Fertility, Welfare and Economic Growth: 
the quantity-quality choice

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I. Introduction
This paper provides a profound understanding of parents’ choice of fertility and examines whether this choice is socially optimal. Whether growth can be sustained in the long run with low levels of population growth is a central question in economics and it is an area dominated by strong opinion. Based on historical evidence Kremer (1993) finds that the growth rate of technology has always been proportional to the rate of population growth except for the last 150 years. Recent endogenous growth models offer some insights which might provide some guidelines as to how growth will evolve in the future.
This paper argues that because parents do not internalise the intergenerational welfare nor the long-run growth rate of the economy, their fertility choice need not be socially optimal. Because there are potential scale effects associated with population as well as diminishing returns to labour, the optimal population size at a given point in time is important when designing policies. While some researchers believe that growth can be sustained even in the absence of population growth, others express more alarmist concerns. The paper is organised as follows: Section II will give a brief account of the quantity-quality choice and the demographic transition. Section III shows that the privately optimal fertility choice may not be socially optimal and Sections IV examines the causes of low population growth and some extreme cases of corner solutions. Section V discusses the implications for long-run growth and finally Section VI concludes.

II. The Quantity-Quality Choice and the Demographic Transition
Most of human history was characterised by people living at subsistence level. Malthus’ (1798) gloomy predictions of a positive correlation between incomes and fertility seem to describe the past surprisingly well. It is only the last 150 years which do not seem to fit this pattern; the trend was reversed and fertility and incomes became negatively correlated. Most of human history saw no significant growth in per capita incomes.
According to Malthus, increases in income induce more marriages and encourage people to have children. However, an increase in the supply of labour depresses wages and eventually income returns to its natural level and population declines to its equilibrium. He believed that population rises geometrically with incomes whereas food supply only increases arithmetically over time; mankind’s biological capacity to reproduce is assumed to exceed its physical capacity.
But the last 150 years saw significant increases in per capita incomes and Malthus’ predictions were considerably weakened by empirical facts following the so-called demographic transition. To account for these stylised trends, endogenous fertility models were developed and this was first formalised by Becker and Lewis (1973). The basic idea is that when income rises above a certain level, parents find it optimal to substitute quality for quantity, that is, choosing to have fewer children and instead investing in their quality by which is understood welfare and education. Several models have been developed to account for the micro foundations underlying this choice amongst many others Becker et. al. (1990) in a model of altruism, Ehrlich and Lui in a model where motives for having children are both altruistic and materialist and Moav (2005) which is based on a comparative advantage for poor households in producing many children and a comparative advantage for wealthy households in producing educated children. Hence, poor people’s relative price of quantity and quality is different to that of rich people as we shall see below.

Optimal population size is not fixed in the long run; it may vary with economic development, technology and education. For any given economic development there are two opposing forces at work. On the one hand, the productivity of labour diminishes when everyone has a smaller share of land and natural resources. On the other hand, human efforts, cooperation, organisation of industry and accumulation of ideas exhibit economies of scale. The optimal population size is obtained when these two forces exactly offset each other1. This paper will argue that population size is determined by parents, and that the outcome of their decisions may not coincide with the socially optimal decision. Imagine a parent with one child: if the former realises that his or her optimal choice is to have another child, and if having another child implies that less quality investment is made in the first-born child, then the parent’s choice of an extra child exerts a negative externality on the welfare of the first-born child. The parent desires more children but the latter prefer fewer siblings. In the reverse case, where a second child would cause a positive externality on the first born, it could be such that parents find it optimal with just one child whereas the child would prefer a sibling.

This bears important implications for policy by posing the challenge on the part of the policy-maker to design the right incentives. The next section will formalise the idea of optimal population.

