Donor policy rules and aid effectiveness

Carl-Johan Dalgaard*

Department of Economics, University of Copenhagen, Studiestraede 6, 1455 Copenhagen K, Denmark

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Abstract

The present paper examines the macroeconomic impact of aid, by introducing endogenous aid allocations into a neoclassical growth framework. On this basis it is shown that donor policies can have important implications for the trajectory of recipients’ GDP per capita. Depending on specific donor policy choices, aid disbursements may lead to faster transitional growth, stagnation or cyclical growth. Moreover, the analysis also suggests that donor policies may be part of the reason why foreign aid is not found to be uniformly effective in raising long-run productivity across recipients.

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1. Introduction

To many poor nations, foreign aid represents a substantial contribution to their Gross National Product (GNI). As Fig. 1 documents, for roughly 20% of the recipients in 2000 aid accounted for more than 15% of GNI. To fully appreciate the relative importance of these transfers, one may observe that the contribution of manufacturing to GNI in the US was 16% in 2000. It is therefore natural to study the macroeconomic consequences of capital transfers on this scale. Specifically, how does foreign aid impact on economic growth?

*Tel.: +45 35 32 44 07; fax: +45 35 32 30 64.
E-mail address: Carl.Johan.Dalgaard@econ.ku.dk

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A large, mainly empirical, literature has pondered this question during the last several decades. Recently, a number of contributions have found a positive impact of aid on growth. However, the estimated impact varies from country to country. In fact, aid never enters linearly, with a significant positive sign, in cross-country regressions. Instead, aid only turns positive and significant once some kind of interaction effect (or non-linearity) is introduced thus potentially allowing for a positive growth impact on average. In this way some researchers conclude that aid only is effective in ‘strong’ policy environments (Burnside and Dollar, 2000), others that aid seems to have worked better outside the geographical tropics (Dalgaard et al., 2004), in places with democratic institutions (Svensson, 1999), or in places where the inflows have been sufficiently modest (the ‘diminishing returns hypothesis’, Hansen and Tarp, 2001).¹ Still, considerable debate surrounds many of these findings. For example, Dalgaard and Hansen (2001) and Easterly et al. (2004)

¹This is a non-exhaustive list of potential explanations for a varying impact of aid on growth. See Clemens et al. (2004) for further references.
re-examine the Burnside and Dollar ‘policy-interaction’ and find it to be non-robust within the original sample, while recent work of Rajan and Subramanian (2005) fail to find any effect of aid at all.

In spite of its diversity this literature shares a common theme: The reason why aid may not be effective is found within the aid receiving nation; in the enacted policies, their climatic circumstances/established institutions, or perhaps in their lack of ‘absorptive capacity’. The present paper takes another approach and explores theoretically whether the apparent variation in aid effectiveness may be ascribed, at least in part, to donor policies.

The basic framework invoked is a two-period overlapping generation’s (OLG) model. As in Glomm and Ravikumar (1997) growth is fuelled by capital accumulation and productive government investments. Beyond this, the theoretical argument rests on three premises or assumptions.

First, foreign aid inflows are used to fund productive government services. Consistent with this assumption a considerable amount of ordinary development assistance (around 30%) is in practise used for investments in infrastructure, or is invested in the production sector (Hjertholm and White, 2000, Table 3.4). Second, borrowing to cover revenue loss from shortfalls of aid is ruled out. This is supported by the cross-country findings of Gemmell and McGillivray (1998) who document that a decline in foreign aid transfers usually is associated with government spending cuts.2 Third, aid inflows are endogenously determined, and decline with (lagged) GDP per capita of the recipient. This assumption is strongly supported by a large literature on the allocation of aid (e.g., Dudley and Montmarquette, 1976; Trumbull and Wall, 1994; Alesina and Dollar, 2000; Alesina and Weder, 2002; Neumayer, 2003; Gates and Hoeffler, 2004). Moreover, this literature also documents convincingly that the strength of the link between aid allocations and recipient income differs from one donor to the next (e.g., Dudley and Montmarquette, 1976; Alesina and Dollar, 2000; Gates and Hoeffler, 2004). The allocation rule, which is estimated by the empirical literature, can be derived from first principles. In particular, MacDonald and Hoddinott (2004) demonstrate that the negative association between aid donations and recipients’ GDP per capita can be interpreted as reflecting inequality aversion of the donor; more inequality adverse donors provide comparatively more aid to the poorest countries. At face value, the empirical findings can then be taken to suggest that the extent of inequality aversion varies from one donor to the next.

Taken together these assumptions imply that aid is productive. Increasing aid flows enable more government investments, which in turn stimulates labor productivity and capital accumulation. In the long run the economy converges to a steady state where both aid flows and GDP per capita are endogenously determined. The long-run impact of aid on growth, including the nature of the transitional path, however, is crucially dependent on the donor policy rule; the feedback loop from growth to aid donations.

2McGillivray (2000) provide further support for both assumptions by showing that aid grants to the Philippines have had no impact on tax and non-tax revenue, while grants have been positively associated with government investments.
Specifically, it is shown that depending on how inequality adverse donors are, when measuring out the amount of aid to donate, the aid receiving nation may prosper, stagnate or end up following a trajectory characterized by (dampened) oscillations. The latter case arises when the donor is sufficiently inequality adverse, though not more so that what is empirically plausible. This mechanism implies, in reduced form, a positive association between volatility in aid per capita growth and GDP per capita growth. As illustrated in Fig. 2 this association does seem to be present in the data. The pure correlation is quite high, at 0.34 (0.37 without Liberia).

More generally, the model predict that, ceteris paribus, countries faced with more inequality adverse donors will not only be more volatile, but also poorer in the long run. This model therefore delivers a novel explanation for the well-known fact that the evolution of GDP per worker is more volatile in poorer places, over the long run (e.g., Ramey and Ramey, 1995).

A second key result of the analysis relates to the empirically detected variation in aid effectiveness. As shown below, the model deliver foundations for parameter heterogeneity in growth regressions along the line of what is detected in Durlauf and Johnson (1995) and others. In the present context, however, this heterogeneity is not caused by differences in the aggregate production function (as Durlauf and Johnson suggest), but is policy induced and relates to the inequality aversion of donors. As argued below, the estimated non-linearity in the aid/growth relationship may be picking up this heterogeneity, rather than capturing fundamental differences in the impact from aid per se. In other words, the theoretical analysis suggests that the empirical literature on aid effectiveness is facing a serious identification problem which so far has not been addressed.

The present paper is related to the theoretical growth literature on aid effectiveness. Two approaches can be identified and distinguished by the way foreign aid is introduced into the model.

