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Is Declining Productivity Inevitable?

CARL-JOHAN DALGAARD

University of Copenhagen

CLAUS THUSTRUP KREINER

University of Copenhagen, EPRU, and CESifo

Fertility has been declining on all continents for the last couple of decades and this development is expected to continue in the future. Prevailing innovation-based growth theories imply, as a consequence of scale effects from the size of population, that such demographic changes will lead to a major slowdown in productivity growth. In this paper we challenge this pessimistic view of the future. By allowing for endogenous human capital in a basic R&D driven growth model we develop a theory of scale-invariant endogenous growth according to which population growth is neither necessary nor conductive for economic growth.

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1. Introduction

According to UN population estimates (United Nations, 1998) major global demographic changes are taking place. During the last 25 years fertility has been declining in all regions of the world leading to a gradual decline in world population growth from its peak rate of 2.04 percent per year in the 1965–1970 period to 1.33 percent between 1995 and 2000. All the leading industrial economies (the G7 group) are currently experiencing below-replacement fertility. Even if immigration is taken into account, the total population in the G7 area in 2050 is expected to be roughly the same as in 1998. Perhaps surprisingly, these developments are quite pervasive—in fact, projections indicate that global *de*population may arise after 2040. How will these demographic changes affect future economic growth?

In the endogenous growth theory initiated by Romer (1986, 1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) (below referred to as R/GH/AH) positive population growth entails accelerating income per capita growth as the size of population directly affects growth of scientific knowledge.[1] This scale effect, however, has not found much empirical support (Dinopoulos and Thompsen, 1999). Consequently, Jones (1995), Kortum (1997), and Segerström (1998) (henceforth J/K/S) have proposed a theory of so-called semi-endogenous growth.[2] Instead of assuming constant returns, this theory imposes diminishing returns to knowledge in creating new ideas. Hence, to achieve a constant growth rate of knowledge one needs to allocate an increasing number of researchers to the R&D sector. This avoids the scale effect on the long-run *growth rate* but implies instead that the *level* of income per capita is increasing in the size of population. As a consequence of this level scale effect, population growth continues to be conducive for economic growth; in fact, in the J/K/S framework, population growth is even necessary for perpetual growth. Thus, according to prevailing theories of economic growth, the aforementioned demographic changes are disturbing; theoretically one should expect a secular decline in productivity growth or perhaps even global economic stagnation.

In this paper we argue that these somewhat gloomy growth prospects hinge on a simplifying, but quite unrealistic, assumption: a fixed individual stock of human capital. Allowing for endogenous skill formation, featuring complementarity between human capital and scientific knowledge, leads to a theory of scale-invariant endogenous growth.[3] That is, a theory where the balanced growth path of income per capita is altogether independent of the size of population, implying that population growth is neither necessary nor conducive for economic growth.[4]

One might conjecture that adding accumulation of human capital would aggravate the scale problems since a larger population allows for more teachers thereby speeding up human capital formation. While this is true, human capital formation also introduces an important congestion effect: more students for a given level of expenditures on education reduces the human capital acquired by the average student. Thus, if both R&D and human capital accumulation are necessary for growth then from a modeling perspective it is important to remember that human capital, contrary to scientific knowledge created through R&D, is a rival input which is linked to the human body.

Our model suggests that it is not the quantity of citizens but solely the skill level of the average citizen that matters for the long-run level of per capita income. The skill level of the average individual is raised through education, which uses both human capital (embodied in teachers) and scientific knowledge. Therefore, the human capital level of the average individual continues to rise but, as a consequence of the congestion effect, at a slower pace than the aggregate stock of scientific knowledge. In this sense each individual becomes relatively more ignorant over time. In a world featuring increased specialization this implication seems eminently reasonable; contrary to, say, the last century, most people today certainly cannot make, and probably do not know the functioning of, the simple equipment they use daily, for example calculators, washing machines, cars, and so on.

When removing the scale effect on the growth rate from the R/GH/AH framework, J/K/S also remove an identifying characteristic of endogenous growth, namely that economic policy may influence the long-run growth rate. Our analysis shows that the endogenization of human capital in the J/K/S framework not only removes all scale effects, it also reestablishes the possibility for policy, and economic incentives in general, to affect growth.[5]

The remainder of the paper is organized as follows. In the next section we use a simple reduced from framework to briefly review some key properties of the R/GH/AH models and the J/K/S models. The third section shows how the simple framework can be augmented with endogenous human capital formation and thereby lead to a theory of ''scale-invariant endogenous growth''. The fourth section demonstrates that the ''toy'' model in the third section can be microfounded in a Romer (1990) type model. The last section contains concluding remarks.

2. Growth Models with Scale Effects

Time, t, is continuous. Final output, Y_t, is produced using human capital augmented labor input, H_t, ideas, A_t, and some fixed factor of production, say, land, denoted by Z. Whereas A_t is assumed non-rival, Z and H_t are conceived to be rival. More specifically, as human capital is inevitably linked to the human body, H_t is given as the total labor force, L_t, multiplied by the quality of labor, h_t. The production function allows for constant returns to rival inputs but increasing returns to rival and non-rival inputs taken together. Formally, we assume that

$$Y_t = H_t^{\alpha} Z^{1-\alpha} A_t^{\beta}, \quad 0<\alpha<1, \quad 0<\beta \leq 1. \tag{1}$$