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1 In the 1930s, John Maynard Keynes became a leading exponent of those population economists that attributed economic stagnation to population decline (cf. Zimmermann (1989) p. 2).
III. Optimal Population Size

We live in a society that consists of individuals – human beings – who maximise their well-being subject to various constraints. At any point in time, there is a certain number of people. The growth rate of population is determined by the number of births $n$ and the number of deaths $d$. Hence, the size of the population at a given point in time is in the long-run determined by decisions made by past generations who in their choice of fertility not only maximised the number of children and their quality but also their own consumption subject to a given resource constraint. The idea is best illustrated in a two-period two-generation model from Razin and Sadka (1995). At the end of period one, parents die and the children inherit the world for one period after which they die as well. Let the utility function of parents be given by:

$$u_i = u_i(c_1, n, u_2(c_2)),$$  \hspace{1cm} (1)

where $c_1$ is the consumption of parents, $n$ is the number of children and $u_2(c_2)$ is the quality of children which in this case depends on their consumption in period two. Following Razin and Sadka, we imagine there is a thing called optimal population size which maximises the utility of the members of society. An optimal population could be brought about if we like Razin and Sadka assume that we have a world which consists of one public good which is characterised by economies of scale and a factor such as labour which exhibits diminishing returns. At small levels of population, people can enjoy increasing welfare by increasing the size of the population by reaping the benefits of scale. At some point in time, however, the benefits of economies of scale are exhausted and diminishing returns to labour set in. This leads to an inverted U-shaped relationship between average utility (welfare) and population (cf. Figure A1 in Appendix A). At the level of population $p^*$, average utility is maximised. The resource constraint of this economy takes the following form:

$$c_1 + nb = k,$$  \hspace{1cm} (2)

where $b$ is a bequest for each child and $k$ is an exhaustible resource. Children are born without any endowment and therefore their consumption is equal to the bequest, $b = c_2$. The exhaustible resource $k$ thus has to suffice for both generations. In a state of laissez-faire, parents ultimately decide how many children to produce; parents choose the number of
children and their quality to maximise (1) subject to the resource constraint (2). The question is whether a benevolent social planner would reach the same outcome. It turns out that the social planner is not only concerned with maximising the welfare of the parents but also that of the children. This is not to say that parents do not care about their children, but the utility of children is only present in the objective function of the parents through the utility parents derive from the utility of children. The social planner would assign more than just an instrumental role to children; it would treat them as individuals. It is important to stress that parents have the best intentions with their children but it is not unrealistic to assume a certain degree of parental selfishness associated with their choice of fertility. Hence, a social planner which maximises the total utility of a society has the following objective function:

\[ u_B = u_1[c_1, n, u_2(c_2)] + nu_2[c_2, n, u_3(c_3)] + \ldots \]  

(3)

Now, since this is a two-generation model \( u_2[c_2, n, u_3(c_3)] = u_2[c_2] \) and all subsequent generations do not exist. In this case (3) becomes

\[ u_B = u_1[c_1, n, u_2(c_2)] + nu_2[c_2] , \]

(4)

where “\( B \)” is short for “Benthamite”. The Benthamite social planner maximises total welfare in society whereas the Millian social planner maximises average welfare. The average welfare is obtained by dividing by the total number of people in the economy – parents and children in this model. If we assume there is only one parent, then there will be \( n \) children and the Millian objective function is obtained by dividing (4) by \( (1 + n) \). This yields:

\[ u_M = \frac{u_1[c_1, n, u_2(c_2)] + nu_2[c_2]}{1 + n} , \]

(5)

where “\( M \)” is short for “Millian”. Hence, the laissez-faire optimal population is obtained by maximising (1) subject to (2), the Benthamite optimal population is obtained by maximising (4) subject to (2) and finally the Millian optimal population is obtained by maximising (5) subject to (2). It is obvious that the population size will be larger under the Benthamite objective than under that of the Millian. Suppose the Millian planner maximises the average utility of the members of society, then adding an extra person to the economy would increase
total utility as long as his or her utility is positive, but average utility declines. The question of interest, however, is whether the population size under laissez-faire $n^L$ is larger or smaller than the population sizes obtained under the Millian and the Benthamite objectives, $n^M$ and $n^B$, respectively. It turns out that in the general case it cannot be proven that $n^L$ is larger or smaller than either of $n^M$ or $n^B$. In Appendix B, I prove that it is ambiguous whether $n^L$ is larger or smaller than $n^M$ and the result is reproduced here:

$$\frac{1 + n^M}{1 + n^L} \leq 1 + \left\{ \frac{n^M u_2[c^M_2]}{u^1(c^M_1, n^M, u_2[c^M_2])} \right\}. \quad (6)$$

The superscript “$M$” denotes the Millian choice variable. Since the second term on the RHS of (6) is strictly positive, we cannot say anything about the ratio on the LHS. The size of the population as brought about by the fertility choice made by the parent may be larger or smaller than the choice made by a social planner which cares about the average utility in the economy.

