Lower Tax Progression, Longer Hours and Higher Wages*

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Final Version: January 1998

Abstract

This paper analyses the impact of tax reforms that decrease income tax progression in an equilibrium search model with wage bargaining and endogenous individual working hours. Working hours are either bargained together with the hourly wage (case 1) or determined solely by workers after bargaining over the wage (case 2). In both cases reducing tax progression increases working hours of employed and, more interestingly, increases unambiguously wages and unemployment. Wages and unemployment rise more and working hours and production less in case 1 compared to case 2; probably making case 2 countries best suited for such tax reforms.

Keywords: Progressive taxes, wage bargaining, labour supply, search model.
JEL codes: H24, J22, J41.

* I like to thank Marta Aloi, Huw Dixon, John Driffield, Marcus Miller, Dale Mortensen, Katrine B. Poulsen, Michele Santoni, Christian Schultz, Troels Østergaard Sorensen, two anonymous referees, an editor of this journal, and seminar participants at University of York, University of Copenhagen, and the EEA’96 Congress for useful comments and suggestions. I am also grateful to Lars Haagen Pedersen for discussions initiating this paper. Correspondence: Institute of Economics, University of Copenhagen, DK-1455 Copenhagen K, Denmark. Ph.: +45 35323020, fax: +45 35323000, e-mail: claus.thustrup.hansen@econ.ku.dk.

† The activities of EPRU (Economic Policy Research Unit) are financed through a grant from the Danish National Research Foundation.
1 Introduction

It is well-known that high income tax progression, defined as high marginal tax relative to average tax, reduces economic efficiency in a perfectly competitive labour market. Reforms that decrease the marginal tax holding fixed the average tax improve efficiency by increasing individual labour supply implying lower wages, higher employment, and higher activity in equilibrium. However, it is also well-known that the opposite result is obtained in models with wage bargaining because high marginal taxes ”punish” wage rises. This is shown in a trade union context in Hersoug (1984), Malcomson & Sartor (1987), Creedy & McDonald (1990), Lockwood & Manning (1993), and Koskela & Vilmunen (1996) and in an equilibrium search framework in Pissarides (1990) ch. 8. These theoretical analyses treat individual working hours as exogenous, thereby disregarding the above beneficial effect on individual labour supply. Thus, the overall theoretical effects on wages and employment are ambiguous.

Empirical evidence from Italy by Malcomson & Sartor (1987), from the United Kingdom by Lockwood & Manning (1993), from Sweden by Holmlund & Kolm (1995), from Finland by Tyrväinen (1995), and from Denmark by Hansen et al. (1996) show that high marginal taxes tend to reduce pretax real wages. If these empirical findings are due to wage bargaining, it
is still theoretically and empirically unclear whether they are results of 1) very inelastic individual labour supply making the wage bargaining effects dominant or 2) wage bargaining effects too strong to be dominated by labour supply effects. It is also unclear whether labour supply effects may reverse the negative effects on activity and unemployment obtained in the wage bargaining models. This paper addresses these questions in an equilibrium search model with wage bargaining and endogenous individual working hours. This combination has been analysed in a slightly more general framework in Pissarides (1990) ch. 6 but without addressing tax issues.

The results are in favour of the second of the above hypotheses regarding wages and unemployment, and the first hypothesis regarding aggregate employment and activity - i.e. reducing the marginal tax holding fixed the average tax results in unambiguous increases in hourly (and yearly) wages, individual working hours, and unemployment leaving the effects on aggregate employment measured in hours and production indeterminate. Thus, the above empirical evidence on wages is not necessarily evidence in favour of a negative effect on aggregate employment and production, but may be interpreted as an indicator of a negative effect on unemployment. Furthermore, evidence of a negative relationship between tax progression and individual labour supply (see e.g. Feldstein 1995) is consistent with decreases in GNP and increases in wages and unemployment following tax reforms that reduce
These conclusions seem fairly robust as the results are *qualitatively* independent of whether working hours are determined together with the hourly wage in a bargain between firm and worker (case 1) or determined solely by the worker after the wage negotiation (case 2). *Quantitatively* the two cases differ: Removing the progression of the tax scheme by reducing marginal taxes implies that wages and unemployment increase more and working hours and aggregate employment less when bargaining is over both wage and working hours (case 1) compared to the case where working hours are determined by the worker after the wage bargain (case 2). Thus, if one believes (which seems to be the received perception) that unemployment is inefficiently high (possibly because of imperfect competition in the labour market) and individual working hours inefficiently low (because of high marginal taxes), then the analysis indicates that case 2 countries suffer less and gain more from tax reforms that reduce marginal taxes.\(^1\)

It is difficult to determine which case is most realistic. It depends, probably, on country traditions as well as the possibility of enforcing contracts. The first case results in Pareto-optimal outcomes of the negotiations but it may be impossible to enforce contracts concerning working hours making

\[^1\text{It seems natural to presume that the average tax cannot be reduced substantially. Thus, we are in the world of second best. It is impossible at the same time to reach an optimal rate of unemployment and an optimal length of the working year with only the marginal tax as instrument.}\]
the second case a relevant alternative. Thus, it seems worthwhile to examine both cases.