Below we abstract from the fixed factor of production by normalizing Z to one.[6] Ideas are produced using units of final output, that is,

$$\dot{A}_t = \sigma_A Y_t, \tag{2}$$

where A_0 is given and where the parameter σ_A is the share of total output invested in R&D.[7] Output not used for investments is consumed. R/GH/AH assume that the production of knowledge is linear in existing knowledge, $\beta = 1$, and that human capital is constant, $H_t = \bar{H} = \bar{h}\bar{L}$. Therefore, the productivity of workers grows at the same pace as knowledge. Using equations (1) and (2) it is easy to show that per capita income, $y \equiv Y/\bar{L}$, increases along the balanced growth path at the rate

$$\frac{\dot{y}_t}{y_t} \equiv g_y = \frac{\dot{A}_t}{A_t} \equiv g_A = \sigma_A (\bar{h}\bar{L})^{\alpha}. \tag{3}$$

Thus, economic growth is increasing in the share of resources devoted to R&D, but is also increasing in the (effective) size of the population. By implication, reduced population growth would lead to a major decline in income per capita growth.

As mentioned in the introduction, a scale effect on the growth rate from the size of population is inconsistent with the available empirical evidence. This raises the question of whether growth can be explained through the accumulation of knowledge without having a positive dependence from population size to growth. J/K/S have shown that this may be accomplished by replacing the linearity of existing knowledge in producing new ideas with diminishing returns, $\beta < 1$, and by assuming that the total stock of human capital, H_t, is rising. In the present lab-equipment variant of the J/K/S models it is also necessary to assume increasing returns to scale to the two growing factors of production, $\alpha + \beta > 1$, in order to ensure positive income per capita growth. In the J/K/S models the average human capital endowment per worker is assumed constant, $h_t = \bar{h}$. Still, the human capital stock rises through time, albeit at an exogenous rate $\dot{H}_t/H_t = \dot{L}_t/L_t \equiv n \geq 0$, where the parameter n denotes the constant growth rate of population and where $H_0 = \bar{h}L_0$ is given. Using equations (1), (2), and the steady state relationship $g_A = n\alpha/(1-\beta)$, it follows that income per capita along the balanced growth path develops according to

$$y_t = \left(\frac{1-\beta}{\alpha} \frac{\sigma_A}{n} \right)^{\frac{\beta}{1-\beta}} \bar{h}^{\frac{\alpha}{1-\beta}} L_0^{\frac{\alpha+\beta-1}{1-\beta}} e^{\frac{\alpha+\beta-1}{1-\beta} nt}. \quad (4)$$

Inspection of this equation reveals that the scale of the economy still matters, albeit in a more subtle fashion; a larger population leads to a higher long run level of income per capita. Moreover, contrary to the R/GH/AH model, population growth is not only conducive, but even necessary for growth in per capita income. Hence, if population ceases to expand so will per capita income. Another important difference with regard to the R/GH/AH framework is that the growth rate is independent of σ_A. The J/K/S framework therefore removes an identifying characteristic of endogenous growth theory, namely that economic policies, or incentives in general, matter for long-run growth.

Observe from equation (4) that in the limiting case of constant returns to the two growing factors of production, $\alpha + \beta = 1$, the scale effect on the level of income is in fact eliminated from the J/K/S framework. However, in this case the long-run growth rate of per capita income is zero. But this result rests on the assumption that the quality of researchers is assumed constant. On the intuitive level, all that is needed to overcome the diminishing technological opportunities is that the skills of researchers rise over time. And this need not entail a growing population as assumed in the J/K/S models. Therefore we investigate the implications of allowing h_t to be endogenous in the next section.

3. A Simple Scale-Invariant Endogenous Growth Model

In this section we reconsider the J/K/S model above, albeit with two modifications. First, we allow the average level of human capital, h_t, to be endogenous in the model. Thus, the aggregate stock of human capital rises, not only because of an increasing number of individuals in the economy, but also because of a rising quality of each individual. Second, we assume, like in the R/GH/AH models, constant returns to reproducible factors of production, in our case A_t and h_t. This requires that $\alpha + \beta$ equals 1.[8] Note, that the model therefore allows for diminishing returns to knowledge as in the J/K/S models. However, increases in the quality and quantity of researchers tend to mitigate this effect. If the number of researchers grows, measured in efficiency units, a constant flow of research results can be obtained. If the quality adjusted number of researchers does not expand, growth will eventually come to a halt. The model thus captures that a high level of A_t is less useful, when it comes to producing the next idea, if the level of human capital is low.

Following Mankiw, Romer and Weil (1992) we assume that the production technology for human capital is $\dot{H}_t = \sigma_H Y_t$, where $H_0 \equiv h_o L_0$ is given, and where σ_H is the share of output used to produce human capital. Although, this formulation appears to be standard, it conceals an important congestion effect. This is evident if we use the definition of the human capital stock, $H_t = h_t L_t$, to eliminate $\dot{H}_t$ in the above equation. This yields

$$\dot{h}_t = \frac{\sigma_H Y_t}{L_t} - n h_t \quad (5)$$

where h_0 is given. The numerator in the first term, $\sigma_H Y_t$, represents the input in the human capital sector which consists of land, ideas, and human capital. The denominator can be interpreted as the number of students (here equal to the entire population). As is apparent from the equation, the smaller the ratio of expenditures on education to the number of students the less quality expands. If a given growth of individual human capital is to be attained expenditures have to be growing relative to the inflow of students; otherwise the sheer number of students will ''crowd out'' quality growth. This congestion effect ensures that the accumulation of human capital does not introduce any new scale effects either on the growth rate of per capita income or on the level of per capita income. The second term on the RHS of equation (5) reflects the costs of bringing the skill level of the newcomers up to the average level of the existing population.[9] This implies that population growth, *ceteris paribus*, tends to reduce quality growth of the average individual in the population.