One set of contributions have analyzed the dynamic consequences of direct transfers of resources, which enters the budget of the consumer. In a two-country world inhabited by OLG, Galor and Polemarchakis (1987) demonstrate that a transfer from a donor to a recipient may, in some cases, induce a transfer paradox whereby the recipient is left worse off, whereas the donor benefits. They also demonstrate, however, that the transfer can be Pareto improving. Focusing on recipients only, Obstfeld (1999) analyze the impact from a pure income transfer within a standard Ramey–Cass–Koopmans (RCK) model. It is demonstrated that such aid flows only increases consumption in the long run, but leaves long-run labor productivity unaffected. Dalgaard et al. (2004), in contrast, show that the long-run impact from the same kind of aid transfer on labor productivity is ambiguous in a two-period OLG model. In general, the impact depends on the distribution of the transfer between investing (young) and

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3Boone (1996) also analyze aid effectiveness within a RCK model. In Boone’s model aid may stimulate productivity if the government chooses to use the inflow to fund tax cuts, which increases the return on investments. In the absence of such cuts aid is ineffective in raising labor productivity.
Finally, Torvik (2001) examines the ‘Dutch disease hypothesis’ within a two-sector framework involving growth via learning-by-doing. The process of capital accumulation is suppressed. Instead growth is ultimately determined by the allocation of labor between sectors, which drive the process of knowledge creation. It is shown that the long-run impact on productivity growth, from a pure resource transfer, is theoretically ambiguous and depends on the extent of knowledge spillovers between the tradable and non-tradable sectors.

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It is interesting that results differ so markedly between the two standard workhorse models. Which framework is more ‘relevant’ in the growth context depends to a large extent on which savings motive is more important in practice. In the RCK model the life-cycle motive is effectively ignored. Instead the bequest motive for saving represents the chief cause of capital accumulation. In the standard two-period OLG model it is the other way around (see Blanchard and Fisher, 1989, Chapter 3). Empirically, both motives seem to be operative however (Dynan et al., 2002). For a general discussion of the broader virtues of the OLG approach, see Cass and Shell (1980).
Another set of contributions focuses on the impact from aid transfers which enters the budget of the government; such transfers are thought to finance productive government investments, or consumption. Accordingly, this strand of literature is more closely related to the present analysis. In particular the works of Chatterjee et al. (2003) and Chatterjee and Turnovsky (2005, 2007) are noteworthy. These contributions examine the structural conditions of the economy which influence aid effectiveness. Such features include the donor policy choice of tying aid to public investments or not, as well as parameters of the model such as the elasticity of labor supply and the elasticity of substitution between inputs in production. It is demonstrated that aid tied to government investments increases long-run growth and welfare. However, untied aid may lower growth (if labor supply is endogenous) albeit tends to increase welfare in the long run. The analysis is conducted within a RCK model featuring endogenous growth.5

The present paper differs from Chatterjee et al. (2003) and Chatterjee and Turnovsky (2005, 2007) in a couple of important respects. First, the analysis is conducted within an OLG framework. Nevertheless the analysis below also suggests that aid may increase long-run productivity and welfare, when aid is used to finance productive government investments. Hence, in contrast to the case involving a pure transfer, the OLG and RCK models lead to similar qualitative conclusions when aid finances government investments. Second, and more importantly, the present analysis introduces an aid allocation policy rule, which creates a dynamic feedback loop from growth to aid donations. In existing work on aid effectiveness it is assumed that aid transfers rise with GDP, so as to ensure the existence of a balanced growth path. Strictly speaking, however, this assumption is not consistent with the empirical literature on aid allocation, as explained above. The present paper therefore contributes to the literature by showing that a realistic allocation rule has important implications for the impact of aid on steady state income, transitional dynamics and long-run welfare.

The political-economy literature which explores the interaction between donors and recipients is also related (e.g., Casella and Eichengreen, 1996; Svensson, 2000; Lahiri and Raimondos-Møller, 2004). These contributions demonstrate how policy differences across donors, like the timing of disbursements (Casella and Eichengreen, Lahiri and Raimondos-Møller) or the degree to which aid is directed toward the poorest (Svensson) may matter for outcomes.6

Accordingly, a common denominator of this literature and the model below is the notion that donor policies are important for aid effectiveness. A limitation of the present analysis (and all existing growth models on the topic), compared with the political-economy literature, is that it ignores strategic interaction between the donor

5See also Lensink and White (2001) for an analysis of aid effectiveness within an endogenous growth RCK model, where aid finances government investments.

6There is also a political-economy literature which argues that ‘windfall gains’, be that aid or natural resource rents, may induce rent-seeking or corruption, which hamper growth. See Dalgaard and Olsson (2006) for a survey.
and the recipient. This allows for a tractable analysis of the implications of donor policies for long-run productivity, within a fully dynamic general equilibrium model. Allowing for strategic interactions between donors and recipients in the present dynamic setting would be an interesting extension of the analysis, but is left for future research.

2. Analysis

Consider an aid dependent nation in the process of development. The economy in question is inhabited by a large number of individuals whose life span is compressed to two periods: youth and old age. Time is discrete \( t = 0, 1, \ldots \), and extends into the infinite future. The economy is closed, and all markets are competitive. There is a unique output good which can be either consumed or invested. Its price is normalized to 1. Capital depreciates fully during a period. Finally, and for convenience only, population growth and exogenous technical change is suppressed; the size of the work force is normalized to 1.\(^7\)

2.1. Production

Productivity is assumed to be affected by government investments. This is captured, following Arrow and Kurz (1970), by allowing government services to enter the production function:

\[
y_t = k_t^z g_t^{1-z}, \quad z \in (0, 1),
\]

where \( k_t \) represents the stock of capital per worker at time \( t \), while \( g_t \) represents government productive services.\(^8\) The representative firm takes \( g_t \) as given when optimizing. As a result, the inverse factor demand equations are

\[
\alpha \frac{y_t}{k_t} = 1 + r_t
\]

and

\[
(1 - \alpha) y_t = w_t,
\]

where \( r_t \) and \( w_t \) are the real rate of return and the wage, respectively.

2.2. Foreign aid flows

In order to capture the allocation decision of the donor in a realistic way, it is assumed that per capita inflow of aid at time \( t \), \( a_t \), is determined by the recipients’

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\(^7\)Appendix C.1 develops a multi-country version of the model where technological progress is admitted. All the results derived below, in a simpler framework, carries over.