The paper is organized as follows. Section 2 presents the general framework common to both cases. Section 3 explores the consequences of reduced progression when firms and workers negotiate both wages and working hours (case 1), whereas Section 4 explores the issue when workers determine working hours after the bargain over hourly wages (case 2). The two cases are compared in Section 6 after some notes on welfare in Section 5. Section 7 contains concluding remarks.

2 The framework

The analysis is confined to steady state values. Let $N$, $u$, and $v$ be labour force, rate of unemployment, and number of vacancies relative to labour force, respectively. The number of job matchings per year is described by $X(uN, vN)$ which is assumed to be homogeneous of degree one and to possess the same properties as a neoclassical production function. This implies that vacant jobs become occupied at the rate $q(\theta) \equiv X\left(\frac{1}{\theta}, 1\right)$, $q'(\theta) \leq 0$, where $\theta \equiv v/u$ is the measure of labour market tightness. Transition from employment to unemployment happens at the constant rate $s$. Assuming
unchanged labour force, steady state equilibrium unemployment equals

\[ u = \frac{s}{s + \theta q(\theta)}, \]  

(1)

where \( \theta q(\theta) \) equals the job finding rate of unemployed. A match \( i \) between a worker and a firm leaves the worker with a yearly after tax income equal to \( w_i\ell_i - T(w_i\ell_i) \), where \( \ell_i \) is hours of work, \( w_i \) is hourly wage, and \( T(\cdot) \) is an increasing tax function. Each employed worker has the indirect flow utility function

\[ S(w_i, \ell_i) = w_i\ell_i - T(w_i\ell_i) + z - \frac{1}{\alpha} \ell_i^\alpha, \]  

(2)

where the last two terms represent utility from leisure. The unemployed get utility from leisure equal to \( z \). Working \( \ell_i \) hours a year decreases the value of leisure by \( \frac{1}{\alpha} \ell_i^\alpha \) where the elasticity of marginal disutility with respect to work \( \alpha - 1 \) is assumed positive. The linearity in net-income and the additive separability of net-income and disutility of work have the convenient implication that the individual labour supply is independent of wealth, it only depends on the marginal net-wage and parameters. This feature simplifies the algebra substantially. However, the absence of wealth effects does not seem crucial to comparative static results involving changes in marginal tax for given average tax as such changes leave net wealth unchanged initially, making any possible wealth effects of second order (see also the discussion in Lockwood & Manning 1993).
The expected present value of unemployment is denoted by $U$. Therefore, the expected present value of being employed equals

$$E (w_i, \ell_i) = \frac{w_i \ell_i - T (w_i \ell_i) + z - \frac{1}{\alpha} V + sU}{\delta + s}, \quad (3)$$

where $\delta$ is the common rate of time preference. Unemployment benefits and taxes levied on unemployed are disregarded. Thus, unemployed derive utility only from leisure. The resulting expected present value of being unemployed is

$$U = \frac{z + \theta q (\theta) E}{\delta + \theta q (\theta)}, \quad (4)$$

where $E$ is the expected present value of a job which arrives at the rate $\theta q (\theta)$. All firms produce the same output which is sold in a perfectly competitive market at a price normalized to one. The average product of a worker equals $\eta$ output units per hour. The expected present value of profits from an occupied job is then

$$J (w_i, \ell_i) = \frac{\ell_i (\eta - w_i) + sV}{\delta + s}, \quad (5)$$

where $V$ is the expected present value of profits from a vacant job. It is assumed that the costs per year of having a vacant job are equal to a fraction $\rho$ of the average yearly productivity of workers $\eta \ell$ giving

$$V = \frac{-\rho \eta \ell + q (\theta) J}{\delta + q (\theta)},$$

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where $J$ is the expected present value of profits if the job becomes occupied. Assuming free entry, new job openings occur until the expected present value of profits from a vacant job equals zero. This gives the equilibrium condition

$$J = \frac{\rho \eta \ell}{q(\theta)} .$$

In symmetric equilibrium $w_i = w$, $\ell_i = \ell$ and $J(w_i, \ell_i) = J$ for all $i$. Thus, we can substitute $J$ in (5) by (6) giving an implicit relationship between labour market tightness $\theta$ and overall wage level $w$ equal to

$$w = \eta - \frac{(\delta + s) \rho \eta}{q(\theta)} .$$

For a given wage, this equation shows that firms enter until the wage equals the net gain equal to the average product minus the expected capitalized value of hiring costs. The equilibrium conditions (1) and (7) are common to both of the two cases considered in the following.