This simple two-period two-generation model serves to illustrate that there may exist a conflict of interest between parents and children (here represented by a social planner) with respect to the size of population. For particular preferences the objective of the planner may coincide with that of parents but this need not be so. The implications for policy-makers would thus be to design policies that are pro-natal (if the objective is to increase the level of population in the long run) or anti-natal (if the objective is the decrease the level of population) or to switch the relative price of quantity and quality in favour of the desired policy objective.

Now, maximising welfare is not tantamount to maximising long-run growth. Consider the Millian planner. Maximising welfare, in this model, corresponds to maximising total average utility but this is not the same as maximising long-run growth. If intergenerational consumption smoothing is desirable, the Millian planner is not interested in maximising the increment to output – growth. Section V briefly considers a problem where there is a discrepancy between maximising long-run growth and the welfare of parents and that section will also review the recent discussions on endogenous growth theory.
IV. Low Population Growth

In the rest of this paper I shall be concerned with the empirically observed low population growth in the rich world and examine its consequences for long-run growth and welfare. Below-replacement fertility rates are an empirical fact in the rich world. According to Eberstadt (2001), 44 percent of the world experiences below-replacement fertility rates including Russia and China. Net population growth is positive but in Europe this is due to immigration and in China it is due to increased longevity. In Japan, population growth is negative. Table A1 presents fertility rates for all European countries. There is not a single European country that can maintain its population with the present fertility rates (maintaining the population requires a fertility rate of 2.1 children per woman). The population growth of persons of Danish origin in Denmark is projected to fall from 4959310 in 2005 to 4707736 in 2050, or by five percent (cf. Figure A2).

As implied in the previous section there may exist a conflict of interest between parents and children depending on the preferences of the parents. Moav (2005) and Becker et. al. (1990) develop models of endogenous fertility that generate club convergence\(^2\). In Moav’s model, the relative price of quantity and quality is determined by parent’s child-raising productivity. Since poor people’s productivity in educating their offspring is low compared to that of wealthy people, the former’s relative price of quantity \(n\) and quality \(q\) is steeper and it is represented by the slope of the line PC (“PC” stands for “poor club”) in Figure A3. The slope of the line RC (“RC” stands for “rich club”) is flatter because of wealthy people’s comparative advantage in producing quality offspring, that is, more educated offspring. The lowest steady state which characterises the poor club is brought about by a corner solution whereby parents choose not to invest in children’s quality; the consumption constraint binds. The number of children is bounded from above and parents choose to be at the corner where they do not invest in education simply because it places them on the highest feasible indifference curve, \(I_1\). In this case there is a conflict of interest between parents and children if having more children implies less human capital investment in each child; parents prefer a lot of children but children prefer fewer siblings. If the Millian planner is able to maximise the welfare of both parents and children then it will place parents on a lower indifference curve (which implies fewer children) if the net change in intergenerational utility is positive. In this case, it would choose a point, say \(A\), in Fig. A3 which is on a lower indifference curve. (notice that this is not pareto improving; the Millian planner cannot make the children better

\(^2\) I shall not present the models as this would be space-consuming and beside the point
off without sacrificing welfare on the part of the parents). Now, Ehrlich and Lui (1991, 1997) demonstrate that this outcome is also possible under laissez-faire in the Becker et al. model if (i) motives for having children are strictly material and (ii) if credible intergenerational contracts are imposed. In this case parents form explicit contracts with their children and they are able to increase their utility in their old age. In this way, multiple equilibria and the poverty trap are eliminated in the long run.