\(^8\)See also Barro (1990) and Glomm and Ravikumar (1997) for endogenous growth applications building on this framework. A previous version of this paper showed how this specification may emerge in reduced form, if government services work so as to lower setup cost of firms, thereby instigating productivity growth via increasing specialization.
level of income per worker at $t - 1^9$:

$$a_t = \phi \cdot y_{t-1}^\lambda, \quad \phi > 0, \quad \lambda < 0. \quad (3)$$

This allocation equation can be derived from first principles, as demonstrated by MacDonald and Hoddinott (2004) and Dudley and Montmarquette (1976). MacDonald and Hoddinott (2004) assume donors are altruistic; the welfare of donors increases in the per capita income of the recipients. In addition they assume (the donor believes) aid can increase income per capita. Given a finite aid budget they proceed to show that the allocation of aid per capita to individual countries fulfill Eq. (3). The parameter $\lambda$ will, under this theory, reflect the degree of inequality aversion on the part of the donor. Technically it is given by (the negative of) the elasticity of substitution between income levels in the social welfare function.\(^{10}\)

From a quantitative perspective Eq. (3) is useful in that it exactly mimics the regression specification implemented in a large number of empirical studies on aid allocation; see Neumayer (2003) for a survey. That is, Eq. (3) implies a log-linear association between aid per capita and income per capita. Essentially every study find a significant negative elasticity of aid per capita with respect to recipients’ GDP per capita. Beyond the microfoundations discussed above, this regularity motivates the assumption of $\lambda < 0$.\(^{11}\)

Another important lesson from the empirical literature is that the numerical size of $\lambda$ varies considerably across donors (e.g., Gates and Hoeffler, 2004; Alesina and Dollar, 2000; Dudley and Montmarquette, 1976). The fact that direct evidence on the size of $\lambda$ is available will be useful below in assessing the viability of various growth trajectories.

\(^9\)It will be clear momentarily that this functional form is not chosen for convenience. It has both a theoretical and an empirical foundation. However, the qualitative nature of the results with regard to steady state comparative statics, and transitional dynamics, are preserved if we instead adopted the general specification $a_t = a(y_{t-1})$, with $a' < 0$, $\lim_{y \to 0} a(y) = a$, $\lim_{y \to \infty} a(y) = a$ and $\lim_{y \to 0} a' < 0$. See Appendix C.2 for details on the dynamical analysis with this general allocation rule.

\(^{10}\)Dudley and Montmarquette (1976) also derive Eq. (3), albeit based on a slightly different setup. They assume donors care about an (unspecified) outcome in each aid receiving country, which is produced subject to diminishing returns by way of foreign aid. In addition, donors care about the supply of a public good which benefits their own population. Hence, the size of the aid budget itself is also determined within the model. Under their theory $\lambda$ would capture (the negative of) the degree of diminishing returns to aid in producing the relevant outcome. As noted below: Empirically, estimates for $\lambda$ frequently exceed 1, which does not sit well with the ‘diminishing returns’ interpretation. I will therefore stick with the ‘inequality aversion’ interpretation of $\lambda$.

\(^{11}\)There is also a literature which examines the cyclical properties of aid. For example, Pallage and Robe (2001) find that aid is procyclical. This, of course, is a statement about how aid moves around a trend. In contrast, the aid allocation literature speaks to the evolution of the trend itself. The general tendency for aid per capita to decline secularly when output per capita rises, supported by the aid allocation literature, is reconcilable with the cyclical properties of aid. Seen through the lenses of the model: A small shock to $\phi$ will instigate more productive investments and thus higher labor productivity in the ‘very short run’ (within a period). Yet, over longer periods of time, the negative association between aid and output would manifest itself. The cyclical movements of aid is therefore unlikely to convey information about the size and sign of $\lambda$.  


While recipient needs do seem to matter for the allocation of aid, donor interests also influences the allocation of aid in practise. For example, political determinants like former colonial status and voting behavior in the UN appears to affect aid allocation (e.g., Alesina and Dollar, 2000). For present purposes such influences are (crudely) captured in Eq. (3) by the parameter $\phi$. Since $\phi$ is independent of the per capita income of the recipient it will be referred to as ‘exogenous’ determinants of $a_t$.

If the allocation of aid is determined in the manner suggested by the theoretical work of MacDonald and Hoddinott, or Dudley and Montmarquette, we can treat Eq. (3) as a stable policy rule and proceed to study its dynamic ramifications. Such stability is assumed in the existing empirical literature on the allocation of aid (i.e. by assuming stable regression parameters), and it is also assumed in the remaining part of the present theoretical analysis.

2.3. Government spending

All aid inflows accrue to the government. At each point in time the government uses the inflow to finance public services. Accordingly, imposing a balanced budget implies:

$$ g_t = a_t, $$

where $a_t$ is the inflow of aid in period $t$. The natural interpretation of Eq. (4) is that the aid transfer is tied to government investments. More specifically, one may view the above specification as a reduced form of the following setup.

Imagine, as is typically done in the literature on the fungibility of aid, that the government derives utility from various kinds of expenditures; in the present case, government consumption and government investments. Foreign aid enters the revenue side of the budget. In addition, we may assume the government can rely on receipts from taxation of labor income; the tax rate on labor income is exogenous and constant through time. Assuming log preferences, the government will spend a constant fraction of the budget on both kinds of expenditures. However, suppose the recipients are faced with terms of conditionality from donors. That is, donors require the recipient to spend at least $\kappa a_t$ on investment, where $\kappa \geq 1$. If the recipient is sufficient poor, and/or sufficiently aid dependant, the government will spend $g_t = \kappa a_t$, and the remaining revenue on government consumption. The formulation (4) is the particular case where $\kappa = 1$ is imposed. Empirically, Feyzioglu et al. (1998) find that aid earmarked for infrastructure investments, like transport and communication, does not seem to be fungible, which supports the assumption $g_t = a_t$.

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12See also Chatterjee et al. (2003), Chatterjee and Turnovsky (2005, 2007).
13See Appendix A for analytical details.
14See e.g. Feyzioglu et al. (1998).
15Of course, suppressing the conditionality parameter $\kappa$ leaves us with one less policy instrument. But since changes in $\kappa$ are ultimately isomorphic to changes in $\phi$, nothing much is lost by this simplification.
16Appendix A does not admit the local government to take the allocation rule into account. If it does, and if the transfer is made in cash, the donor is likely to be faced with the ‘Samaritan’s dilemma’ (e.g.,
To make the analysis more closely comparable to that of Chatterjee et al. (2003) and Chatterjee and Turnovsky (2005, 2007) one could treat $g_t$ as public capital, rather than government services. In that case it would be more meaningful to assume, symmetrically with what holds for the private capital stock, that $g_{t+1} = a_t$, assuming full capital depreciation. This would make parts of the formal analysis slightly more complicated, though the qualitative nature of the results, with respect to steady state comparative statics and transitional dynamics, carry over. Consequently, the main analysis will focus on the simpler specification (4).\textsuperscript{17}

2.4. Consumption and savings

The individual lives for two periods; youth (denoted by subscript $y$) and old age (subscript $o$). The preferences of an individual born at time $t$ is

$$U(c_{y,t}, c_{o,t+1}) = \ln c_{y,t} + \frac{1}{1 + \rho} \ln c_{o,t+1},$$

where $\rho$ is the rate of time preference. During youth the individual works, saves for old age, and consumes. Hence

$$w_t = c_{y,t} + s_t.$$ \hspace{1cm} (5)

During old age the individual consumes his accumulated wealth

$$c_{o,t+1} = (1 + r_{t+1}) s_t.$$ \hspace{1cm} (6)

The problem of maximizing life time utility, subject to Eqs. (5) and (6) implies the following closed form solution for optimal savings

$$s_t = \bar{s} w_t,$$ \hspace{1cm} (7)

where $\bar{s} \equiv 1/(2 + \rho)$.

2.5. The evolution of income per worker

As the old consume their total savings the capital stock in period $t + 1$ is determined by the savings of the young. Combining this with Eqs. (2) and (7) leads to the following law of motion for capital:

$$k_{t+1} = s w_t = \bar{s}(1 - \alpha)y_t.$$ \hspace{1cm} (8)

Next, using Eqs. (3) and (4) in Eq. (1) we can express output per worker in period $t$ as

$$y_t \equiv k_t^{\alpha} \phi^{1-\alpha} y_{t-1}^{(1-\alpha)}.$$ \hspace{1cm} (9)

\textit{(footnote continued)}

Pedersen, 2001). However, the donor may respond by offering tied aid (transfers in kind) instead, as assumed above, thereby alleviating the commitment problem. See Bruce and Waldman (1991) and Coate (1995).

\textsuperscript{17}See Appendix C.3 for details on how the analysis is modified if aid investments fuels government capital accumulation.
Substituting for $k_t$ in Eq. (9), using the lagged version of Eq. (8), and taking logs leaves us with the following law of motion for log income per worker:

$$
\tilde{y}_t = \Omega + \beta \tilde{y}_{t-1} \equiv \Phi(\tilde{y}_{t-1}; \Omega), \quad \tilde{y}_0 \text{ given,}
$$

(10)

where

$$
\tilde{y}_t = \ln y_t, \quad \Omega \equiv \ln[(1 - \alpha)^2 \sigma^a \phi^{1-\sigma}] \text{ and}
\beta \equiv \alpha + \lambda(1 - \alpha).
$$

(11)

Since Eq. (10) is a linear first order difference equation we can solve for the entire time path of log income per worker $\{y_t\}_{t=0}^{\infty}$ in closed form:

$$
\tilde{y}_t = \tilde{y}^* - \beta'(\tilde{y}^* - \tilde{y}_0), \quad \text{for } |\beta| < 1,
$$

(12)

where

$$
\tilde{y}^* = \frac{\Omega}{1 - \beta} > 0,
$$

(13)

is the steady state level of (log) income per worker.

Before examining the impact of donor policies on the transitional dynamics of the economy the following results can be stated:

**Proposition 1.** Donor policy rules and steady state income per worker.

(i) An exogenous increase in foreign aid increases steady state income per worker, $\partial \tilde{y}^*/\partial \phi > 0$.

(ii) Increased poverty bias reduces long-run income per worker $\partial \tilde{y}^*/\partial \lambda > 0$.

**Proof.** Follows immediately from Eq. (13) and the definition of $\Omega$. □

Hence exogenous aid inflows increases income per worker in the long run. The intuition for the result is straightforward; an additional (exogenous) inflow of foreign aid yields more productive government services, which translates into increasing wages, savings and ultimately spurs capital accumulation. Enhanced inequality aversion on the part of the donor, i.e. a reduction in $\lambda$, works so as to decrease the aggregate returns to scale in broad capital. As a result, any given amount of domestic savings and foreign aid donations ‘buys’ less productivity in the long run. But aside from these aspects, the model otherwise has the steady state properties of a standard Diamond model with Cobb–Douglas production technology and preferences (e.g., Romer, 2001, Chapter 2).

Turning to the issue of transitional dynamics, it should be clear that the characteristics of the trajectory followed by the economy hinges on the sign and size of $\beta$. Under the assumption that all countries share a common production technology, and thus share the parameter $\alpha$, the degree of inequality aversion among donors, $\lambda$, becomes the key parameter. Indeed, if donors differ in this respect, as the empirical literature on aid allocation suggests, it can rightly be considered a country specific parameter, since the composition of donors differs from one recipient to the next.

Consider first a case which can be labeled weak inequality aversion, where the degree of inequality aversion is bounded from below:

$$
0 < \beta < 1 \iff -\frac{\alpha}{1 - \alpha} < \lambda < 1.
$$

(14)
The transitional dynamics of an economy fulfilling this parameter restriction is illustrated in Panel A of Fig. 3. In this case the transition to steady state is monotonic, and the steady state is globally stable. Thus, if the tendency to reduce aid as the economy grows is not ‘too great’ the trajectories of income per worker, for an aid recipient and a non-recipient nation, are qualitatively indistinguishable.

Consider next the case we may label strong inequality aversion:

\[-1 < \beta < 0 \iff \frac{1 + \alpha}{1 - \alpha} < \lambda < \frac{-\alpha}{1 - \alpha}. \tag{15}\]

The associated transitional dynamics are depicted in Panel B of Fig. 3. While the stability properties of this economy are identical to the case of weak inequality
aversion, the trajectory followed by the economy is very different in nature. The transitional dynamics are characterized by dampened oscillations. The intuition is as follows. Suppose donors reduce aid in the face of an increase in economic activity in the previous period. A sufficiently marked reduction in aid inflows lowers productive government investments, reduces aggregate labor productivity and thereby wages. Consequently, savings decline, which instigates a decline in future labor productivity. In the following period however, donors will tend to react to the contraction in economic activity, and after an infusion of aid, growth will be revitalized. Then the process may begin anew.

The remaining cases, illustrated in Panels C and D, correspond to two different ‘knife-edge’ conditions. Panel C illustrates the case where

\[ \frac{1}{1 - \frac{\alpha}{\gamma}} = \lambda. \]

Hence the economy immediately reaches \( \Omega \), and absent disturbances to the system, stays there. In other words, the economy stagnates, even though it receives foreign aid in every period. In Panel D the elasticity of income per worker with respect to aid disbursements fulfill

\[ \frac{1 + \frac{\alpha}{\gamma}}{1 - \frac{\alpha}{\gamma}} = \lambda. \]

In this case the economy is characterized by a two-period cycle. Convergence to steady state does not occur; the growth rate fluctuates. Finally, for completeness, we may note that if \( -\beta > 1 \) (corresponding to \( \lambda < -\frac{1+\alpha}{1-\alpha} \)), the economy ventures along a divergent path where \( \lim_{t \to \infty} |y_t| = \infty \).

The relationship between donor policies and the transitional dynamics of the economy is summarized in Proposition 2.

**Proposition 2.** Donor policy rules and transitional dynamics. Assume \( \lambda \) is chosen such that \( -1 \leq \beta < 1 \). The degree of inequality aversion determines the characteristics of the transitional path. Convergence is (A) monotonic if \( -\frac{\alpha}{\gamma} < \lambda < 1 \), (B) characterized by dampened oscillations if \( -\frac{1+\alpha}{1-\alpha} < \lambda < -\frac{1+2\alpha}{1-2\alpha} \), (C) instantaneous (stagnation) if \( -\frac{\alpha}{1-\alpha} = \lambda \) or (D) absent (two-period cycle) if \( -\frac{1+3\alpha}{1-3\alpha} = \lambda \).

From an empirical standpoint case (A) to (C) can be regarded as the more plausible scenarios. Alesina and Dollar (2000) provide estimates for \( \lambda \) across 12 major donors. In their study \( \lambda \) is estimated to fall in the interval \([-2.08, -0.25]\), and for half of the donors \( \lambda < -1 \) is obtained. This group includes major contributors like the US and Canada. Gates and Hoeffler (2004) find \( \lambda \in [-1.4, -0.29] \) in a sample comprising 11 donors. Donors with a \( \lambda \) below \(-1\) are USA, Canada, Norway and Denmark. Interestingly, Gates and Hoeffler also examine aid disbursed by all multilateral agencies. Here they find an estimate for \( \lambda \) of \(-1.5\). Finally, in Dudley and Montmarquette (1976) \( \lambda \in [-1.02, -0.32] \).

Hence, the empirical evidence on \( \lambda \) does not dramatically limit the viable set of outcomes from Proposition 2; plausible values for \( \lambda \) may generate monotonic convergence, oscillations as well as stagnation. For example, for oscillations we
require \( \lambda \) to be smaller than \(-\frac{2}{3}\), if \( \alpha = \frac{1}{3} \). If \( \alpha \) is assumed to be as high as 0.6, which Kraay and Raddatz (2007) has recently advocated in favor of (in the context of poor countries), the requirement is \(-1.5\). Inequality aversion of this magnitude cannot be ruled out on the basis of estimates from the empirical literature. In contrast, if capital’s share indeed is around 0.6 the two-period cycle requires a \( \lambda \) around \(-4\). Inequality aversion of this magnitude can be rejected on empirical grounds.

Regardless, however, of whether oscillations occur or not, the model suggest a clear relationship between growth volatility, suitably measured, and donor policies. Indeed, it can be shown that the coefficient of variation of the growth rate, \( CV_{\gamma} \), between \( t = 1 \) and \( t = T \), is given by\(^{18}\):

\[
CV_{\gamma} = \sqrt{\frac{T[1-(\beta^2)^T](1-\beta)^2}{(1-\beta^2)(1-\beta^T)^2}} - 1
\]

for \( |\beta| < 1 \). Since the parameter \( \beta \) is determined by the degree of inequality aversion of donors, it follows that differences in measured growth volatility are determined by variations in donor policy rules. The implied relationship between growth volatility and \( \beta \) (thus, implicitly, the parameter \( \lambda \)) is depicted in Fig. 4. Consequently, we have the following result:

**Proposition 3.** Donor policy rules and growth volatility. Let growth volatility be measured by the coefficient of variation, and assume \( \lambda \) is such that \( |\beta| < 1 \). Then the volatility of the growth rate is higher in countries submitted to more inequality averse donors.

\(^{18}\)\( CV_{\gamma} = \frac{\sigma_{\gamma,T}}{\bar{\gamma}_T} \), where \( \sigma_{\gamma,T} \) is the standard deviation of growth over the period \( t = 1, \ldots, T \), whereas \( \bar{\gamma}_T \) is the average growth rate over the same period.
There is an interesting corollary to this result. Proposition 1 says that countries exposed to more inequality averse donors will be poorer in the long run, \textit{ceteris paribus}. Combining this fact with Proposition 3 implies the following:

**Corollary.** Poorer countries will tend to have more volatile growth.

This correlation, which is documented e.g. by Acemoglu and Zilibotti (1997), is in the present context the result of (variations in) donor policy rules.19

A final issue worth examining is the welfare impact from donor policies. The welfare criterion adopted in the present context is the discounted sum of utility across an infinite sequence of generations (see De la Croix and Michel, 2002, p. 91). That is

\[ W = \sum_{t=0}^{\infty} U_t \Delta^t, \]

where \( U_t = \ln(c_t) + \frac{1}{1+\rho} \ln(c_{t+1}) \) is the utility of a generation born at time \( t \). Accordingly, \( \Delta \) is the discount factor for the social planner, which weights together the utility of different generations.

Since the model admits an analytical solution to the entire growth trajectory (cf. Eq. (12)) it is feasible to calculate \( W \). The results are summarized in the following proposition:

**Proposition 4.** Donor policy rules and welfare. Assume the welfare function (16), and let the growth trajectory be described by (10). Then

\[ W \propto \left\{ \frac{2 + \rho}{1 - \Delta} + \frac{1 + \rho + \beta}{1 - \beta \Delta} \right\} \ln(y^*) \ln(y^*_{0})/C_{0} + \frac{1 + \rho + \beta}{1 + \rho + \beta} \ln(y^*_{0})/C_{0} - \beta \Delta. \]  

Social welfare increases in exogenous aid (\( \phi \)) and decreases in the degree of inequality aversion of the donor (\( -\lambda \)).

**Proof.** For a derivation of Eq. (17), see Appendix B. The comparative statics follow immediately from the definition of \( \beta \), Eq. (11), and from Proposition 1.

