Second-Best Antitrust in General Equilibrium - A Special Case*

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Abstract

(i) Regulation of only some monopolistic industries decreases welfare if a large fraction of industries are monopolistic. (ii) The difference between the conventional dead-weight loss and the true welfare loss grows progressively as the monopolistic part of the economy grows.

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1 Introduction

This paper addresses two second-best antitrust issues: (i) what is the welfare gain of regulating some but not all monopolistic industries in the economy and (ii) how well does the conventional dead-weight loss based on the area between the partial equilibrium demand curve and the marginal cost curve approximate the true welfare loss? The first issue is relevant for practical antitrust policy; there are always some unregulated industries in an ever changing world where old industries are replaced by new ones and where regulation takes time. The second issue is relevant for empirical estimates of the aggregate welfare loss from monopolistic industries which rely on summation of partial dead-weight losses.\footnote{Scherer & Ross 1990 ch. 18 reviews the empirical evidence and discusses also problems of neglecting general equilibrium repercussions.}

The analysis considers only one inefficiency of monopolistic price setting; the misallocation of fixed resources between industries. The basic framework is adopted from Dixit & Stiglitz (1977) which has been used in many fields of economics in recent years. In this framework everything is symmetric, e.g., each industry uses monopoly pricing. This paper deviates by having a mixed market structure consisting of both monopolistic and competitive industries. By using an otherwise symmetric setting, the market structure is fully described by a scalar, $\alpha$, denoting the share of monopolistic industries in the economy.

In this setting, the welfare gain of regulating the marginal monopolistic industry decreases as $\alpha$ increases and turns negative after a certain threshold, $\hat{\alpha}$. The reason is a negative welfare effect not present in partial models; forcing a monopolistic industry to use marginal costs pricing removes some of the input endowment from other monopolistic industries implying that production in these industries moves further away from the first best. This negative effect dominates the standard positive effect if $\alpha$ is large. The result has two implications: First, regulation may reduce welfare if authorities can regulate only a fraction of the monopolistic industries in the economy. Sec-
ond, both the partial dead-weight loss of the marginal monopolistic industry and the aggregation of such losses overstate the ’true’ welfare losses\textsuperscript{2} and the errors grow progressively when $\alpha$ increases.

The next section presents the model. Section 3 states the result regarding the welfare consequences of regulation. Section 4 compares the partial dead-weight loss and the summation of such losses with the ’true’ welfare losses established from the model. Section 5 concludes.

2 A Model of a Mixed Market Structure

The economy consists of a continuum of industries each producing a final good. An identical production technology is available to all industries and they all use one common input hired in a perfectly competitive market. A share $\alpha \in [0, 1]$ of the industries are characterized by monopolistic price setting whereas the rest are characterized by price taking behavior. The representative household consumes the different final goods which are imperfect substitutes and earns income from two sources; supply of input to the industries and ownership of company shares. The household supplies inelastically the entire endowment of input which is normalized to unity and serves as numeraire.

2.1 The Representative Household

The representative household has the utility function

$$U \equiv \left( \int_{j=0}^{1} c(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \ (1)$$

\textsuperscript{2}This is compatible with Friedland (1978) that concluded from a simulation exercise using a model with only two industries that the true welfare loss is uniformly lower than the partial dead-weight loss.
where the elasticity of substitution between any two final goods equals $\varepsilon > 1$. The household maximizes utility subject to the budget constraint

$$\frac{1}{j=0} p(j) c(j) \, dj \leq l^a + \frac{1}{j=0} \pi(j) \, dj \equiv I, \quad (2)$$

where $p(j)$ is the price of good $j$, $l^a$ is supply of input, $\pi(j)$ is a lump sum profit transfer from industry $j$. The first order conditions are

$$c(j) = \left(\frac{p(j)}{p}\right)^{-\varepsilon} \frac{I}{p} \quad \forall j, \quad (3)$$

where $p$ is a price index defined as the minimum price of one unit of composite good/utility equal to

$$p = \left(\frac{1}{j=0} p(j)^{1-\varepsilon} \, dj\right)^{\frac{1}{1-\varepsilon}}.$$

The supply of input, $l^a$, equals the endowment, 1, as long as the real factor price, $1/p$, is positive.

### 2.2 Firms

The production possibilities of industry $j$ are given by

$$y(j) \leq l(j)^\gamma, \quad 0 < \gamma \leq 1, \quad (4)$$

where $y(j)$ is the number of output units and $l(j)$ is the amount of input hired in the common input market at the factor price 1. The objective demand curve for final good $j$ is identical to (3). In the following subscript $h$ denotes variables in monopolistic industries and subscript $k$ denotes variables in competitive industries. Monopolistic industries maximize profits subject to (3), (4), and $(p, I)$ yielding

$$\frac{\varepsilon - 1}{\varepsilon} \gamma p(h) l(h)^{\gamma - 1} = 1 \quad \forall h \in [0, \alpha], \quad (5)$$

where $(\varepsilon - 1)/\varepsilon$ is the mark-up factor. Competitive industries maximize profits subject to (4) and $(p(k), I)$ which gives

$$\gamma p(k) l(k)^{\gamma - 1} = 1 \quad \forall k \in (\alpha, 1]. \quad (6)$$
2.3 General Equilibrium

All monopolistic industries are symmetric and all competitive industries are symmetric. Thus, \( p_h = p^m, \ l_h = l^m, \ y_h = y^m \ \forall h = [0, \alpha] \) and \( p_k = p^c, \ l_k = l^c, \ y_k = y^c \ \forall k = (\alpha, 1) \) where superscript \( m \) denotes variables in monopolistic industries and superscript \( c \) variables in competitive industries. The model is closed by a market clearing condition in the factor market:

\[
\alpha l^m + (1 - \alpha) l^c = 1. \tag{7}
\]

3 Welfare Loss/Gain of Regulation

The symmetry of the model implies that all sectors use the same amount of input both in the case of no monopolistic industries (\( \alpha = 0 \)) and in the case of no competitive industries (\( \alpha = 1 \)). In fact, the allocations of inputs are identical in the two cases as the aggregate supply of input is inelastic. If \( \alpha \in (0, 1) \) then (3), (4), (5), and (6) yield

\[
\Psi \equiv l^c/l^m = \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{\frac{\gamma - 1}{\gamma + \varepsilon}} > 1, \tag{8}
\]

which reveals that the aggregate input endowment is misallocated in all intermediate cases; too much input is allocated to competitive industries and too little to monopolistic industries.

Household utility may be derived as a function of the market structure parameter, \( \alpha \), by the use of equations (1) to (8). This gives

\[
U = \Omega(\alpha) \equiv \frac{(\alpha + (1 - \alpha) \frac{\varepsilon - 1}{\varepsilon} \Psi)^{\frac{\gamma}{\gamma - 1}}}{(\alpha + (1 - \alpha) \Psi)\gamma}. \tag{9}
\]

Solving the equation \( \Omega'(\hat{\alpha}) = 0 \) yields

\[
\hat{\alpha} \equiv \frac{(\gamma - \omega^2) \Psi + (\omega - \gamma) \Psi^2}{(\omega - \gamma) (\omega - \omega \Psi - \Psi^2)}, \quad \omega \equiv \frac{\varepsilon}{\varepsilon - 1}, \tag{10}
\]

which lies in the set \([0, 1]\).
Proposition 1 (i) Utility is identical for $\alpha = 0$ and $\alpha = 1$. Utility is lower in all intermediate cases $\alpha \in (0, 1)$. (ii) Utility decreases (increases) if the marginal monopolistic industry is forced to use marginal cost pricing when $\alpha$ is larger (lower) than $\hat{\alpha}$.

Proof. (i) $\Omega(0) = \Omega(1) = 1$. $\Omega'(0) < 0$ and $\Omega'(1) > 0$. (ii) $\Omega'(\alpha) < 0$ for $\alpha \in [0, \hat{\alpha})$ and $\Omega'(\alpha) > 0$ for $(\hat{\alpha}, 1]$.

The first part of Proposition 1 states that a market structure characterized by either purely competitive industries or purely monopolistic industries yields largest utility. The household receives a lower factor income in the monopolistic case but this loss is exactly offset by larger dividends on shares giving the same aggregate income. Any mixed market structure results in misallocation of inputs such that the household consumes too much (little) of final goods made in competitive (monopolistic) industries.

The second part of Proposition 1 states that authorities that control only a small fraction of the monopolistic industries in the economy reduce (increase) utility by forcing these industries to use marginal cost pricing if the market structure is characterized by a large (small) fraction of monopolistic industries. Proposition 1 is illustrated by the gray curves in Figure 1 displaying the utility loss of a given market structure, $\alpha$, measured in per cent of the maximal attainable utility level:

$$\Gamma(\alpha) = 1 - \Omega(\alpha). \quad (11)$$

The figure reveals that the loss is a hump shaped function of the market structure parameter, $\alpha$, as stated in Proposition 1. The three gray curves are drawn for the case of a constant returns production function ($\gamma = 1$) and 3 different values of the elasticity of substitution, $\varepsilon$. The maximal possible loss with an elasticity of substitution equal to 2 occurs at $\hat{\alpha} = 0.67$ giving a loss equal to 11 percent. The mark-up factor is in this case 100 percent. Reducing the mark-up to 25 percent ($\varepsilon = 5$) gives $\hat{\alpha} = 0.66$ and a loss equal
to 3 percent. A lower value of the technology parameter $\gamma$ reduces both $\hat{\alpha}$ and the maximal loss; e.g. for $\gamma = 0.6$ and $\varepsilon = 2$ the largest possible loss is 5 percent and occurs at $\hat{\alpha} = 0.61$. The lower bound of $\hat{\alpha}$ is $\frac{1}{2}$ which occurs when $\gamma \to 0^+$ and $\varepsilon \to 1^+$. Thus, regulation does always increase utility as long as monopolistic industries do not dominate the market structure.

4 Evaluating the Dead-Weight Loss

An important general equilibrium effect neglected when adding partial dead-weight losses works through the factor price. This is obvious from the model when considering the market structure with no competitive industries ($\alpha = 1$). The aggregate dead-weight loss is positive as all prices in the economy are larger than marginal costs (cf. (5)) whereas the ‘true’ welfare loss is zero according to Proposition 1. The following analysis compares the conventional partial dead-weight loss for this specific model with the true loss in order to evaluate how well the summation of partial dead-weight losses approximate the true loss. In the comparison, we concentrate on the case of constant returns to scale ($\gamma = 1$) as normally done in empirical studies of the welfare loss. Using the demand function (3) the partial dead-weight loss of one monopolistic industry equals

$$D = \int_{y_m}^{y_c} \left( \left( \frac{1}{p} \right)^{\frac{1}{\varepsilon}} p z^{-\frac{1}{\varepsilon}} - 1 \right) dz = I \left( \frac{1}{p} \right)^{1-\varepsilon} \left( 1 - 2\varepsilon \right) \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{-\varepsilon} + \varepsilon - 1 \left( \varepsilon - 1 \right)^2.$$  \hspace{1cm} (12)

Symmetry implies that the aggregate dead-weight loss equals $\alpha D$. Measured relative to GNP the aggregate dead-weight loss equals

$$\Lambda(\alpha) = \frac{\alpha D}{p^c y^c} = \alpha \frac{(1 - 2\varepsilon) \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{-\varepsilon} + \varepsilon - 1}{(\varepsilon - 1)^2}.$$  \hspace{1cm} (13)

**Proposition 2** \(\textbf{(i)}\) The conventional measure of dead-weight loss, $\Lambda(\alpha)$, is larger than the true loss, $\Gamma(\alpha)$, for all $\alpha \in (0,1]$. \(\textbf{(ii)}\) The measurement error $\Delta(\alpha) = \Lambda(\alpha) - \Gamma(\alpha)$ grows progressively as the share of monopolistic industries in the economy, $\alpha$, increases.
Proof. (i) \( \Gamma'(0) = \Lambda'(\alpha) \). Since \( \Lambda(\alpha) \) is linear, \( \Gamma(\alpha) \) is strictly concave, and \( \Gamma(0) = \Lambda(0) = 0 \), it follows that \( \Lambda(\alpha) > \Gamma(\alpha) \forall \alpha \in (0, 1] \). (ii) From the strict concavity of \( \Gamma(\alpha) \), it follows that \( \Delta'(\alpha) = \Lambda'(\alpha) - \Gamma'(\alpha) > 0 \) and \( \Delta''(\alpha) = -\Gamma''(\alpha) > 0 \forall \alpha \in (0, 1] \).

Proposition 2 is illustrated in Figure 1. Linear curves represent \( \Lambda(\alpha) \) and hump shaped curves represent \( \Gamma(\alpha) \) both depicted for three different values of the elasticity of substitution. It is apparent from Figure 1 that \( \Lambda(\alpha) \) systematically overestimates the true loss and that the error grows progressively as \( \alpha \) increases. Note also the growing difference in slopes meaning that the measurement error on the marginal monopolistic industry increases when \( \alpha \) increases. On the other hand, one may also find that the measurement errors are relatively small if \( \alpha \) is not too large; for \( \varepsilon \in [2, 5] \) the relative measurement error \( [\Lambda(\alpha) - \Gamma(\alpha)] / \Gamma(\alpha) \) is in the range \([8\%, 11\%]\) for \( \alpha = 0.3 \) and in the range \([13\%, 17\%]\) for \( \alpha = 0.4 \).

5 Concluding Remarks

Two limitations of the analysis are important to acknowledge: Firstly, it only considers one type of inefficiency. Secondly, the results rely on a very specific model which is not unusual when dealing with second best problems. The results may be slightly more general than they appear; e.g., the assumption of a constant elasticity of substitution between final goods is not crucial. Furthermore, part (i) of Proposition 1 holds for any utility function where consumption of final goods enter symmetrically.\(^3\) The assumption of a mixed market structure is the only deviation from the symmetry assumptions of a basic Dixit & Stiglitz framework. This simplifies the analysis considerably as the market structure is fully characterized by one parameter. It may though be possible to obtain results similar to Proposition 1 in more general settings without symmetry; e.g., show that there always exists a market structure where regulation of one of the monopolistic industries reduces welfare.

\(^3\)This presumes that the aggregate factor demand curve is bounded away from the x-axis. This needs not be the case with imperfect competition (see Silvestre 1990).
References


Fig. 1. Welfare loss from monopolies.