### 3 Bargaining over wage and working hours (case 1)

The bargaining outcome solves the asymmetric Nash-bargaining problem

$$\max_{w_i, \ell_i} \Omega(w_i, \ell_i) = (E(w_i, \ell_i) - U)^\beta (J(w_i, \ell_i) - V)^{1-\beta} ,$$

where $\beta \in (0, 1)$ measures the bargaining strength of the worker relative to the firm. The disagreement point $(U, V)$ is identical to the market values of
being unmatched and is therefore independent of the bargaining outcome.

The first order conditions are

\[
\frac{\partial \log \Omega(w_i, c_i)}{\partial w_i} = \frac{\beta}{E(w_i, c_i) - U} \frac{\partial E(w_i, c_i)}{\partial w_i} + \frac{1 - \beta}{J(w_i, c_i) - V} \frac{\partial J(w_i, c_i)}{\partial w_i} = 0,
\]

\[
\frac{\partial \log \Omega(w_i, c_i)}{\partial c_i} = \frac{\beta}{E(w_i, c_i) - U} \frac{\partial E(w_i, c_i)}{\partial c_i} + \frac{1 - \beta}{J(w_i, c_i) - V} \frac{\partial J(w_i, c_i)}{\partial c_i} = 0.
\]

Using (3) and (5), the conditions can be written as

\[
\beta (\ell_i (\eta - w_i) - \delta V) (1 - T'(w_i, \ell_i)) = (1 - \beta) \left( w_i \ell_i - T(w_i, \ell_i) + z - \frac{1}{\alpha} \ell_i^{\alpha - 1} - \delta U \right),
\]

\[
\ell_i^{\alpha - 1} = \eta (1 - T'(w_i, \ell_i)).
\]  

(Figure 1 here)

In the following, we look at a parameterized tax system with marginal tax \(MT\) and average tax \(AT\) as the parameters.\(^2\) This simplification makes it possible to find closed form solutions to the variables and is not crucial to the following comparative static results. Equilibrium wage and working hours are obtained as functions of the tax parameters by evaluating (9) and (10) in symmetric equilibrium and using (4), (6), and the free entry condition \(V = 0\). This yields

\[
w^1 = \eta \frac{\beta \nu \left( 1 + \rho \theta^1 \right) + (1 - \beta) \frac{\nu}{\alpha}}{\beta \nu + 1 - \beta}, \quad 0 < \nu \equiv \frac{1 - MT}{1 - AT} \leq 1, \quad (11)
\]

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\(^2\)This is identical to a linear tax scheme with deductibility allowances being endogenous: If \(T(w_i, \ell_i) = tw_i \ell_i - t_0\) then \(t = MT\) and \(t_0 = (MT - AT) w\ell\) where \(w\) and \(\ell\) are average values beyond influence of a single individual. Note, that it is possible to approximate any non-linear tax scheme in equilibrium with a linear scheme that has the same marginal and average tax.
\[ \ell^1 = \left( (1 - MT) \eta \right)^{\frac{1}{\alpha - 1}}, \] (12)

where superscript 1 refers to case 1. \( \nu \) is the residual income progression coefficient (see e.g. Musgrave & Musgrave 1989 p. 359) which is equal to one for a proportional tax scheme and less than one for a progressive tax scheme.\(^3\)

The bargaining outcome is illustrated in Figure 1 where \( \ell^* \) denotes the individual labour supply as function of the pre-tax wage. Equations (10) and (12) show that working hours are set where marginal disutility of work equals the marginal gain to the two parties resulting in a Pareto-optimal outcome. This is illustrated by the contract curve \( \text{CC} \) (the tangency-points of the iso-utility curves \( \bar{S} \) and the iso-profit curves \( \bar{\pi} \)) which is vertical because utility is linear in consumption.\(^4\) The total surplus is the area bounded by the line \( \text{O}, \text{A}, \text{C} \) and the \( \ell^* \)-curve. The division of the surplus is determined by the hourly real wage which is less than \( \eta \) giving an equilibrium like \( \text{E}_1 \) in Figure 1.