The key insights from Propositions 1–4 is that donor policy choices matter for long-run productivity, the nature of the transitional dynamics, the amount of growth volatility over the long run, and long-run welfare. The next section shows that donor policy rules also have bearing on aid effectiveness as measured in the empirical literature on the topic.

19Aside from documenting that the variability of growth in current-day poor economies is larger than in their richer counterparts, Acemoglu and Zilibotti proceed to develop a theory of how financial development may facilitate the diversification of risk related to high-return investment projects. Such a theory is also, as pointed out by the authors, consistent with historical evidence suggesting that current-day rich economies faced more volatile growth around the time of take-off compared with today. The present ‘aid explanation’ will only have bearing on the experiences of current-day poor.
2.6. Donor policies and aid effectiveness

The empirical literature on aid effectiveness typically focuses on growth, rather than on long-run income per worker. In a neoclassical growth framework, however, steady state income per worker and (transitional) growth are two sides of the same coin. Indeed, at any point in time, we may write the growth rate of income per worker, \( \gamma_t \), as

\[
\tilde{y}_t - \tilde{y}_{t-1} = \gamma_t = \left( \frac{\tilde{y}^* - \tilde{y}_0}{1 - \beta} \right) \beta^{t-1}.
\]

The discussion which follows focuses for convenience on the relationship between aid transfers and \( \tilde{y}^* \), but it should be clear that any point made relates directly to how empirical work is done in this field.

In the steady state the level of GDP per worker and the inflow of aid per capita are jointly determined. Using Eqs. (3) and (13) the system of equations can be written

\[
\tilde{y}^* = \ln[(1 - \alpha)\tilde{s}] + \frac{\alpha}{1 - \alpha} \ln \tilde{s} + \ln a^* \equiv \tilde{y}(a),
\]

\[
\ln a^* = \ln \phi + \lambda \tilde{y}^* \equiv a(\tilde{y}^*),
\]

which pins down \( a^* \) and \( \tilde{y}^* \). If we so desire, noise terms can be added to both equations thus making it an ‘empirical’ model. The system is depicted in Fig. 5.

Notice that the ‘pure’ impact of aid on long-run productivity is given by the slope of the \( y(a^*) \) schedule. As should be clear, the slope is 1 which implies that aid inflows

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Fig. 5. Steady state income per worker and aid per capita.
should increase GDP per worker to the same extent in all countries. From the perspective of aid effectiveness the empirical challenge is that of identifying the slope of $y(a^*)$.

It is immediately obvious, however, that a standard OLS regression analysis is unlikely to produce this slope, due to the presence of the feedback loop represented by the allocation equation, $a(\hat{y}^*)$. Indeed, seen through the lenses of the model one would expect an OLS regression to produce a slope estimate for $y(a^*)$ which is biased toward zero. This basic identification problem is well recognized in the empirical literature on the topic; instrumental variable techniques is therefore typically invoked.20

Unfortunately, the identification problem is so complicated that it cannot be solved by invoking standard instrumental variables techniques. The problem is that the slope of the $a(\hat{y}^*)$ schedule is most likely country specific. As mentioned above, $\lambda$ varies from one donor to the next, and the composition of donors differ from one recipient to the next.

To assess the consequence of this heterogeneity for the results stemming from cross-country examinations of the aid/growth nexus, it is useful to look at the reduced form solution for income per capita:

$$\hat{y}^* = -\frac{\alpha}{1 - \alpha - \lambda(1 - \alpha)} \ln(1 - \alpha) + \frac{\alpha}{1 - \alpha - \lambda(1 - \alpha)} \ln 5 + \frac{1}{1 - \lambda} \ln \phi.$$ 

In theory the elasticity of income per worker with respect to an exogenous increase in aid per capita is $\frac{1}{1 - \lambda}$. This elasticity will be smaller in countries receiving aid from highly inequality adverse donors. Therefore, although the true impact of aid on growth is the same across countries (i.e. 1), the measured impact from an exogenous increase in aid is likely to differ depending on donor policies, and therefore donor selection.21

This heterogeneity can motivate many of the empirically detected non-linearities. Consider, for example, the regularity that aid features diminishing returns (e.g., Hansen and Tarp, 2001). It is possible, as commonly argued, that the cause of this finding is a lacking ability to absorb aid in highly aid dependent nations. But the same regularity could also be rationalized by donor selection. The argument runs as follows. Aid donors tend to be inequality adverse, for which reason one would expect that the poorest countries in the world, ceteris paribus, are the ones that receives the most aid. Moreover, the presence of relatively inequality adverse donors (like the Scandinavian ones) should tend to be greater in the poorest places. As a result, we would expect to find a smaller impact of aid on productivity in highly aid dependent

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20 See e.g. the discussion in Dalgaard et al. (2004, Section 2).

21 Actually, the model suggests that the impact of all the right-hand side variables in the growth regression could be country specific. Such heterogeneity has been detected in a number of studies, following Durlauf and Johnson (1995). However, whereas the finding of various growth ‘regimes’ in this literature typically is seen as suggestive evidence in favor of differences in the aggregate production technology, the present model demonstrate that the observed heterogeneity could (also) be policy induced, and associated with aid disbursements.
countries (due to the concentration of relatively inequality adverse donors), compared to less aid dependent countries.

As another suggestive example, consider Fig. 6. The figure shows the correlation between selected coefficients from the aid allocation study by Alesina and Dollar (2000).

As can be seen, the coefficient associated with income per capita (i.e. $\lambda$) is highly and negatively correlated with the coefficient attached to whether the recipient is democratic or not. The pure correlation is $-0.7$. Hence, inequality adverse donors are apparently also more sensitive to whether recipients are democratic or not. At the intuitive level, this could also motivate an interaction effect between aid and democracy.

Clearly, other interactions could be explained insofar as the variable being interacted with aid plausibly is picking up donor selection, which in turn will determine the ‘effective’ $\lambda$ faced by individual recipients.

The bottom line is that the observed non-linearity in the aid/growth relationship may not be caused (solely) by variation in the ‘true’ impact of aid on growth. Instead, it could (in part) be caused by the way aid is given, the policy rule of donors, and therefore be the result of donor selection.
3. Conclusion

The present paper has examined the dynamic implications of endogenous aid per capita disbursements for the growth process in aid dependent nations. The analysis offers an explanation for why GDP per capita growth is more volatile in poor countries, and links this phenomenon to foreign aid disbursements and donor policy rules.

More generally it delivers an example of policy-induced bifurcations of the dynamical system. If the donor composition of an aid dependent nation changes, and the associated policy rule therefore changes, the nature of the dynamical system may change; cyclical adjustment may replace stagnation or smooth convergence, for example. Likewise, when comparing different aid dependent nations the observed trajectory of GDP per capita may look very different, even though time averaged aid flows (and investment rates, etc.) are fairly similar.