Notice that scientific progress allows for perpetual growth in the human capital stock. This does not necessarily mean that the quantity of knowledge per individual (the number of ''facts'') increases through time. It may just imply that the quality of knowledge increases because science progresses and new insights are gathered and transferred to individuals through education.[10]

To solve the model we define $\chi_t \equiv (h_t L_t)/A_t$. The dynamic evolution of χ_t can subsequently be derived from equations (1), (2), and (5):

$$g_\chi \equiv g_h + n - g_A = \sigma_H \chi_t^{\alpha - 1} - \sigma_A \chi_t^{\alpha}. \tag{6}$$

Along a steady growth path, where g_y is constant, human capital and knowledge have to grow at the same rate, that is $g_\chi = 0$. Therefore, the steady state ratio of human capital to knowledge is

$$\chi = \frac{\sigma_H}{\sigma_A}. \tag{7}$$

It is easy to see from equation (6) that this steady state is indeed stable. Using equations (2), (5) and (7) it is possible to derive the growth rates of human capital, g_h, and knowledge, g_A, along the balanced growth path

$$g_A = g_h + n = \sigma_H^{\alpha} \sigma_A^{1-\alpha}. \tag{8}$$

By applying equation (1) it can be confirmed that the growth rate of per capita income equals

$$g_y = \sigma_H^{\alpha} \sigma_A^{1-\alpha} - n, \tag{9}$$

which is independent of the population size. To see that there is no scale effect on the level of income either, we compute per capita income along the balanced growth path. Using equations (1), (7), (8), and (9), we get

$$y_t = \frac{(h_t L_t)^{\alpha} A_t^{1-\alpha}}{L_t} = \left(\frac{A_t}{h_t L_t}\right)^{1-\alpha} h_t = \left(\frac{\sigma_A}{\sigma_H}\right)^{1-\alpha} h_0 e^{g_y t}, \tag{10}$$

which is independent of L_0 for given h_0. From equations (9) and (10) it now readily follows that the balanced growth path has the following properties:

Proposition *The long-run level of per capita income is proportional to the skill level of the average individual but is, for a given skill level of the average individual, independent of population size. Population growth is neither necessary nor conducive for long-run growth in income per capita. Economic incentives and policy may in general affect both the growth rate and the level (through σ_A and σ_H).*

Thus, the long-run difference in income per capita between two economies with same structural features, i.e. parameters, and same population growth rate is only due to differences in the average human capital level of the populations — the simple size of the populations does not matter in itself. This is in contrast to the R/HG/AH/J/K/S frameworks where both the average human capital level and the population size matter. Hence, the present model demonstrates how endogenous human capital accumulation in combination with diminishing returns to scientific knowledge allow for the simultaneous removal of the scale effect on the growth rate and on the level of income while still allowing for positive per capita income growth. As a consequence, even a dramatic decline in population growth will not lead to a long-run productivity slowdown. On the contrary, in the present model a lower population growth rate increases long-run growth in income per capita. In fact, an economy on its balanced growth path, which suddenly experiences a permanent fall in the population growth rate, will continue immediately on a new balanced growth path with higher income per capita growth.[11] It should be mentioned though that a decline in population growth in the microfounded version of the model, developed in the next section, may have either none or a positive effect on long-run income per capita growth depending on household preferences. Note also that an economy does not continue on its balanced growth path after, say, a once and for all positive shift in the population size (for given A and h). Instead, income per capita falls instantly and converges afterwards to a new balanced growth path featuring a permanent lower level of income per capita.[12]

The proposition reveals another interesting implication of the model, namely that economic policy may play a role in shaping long-run growth. Hence, the endogenization of human capital in the J/K/S framework reestablishes this central result of the original R/GH/AH endogenous growth theories.[13]

Finally, it is worth noting that $g_A - g_h > 0$ in the model. Accordingly, the ''knowledge frontier'' grows faster than the average knowledge of any given individual. Or in other words, individuals tend to become relatively more ignorant over time. In a world featuring increasing specialization this implication seems eminently reasonable. This will become more evident when we, in the next section, reconfirm the conclusions of the above simple analysis in a model of growth through specialization.

4. The Decentralized Model

In this section, we develop a scale-invariant endogenous growth model where technical progress manifests itself as increasing specialization, that is through an increasing variety of intermediate inputs. The structure follows Rivera-Batiz and Romer (1991), but we depart from the basic framework by allowing the stock of human capital to be endogenously determined as in the toy model above. Therefore, growth persists in our model for two reasons—R&D and human capital accumulation.

At the more detailed level, the model comprises three sectors; a final goods sector, an intermediate good sector, and finally, an R&D sector. While the final goods sector and the R&D sector are competitive, we assume that the intermediate goods sector is monopolistic. Final goods are produced using intermediate goods, labor, and a natural resource in fixed supply. The output from the final goods sector is used for consumption and investment. Furthermore, we assume that investments can be made in patents, that is funding for R&D and intermediate goods production, and human capital. This implies that final goods are used for three kinds of production purposes; R&D, intermediate good production, and production of human capital. While firms decide on how many resources to employ in R&D and in production of intermediate goods, it is the households that decide on investing in human capital.

We start by examining the final goods sector in Section 4.1. In Section 4.2 we solve the monopolists' problem in the intermediate goods sector, and in Section 4.3 we characterize the incentives to innovate. Then, in Section 4.4, we solve the households' problem. Lastly, we derive the balanced growth path and state our main results in Section 4.5.

4.1. The Final Goods Sector

We assume that final goods, Y_t, are produced using human capital augmented labor input, $H_t \equiv h_t L_t$, a fixed factor Z, and specialized inputs x_{jt}. The latter is indexed by j. We denote by A_t the total number of varieties used in production at time t. Specifically, the production function of the representative firm is given by

$$Y_t = \left(\frac{H_t}{A_t}\right)^{\alpha} \int_{j=0}^{A_t} x_{jt}^{\gamma} dj \cdot Z^{1-\alpha-\gamma}, \tag{11}$$

where α and γ are positive parameters fulfilling $\alpha + \gamma < 1$. The term $(H_t/A_t)^{\alpha}$ is thought to capture that production tends to become more human capital intensive through time as production complexity increases (Howitt, 1999). Technically, it allows for an aggregate production function which exhibits constant returns both to rival inputs (H_t, x_{jt}, and Z) and to reproducible factors (h_t and A_t). The former property ensures that the aggregate production function is consistent with the well-known replication argument whereas the latter represents a sufficient condition for endogenous growth. Like in the toy model we normalize Z to one.

The representative firm maximizes profits. The price of final goods acts as numeraire. Therefore, each firm employes intermediate goods and labor until the point where the marginal product equals the price of intermediate goods, p_{jt}, and the wage, w_t, respectively. Thus,

$$\frac{\partial Y_t}{\partial x_{jt}} = \gamma \left(\frac{h_t L_t}{A_t}\right)^{\alpha} x_{jt}^{\gamma - 1} = p_{jt}, \tag{12}$$

$$\frac{\partial Y_t}{\partial L_t} = \alpha h_t^{\alpha} L_t^{\alpha-1} A_t^{-\alpha} \int_{j=0}^{A_t} x_{jt}^{\gamma} dj = w_t. \tag{13}$$

Note for future reference that the rate of return on human capital is

$$r_t^H \equiv \frac{\partial Y_t}{\partial H_t} = \frac{w_t}{h_t}. \tag{14}$$

4.2. *The Intermediate Goods Sector*

The intermediate goods sector operates under monopoly. We assume that once monopoly status is acquired, by purchasing a blueprint from the R&D sector, it lasts indefinitely. Additionally, it is assumed that the production of one unit of intermediary input costs one unit of final output. Thus, the jth monopolists problem of maximizing profits, π_{jt}, can be stated as

$$\max_{x_{jt}} \pi_{jt} = (p_{jt} - 1)x_{jt}, \tag{15}$$

subject to the demand schedule, equation (12). On this basis it can readily be shown that the profit maximizing price and output level are given by

$$p_{jt} = p_t \equiv 1/\gamma, \qquad x_{jt} = x_t \equiv \gamma^{\frac{2}{1-\gamma}} \left(\frac{h_t L_t}{A_t}\right)^{\alpha/(1-\gamma)}. \tag{16}$$

As output and prices are identical for all j, it follows that profits are the same for all j:

$$\pi_{jt} = \pi_t \equiv (1/\gamma - 1)\gamma^{\frac{2}{1-\gamma}} \left(\frac{h_t L_t}{A_t}\right)^{\alpha/(1-\gamma)}. \tag{17}$$

Notice, that insofar as $(h_t L_t)/A_t$ is constant, as it will be in steady state, x_t, p_t, and π_t will be constant.

4.3. *The R&D Sector*

Anybody can engage in R&D and will do so as long as the benefits exceed the costs. Assuming that spending one unit of output (deterministically) leads to the discovery of a new variety, there will be entry until

$$1 = V_t = \int_{s=t}^{\infty} \pi_s e^{-\int_{\tau=t}^{s} r_\tau^A d\tau} ds, \tag{18}$$

where V_t is the benefits from engaging in production of intermediary inputs and r_t^A is the required rate of return on research and development. Differentiating equation (18) one can derive the condition on flow form:

$$1 = \frac{\pi_t}{r_t^A}. \tag{19}$$

4.4. The Households

The total number of households is constant through time and normalized to unity. However, the size of the household increases through time at the rate of population growth, n. The representative household maximizes

$$U_0 = \int_{t=0}^{\infty} \frac{c_t^{1-\varepsilon} - 1}{1-\varepsilon} e^{-\theta t} dt, \tag{20}$$

where c_t is the consumption level of each individual in the household and where θ and ε are the rate of time preference and the coefficient of relative risk aversion, respectively.[14] The optimization problem of the representative household consists of dividing income, Q_t, between consumption, $c_t L_t$, and investments, and furthermore, in allocating investments between human capital, I_t^H, and patents, I_t^A. Thus,

$$Q_t = I_t^H + I_t^A + c_t L_t, \tag{21}$$

where $\dot{A}_t = I_{A_t}$ and $\dot{H}_t = I_t^H$. Note, that the latter equation implies $\dot{h}_t = I_t^H / L_t - n h_t$ which is the mirror expression of equation (5) in the toy model.

Total household income derives from labor income $w_t L_t = r_t^H h_t L_t$, and from investing in the production of ideas, the proceeds of which are returned to the households in the form of dividends, $r_t^A A_t$. In order to parameterize policy, we allow for subsidies to human capital and R&D investments at the proportional rates τ^H and τ^A (which may be negative corresponding to a tax). The subsidies are financed trough a lump sum tax, T_t, and we assume that the government balances the budget at all times. Hence,

$$Q_t = (1+\tau^H) r_t^H h_t L_t + (1+\tau^A) r_t^A A_t - T_t. \tag{22}$$

The representative household chooses $\{c_t, I_t^A, I_t^H\}_{t=0}^{\infty}$ in order to maximize (20) subject to (21), (22), and the non-negativity constraints $c_t \geq 0, I_t^A \geq 0$, and $I_t^H \geq 0$. The solution to this problem is provided in Appendix A.

4.5. The Balanced Growth Equilibrium

The above non-negativity constraints on the household may give rise to transitional dynamics, that is the household may choose temporarily to invest in only one of the two assets.[15] However, the economy converges to a unique balanced growth path in finite time (see Appendix B). On this path the household invests in both human capital and patents and the returns on these two investments are equalized, that is

$$(1+\tau^H) r_t^H = (1+\tau^A) r_t^A \equiv r, \tag{23}$$

which corresponds to a standard no-arbitrage condition.[16] Additionally, the path is characterized by the Keynes–Ramsey rule

$$\frac{\dot{c}_t}{c_t} = g_c = \frac{1}{\varepsilon}(r - \theta - n). \tag{24}$$

The research arbitrage equation (19), the equilibrium condition for the asset market (23), along with the expression for the rate of return on human capital (14), and the expression for profits (17), pin down the $(h_t L_t)/A_t$ ratio along the balanced growth path:

$$\frac{A_t}{h_t L_t} = \Phi \equiv \frac{\gamma(1-\gamma)}{\alpha} \frac{1+\tau^A}{1+\tau^H}. \tag{25}$$

The immediate implication of this equation is that $g_A = g_h + n$ along the balanced growth path. From the accounting equations (21) and (22), it can readily be confirmed that aggregate output and consumption grow at the same rate. Hence, the Keynes–Ramsey rule pins down growth in total (and per capita) income.[17]

To solve for the growth rate in per capita income, we derive the equilibrium interest rate, r, from equations (13) and (16), and insert the expression in equation (24). This yields

$$g_y = \frac{1}{\varepsilon}\Big((1+\tau^H)\alpha\Phi^{(1-\alpha-\gamma)/(1-\gamma)}\gamma^{2\gamma/(1-\gamma)} - \theta - n\Big), \tag{26}$$

which we assume is positive. The entire balanced growth path of per capita income can now be derived from equation (11) and the equilibrium expressions for x_t and $(h_t L_t)/A_t$, which gives

$$y_t = y_0 e^{g_y t} = \left(\frac{A_0}{h_0 L_0}\right)^{1-\alpha} x_0^{\gamma} h_0 e^{g_y t} = \gamma^{2\gamma/(1-\gamma)}\Phi^{(1-\alpha-\gamma)/(1-\gamma)} h_0 e^{g_y t}. \tag{27}$$

Visual inspection of equations (25)–(27) reveals that the Proposition holds without qualifications (except that policy now works directly through the parameters τ_A and τ_H).

Observe from equation (26) that increases in population growth, n, decrease per capita income growth like in the toy model of Section 3. However, this result is not robust as it depends crucially on the specification of preferences. Suppose that the household, instead of using the current welfare criteria, had used the Benthamite welfare criteria.[18] Under such circumstances θ would be replaced by $\rho - n$, where ρ reflects the ''pure'' rate of time preference. It is easy to see from equation (26) that growth of per capita income in this case would be independent of n. The intuition is simply that, under total utility maximization (the Bentham criteria), the household becomes more patient, because the size of the family in the future is taken into account. This tends to increase the propensity to save, and as a result, exactly cancels the detrimental effect of population growth on the rate of income expansion, which is present in equation (26). This is, however, the only difference in results between the general equilibrium model and the toy model.

Note finally that $g_h = g_A - n$. Thus, in a relative sense, each person in the economy becomes increasingly ignorant as time passes by. This feature is a product of the balanced growth property, that scientific knowledge grows at the same rate as the effective amount of labor input.

5. Concluding Remarks

This paper has emphasized the importance of distinguishing between non-rival knowledge in the shape of ''blueprints'' (i.e. scientific knowledge) and rival knowledge embodied in humans (i.e. human capital). Whereas existing innovation-based growth theories tend to focus on the former, the latter is demonstrated to be important *vis-a-vis* the relationship between population growth and long-run income growth. In particular, in the present paper it is argued that the size and growth of population is of secondary importance for long-run income growth. This result rests on two assumptions about the nature of human capital.

First, human capital is, contrary to ideas, innately rival. Human capital production will therefore be associated with an important congestion effect: an increase in the number of students will, ceteris paribus, crowd out the quality of the average student. This simple congestion effect dissipates the scale effects from the size of population.

Second, human capital represents scientific knowledge embodied in individuals. Hence, as scientific discoveries are diffused through the educational system the quality of the labor force rises. Since increases in the skills of individuals are likely to promote further scientific progress, a virtuous circle of increasing scientific knowledge and human capital can be envisioned. This complementarity between scientific knowledge and human capital is what in our model allows for perpetual growth in income per capita without any positive influence from population growth.

These results are important when it comes to assessing the prospects for future income growth. The dramatic reductions in fertility already taking place on all continents of the globe should, according to the existing literature, ultimately entail a productivity slowdown. Conversely, our theory of scale-invariant endogenous growth implies that a reduction in fertility might even lead to higher growth in income per capita. As pervasive fertility reductions, according to UN projections, are expected to continue in the future, there seems to be a pressing need for empirical investigations of these issues.

Appendix

A. The Households' Problem

The households' problem is to

$$\max_{\{c_t, I_t^A, I_t^H\}_{t=0}^{\infty}} U_0 = \int_{t=0}^{\infty} \frac{c_t^{1-\varepsilon} - 1}{1-\varepsilon} e^{-\theta t} dt,$$

subject to the following set of constraints:

$$\dot{h}_t = \frac{I_t^H}{L_t} - nh_t, \quad h_0 \text{ given},$$

$$\dot{A}_t = I_t^A, \quad A_0 \text{ given},$$

$$c_t \geq 0, I_t^H \geq 0, I_t^A \geq 0,$$

$$I_t^H + I_t^A = (1 + \tau^H) r_t^H h_t L_t + (1 + \tau^A) r_t^A A_t - T_t - c_t L_t,$$
$$A_t > 0 \quad \text{for all } t,$$
$$h_t > 0 \quad \text{for all } t.$$

The discounted value Hamiltonian, J_t, after insertion of c_t, is given by

$$J_t = \frac{\left(\frac{(1+\tau^H) r_t^H h_t L_t + (1+\tau^A) r_t^A A_t - T_t - I_t^H - I_t^A}{L_t}\right)^{1-\varepsilon} - 1}{1-\varepsilon} e^{-\theta t} + \lambda_{Ht}\left(\frac{I_t^H}{L_t} - n h_t\right) + \lambda_{At} I_t^A.$$

Due to the inequalities $I_t^A \geq 0$ and $I_t^H \geq 0$, the first order conditions with respect to I_t^A and I_t^H are

$$\frac{\partial J_t}{\partial I_t^A} \cdot I_t^A = 0, \tag{28}$$

$$\frac{\partial J_t}{\partial I_t^H} \cdot I_t^H = 0, \tag{29}$$

and

$$\frac{\partial J_t}{\partial I_t^A} = \lambda_{At} - c_t^{-\varepsilon} e^{-\theta t} L_t^{-1} \leq 0, \tag{30}$$

$$\frac{\partial J_t}{\partial I_t^H} = \lambda_{Ht} L_t^{-1} - c_t^{-\varepsilon} e^{-\theta t} L_t^{-1} \leq 0. \tag{31}$$

The first order conditions with respect to the state variables are

$$\frac{\partial J_t}{\partial A_t} = L_t^{-1} c_t^{-\varepsilon} e^{-\theta t} (1 + \tau^A) r_t^A = -\dot{\lambda}_{At}, \tag{32}$$

$$\frac{\partial J_t}{\partial h_t} = c_t^{-\varepsilon} e^{-\theta t} (1 + \tau^H) r_t^H = -\lambda_{Ht} n = -\dot{\lambda}_{Ht}. \tag{33}$$

Finally, the solution has to fulfill the two transversality conditions:

$$\lim_{t \to \infty} \lambda_{At} A_t \leq 0,$$

$$\lim_{t \to \infty} \lambda_{Ht} h_t \leq 0.$$

A.1.

If the returns on the two assets are identical, it follows from equations (13), (16), (17), and (19) that

$$(1 + \tau^H) r_t^H = (1 + \tau^A) r_t^A = r \equiv (1 + \tau^H) \alpha \Phi^{(1-\alpha) - \gamma/(1-\gamma)} \gamma^{2\gamma/(1-\gamma)},$$

where Φ is a constant given by equation (25). We now rule out that (a) $I_t^A > 0 \wedge I_t^H = 0$, (b) $I_t^A = 0 \wedge I_t^H > 0$, and (c) $I_t^A = I_t^H = 0$ thus implying that the solution is characterized by $I_t^A > 0 \wedge I_t^H > 0$.

First, note that the transversality conditions and $A_0 > 0$ and $h_0 > 0$ and $I_t^A \geq 0$ and $I_t^H \geq 0$ imply

$$\lim_{T\to\infty} \lambda_{AT} = 0, \tag{34}$$

$$\lim_{T\to\infty} \lambda_{HT} = 0. \tag{35}$$

Next, integrate equation (32) from time 0 to time T to yield

$$\int_{t=0}^{T} \dot{\lambda}_{At} dt = \lambda_{AT} - \lambda_{A0} = -\int_{t=0}^{T} L_t^{-1} c_t^{-\varepsilon}(1 + \tau^A) r_t^A e^{-\theta t} dt. \tag{36}$$

Correspondingly, integrating equation (33) and using $L_t^{-1} = L_0^{-1} e^{-nt}$ yield

$$(\lambda_{HT} e^{-nT} - \lambda_{H0}) L_0^{-1} = \int_{t=0}^{T} L_t^{-1} c_t^{-\varepsilon}(1 + \tau^H) r_t^H e_t^{-\theta t} dt. \tag{37}$$

We next subtract equation (37) from (36) and let $T\to\infty$. After using equations (34) and (35), we obtain

$$\lambda_{A0} - \lambda_{H0} L_0^{-1} = \int_{t=0}^{\infty} L_t^{-1} c_t^{-\varepsilon}\left[(1 + \tau^A) r_t^A - (1 + \tau^H) r_t^H\right] e^{-\theta t} dt. \tag{38}$$

When $(1+\tau^A)r_t^A$ and $(1+\tau^H)r_t^H$ are identical, so are λ_{A0} and $\lambda_{H0}L_0^{-1}$. This implies from equations (30) and (31) that $\partial J_t/\partial I_t^A = \partial J_t/\partial I_t^H$, which rules out case (a) and (b). The assumption of positive growth, that is $r > \theta + n$, implies that the household has an incentive to invest thereby ruling out case (c) directly. Hence, $I_t^A > 0$ and $I_t^H > 0$. It is now straightforward to derive the Keynes–Ramsey rule, equation (24), from the equations (28) to (33).

A.2.

Consider the case where $(1+\tau^A)r_0^A > (1+\tau^H)r_0^H$ at time zero. We wish to rule out (a) $I_0^H > 0 \wedge I_0^A = 0$, (b) $I_0^H > 0 \wedge I_0^A > 0$, and (c) $I_0^H = 0 \wedge I_0^A = 0$, thus demonstrating that $I_0^A > 0$ and $I_0^H = 0$ holds.

Consider, first case (a). When $I_t^H > 0$ inequality (31) holds with equality according to equation (29), that is

$$L_t^{-1} c_t^{-\varepsilon} e^{-\theta t} = L_t^{-1} \lambda_{Ht} > 0.$$

Using this equation and inequality (30), it follows that

$$\lambda_{At} - L_t^{-1} c_t^{-\varepsilon} e^{-\theta t} = \lambda_{At} - L_t^{-1} \lambda_{Ht} \leq 0. \tag{39}$$

If $I_t^H > 0$ and $I_t^A = 0$ then the production technology implies that $(1+\tau^H)r_t^H$ will be decreasing through time whereas $(1+\tau^A)\tau_t^A$ will be increasing through time. Hence, if $(1+\tau^A)r_0^A > (1+\tau^H)r_0^H$ implies $I_0^H > 0 \wedge I_0^A = 0$ then $(1+\tau^A)r_t^A > (1+\tau^H)r_t^H$ for all $t > 0$. It then follows from equation (38) that $\lambda_{At} - \lambda_{Ht}L_t^{-1} > 0$, which contradicts equation (39).

Consider case (b). Equations (28) and (29) imply that $\partial J_t/\partial I_t^A = \partial J_t/\partial I_t^H = 0$. Hence, inequalities (30) and (31) imply that

$$\lambda_{At} = \lambda_{Ht}L_t^{-1} \text{ for all } t,$$

from which it follows that $\dot{\lambda}_{At} = \dot{\lambda}_{Ht}L_t^{-1} - \lambda_{Ht}nL_t^{-1}$ for all t. It then follows from equations (32) and (33) that $(1+\tau^A)r_t^A = (1+\tau^H)r_t^H$ $\forall t$ contradicting that $(1+\tau^A)r_0^A > (1+\tau^H)r_0^H$.

Finally, consider case (c). If $I_t^A = I_t^H = 0$ then H_t and A_t are constant implying that total income is constant and equal to total consumption, that is $Y = \bar{Y} = c_tL_t$. Thus,

$$\frac{\dot{c}_t}{c_t} = -n.$$

If $I_t^A = I_t^H = 0$, then A_t/H_t is constant implying that the returns to the two assets are constant, that is $r_t^A = \bar{r}^A$ and $r_t^H = \bar{r}^H$. Using these facts, equation (36), and the transversality condition, we have

$$\begin{aligned}\lambda_{A0} &= \int_{t=0}^{\infty} L^{-1}c_t^{-\varepsilon}(1+\tau^A)\bar{r}^A e^{-\theta t}dt \\ &= L_0^{-1}c_0^{-\varepsilon}(1+\tau^A)\bar{r}^A \int_{t=0}^{\infty} e^{-(\theta - n(\varepsilon-1))t}dt.\end{aligned} \tag{40}$$

For the parameter constellation $\theta - n(\varepsilon - 1) < 0$, this equation implies that λ_{A0} becomes infinite. This leads to a contradiction since inequality (30) at $t = 0$ implies that

$$\lambda_{A0} \leq c_0^{\varepsilon} \cdot L_0^{-1} \tag{41}$$

in case (c). Consider instead the parameter constellation $\theta - n(\varepsilon - 1) > 0$. Then equation (40) yields

$$\lambda_{A0} = \frac{L_0^{-1}c_0^{-\varepsilon}(1+\tau^A)\bar{r}^A}{\theta - n(\varepsilon - 1)}. \tag{42}$$

To see that this contradicts (41), insert (42) into (41). This gives

$$\frac{L_0^{-1}c_0^{-\varepsilon}(1+\tau^A)\bar{r}^A}{\theta - n(\varepsilon - 1)} \leq c_0^{\varepsilon} \cdot L_0^{-1},$$

or, equivalently,

$$(1+\tau^A)\bar{r}^A - \theta - n \leq -\varepsilon n < 0. \tag{43}$$

Note that $(1+\tau^A)\bar{r}^A > (1+\tau^H)\bar{r}^H$ implies that $(1+\tau^A)\bar{r}^A > r > (1+\tau^H)\bar{r}^H$ where r corresponds to the value of A_t/H_t where $(1+\tau^A)r_t^A = (1+\tau^H)r_t^H \equiv r$. Hence,

$$(1+\tau^A)\bar{r}^A - \theta - n > r - \theta - n > 0,$$

where the last inequality follows from the assumption of positive steady state growth. The above inequality contradicts the inequality (43), thereby ruling out case (c).

