionless revenue.

In Figure 2 we illustrate the results for $\alpha = 5$ and $\psi = 0.7\%$, which is the menu-cost estimate given by Levy et al. (1997). The lower solid curve shows the output loss without any fixed cost of production, corresponding to the standard case without a fixed capacity. The curve confirms the conclusions of Kuran (1986) and Naish (1986): the loss of output without a capacity constraint is non-negligible, although not large for moderate inflation rates, and increases with inflation. For example, the loss is 0.9% for an inflation rate of 5%, and 2.5% for an inflation rate of 25%. The upper solid curve shows the loss of output without any variable cost of production, corresponding to our analysis in Section 3. The loss in this situation is several times higher: the loss of output is 6.2% for an inflation rate of 5% and 13.9% for an inflation rate of 25%. The dashed curves illustrate intermediate cases with both fixed and variable costs of production. The most interesting conclusion from these simulations is that the quantitative impact of the capacity constraint is significant even when the fixed cost is only a minor fraction of total cost. For instance, even if the variable cost is 99% of total cost, the quantitative impact of the capacity constraint is a doubling of the output loss for an inflation rate of 5%. When the variable-cost share is lowered, the output loss increases fast. As is clear from Figure 2, the case of fifty-fifty cost spliting resembles closely the case of only fixed costs of production. Thus, a predetermined production capacity significantly enlarges the negative consequences of a price-adjustment cost.

< Figure 2 >

Figure 2 can also be used for other sizes of the menu cost, since the losses depend on only $\mu\psi$ for a given α. Simultaneously changing the menu cost by a factor of $\gamma > 0$ and the inflation rate by a factor of $1/\gamma$ leaves the loss of output unchanged. In terms of Figure 2, halving the menu cost is equivalent to rescaling the horizontal axis by doubling all inflation rates. Thus, for a menu cost equal to 0.35% and an inflation rate equal to 10%, the loss of output is 5.9% for $\beta = 1$ and only 0.9% for $\beta = 0$.

To examine whether our results are sensitive to the choice of α, we have calculated the output loss also for $\alpha = 2$ and $\alpha = 8$. As shown in Table I, while the loss increase with α, it is true for the other values of α as well that the loss with the capacity constraint is several times higher than the loss without.

Table I. Loss of Output for $\psi = 0.7$ and Different Values of α, μ, and β.

α	μ	$\beta = 0$	$\beta = 0.9$	$\beta = 1$
2	5%	2.8%	1.5%	0.22%
	25%	6.2%	3.2%	0.64%
5	5%	6.2%	4.1%	0.9%
	25%	13.9%	9.1%	2.5%
8	5%	10.1%	7.3%	1.7%
	25%	22.5%	16.3%	4.95%

5 Concluding Remarks

This paper analyzes the impact of inflation on the price and production decisions of a firm that faces a fixed price-adjustment cost and has to commit to a fixed output capacity. Under our specific assumptions it is shown that, as in the case of frictions in only price adjustments, the initial real price increases and the terminal real price decreases with inflation, and the output-inflation relationship is negative. Moreover, simulations reveal that the fixed output capacity significantly amplifies the negative impact of inflation on output.

Due to tractability we have relied on rather specific assumptions and the generality of the results is therefore an open question. One assumption is that there is no discounting, the consequences of which are analyzed in Danziger (2001). There, it is shown for a general profit function and sufficiently low inflation rates that discounting causes a predetermined output level to be lower than the frictionless level, thus providing an alternative explanation for a negative output-inflation relationship.

Another assumption is that the elasticity of demand and the real unit cost are constant, and other demand and cost functions may lead to a different result. For instance, simulations with a linear demand and a constant real unit cost show that it is possible for output to increase with inflation. However, this does not change the overall conclusion that a menu cost matters even when the production capacity is completely fixed due to large adjustment costs.