Some of these predictions may be submitted to empirical testing. For example, whereas previous research has argued that growth volatility is determined by e.g. lack of financial development (Acemoglu and Zilibotti, 1997) and terms of trade volatility (Kose et al., 2003), the analysis in the present paper suggests that donor composition should be added to the list of explanatory variables.

The analysis also provides an interpretation of the complexity of empirically inquiring into the aid growth nexus. Previous research has discovered significant variation across recipients in the effectiveness of aid in raising productivity. The usual interpretation is that domestic structural characteristics are to blame; geography, political institutions, policies or the magnitude of the inflow, etc. However, as the analysis shows, the interaction effects underlying this conclusion might proxy for donor composition, and ultimately, for variations in donor policies. Whether variation in aid effectiveness is caused by domestic conditions, or reflect the influence of donors, is obviously critically important to the issue of how to elevate the impact of aid. Hence, future empirical work on aid effectiveness should attempt to control for such composition effects directly when examining the inflows macroeconomic consequences.

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Appendix A. Some microfoundations for government behavior

Consider a government with the following utility function:

$$\max U(g_I, g_c) = \beta \ln g_I + \ln g_c,$$

where $g_I$ is the inflow of foreign aid and $g_c$ is the government consumption.
where \( g_I \) is government investments, \( g_c \) is government consumption. Aid enters the budget:

\[
\tau w_t + a_t = g_{I,t} + g_{c,t},
\]

where \( \tau w_t \) is revenue from labor income, \( \tau \) is assumed to be constant over time. The problem then is to select

\[
g_I = \arg \max \{ \beta \ln g_I + \ln(\tau w_t + a_t - g_{I,t}) \}
\]

yielding the following first order conditions:

\[
g_I = \beta g_c.
\]

So

\[
g_{I,t} = \frac{1}{1 + \beta} (\tau w_t + a_t), \quad g_{c,t} = \frac{\beta}{1 + \beta} (\tau w_t + a_t).
\]

**Conditionality.** Suppose the donor ties aid to investments. That is, requires the government to spend at least \( g_{I,t} = \kappa a_t \) on \( g_{I,t} \). If so, then (when \( a_t > 0 \)):

\[
g_{I,t} = \begin{cases} \kappa a_t & \text{if } \kappa a_t \geq \frac{\tau w_t}{(1 + \beta)} \\ \frac{1}{1 + \beta} (\tau w_t L + a_t) & \text{otherwise}, \end{cases}
\]

implying

\[
g_{c,t} = \begin{cases} \tau w_t - \kappa a_t & \text{if } \kappa a_t \geq \frac{\tau w_t}{(1 + \beta)} \\ \frac{\beta}{1 + \beta} (\tau w_t + a_t) & \text{otherwise}. \end{cases}
\]

So the ‘constrained economy’, with sufficiently low income and large aid inflows will be characterized by \( g_{I,t} = \kappa a_t \).

The consequence for the dynamics of the economy is minor, given this assumption. The evolution of GDP per worker becomes

\[
y_t = \hat{\Omega} + \beta y_{t-1} = \hat{\Phi}(y_{t-1}, \hat{\Omega}),
\]

where \( \hat{\Omega} = \Omega + (1 - z) \ln \kappa + \ln(1 - \tau) \). The term \( \ln(1 - \tau) \) follows from the fact that the period 1 budget constraint of the household now becomes \( c_{y,t} + s_t = (1 - \tau)w_t \). The parameter \( \Omega \) is as defined in the text. Accordingly, the only effect from this extension is to modify the constant term \( \Omega \), which is not important for Propositions 1–4.
Appendix B. Welfare

For any given generation $t$ life-time utility is

$$U_t = \ln c_t + \frac{1}{1 + \rho} \ln c_{t+1}.$$  

If we insert the solutions to the optimization problem it follows that

$$U_t \propto \ln(y_t) + \frac{1}{1 + \rho} \ln(y_{t+1}),$$

where constant terms have been suppressed. Next, since the law of motion for output per worker is

$$\log(y_{t+1}) = (1 - \beta) \log(y^*) + \beta \log(y_t)$$

we get

$$U_t \propto \left(\frac{1 + \rho + \beta}{1 + \rho}\right) \ln y_t + \frac{1 - \beta}{1 - \Delta} \ln(y^*).$$

Define the social welfare criterion as the discounted sum of life-time utilities across generations

$$W = \sum_{t=0}^{\infty} U_t A^t \propto \left(1 + \frac{\beta}{1 + \rho}\right) \sum_{t=0}^{\infty} \ln y_t A^t + \frac{\ln(y^*)}{1 + \rho} \frac{1 - \beta}{1 - \Delta}.$$  

To solve this expression, note that $\ln(y_t) = \ln(y^*) - \beta[\ln(y^*) - \ln(y_0)]$. Upon substitution the sums can be worked out. After some rearrangements we find

$$W \propto \left\{2 + \frac{\rho + \beta}{1 - \Delta} \right\} \ln(y^*) + \frac{1 + \rho + \beta}{1 + \rho} \frac{\ln(y_0)}{1 - \beta A},$$

as stated in the text.

Appendix C. Extensions

C.1. Steady state growth and many aid recipients

Suppose the production technology in the final goods sector, Eq. (1), is modified to

$$y_t = k_t^2 E_t g_t^{1 - z},$$

where $E_t$ represent efficiency gains resulting from exogenous technological progress. Let $E_{t+1} = (1 + \varepsilon)E_t$, $\varepsilon > -1$. Moreover, define GDP per efficiency unit of labor as $\hat{y}_t$, and capital per efficiency unit as $\hat{k}_t$. Then

$$\hat{y}_t = \hat{k}_t g_t^{1 - z}.$$  

Capital will still be accumulated in accordance with

$$k_{t+1} = \bar{s}(1 - \alpha)y_t.$$
so in efficiency units $\hat{k}_{t+1} = \frac{3(1-\alpha)}{1+\varepsilon} \hat{y}_t$. Inserted into the production function

$$\hat{y}_t = \left[ \frac{\bar{s}}{1+\varepsilon} (1-\alpha) \right]^x \hat{y}_{t-1}^x g_{t-1}^{1-x}. \quad (18)$$

Finally, we need to add aid allocation. In order to ensure that aid inflows does not asymptote to zero it is assumed that

$$a_t = \phi \cdot \left( \frac{y_{t-1}}{y_{R,t-1}} \right)^{\lambda}, \quad \phi > 0,$$

where $y_{R,t-1}$ is income per capita of a reference country. In principle, the reference country could be the donor, another less developed country (LDC), or an average of LDCs. In the remaining it will be assumed the reference country is another LDC.