Equations (11) and (12) together with (1) and (7) describe the case 1 equilibrium \( (w^1, \ell^1, \theta^1, u^1) \). These four equations establish the following re-

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\(^3\)The possibility of a regressive tax scheme is disregarded in order to secure existence of an interior solution for all possible values of the different parameters. The results generalize, however, to the case of a regressive tax scheme provided that an interior solution exists.

\(^4\)These features are well-known from the efficient bargaining model of McDonald & Solow (1981). Note though, that bargaining is over working hours instead of aggregate employment. Thus, firms have still the ‘right-to-manage’ new job openings. Furthermore, it is workers (not firms) who have an interest in deviating from the agreement.
sult where $\xi_{y,x}$ denotes the elasticity of $y$ with respect to $x$.

**Proposition 1** A lower tax progression implies longer working hours, higher wages, and higher unemployment when bargaining over both wages and hours.

Formally: $\xi_{l^1,(1-MT)} \geq 0$, $0 \leq \xi_{w^1,(1-MT)} \leq 1$, $\xi_{u^1,(1-MT)} \geq 0$.

**Proof.** See appendix.

A lower tax progression influences the bargaining outcome through two different channels. First, it increases the worker’s share of the pie because the marginal gain of claiming a higher wage is increased. Second, it increases the size of the pie because both parties benefit from a marginal increase in working hours (an outward shift in the contract curve): The firm gains because the marginal product is higher than the wage, and the worker gains because the net wage at the margin is higher. Thus, matched workers and firms agree on higher wages and longer hours as illustrated by the movement from $E_1^A$ to $E_1^B$ in Figure 2. The total effect on profits is negative which induces less entry of new firms, fewer job openings, and higher unemployment in steady state. It also follows that yearly wages increase even more than hourly wages whereas the signs on aggregate employment and activity are

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5 Proposition 1 and 2 do not contain any effects of changing the average tax as such effects are of minor interest (c.f. footnote 1). It is, however, easy to verify that the elasticities of wage and unemployment with respect to $1-MT$ equal $-\xi_{w^1,(1-MT)}$ and $-\xi_{u^1,(1-MT)}$, respectively, whereas working hours are independent of any changes in the average tax. This feature of the model is caused by the non-existence of wealth effects implied by the utility function (2).
indeterminate.\footnote{The production function implies that the relative effect on aggregate production equals the relative effect on aggregate employment.}

(Figure 2 here)

\section{Bargaining over wage only (case 2)}

The equilibrium $E_1$ in Figure 1 requires some sort of enforcement mechanism (e.g. legally binding contracts or reputation effects) as it is in the workers interest to work less at the going wage. If the worker could change the amount of work after the bargain, he would choose to set marginal disutility of work equal to net hourly real wage (cf. (3) and point $E_2$ in Figure 1). It is now assumed that the worker cannot precommit to a given amount of work per year. Thus, only the hourly wage is subject of negotiation.\footnote{This bargaining structure is similar to the well-known 'right-to-manage' assumption except that it is the \textit{worker} who has the 'right-to-manage' his working hours.} The problem is solved backwards. The first step is to maximize (3) with respect to $\ell_i$ taking $w_i$ as given. This yields