Thus, if $(1+\tau^A)r_0^A > (1+\tau^H)r_0^H$ then $I_0^A > 0$ and $I_0^H = 0$. A similar argument as above can be used to show that $(1+\tau^A)r_0^A < (1+\tau^H)r_0^H$ implies $I_0^A = 0$ and $I_0^H > 0$.

B. The Transition to the Balanced Growth Path

If $A_t/(h_t L_t) = \Phi$, where Φ is given by equation (25), it follows from equations (13), (16), (17), and (19) that $(1+\tau^A)r_t^A = (1+\tau^H)r_t^H = r$. In this case the economy follows the balanced growth path which exhibits the properties summarized in the Proposition. We now show that the economy converges to this path if $A_t/(h_t L_t) \neq \Phi$. Consider first the case where $A_t/(h_t L_t) < \Phi$. It then follows from equations (13), (16), (17), and (19) that $(1+\tau^A)r_t^A > r > (1+\tau_t^H)r_t^H$. In Appendix A we have established that this implies that $I_t^A > 0$ and $I_t^H = 0$ ensuring that $A_t/(h_t L_t)$ is rising. This process will continue until $A_t/(h_t L_t) = \Phi$ corresponding to the balanced growth path. A similar argument applies to the case when $A_t/h_t L_t) > \Phi$.

The economy reaches the balanced growth path in finite time. To see this, remember that the households only invest in A_t as long as $A_t/(h_t L_t) < \Phi$, and that the amount of investment needed to reach the steady state ratio Φ equals $\Phi h_0 L_0 - A_0$, which is finite. Thus, the economy will eventually reach the balanced growth path and continue along this path afterwards.

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Notes

1. It should be noted that Grossman and Helpman (1991) mention that the scale effect need not be positive if the increase in question is just an increase in unskilled labor. Our paper elaborates on this argument by distinguishing between the number of individuals and the quality of individuals and by endogenizing the accumulation of skills.
2. A model exhibits semi-endogenous growth if the growth rate in per capita income is determined by a (some) exogenous—non-technological—growth rate(s).
3. Groth (1997) also explores the consequences of the interaction between endogenous R&D and human capital accumulation, complementary to each other. Groth shows—in a model without population growth—that the scale effect on the growth rate is dampened considerably.
4. It is worth noting that the beneficial effect of population growth on economic growth has not found much

empirical support. Kremer (1993) does give some indication of a beneficial effect of population growth on long-run growth in income per capita in the pre-industrial era. But, many empirical studies following Barro (1991) have documented a negative effect using post-world war two data. It is, however, unclear whether this negative effect is temporary, as predicted by standard neoclassical growth theory, or permanent as it can be in our model. Hall and Jones (1998) study the relationship between the long-run level of income and the size of the population. Their analysis reveals no significant effect of population size on the level of income per worker.

5. Several recent papers (Young, 1998; Dinopoulos and Thompson, 1998; Peretto, 1998; Aghion and Howitt, 1998, ch. 12; Howitt, 1999) have explored a new framework which combines vertical (quality improvements) and horizontal (new product lines) innovations. The main result is that, contrary to the semi-endogenous growth framework, economic policy can affect the long-run growth rate. However, the approach also features a scale effect on the level of income per capita, and as a result, population growth stimulates growth in per capita income.
6. It should be noted that all models discussed below, including ours, exhibit a scale effect from Z. Below we demonstrate that a growth equilibrium can be completely scale invariant with respect to population—the empirical worrisome prediction.
7. Hence, we are applying the so-called ‘lab-equipment’ framework. See Rivera-Batiz and Romer (1991).
8. The J/K/S parameter constellation $\alpha + \beta > 1$ would entail explosive growth. Conversely, if $\alpha + \beta < 1$ long-run growth in income per capita becomes negative.
9. It should be noted though that the Proposition holds if the second term of equation (5) is omitted.
10. As an extreme example, the scientific discovery of the earth being round, and not flat, obviously leads to a quality improvement of individuals’ human capital.
11. Thus, the effect is permanent—contrary to, say, standard neoclassical growth theory where a reduction in n only has a temporary positive impact on growth.
12. To see this note first that the production function directly implies that y_t falls instantly after the change. Second, note that χ_t increases by definition and converges back to its previous level according to (7). The development of χ_t implies according to (5) that all future values of h_t decrease after the population increase. Finally, note that y_t is proportional to h_t on the balanced growth path according to (10).
13. However, the growth rate is not necessarily increasing in the share of resources used on R&D as in the R/GH/AH models. If the share is raised at the expense of resources for human capital formation growth may decrease.
14. As usual we assume that discounted utility is bounded implying that $\theta > (1 - \varepsilon)g_y$, where g_y is the long-run growth rate in per capita income derived below.
15. Such dynamics arising from combining accumulation of both R&D and human capital are explored in greater detail in Sørensen (1999).
16. Psacharopoulos (1994) provides a survey on the return on education in different countries. It appears that the rate of return varies across countries from approximately 8–14 percent. Jones and Williams (1998) mention that the private return on R&D typically is found to be around 10–15 percent. Thus, the equilibrium equalization of private returns on R&D and human capital does not seem to be a bad approximation.
17. Notice, that the ratio of R&D expenditures to GDP will be constant through time, as $g_A = g_Y$; an implication which conforms with the available empirical evidence (see e.g. Howitt, 1999).
18. That is, if we had used $L_t(c_t^{1-\varepsilon} - 1)/(1 - \varepsilon)$ as instantaneous utility function in equation (20).

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