In what follows, I will speculate in a theoretical curiosity which is not based on any particular economic model of endogenous fertility but, rather, it derives from the aforementioned empirical observations. The reason why I included the analogue from Moav’s and Becker’s poverty traps should now become clear. Because if the purpose of writing is to convince the reader of a particular objective then it is better to start with something which might be conventionally accepted – i.e. the corner solution and the poverty trap. What if there exists a wealth trap in the developed world, this wealth trap being a corner solution with below-replacement fertility? In this case, however, it is not the consumption constraint but the time endowment that binds. Suppose children need to be endowed with so much human capital that parents cannot afford raising them without sacrificing more consumption than is optimal because they do not have enough time. We will then have a corner solution in quality as in Fig. A4, where fertility rates are below replacement.

It seems hard to imagine that below-replacement fertility rates can ultimately be a long-run outcome under laissez-faire. Chesnais (2001) summarises the conventionally accepted sociological causes of the low fertility rates. He argues that increased longevity tends to create a new psychological context that confers a feeling of eternity, which in turn dampens the desire for self-replication and makes people more selfish. This corresponds to consumption taking a greater weight in the objective function of the parents. That people will continue to choose the corner forever seems unlikely as this implies that the human race becomes extinct. But even if below-replacement fertility is only a temporary trend, then if there are scale effects associated with population, the fact that population levels are shrinking has serious consequences for growth and welfare. If there is some truth in Chesnais’ arguments then perhaps a correct model is one which endogenises the changing preferences of the parents to take account of the fact that with higher incomes preferences for consumption rises.

3 Obviously, this is not a strict corner solution as this would imply no children at all. I assume that the corner solution only implies below-replacement fertility.
The question is whether the Millian social planner in this wealth trap would choose a different quantity-quality choice than the parents. The Millian planner could choose to have more children but notice that this implies that each one of them will be endowed with less quality. This might have consequences for growth if human capital drives long-run economic growth. But the Millian planner has a different policy objective at its disposal. It may also sacrifice consumption thereby shifting out the budget line RC, (Cf. Fig. A4). This allows for more children without necessarily sacrificing quality. This was not possible in the poverty trap as it was the consumption constraint that bound.

Now, if there is indeed a corner solution, could optimal intergenerational transfers improve the outcome in this so-called wealth trap just as Ehrlich and Lui were able to do it in Becker’s poverty trap? In this line of thought, pay-as-you-go pension systems are bad for growth as it leaves little incentive for parents to invest in more children as an old-age security motive. However, the motive for having children is in the rich world less one of securing oneself for the old age financially; it is more one of an emotional motive.

**Empirical Issues**

Kosai et al. (1998) provide empirical evidence showing that as the wage gap of men and women narrows, fertility rates increase. Their regression is reproduced in Figure A5. The wage gap of men and women is depicted along the horizontal axis and fertility rates along the vertical axis. Initially, as the wage of women rise relative to that of men, the opportunity cost of women’s time rises and they choose to raise fewer children. However, as the wage gap narrows fertility rises again. Intuitively, this might be explained with the fact that as wages of men and women approach each other, the opportunity cost of childrearing of men and women also approach each other. Because of this, men take part in childrearing activities to a larger extent, and the time endowment for raising children of the entire family increases – allowing for more children. It might also reflect deliberate pro-natal government policies allowing women to combine childrearing and work which, by itself, also has an impact on the gender wage gap. Indeed, France and the Nordic countries have invested heavily in childcare facilities increasing the time endowment of the modern family with the positive result that these countries have the highest fertility rates in Europe (Cf. Table A1).

The role of the gender wage gap for fertility was previously formalised by Galor and Weil (1996) where opportunity costs of women’s time in part explains the demographic transition. However, there is no evidence that the outcome of the closing of the gender wage gap will ultimately lead to a path of fertility rates at the replacement level. Indeed, in an empirical
exercise, Kögel (2004) finds no evidence across the OECD of a sign change in the association between income and fertility. He does, however, find that the negative impact of income on fertility is reduced for high levels of income. Romer (2001) puts forward a more optimistic argument. He notes that it seems likely that genetic engineering will progress to the point where the concept of a person is no longer well-defined. When that occurs, he concludes, a different model might be needed to capture the growth process. Below-replacement fertility rates might have consequences for the welfare of current generations but also on long-run growth. This will be examined in depth in the next two sections.