Appendix

Proof of (i) The time in the Keynesian regime, i.e., where demand is below capacity, as a fraction of the total length of a time period equals

$$\frac{\tilde{T}}{T} = \frac{\ln \frac{S}{\tilde{z}}}{\ln \frac{S}{s}}.$$

>From conditions (2) and (3), we get $S^{1-\alpha} = s\tilde{Y}$ implying that

$$\frac{S}{s} = \left(\frac{S}{\tilde{z}}\right)^{\alpha}.$$

Inserting this in the above formula gives

$$\frac{\tilde{T}}{T} = \frac{\ln \frac{S}{\tilde{z}}}{\ln \left(\left(\frac{S}{\tilde{z}}\right)^{\alpha}\right)} = \frac{1}{\alpha}.$$

Proof of (ii) and (iii) We start be proving that $s < \tilde{z} < S$. Conditions (2) and (3) can be written as $V = \Pi\left(S, \tilde{Y}\right) = \Pi\left(s, \tilde{Y}\right)$. Since a solution must satisfy the second-order condition, $\partial \Pi\left(S, \tilde{Y}\right)/\partial S < 0$ and $\partial \Pi\left(s, \tilde{Y}\right)/\partial s > 0$, it follows that $s < S$. Condition (4) then shows that $s < \tilde{z}$.

>From condition (2), we get

$$-V + \left[\left(\frac{S}{\tilde{z}}\right)^{-\alpha} S - k\right]\tilde{Y} = 0,$$

and comparing this with condition (3) shows that $\tilde{z} < S$. Hence, $s < \tilde{z} < S$.

To establish that $s < \hat{z}$, we substitute condition (3) in equation (1) to obtain

$$\frac{1}{\ln \frac{S}{s}}\left(\frac{\alpha\tilde{Y}^{1-1/\alpha} - S^{1-\alpha}}{\alpha - 1} - s\tilde{Y} - \mu c\right) = s\tilde{Y}.$$

Using condition (4) to substitute for $\ln \frac{S}{s}$ gives

$$\frac{k}{\tilde{z}-s}\left(\frac{\alpha \tilde{Y}^{1-1/\alpha}-S^{1-\alpha}}{\alpha-1}-s\tilde{Y}-\mu c\right)=s\tilde{Y}.$$

Conditions (2) and (3) imply that $S^{1-\alpha}=s\tilde{Y}$, from which it follows that

$$-k\mu c=\tilde{Y}\left(\tilde{z}-s\right)\left(s-\frac{\alpha k}{\alpha-1}\right) \quad \Rightarrow \quad s<\frac{\alpha k}{\alpha-1}=\hat{z}.$$

Now, we derive $\frac{dS}{d\mu}$, $\frac{ds}{d\mu}$, and $\frac{d\tilde{z}}{d\mu}$. Total differentiation of conditions (2)-(4) yields

$$\begin{aligned}
\frac{dS}{d\mu} &= -\frac{1}{D}\frac{c\tilde{Y}}{\tilde{z}\ln\frac{S}{s}}\left[\tilde{z}-\alpha\left(s-k\right)\right], \\
\frac{ds}{d\mu} &= -\frac{1}{D}\frac{cS^{-\alpha}}{\tilde{z}\ln\frac{S}{s}}\left[\tilde{z}-\alpha\left(\tilde{z}-k\right)\right], \\
\frac{d\tilde{z}}{d\mu} &= -\frac{1}{D}\frac{cS^{-\alpha}}{s\ln\frac{S}{s}}\left[s-\alpha\left(s-k\right)\right],
\end{aligned}$$

where D is the Hessian determinant, which is negative due to the second-order condition. It follows from $s<\hat{z}$ that $\frac{d\tilde{z}}{d\mu}>0$. For $\mu\rightarrow 0$ it follows from the first-order conditions that $\tilde{z}\rightarrow\hat{z}$, implying that $\tilde{z}>\hat{z}$. Hence, it has been proved that $s<\hat{z}<\tilde{z}<S$ and $\frac{d\tilde{z}}{d\mu}>0$.