Using that $g_t = a_t$ along with the modified policy rule in Eq. (18) we get

$$\hat{y}_t = (1-\alpha)^x \left( \frac{\bar{s}}{1+\varepsilon} \right)^x \hat{y}_{t-1}^{\lambda(1-\alpha)} \phi^{1-x} \left( \frac{\hat{y}_{t-1}}{y_{R,t-1}} \right)^{\lambda(1-\alpha)},$$

where it has been assumed that both the reference country and the one under consideration share the same underlying rate of exogenous technical progress. The law of motion for income per efficiency unit of labor becomes

$$\hat{y}_t = (1-\alpha)^x \left( \frac{\bar{s}}{1+\varepsilon} \right)^x \hat{y}_{R,t-1}^{\lambda(1-\alpha)} \phi^{1-x} \hat{y}_{t-1}^{\lambda(1-\alpha)},$$

which now obviously depends on the path of $\hat{y}_R$.

Consider the law of motion for income per efficiency units of labor in the reference nation

$$\hat{y}_{R,t} = (1-\alpha)^x \left( \frac{\bar{s}_R}{1+\varepsilon} \right)^x \phi^{1-x} \hat{y}_{R,t-1}^{\lambda(1-\alpha)}.$$

Accordingly, this nation will be converging smoothly toward steady state, where

$$\hat{y}^*_R = \left[ (1-\alpha)^x \left( \frac{\bar{s}_R}{1+\varepsilon} \right)^x \phi^R \right]^{1/(1-\alpha)}.$$

When the reference country is in steady state the law of motion for log income per efficiency units of labor in all other aid receiving nations becomes

$$\ln \hat{y}_t = \bar{Q} + \beta \ln \hat{y}_{t-1},$$

where $\bar{Q} = \Omega - \lambda \alpha \bar{y}^*_R - \alpha \ln(1+\varepsilon)$. $\Omega$ and $\beta$ are defined in the text above.

Therefore, Propositions 1–4 continues to hold. Of course, in this version of the model individual countries’ growth rate will be oscillating (if $-1 < \beta < 0$) around its steady state growth rate, $\varepsilon$. 
C.2. A generalized aid allocation rule

Suppose we, instead of Eq. (3), adopt

\[ a_t = a(y_{t-1}), \]

where \( a'_t < 0 \) for all \( y \) (based on empirical considerations from a qualitative angle) and that \( \lim_{y \to 0} a(y) = \bar{a} \), \( \lim_{y \to \infty} a(y) = \underline{a} \), \( \lim_{y \to 0} a'_t < \infty \); \( \bar{a} > \underline{a} \). But other than that we leave the functional association unrestricted.

In this case the dynamical system for GDP per worker becomes

\[ y_t = \left[ \bar{s}(1 - \alpha) \right]^2 y^\alpha_{t-1} a(y_{t-1})^{1-\alpha} = F(y_{t-1}). \]

We may note that \( F(0) = 0 \), and that the law of motion for output is non-linear. Specifically, the slope of \( F \), evaluated in the vicinity of the steady state, is

\[
\frac{\partial y_t}{\partial y_{t-1}} \bigg|_{y^*} = F' = \left[ \bar{s}(1 - \alpha) \right]^2 \left\{ \frac{\alpha}{y^*} \frac{F(y^*)}{y^*} + (1 - \alpha) \frac{F'(y^*)}{a'(y^*)} \right\}
\]

\[
= \left[ \bar{s}(1 - \alpha) \right]^2 \frac{F(y^*)}{y^*} \left[ \alpha + (1 - \alpha) \left( \frac{y^* d'(y^*)}{a'(y^*)} \right) \right] \equiv 0,
\]

depending on whether \( \frac{\partial y_t}{\partial y_{t-1}} \bigg|_{y^*} = -\lambda(y^*) \equiv \frac{\partial}{\partial y} \mid_{y^*} \). However, \( \lim_{y \to 0} F' = \infty \). Hence, \( F \) ‘starts’ at the origin, begins with a positive slope which may turn negative at some point depending on the curvature of \( a(\cdot) \). Note that \( \lambda(y^*) \) is the elasticity of disbursements with respect to GDP per worker, evaluated in the steady state.

It should be clear that all the cases discussed in Proposition 2 are still theoretically viable. For example, oscillations require \( \frac{\partial y_t}{\partial y_{t-1}} < 0 \), and \( \frac{\partial y_t}{\partial y_{t-1}} < 1 \) for stability. Hence, as long as \( \lambda(y^*) \) is sufficiently large (in absolute value), this is a viable outcome. The conditions stated in the paper for this outcome (invoking Eq. (3)) are therefore still valid; in a more general setting the same condition would apply \textit{locally} (i.e. in the vicinity of steady state GDP per worker).

C.3. Government capital investments

Suppose \( g \) is publicly financed \textit{capital}. In this case Eq. (4) should be replaced by

\[ g_{t+1} = a_t. \]

Accordingly, the stock of public capital available in period \( t + 1 \) depend solely on (aid financed) investments in period \( t \), maintaining the assumption that capital depreciates fully during a period. In all other respects the model remains unaltered.

The consequence of this reformulation, is that Eq. (10) is replaced by a second order difference equation:

\[ y_t = \Omega + \alpha y_{t-1} + \lambda(1 - \alpha)y_{t-2}. \]

For dampened oscillations to occur we require (Sargent, 1987, p. 189):

\[ \alpha^2 + 4\lambda(1 - \alpha) < 0. \]
Clearly, this condition is less likely to be fulfilled if \( x \) is ‘large’. Accordingly, suppose \( x = 0.6 \). Then oscillations in the growth rate during transition to steady state requires:

\[
\frac{0.6^2}{6 \cdot (1 - 0.6)} \approx 0.2 < -\lambda.
\]

Consulting the empirical estimates for \( \lambda \) in Alesina and Dollar (2000) or Dudley and Montmarquette (1976) reveals that this condition is met for all donors in their samples.

Technically speaking, oscillations may become explosive if \( -\lambda(1 - x) > 1 \). Given \( x = 0.6 \), however, this would require \( -\lambda > 2.5 \), which can be rejected on empirical grounds.

In sum, scenarios involving oscillations arise for empirically reasonable parameter configurations in the baseline model, as well as in a reasonable extension of the model.

References