V. Long-run Economic Growth

Low population growth is obviously bad news for proponents of the so-called endogenous growth theory where there are scale effects associated with population growth. This implication flows naturally from the non-rivalry of knowledge. Jones (2001) constructs a model where long-run growth is lower under laissez-faire than that achieved under a social planner. In his model, the representative dynasty chooses \( c_t \) and \( N_t \) to maximise the following objective function

\[
U(c_t, N_t) = \log c_t + \log N_t, \quad (8)
\]

subject to a resource constraint. \( c_t \) is consumption and \( N_t \) is the size of the dynastic family living at time \( t \). The social planner maximises, with the same choice variables, the welfare of a representative generation (in this case, the generation alive at time zero); the objective function is given by:

\[
\max_{[c_t, N_t]} U_0 = \int_0^\infty e^{-\rho t} U(c_t, N_t) dt, \quad (9)
\]

where \( \rho \) is a discount rate. The two maximisation problems yield different results. The decentralised solution ignores the effects of scale on long-run growth, and population growth is too slow. This model, however, suffers from a serious drawback as it ignores the quantity-quality trade-off in the optimisation problem. Sacrificing quality might be inevitable at least

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4 Cf. footnote 23 on page 132.

5 The derivation of these optimisation problems cannot be presented here for reasons of space.
in the short run if the education sector exhibits diminishing returns for a given number of teachers as assumed in Dalgaard and Kreiner (2001). If producing more offspring implies sacrificing the quality of offspring the net effects on economic growth are ambiguous. Here we have a trade-off between maximising welfare in the short run and maximising long-run growth.

In the endogenous growth literature, the accumulation of knowledge is the engine of economic growth because with a larger population the number of potential innovators is larger. Indeed, in the endogenous growth theory of Romer (1986), Grossman and Helpman (1991) and Aghion and Howitt (1992), positive population growth actually accelerates economic growth. Jones (1995), Kortum (1997) and Segerström (1998) (J/K/S) propose a theory of so-called semi-endogenous growth where the only way to generate constant growth rates is by allocating an increasing number of workers to the R&D sector. This avoids the empirically false implication that population growth accelerates economic growth, but it still implies that a positive rate of population growth is conducive and even necessary for sustained economic growth in the long run.

Kremer (1993) provides empirical evidence based on historical data that initial population size matters for economic growth. Technologically separate regions are tested and those regions with a larger initial population experienced higher subsequent growth. For evident reasons, the level of population cannot be the only factor that matters. For if this were true, China should be significantly richer than Europe and the United States put together. On the other hand, scale matters for countries of equal income. For example, Denmark and the United States have roughly the same per capita income but the United States produce more technological innovations because there are more people.

Jones (2002) develops a model that depends on population growth and tests it empirically. The model assumes, like the models of (J/K/S), that the only way to sustain economic growth is by allocating an increasing number of people to the R&D sector and increasing human capital accumulation. Because it is true that research intensity and educational attainment rise steadily over time, we should expect that these factors explain at least some fraction of the growth rate. He finds that transitional dynamics of research intensity and education explain 80 percent of US economic growth while only 20 percent can be attributed to population growth, the latter being a measure of steady-state growth. According to Jones, since educational attainment and research intensity are bounded from above, long-run growth will slow once this upper bound is reached and diminishing returns set in.
However, Jones assumes that the accumulation of knowledge is characterised by the equation
\[ \dot{A} = \delta H \] where \( H \) is effective research effort and \( \delta \) is a constant. This implies that the research quality stays constant over time. This may not be a realistic assumption as past discoveries might make future research more productive. Dalgaard and Kreiner (2001) overcome this problem by allowing for endogenous human capital formation, whereby the quality of research increases over time and growth becomes scale-invariant. Even if Jones is right, one must say that although the accumulation of human capital and research intensity is bounded from above, we are very far from this upper bound and if the third world manages to escape the poverty trap, a very large population of potential researchers emerges. Consequently, it should take a long time before diminishing returns set in and growth slows down.