$$\ell_i = (w_i (1 - MT))^{\frac{1}{\alpha - 1}}. \quad (13)$$

The second step corresponds to maximizing a Nash-function identical to (8), this time only with respect to the wage and taking into consideration the impact on the subsequent decision of working hours in (13). The first order
condition is
\[
\beta \left( \frac{\partial E(w_i, \ell_i)}{\partial w_i} + \frac{\partial E(w_i, \ell_i)}{\partial c_i} \frac{\partial c_i}{\partial w_i} \right) + \frac{1 - \beta}{E(w_i, \ell_i) - U} \left( \frac{\partial J(w_i, \ell_i)}{\partial w_i} + \frac{\partial J(w_i, \ell_i)}{\partial c_i} \frac{\partial c_i}{\partial w_i} \right) = 0,
\]
giving two new terms compared to case 1. It follows, however, from the envelope theorem that \( \frac{\partial E(w_i, \ell_i)}{\partial c_i} \) equals zero (the worker needs not to take into account the indirect influence of the wage setting on the subsequent decision of working hours as this decision is made by himself) giving only one new term compared to case 1. The new term \( \frac{\partial J(w_i, \ell_i)}{\partial c_i} \frac{\partial c_i}{\partial w_i} \) states that the firm must take into consideration that a higher hourly wage increases working hours thereby increasing profits to the firm, ceteris paribus. Using (3) and (5) the first order condition becomes
\[
\beta (\ell_i (\eta - w_i) - \delta V) (1 - MT) = (1 - \beta) \left( 1 - \frac{1}{\alpha - 1} \left( \frac{\eta}{w_i} - 1 \right) \right)
\]
\[
\left( w_i \ell_i (1 - AT) + \frac{1}{\alpha} \ell_i^\alpha - \delta U \right).
\]
(14)
The only distinction from (9) is the term \( -\frac{1}{\alpha - 1} \left( \frac{\eta}{w_i} - 1 \right) \) on the RHS. This term is always negative implying that the wage is larger for a given number of hours worked in this case. The equilibrium wage and working hours can now be obtained as functions of the tax parameters by evaluating (14) and (13) in symmetric equilibrium and using (4), (6), and the free entry condition \( V = 0 \). This yields
\[
w^2 = \eta \frac{\beta \nu \left( 1 + \rho \theta^2 \right) + \frac{1 - \beta}{\alpha - 1} \left( 1 - \frac{\nu}{\alpha} \right)}{\beta \nu + \alpha \frac{1 - \beta}{\alpha - 1} \left( 1 - \frac{\nu}{\alpha} \right)}.\]
(15)
\[ \ell^2 = \left( w^2 (1 - MT) \right)^{\frac{1}{\alpha - 1}}, \tag{16} \]

where superscript 2 refers to case 2, and \( \nu \) is the progressivity parameter as before. Equation (16) shows that the bargaining outcome lies on the individual labour supply curve \( \ell^s \) in Figure 1, and as the wage is always less than average productivity \( \eta \), it must be at a point to the left of the contract curve \( CC \). Thus, the bargaining outcome is not Pareto optimal. E.g. if the equilibrium is \( E_2 \) then the parties lose the area bounded by the line \( E_2, B, C \) and the \( \ell^s \)-curve.

Equations (15) and (16) together with (1) and (7) describe the case 2 equilibrium \( (w^2, \ell^2, \theta^2, u^2) \). The four equations establish the following result

**Proposition 2** A lower tax progression implies longer working hours, higher wages, and higher unemployment when bargaining over wages only. Formally:

\[ \xi_{\ell^2,(1-MT)} \geq 0, \quad 0 \leq \xi_{w^2,(1-MT)} \leq 1, \quad \xi_{u^2,(1-MT)} \geq 0. \]

**Proof.** See appendix.

The effects of lower progression are qualitatively similar to case 1 but the mechanisms are a bit different. The negotiations are only over hourly wages which increase because workers marginal gain of claiming higher wages is increased as in case 1. Afterwards, workers individually choose working hours which increase for two reasons. First, they increase because a decrease in marginal tax increases the net wage at the margin. Second, they increase
because negotiated hourly wages are higher making work more attractive to leisure at the margin. The change is illustrated as the movement from $E_2^A$ to $E_2^B$ in Figure 2. Unemployment increases for the same reason as in the previous case, i.e. profits decrease which depress new job creation. Again, it follows that yearly wages increase more than hourly wages whereas the signs on aggregate employment and activity are indeterminate.

5 Welfare

In general, it is not possible to reach the social optimal choice of unemployment and working hours with only the marginal tax as instrument. In Pissarides (1990) ch. 6 it is argued that the choice of hours in case 1 without any taxes is efficient. This implies that hours are too low in case 1 in the presence of a positive marginal tax and that hours are too low in case 2 even without taxes. If, at the same time, $\beta$ is sufficiently large then unemployment is too large in both cases\(^8\) and a marginal tax reduction will move hours in the right direction but unemployment in the wrong direction.

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\(^8\) Equation (1), (7), (11), and (15) reveal that $\theta \to 0$ and $u \to 1$ when $\beta \to 1$ in both cases independent on the tax structure. For a given value of $\ell$, the social optimal values of $\theta$ and $u$ arise as market values when $\beta = \varepsilon(\theta) \equiv -\frac{q'(\theta)\theta}{q(\theta)} \in (0, 1)$ (see Pissarides 1990 ch. 7).
6 Comparing case 1 and case 2

A little more can be said about the size of these effects and the consequences on aggregate employment (measured in hours) and activity if we compare the two cases.

**Proposition 3** Consider a case 1 economy and a case 2 economy characterized by the same function $q(\cdot)$ and identical values of $\alpha$, $\beta$, $\delta$, $\eta$, $\rho$, $s$, $AT$, and $MT$. Going from a progressive ($MT_A > AT$) to a proportional tax scheme ($MT_B = AT$) increases hourly wages and rate of unemployment more and working hours, aggregate employment, and aggregate production less in case 1 than case 2.

**Proof.** See appendix.

The proposition shows that the quantitative effects of removing progression depend on whether working hours are included in the bargaining process (case 1) or not (case 2). This is illustrated in Figure 2: The two cases have same equilibrium wage but different working hours when the tax scheme is proportional ($E_1^B$ and $E_2^B$). $E_1^A$ and $E_2^A$ correspond to a larger marginal tax implying different wages and a larger difference in working hours. Thus, the proposition implies that going from a progressive tax scheme to a proportional one attenuates the difference between the two cases.
The different responses of removing progression in the two cases can be interpreted in the following way. First, look at working hours. They increase in both cases because of the direct effect on the marginal gain of work. In the second case, they increase further because of an indirect effect stemming from the increase in negotiated wage, which has a positive impact on the subsequent decision of working hours. Second, look at negotiated wages. In both cases, they increase because of a direct effect on the workers incentive to increase the wage claim. In case 2, however, there is an indirect effect making firms more reluctant to accept higher wages. This effect stems from the fact that firms in case 2 take into consideration that higher wages increase working hours which increase profits, ceteris paribus. When the marginal tax is reduced, working hours increase and this lowers the need for firms to increase hours through the wage mechanism. The smaller wage response in case 2 implies a smaller increase in unemployment too, whereas the larger effects on aggregate employment and activity in case 2 are due to a larger increase in working hours of employed and a smaller increase in persons being unemployed.\footnote{The differences between the two cases narrow as individual labour supply becomes more inelastic and disappear in the limit with perfectly inelastic labour supply ($\alpha \to \infty$). In the limit both cases resemble Pissarides (1990) ch. 8 and the beneficial effect on individual labour supply of lowering the marginal tax disappears.}
7 Concluding remarks

Tax reforms have reduced income tax progressivity in many OECD countries during the 1980s and 1990s. The reforms reflect a change of focus from distributional considerations to efficiency considerations among policy makers and before that among economists (see Sandmo 1991); e.g. lower progressivity reduces the distortionary effects on labour supply which reduce wages and increase aggregate employment/activity. Since the mid 1980s new research has showed that high progressivity tends to reduce the impact of labour market imperfections, e.g. wage bargaining, thereby leading to the opposite consequences. This paper includes both types of effects in an equilibrium search framework.

The similar results obtained in Proposition 1 and 2 seem to picture a fairly robust prediction: Tax reforms that reduce progressivity increase hourly real wages and working hours of the employed but reduce the number of persons having a job making the effect on aggregate employment (measured in hours) and aggregate production indeterminate.

Aggregate welfare increases only if the welfare gain of higher working hours is larger than the welfare loss from larger unemployment. An optimal tax progression will have to balance these effects which is beyond the scope
of this paper.\textsuperscript{10} However, Proposition 3 indicates that a country planning to lower marginal taxes in order to expand aggregate employment and activity has a bigger chance of success if workers choose the number of working hours themselves. Furthermore, the costs in terms of increased unemployment are lower.

Empirical evidence on wages (c.f. the Introduction) and on individual working hours (see surveys of US evidence in Feldstein (1995) and of Northern European evidence in Atkinson & Mogensen (1993) and more recent evidence in Aaberge et al. (1995) and Klevmarken et al. (1995)) seems to confirm the above prediction concerning the recent tax reforms.\textsuperscript{11} The size of the effects differs depending on methods and country under consideration.\textsuperscript{12} However, the evidence does not in general point to large significant effects on neither wages nor working hours. This indicates that welfare may go in either direction but the size of the effect is probably not large. Maybe economic efficiency have received to much focus on behalf of distributional preferences in the last tax reforms?

\textsuperscript{10}This would imply calibration. Development along these lines is found in Sørensen (1997).
\textsuperscript{11}The evidence on working hours differs a lot between groups. Typically, the effect on married women is large whereas the effect on prime-age male workers is around zero.
\textsuperscript{12}E.g. Aaberge et al. (1995) finds relatively large effects on working hours for Norway compared to the other studies.
References


From equation (11), we get

\[
\frac{d w^1/w^1}{d \nu/\nu} = \frac{\partial w^1/w^1}{\partial \nu/\nu} + \frac{\beta \rho \theta}{(1-\beta) \frac{1}{\alpha} + \beta (1+\theta \rho)} \frac{d \theta/\theta}{d \nu/\nu}.
\] (17)

Equation (7) yields

\[
\frac{\partial \theta/\theta}{\partial w/w} = -\frac{1}{\xi(\theta)} \frac{w}{\eta - w} < 0, \quad \xi(\theta) = \frac{q'(\theta)}{q(\theta)}.
\] (18)

Inserting equation (11) into the above equation gives

\[
\frac{\partial \theta^1/\theta^1}{\partial w^1/w^1} = -\frac{1}{\xi(\theta)} \frac{\nu \left( (1-\beta) \frac{1}{\alpha} + \beta (1+\theta \rho) \right)}{(1-\beta) \left(1 - \frac{\nu}{\alpha} \right) - \nu \beta \theta \rho} < 0.
\]
Inserting this into (17) and simplifying yield
\[
\frac{dw^1/w^1}{d\nu/\nu} = \frac{\partial w^1/w^1}{\partial \nu/\nu} \left( 1 + \frac{\nu^\beta \theta^1}{(1 - \beta)(1 - \frac{\nu}{\alpha}) - \nu^\beta \theta^1 \xi(\theta^1)} \right)^{-1},
\]
where it follows from equation (11) that
\[
0 \leq \frac{\partial w^1/w^1}{\partial (1 - MT)/(1 - MT) / (1 - MT)} = \frac{\partial w^1/w^1}{\partial \nu/\nu} = \frac{1 - \beta}{1 - \beta (1 - \nu)} \leq 1
\]
Thus, the total effect equals a direct effect between 0 and 1 divided by an indirect effect above 1 giving a total effect between 0 and 1.

Differentiate (1) with respect to \( \theta \) and insert (18) to obtain
\[
\frac{du^1/u^1}{d(1 - MT)/(1 - MT)} = \frac{du^1/u^1}{d\nu/\nu} = \frac{q(\theta)(1 - \xi(\theta))}{s + \theta q(\theta)} \frac{1}{\xi(\theta) \eta - w^1} \frac{dw^1/w^1}{d\nu/\nu} > 0.
\]
Finally from (12)
\[
\frac{d\ell^1/\ell^1}{d(1 - MT)/(1 - MT)} = \frac{1}{\alpha - 1} \geq 0.
\]
QED.

**B Proof of Proposition 2**

Using the same method as outlined in the proof of Proposition 1, we obtain
\[
\frac{dw^2/w^2}{d\nu/\nu} = \frac{\partial w^2/w^2}{\partial \nu/\nu} + \frac{\beta \nu \theta}{\beta \nu (1 + \rho \theta) + \frac{1 - \beta}{\alpha - 1} (1 - \frac{\nu}{\alpha})} \frac{d\theta/\theta}{d\nu/\nu} \frac{dw^2/w^2}{d\nu/\nu},
\]
Insert (15) into (18)
\[
\frac{d\theta^2/\theta^2}{dw^2/w^2} = -\frac{1}{\xi(\theta)} \frac{\beta \nu (1 + \rho \theta) + \frac{1 - \beta}{\alpha - 1} (1 - \frac{\nu}{\alpha})}{(1 - \beta)(1 - \frac{\nu}{\alpha}) - \beta \nu \theta}
\]
\[ \Rightarrow \]
\[
\frac{dw^2/w^2}{d\nu/\nu} = \frac{\partial w^2/w^2}{\partial \nu/\nu} \left( 1 + \frac{\beta \nu \theta^2}{(1 - \beta) \left( 1 - \frac{\nu}{\alpha} \right)} - \beta \nu \theta^2 \xi \left( \theta^2 \right) \right)^{-1}, \tag{20}
\]
where we have from equation (15)
\[
\frac{\partial w^2/w^2}{\partial \nu/\nu} = \nu \left[ \frac{\beta (1 + \rho \theta) - \frac{1 - \beta}{(\alpha - 1) \alpha}}{\beta \nu (1 + \rho \theta) + \frac{1}{\alpha - 1} \left( 1 - \frac{\nu}{\alpha} \right)} - \nu \left( \frac{\beta - \frac{1 - \beta}{\alpha - 1}}{\beta \nu + \alpha \frac{1 - \beta}{\alpha - 1} \left( 1 - \frac{\nu}{\alpha} \right)} \right) \right].
\]
\[\frac{\partial w^2/w^2}{\partial \nu/\nu}\] is positive if
\[0 < \nu \left( \beta (1 + \rho \theta) - \frac{1 - \beta}{(\alpha - 1) \alpha} \right) \left( \beta \nu + \alpha \frac{1 - \beta}{\alpha - 1} \left( 1 - \frac{\nu}{\alpha} \right) \right) - \nu \left( \beta - \frac{1 - \beta}{\alpha - 1} \right) \left( \beta \nu (1 + \rho \theta) + \frac{1 - \beta}{\alpha - 1} \left( 1 - \frac{\nu}{\alpha} \right) \right) \]
\[\Leftrightarrow \]
\[(1 - \beta) (\alpha (1 + \rho \theta) - 1) \frac{\beta \nu}{\alpha - 1} > 0, \]
which is always satisfied. To prove that the total effect is between 0 and 1, we need to prove that the partial effect is between 0 and 1. From the limits
\[\lim_{\alpha \to 1} \frac{\partial w^2/w^2}{\partial \nu/\nu} = 0, \]
\[0 \leq \lim_{\alpha \to \infty} \frac{\partial w^2/w^2}{\partial \nu/\nu} = \frac{1 - \beta}{1 - \beta (1 - \nu)} \leq 1, \]
it follows that the partial effect is between 0 and 1 as \[\frac{\partial w^2/w^2}{\partial \nu/\nu}\] is a continuous increasing function of \( \alpha \in (1, \infty) \).

The change in unemployment is again found from (1) and (18)
\[
\frac{du^2/u^2}{d (1 - MT) / (1 - MT)} = \frac{du^2/u^2}{d \nu/\nu} = \frac{q (\theta) (1 - \xi (\theta))}{s + \theta q (\theta)} \frac{1}{\xi (\theta) \eta - w^2} \frac{dw^2/w^2}{d \nu/\nu} > 0
\]
Finally from (16)

\[
\frac{d\ell^2/\ell^2}{d (1-MT) / (1-MT)} = \frac{1}{\alpha - 1} \left( 1 + \frac{dw^2/w^2}{d(1-MT) / (1-MT)} \right) \geq 0
\]

QED.

C Proof of Proposition 3

From (11) and (15) it is seen that \( w_1 = w_2 \) for \( \nu = 1 \) (\( MT = AT \)) implying that \( \theta_1 = \theta_2 \) and \( u_1 = u_2 \) for \( \nu = 1 \). In the proof of Proposition 2 it was shown that the partial elasticities obtained from (11) and (15) obey

\[
\frac{\partial w^2/w^2}{\partial \nu/\nu} \leq \lim_{\alpha \to \infty} \frac{\partial w^2/w^2}{\partial \nu/\nu} = \frac{1 - \beta}{1 - \beta (1 - \nu)} = \frac{\partial w^1/w^1}{\partial \nu/\nu},
\]

implying that the function (15) always lies above (or on) the function (11) in the \((\theta, w)\) space. It then follows that \( w_1 \leq w_2, \theta_1 \geq \theta_2, \) and \( u_1 \leq u_2 \) for \( \nu < 1 \). Thus,

\[
\left| \frac{w_1}{w_2} \right|_{MT>AT} \leq \left| \frac{w_1}{w_2} \right|_{MT=AT}, \quad \left| \frac{u_1}{u_2} \right|_{MT>AT} \leq \left| \frac{u_1}{u_2} \right|_{MT=AT}.
\]

The result regarding working hours is obtained from (12) and (16) giving

\[
\frac{d\ell^1/\ell^1}{d (1-MT) / (1-MT)} - \frac{d\ell^2/\ell^2}{d (1-MT) / (1-MT)} = -\frac{1}{\alpha - 1} \frac{dw^2/w^2}{d(1-MT) / (1-MT)} < 0,
\]

which implies that

\[
\left| \frac{\ell_1}{\ell_2} \right|_{MT>AT} \geq \left| \frac{\ell_1}{\ell_2} \right|_{MT=AT},
\]

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where the average tax rate is fixed. Finally, look at aggregate employment (and production)

\[
\frac{L_1}{L_2} = \frac{N(1 - u_1)\ell_1}{N(1 - u_2)\ell_2} = \frac{(1 - u_1)\ell_1}{(1 - u_2)\ell_2}.
\]

From the above results on \( u \) and \( \ell \), it is clear that

\[
\left| \frac{L_1}{L_2} \right|_{MT>AT} \geq \left| \frac{L_1}{L_2} \right|_{MT=AT}.
\]
Figure 1. Equilibrium in case 1 and case 2
Figure 2. Removing progression (A→B)