$>$From $\tilde{z}>\hat{z}$ it follows that $\frac{dS}{d\mu}>0$ and $\frac{ds}{d\mu}<0$. Finally, $\tilde{z}>\hat{z}$ and $\frac{d\tilde{z}}{d\mu}>0$ imply that $\tilde{Y}<\hat{Y}$ and $\frac{d\tilde{Y}}{d\mu}<0$. $\square$

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Figure 1. Real Profit with and without Fixed Output Capacity

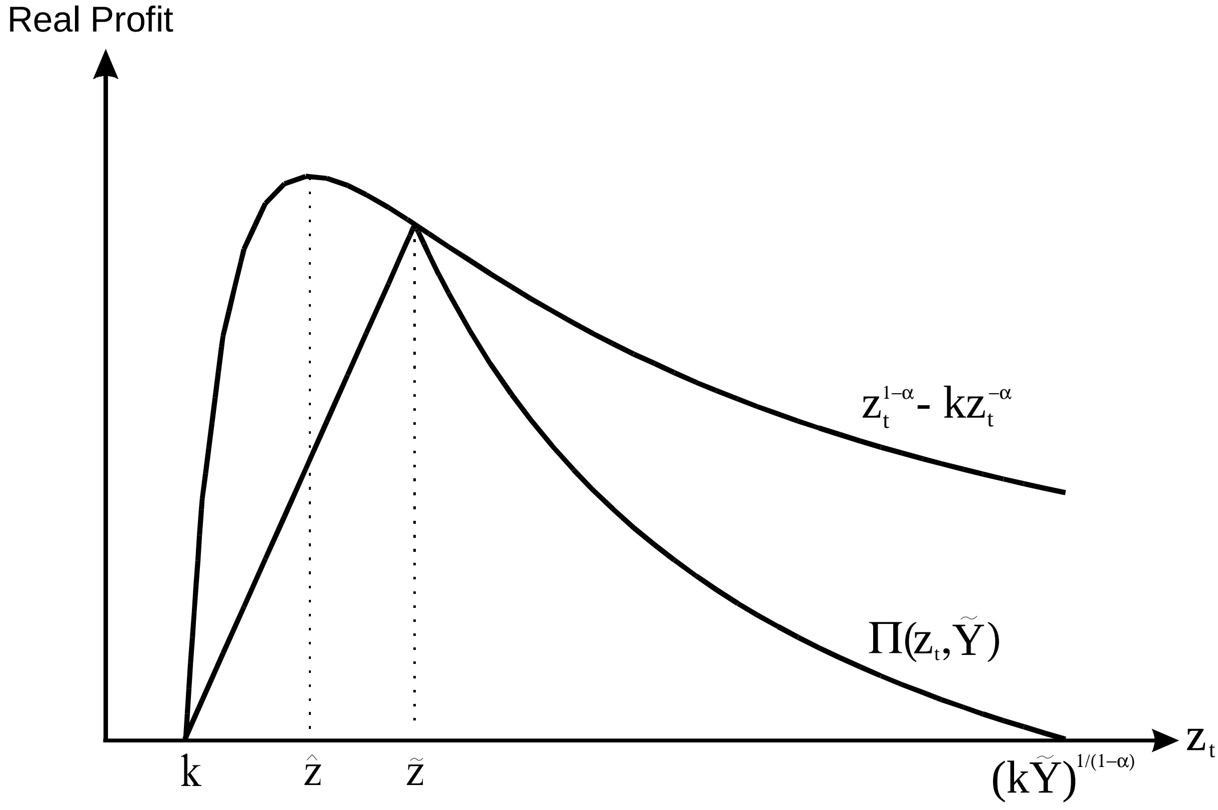

Figure 2. Loss of Output for α=5, ψ=0.7%, and Different Values of β