Finally, recall from Section II that optimal population may not be fixed in the long run. It may vary with economic development, technology and education. Consider Fig. A6 where \( g \) is long-run growth and \( g^* \) is the maximum growth rate. Imagine that optimal population \( p^* \) shifts inward to \( p^{**} \) as people become more educated or because the quality of education and research increases over time as demonstrated in Dalgaard and Kreiner (2001). In this case, parents are optimally responding to the fact that there are too many people in the world and because of their choice of fertility, growth is sustained.

However it may be: this discussion highlights the important fact that it is not unambiguous that long-run growth is characterised by a steady state; a regime of perpetual and sustained economic growth is a qualified candidate as well.

**VI. Concluding Remarks**

Can growth be sustained in the long run? It seems likely that there are scale effects associated with long-run growth; a higher population growth spurs technological change. But in order to increase population growth some investment on the part of current generations is needed in terms of time spent in the childrearing sector and increased expenditure on education. The work by Dalgaard and Kreiner (2001) demonstrating that the quality of education and researchers can increase over time is promising. However, a major challenge is for the policy-maker to accommodate the threat to welfare imposed by an aging population and lower labour force participation rates in the near future.

Because parents do not internalise the intergenerational welfare or the long-run growth when they decide how many children to set into the world, it is a challenge for the policy-maker to
design the right incentives. The European countries have below-replacement fertility rates and this cannot be a long-run equilibrium. The fact that the fertility rates are higher in France and the Nordic countries is due to deliberate government policies that increase the time endowment available to families. Too little work has been carried out in estimating the actual optimal population size\textsuperscript{6}. Since optimal population varies over time, it makes sense to view this in a dynamic context; optimal population should be seen as the correct movement, i.e. that increase or decrease in population that brings about the highest increment to growth or welfare. Work should be carried out to estimate the optimal population and policy-makers should design policies accordingly. It is important to understand why the decentralised choice of fertility is below replacement and examine whether it is socially optimal.

\textsuperscript{6} Cf. Zimmermann (1989)
Appendix A

Figure A1

Figure A2 - from Statistikbanken
Figure A3

Corner solution

Figure A4

Corner solution
Figure A5 From Kosai, Saito and Yashiro (1998)

Figure 1. Female–Male Wage Ratio and Fertility Rate

Figure A6
### Table A1

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Source: Eurostat

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### Appendix B

Since the choice made by the Millian social maximises its objective function, it follows that

\[
M(c^M_1, c^M_2, n^M) \geq M(c^L_1, c^L_2, n^L),
\]

where \(M(\cdot)\) is the Millian optimal allocation and the superscript “\(M\)” stands for “Millian” and “\(L\)” stands for “laissez-faire”. Since \(M = \frac{B}{1+n}\), we have from (5) that

\[
B(c^M_1, c^M_2, n^M) \geq \left[ \frac{1+n^M}{1+n^L} \right] B(c^L_1, c^L_2, n^L),
\]

where \(B(\cdot)\) is the Benthamite optimal allocation. Since \(u_2 > 0\), it follows that

\[
B(c^L_1, c^L_2, n^L) = u_1(c^L_1, n^L, u^2[c^L_2]) + nu_2[c^L_2] \geq u_1(c^L_1, n^L, u_2[c^L_2]) \geq u_1(c^M_1, n^M, u_2[c^M_2])
\]

We are now able to conclude that (from (B2) and (B3))
\[ B(c_1^M, c_2^M, n^M) \geq \left[ \frac{1 + n^M}{1 + n^L} \right] u_1(c_1^M, n^M, u_2[c_2^M]), \]

such that

\[
\frac{1 + n^M}{1 + n^L} \leq \frac{B(c_1^M, c_2^M, n^M)}{u_1(c_1^M, n^M, u_2[c_2^M])} = \left\{ \frac{u_1(c_1^M, n^M, u_2[c_2^M]) + n^M u_2[c_2^M]}{u_1(c_1^M, n^M, u_2[c_2^M])} \right\} = 1 + \left\{ \frac{n^M u_2[c_2^M]}{u_1(c_1^M, n^M, u_2[c_2^M])} \right\},
\]

which is the same as (6).
References:


