Resource and Energy Economics 33 (2011) 782–797

Contents lists available at ScienceDirect

Resource and Energy Economics

journal homepage: www.elsevier.com/locate/ree

Energy distribution and economic growth☆

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ARTICLE INFO

Article history:
Received 30 September 2008
Received in revised form 10 June 2010
Accepted 27 April 2011
Available online 27 May 2011

JEL classification:
O11
O13
Q43

Keywords:
Economic growth
Energy
Power laws
Networks

ABSTRACT

This research examines the physical constraints on the growth process. In order to run, maintain and build capital energy is required to be distributed to geographically dispersed sites where investments are deemed profitable. We capture this aspect of physical reality by a network theory of electricity distribution. The model leads to a supply relation according to which feasible electricity consumption per capita rises with the size of the economy, as measured by capital per capita. Specifically, the relation is a simple power law with an exponent assigned to capital that is bounded between 1/2 and 3/4, depending on the efficiency of the network. Together with an energy conservation equation, capturing instantaneous aggregate demand for electricity, we are able to provide a metabolic-energetic founded law of motion for capital per capita that is mathematically isomorphic to the one emanating from the Solow growth model. Using data for the 50 US states 1960–2000, we examine the determination of growth in electricity consumption per capita and test the model structurally. The model fits the data well. The exponent in the power law connecting capital and electricity is 2/3.

☆ We owe a special thanks to Jayanth Banavar for generously sharing his expertise on the subject of energy distributing networks. We also thank Michael Burda, Henrik Hansen, Martin Kaae Jensen, seminar participants at the Universities of Birmingham, Copenhagen, Gothenburg, Hannover, Humboldt University Berlin and the 2008 SURED Conference, two anonymous referees and the editor, Sjack Smulders, for helpful comments.

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0928-7655/$ – see front matter
doi:10.1016/j.reseneeco.2011.04.004

1. Introduction

The work of Domar (1946) and Solow (1956) marked the beginning of the formal analysis of the growth process where physical capital accumulation is seen a key growth engine. This body of research has unveiled fundamental structural characteristics which impinge on the determination of labor productivity in the long run: savings, population growth, technological change and more. These factors share the common feature that they importantly affect the ability of an economy to mobilize resources for the purpose of capital accumulation.

At the same time, the basic neoclassical growth theory abstracts from the fact that it makes little sense to acquire a piece of machinery, at a particular time and place, unless the machine can be supplied with electricity and put to use. Surely, it is of first order importance that an economy is able to *distribute* electricity across the economy to the sites where investments are deemed profitable.[1] Omitting the *physical* preconditions for growth may be highly problematic as such factors can represent a binding constraint on economic growth. Hence, in the present study we provide an attempt to model electricity distribution, and take a first step towards examining its implications for the growth process.[2]

Electricity networks are highly complex systems; too complex, one might think, for macroeconomic modeling. Fortunately, progress has been made in the natural sciences in describing and modeling the aggregate properties of similarly complex networks. The work of West et al. (1997, 1999) and Banavar et al. (1999, 2002) is a case in point. By modeling biological organisms as an energy distributing networks these authors have been able to show, in keeping with the evidence, why the energy needs of an organism as a whole rises with the size of the organism in accordance with $B = B_0 \cdot m^b$, where B is basal metabolism, m is body mass, B_0 is a constant, and $b = 3/4$.[3] In the present context one might wonder if a similar sort of result could be derived in the case of an electricity network, thus linking electric power consumption and a measure of the "size" of an economy, such as the capital stock.

At first it may seem far-fetched to believe that empirical laws and mathematical theories pertaining to biological organisms should have any sort of bearing on man-made networks. But upon reflection the link is perhaps not improbable, for three reasons.

First, the cardiovascular system and the power grid share the feature of being energy distributing networks (of nutrients in the former case, electricity in the latter). Conceptually they are therefore highly related. Of course, the networks are very different at the more detailed level: in appearance and in terms of the matter being distributed. Nevertheless, and in spite of being developed for the investigation of biological systems, the inventors of the network theory that we employ to the study of electricity distribution below suggest themselves that their theory could be adapted to the case of electrical currents (Banavar et al., 1999, p. 132).

Second, there is a good reason why biological networks and man-made networks would come to have similar aggregate properties. Both biological and man-made networks are developed over time through a process of gradual optimization. In the former case this occurs through natural selection; in the latter it is the result of deliberate decisions to rework, extend and improve the efficiency of the network in question. Undoubtedly, the process of natural selection has worked to produce efficient networks. Quite possibly, man-made networks have moved in a similar direction. As efficient networks have certain unique properties (e.g., minimal transmission losses), biological and man-made networks should have some aggregate characteristics in common.

[1] Empirically, the close link between electricity and growth is well documented. For instance, in Henderson et al. (2009) the authors propose to use satellite data on lights at night as an alternative proxy for GDP.

[2] The present analysis is therefore related to the literature on infrastructure and economic growth. The traditional approach essentially consists of adding another input into the production function, thus capturing infrastructure capital (e.g., Arrow and Kurz, 1970 and many others since). Our approach to model the influence from the electricity infrastructure will differ in that we will provide a model for the network itself, and derive the critical associations between electricity use and capital. At the same time we abstract from other dimensions of infrastructure, like roads, ports, etc. Also, we abstract from the problems associated with the production of electricity, which in practise requires the use of natural resources. See e.g. Stiglitz (1974) and Suzuki (1976) for a discussion of sustainable growth in the presence of exhaustible natural resources.

[3] This formula is known as "Kleiber's law" (Kleiber, 1932), which, remarkably, holds across biological organisms spanning 27 orders of magnitude in mass; from the molecular level up to whales (West and Brown, 2005).

Third, physicists have already started to apply the empirical methods from the study of biological organisms to the case of artificial networks in an effort to uncover universal scaling laws with bearing on human societies (e.g., Bettencourt et al., 2007).[4] This research strategy should be seen and appreciated through the lens of the remarks above. Biological networks and man-made networks have similar objectives, namely to distribute resources for final application (be it cell maintenance or the charging of an ipad) as efficiently as possible. Both networks are under constant selective pressure through which they edge towards optimality. In the end it would actually be rather surprising if biological networks and man-made networks did *not* have many common characteristics.

Accordingly, in this study we begin by developing a model of an economy viewed as a transportation network for electricity. The model predicts that electricity consumption per capita (e) can, loosely speaking, be seen as the economic counterpart to metabolism, and capital per capita (k) as the counterpart to body size; the association between electricity and capital is concave, and log-linear as Kleiber's law. The relevant interpretation of this power law association is as a *supply relation*: it captures the ability of an economy to make electricity available at geographically dispersed sites for the purpose of final use.

The model delivers the above mentioned power law association between e and k and offers predictions regarding the size of the key elasticity linking capital and electricity consumption. Specifically, it is demonstrated that depending on the efficiency of the economy in the context of electricity distribution (in a sense to be made precise below), the elasticity should fall in a bounded interval ranging from 1/2 to 3/4; the more efficient the economy the larger the elasticity. By implication, economies that are more efficient in electricity distribution will be able to make more electricity available for final use. *Ceteris paribus*, such economies should be able to accumulate capital at a faster rate than less efficient economies.

Needless to say, our characterization of electricity distribution neglects a lot of the details of actual power supply. It remains a crude aggregate representation of the network, for which we can provide some micro foundations. In this sense the equation bears some similarity to the aggregate production function. When macroeconomists write down a production function it is not because they believe this to be an *accurate* description of the full complexity of the myriads of production processes that ultimately generate GDP. It is viewed as a useful short cut, which is justified by the need to gain some understanding of how the economy operates. Our network representation should be viewed in a similar light, and is subject to similar limitations.

With an equation summarizing power distribution in hand we subsequently add a representation of electricity demand. From an accounting perspective electricity can be viewed as being used for three basic purposes: running and maintaining existing capital and creating new capital. The notion of capital is broad, including both electricity consuming capital used by households (e.g., an air conditioner in a living room) as well as capital used by firms (e.g., an air conditioner in a factory hall).[5] If a "characteristic" machine consumes a certain amount of electricity in use and requires a certain amount of electricity to be created, total electricity demand (at an instant in time) is simply the sum of the electricity requirements related to use, maintenance, and construction of capital. Assuming the demand for electricity (thus determined) equals supply (as reflected in the power law association) we can derive a simple first order differential equation governing the evolution of capital per capita over time. We can then proceed to study the implied dynamics and characterize the steady-state.

The law of motion for capital is mathematically isomorphic to the one emanating from a Solow (1956) model, where the aggregate production function is assumed to be Cobb–Douglas. But in

[4] "Allometric scaling" is a technique used in biology to study how selected biological variables of an organism correlate with the size of the organism. A fundamental allometry is the one mentioned in the text, between energy consumption B and body mass m of a mammal ("Kleiber's law"). In Bettencourt et al. (2007) the authors examine the links between various urban indicators (like, total number of patents, housing, crime and energy consumption) and city size measured by population.

[5] As a result, our concept of capital is not the same as the national accounts concept, which only classifies an air conditioner as capital if it is used by firms; if the (same) air conditioner is placed in a household it is classified as a durable consumption good in national accounts. In many contexts this distinction is important. In present situation, however, it makes no difference to electricity requirements whether the machine is placed in a home, or in a factory. As a result, we abstract from the distinction in the formal analysis below.

contrast to the Solow model the structure developed below holds predictions for the amount of capital (usable in consumption or production) that an economy can sustain in the long run from a physical perspective. This level of capital is determined by the efficiency of the electricity distributing network, as well as the energy cost associated with running, maintaining, and creating capital.

Below we argue that technological change is the key driver behind changes in this physical limit to capital accumulation per capita in the long run. Hence, one should not view the steady-state derived below as an absolute boundary for capital but rather as a constraint which continually is being modified due to technological change.

From an empirical perspective, however, the law of motion for capital is difficult to confront with data since our definition of capital necessarily is broader than the national accounts concept (cf. footnote 5). However, we demonstrate below that the central law of motion for capital per capita can also be restated in terms of electricity consumption per capita, which is a directly observable variable.

If economies – on average perhaps – are operating at the boundary of physical feasibility, the model should be able to match the data on electricity consumption per capita. In order to examine whether this is the case or not, we examine the model's implications using cross-state data for the 50 US States, 1960–2000. In terms of electricity consumption per capita the model holds several strong predictions. First, conditional on structural characteristics of an economy (notably population growth) one would expect to see β-convergence in electricity consumption per capita. Second, the model can be structurally estimated, which allows for the identification of the networks parameter for which we have a theoretical prior.

Using cross-sectional data on electricity sales for the 50 US states we find strong support for the model. Over the period 1960–2000 there is marked tendency for conditional β-convergence in electricity consumption per capita. Moreover, the 95% confidence interval for the networks parameter conforms with the theoretical predictions: 1/2 to nearly 3/4. The point estimate is about 2/3.

The remaining part of the paper proceeds as follows. Section 2 lays out the model of the economy, viewed as an electricity distributing network, and derives the power law association between electricity consumption and capital. We then proceed, in Section 3, to add electricity demand, and derive the law of motion for capital and electricity consumption, respectively. Section 4 discusses the model's implications for long-run development and Section 5 contains the empirical analysis. Finally, Section 6 concludes.

2. The economy as a network

In the biological context, the power law (i.e., Kleiber's law mentioned above) is derived from the notion of living organisms as energy transporting networks whereby the size of the organism is given by the number of energy consuming units, i.e. the number of cells that have to be fed.[6] By way of analogy we view an economic "organism" as an electricity transporting network whereby the size ultimately is related to the amount of capital which needs to be "fed". This section therefore extends the model due to Banavar et al. (1999) so as to analyze an economy distributing electricity.[7] Since Banavar et al. mentioned the potential applicability of the originally biological theory to man-made networks, a couple of articles were published taking inspiration from these ideas to understand the "metabolism" of cities, regions, or countries and to investigate scaling laws between population size and human consumption (see e.g. Kühnert et al., 2006; Bettencourt et al., 2007; Isalgue et al., 2007; Samaniego and Moses, 2008).

Fundamentally, the purpose of the network is to deliver (non-human) energy to the electricity consuming units of the economy. Electricity is assumed to originate from a source (a power plant) and is diffused across the economy via a power grid to the sites at which it is used. In keeping with the established terminology (Banavar et al., 1999), we refer to each site as "a transfer site"; this is where energy is converted into work effort. Each transfer site is assumed to be scale-invariant; as the network expands the geometric size of the individual transfer site does not change. A reasonable way

6 West et al. (1997), West and Brown (2005) and Banavar et al. (1999, 2002).

7 For other applications of the theory to drainage basins of rivers, see Maritan et al. (2002) and Rinaldo et al. (2006).

to think about the transfer sites is as electricity outlets, which arguably fulfill this requirement.[8] Moreover, all transfer sites are locally connected, and thus linked to the source either directly or indirectly via transmission lines.

The size of the network is defined by the geometric size of the shape which defines its outer contours. In biology the outer contour of the network is tangible – the body. In the present case it is abstract; i.e., the geometric shape which *would* be able to engulf the network. The fact that this geometric shape is not tangible is immaterial for the argument.

Let the linear size of the network be denoted by L so that the total size of the network is proportional to L^D, where D is the dimension of the network. Since electricity is distributed within three-dimensional systems like cities and through three-dimensional structures like houses, plants, and machines, the notion of $D=3$ is certainly most appropriate. However, the theory nowhere crucially hinges on the numerical specification of dimensionality and we can, in fact, let the data decide about the appropriate notion of dimensionality.[9]

The distance between a transfer site and the source i is defined by the number of transfer sites one will have to pass to get from i to the source. A bigger network nests more transfer sites. Hence, given the distance convention it is therefore inevitable that the mean distance between the transfer sites and the source rises as the network becomes larger. However, the specific nature of the network matters for how large the increase is, as will be seen momentarily.[10]

Generally, assuming a space-filling network and scale-invariant transfer sites, the number of transfer sites must rise with the (geometric) size of the network. As each transfer site uses energy, one may anticipate an association between the size of the network and the total electricity consumed at the transfer sites, E. More specifically, we assume that for a given size of the network L^D total electricity consumption is linear in population size P.

$$E \propto L^D \cdot P. \tag{1}$$

As a result, a change in *per capita* electricity consumption requires a proportional change in the size of the network, $e \propto L^D$. That is, per capita electricity consumption, e, is ultimately attributable to the number of devices (e.g. television sets, washing machines, computers and so on), which a given population utilizes. The notion is that every time a new piece of equipment is connected to an electricity outlet, a new transfer site emerges, and the network expands allowing for more electricity consumption per capita.

In order to understand the proportionality in (1) it is helpful to recall that the linear correlation between population size and total electricity consumption holds for a *given* size of the network. The assumption is that doubling the number of persons in a household will double electricity consumption *ceteris paribus* because twice as many people use the appliances connected to the electricity grid. For example, a household's washing machine is used twice as often than before. The effect originating from a change of the number of persons *using* the electricity grid is thus fundamentally different from the effect that originates from *adding* new machines and appliances to the grid. For example, if households complement their washing machine with a dryer, this will enlarge the size of the electricity distributing network.

In principle one could argue that the E – P association need not be linear; there could be scale effects. For example, new household members could use an existing radio without consuming any extra electricity (if they are listening at the same time as the original household member). Whether the association is indeed linear is thus an empirical question. Recent empirical studies investigating this particular question confirmed a linear association between E and P for a cross-section of German

[8] The size of electricity outlets are in practise independent of the size of the associated building, and the surrounding network. They also tend to have the same size across countries; electricity outlets are no bigger in rich countries than in poor countries (though they surely are more plentiful). As a result, maintaining scale invariance of the transfer site may not be unreasonable in an economic context.

[9] Recently, an article has been published that complements our macro theory on the micro-level. Moses et al. (2008) investigate electricity distribution within micro-processors and argue that these objects could be conceptualized as being 2.5-dimensional because of the relatively thin third dimension originating from the metal layers of wires on micro chips.

[10] See Banavar et al. (1999) for sketches of networks.

cities (Kühnert et al., 2006) and Chinese urban administrative units (Bettencourt et al., 2007). In the empirical section of this paper we provide additional support using cross-state data for the US.

A key result in Banavar et al.'s network theory is the proof of an association between *total* flow of energy in a network, F, and the size of the network given by $F \propto E \cdot L^x$, where x depends on the efficiency of the network. Specifically, $x=1$ in directed networks that minimize total energy requirements needed to fuel the economy (or the organism in biology) subject to the requirement that all sites are served. In the most inefficient network, the space-filling spiral, $x=D$. Inserting (1) we get $F \propto L^{D+x} \cdot P$. Notice, by comparison with Eq. (1), that an increasing size of the network implies that a greater fraction of total energy supply (F) is used to "fuel" the system, as opposed to being available for consumption at the sites (E). This result reflects the fact that when the size of the network rises the energy flow per capita (F/P) expands at least in proportion to L^{D+1}, and at most in proportion to L^{2D}.

The basic intuition for this result the following. With a directed network, the average distance between the transfer sites and the source is minimized, which implies that the total amount of energy needed to serve the system (F) of a given size is minimized. By contrast, with the space filling spiral, energy needs to be transported (physically speaking) farther to serve each transfer site in the network. Consequently, the amount of energy needed to serve the system in its totality, at any given instant in time, is larger. Hence, the organization of the network may be more or less efficient, in the sense of the total amount of energy needed relative to the size of the network (or the number of appliances that needs to be fed with electricity).

Finally, we assume proportionality between the total capital stock and total energy in the system $F \propto K$. This assumption is thought to capture that capital is nested at the transfer sites *and* in the network itself, in the form of the transmission lines that connect the transfer sites. Hence capital is needed to transfer electricity (and "hosts" electricity in the process) to the sites where capital uses energy. Energy conservation in the system at large (at any given instant in time) would then suggest proportionality between the capital stock and the total flow of energy in the system.

We impose *exact* proportionality (unit elasticity) based on the following long-run consideration. Suppose we instead assumed $F \propto K^{\phi}$, where ϕ may differ from 1. If $\phi < 1$ this would mean that as the capital stock gets larger, F/K drops, which implies that capital in the limit can be applied without the use of energy. This seems like an undesirable property. Conversely, if $\phi > 1$ it implies that energy can be diffused throughout the system, in the limit, without the use of capital. This does not seem plausible either (currently at least). That is not to deny that there could be periods during which F/K rises or declines, which could be captured by allowing for a specification such as $F \propto K^{\phi}$, where ϕ differs from 1. However, we doubt such a state of affairs could be maintained in the long run. As a result, we opt for the specification where $\phi=1$.

The established associations between the capital stock, size of the network, and electricity consumption per capita, $F \propto K$, $F \propto L^{D+x} \cdot P$, $e \propto L^D$ can be summarized as a log-linear association between electricity use per capita and the amount of capital per capita k, determined up to a constant ϵ. It represents the reduced form of the economy as a network:

$$e = \epsilon k^a, \qquad a \equiv \frac{D}{D+x}, \quad x \in [1, D]. \tag{2}$$

The intuition for the concave scaling association is the following. When K rises, new transfer sites emerge and the size of the network expands. As a result, the mean distance between the source and the transfer sites increases. As explained above, a greater mean distance between the sites and the source implies a smaller fraction of electricity being available for consumption. Accordingly, the concave association between capital and electricity consumption reflects the difficulty in delivering increasing amounts of electricity to machines connected to the power grid, when the size of the network expands. In essence it implies that the *supply* of electricity inevitably will run into "diminishing returns" as the capital stock is expanded. This will ultimately be the reason why the physically sustainable stock of capital per capita is bounded from above, *absent* technological progress.

Notice that the scaling coefficient, a, should fall in a $[1/2,\ D/(D+1)]$ interval. For a more precise prior, we need to pin down D. As argued above, the most natural notion is probably that $D=3$. In this case the scaling exponent a should fall in the interval [1/2, 3/4], depending on the efficiency of the system.

Before we turn to dynamics some remarks on a central assumption, the existence of a unique power source, are in order. If networks are efficient, then the theory is straightforwardly extended towards the existence of *multiple* power sources. In this case we can invoke a standard replication (constant returns to scale) argument. Efficiency requires that each transfer site is fed by a unique source and that the existing sub-networks are similar (of equal size). The total network and the implied total energy flow can be obtained by simply adding up sub-networks and their energy flows. Admittedly, real electricity-distributing networks are more complicated (see Lakervi and Holmes, 1995). In particular, they display purposeful redundancy: larger subsystems (like cities) are connected to multiple sources in order to avoid or limit the damage done by system failures (power cuts). In other words, real networks are inefficient by design. At a smaller scale of sub-networks, however, there is no redundancy. For example, usually there is just one electricity line to each house and apartment, just one cable connecting outlets with machines, and just one wire feeding any transistor in a micro-chip. We thus expect that redundancy from interconnecting lines between large-scale subsystems to have only mild impact on aggregate efficiency of the network.

3. A theory of capital accumulation and electricity consumption

Electricity is used to run, maintain, and create capital. Here we use a notion of capital that is broader than the national accountants' definition. It includes all electricity consuming appliances. That is, the k appearing here also includes durable consumption goods. As a result, we do not distinguish whether, for example, an air-conditioning system is placed in a firm or a private household. Assume time is continuous, and let μ be the energy requirement to operate and maintain the generic capital good while ν is the energy costs to create a new capital good. In that case energy conservation implies

$$E(t) = \mu K(t) + \nu \dot{K}(t). \tag{3}$$

We may think of Eq. (3) as capturing *demand* for electricity at any given instant in time; from an accounting perspective the right hand side of the equation summarizes the instantaneous electricity requirements.[11]

Note that Eq. (3) provides a new metric for aggregation of capital. Here we measure aggregate capital in energy-units, i.e. by the amount of electricity needed to create, use, and maintain the aggregate capital stock, While this thermodynamic measurement of capital is certainly superior to the conventional economic measurement of capital because it is not plagued by the well-known aggregation problem (see Cohen and Harcourt, 2003 for a survey) it has also its drawbacks. In particular we cannot distinguish between capital in the conventional accounting sense and consumer durables since both are connected to the same electricity network. We thus cannot (yet) integrate investment demand in the conventional sense into the model, a feature, which in turn prevents inferences about economic growth in terms of GDP. We can, however, derive inferences about economic growth in terms capital accumulation and electricity consumption.

Inspection of (3) shows that that if we were to shut off electricity use, the capital stock would be declining over time at the rate μ/ν, due to lack of maintenance and replacement. Hence, regardless of the missing link to investment in the conventional sense, we observe a correspondence to the rate of depreciation in the conventional sense. It is given by the ratio μ/ν at which the capital stock deteriorates without electricity supply.

[11] One may object to the conservation equation from the perspective that human energy is also used to run, maintain and build capital. This would call for an additional term on the left hand side, capturing metabolic energy supply by the labor force. In per capita terms, however, the human contribution is in contemporary societies minuscule compared to the non-human energy use. Moses and Brown (2003) puts it nicely into perspective (p. 296): "*The per capita energy consumption in the United States is* 11,000 W ... *which is approximately* 100 *times the rate of biological metabolism and,* ... [it] *is the estimated rate of energy consumption of a* 30,000-kg *primate*". From this perspective nothing much lost by ignoring the human *physical* (as opposed to intellectual) contribution to the growth process.

Dividing through by population size P in the energy conservation Eq. (3) we get

$$e(t) = \mu k(t) + \nu \frac{\dot{K}(t)}{P(t)}.$$

Assume that the population grows at a constant rate of n. Then $\dot{K}(t)/P(t) = \dot{k}(t) + nk(t)$. Inserting this and the power law association (2) into the above equation provides the law of motion for capital:

$$\dot{k}(t) = \frac{\epsilon}{\nu} k(t)^a - \left(\frac{\mu}{\nu} + n\right) k(t). \tag{4}$$

The dynamical analysis is straightforward. Formally, the model shares the technical properties with the Solow model, where technology is assumed to be Cobb–Douglas. In particular, there exists a unique globally stable steady-state to which the economy adjusts.

Mechanically, the adjustment process works as follows. At any given instant in time k is predetermined. Given k, a size of the underlying network is implied and consequently the supply of electricity (which is assumed to adjust), e, is determined. If e is sufficiently large, i.e. it exceeds the energy needs required to maintain and run existing capital, the stock of capital can expand further. However, as the network expands the amount of additional energy which can be made available for direct use starts to diminish (i.e. E/F declines). Eventually, therefore, the system settles down at a steady-state level of $k = k^* \equiv [(\mu + n\nu)/\epsilon]^{1/(a-1)}$. This can be seen as the maximum sustainable stock of capital, given the parameters. Below we argue that ν, a and μ should be given a technological interpretation. Hence, if these parameters change then k^* changes as well. Accordingly, the steady-state is to be viewed as a moving "target".

By combining the power law (1) with (3) we have in effect assumed that demand for electricity equals supply at any given instant in time. As a result we obtain a law of motion for *feasible* capital accumulation from a *physical* perspective. In practise, households and firms decide on how much capital to accumulate. Standard neoclassical growth theory provides a basis for analyzing this process of *economic* capital accumulation. The law of motion we have derived represents the upper contour of what an economy at most can sustain if the energy conservation law is to be fulfilled and electricity is to be distributed to serve the (increasing number of) machines. In general, therefore, the above equation may not hold positive implications for economic accumulation and growth. For example, firms and households could, in principle. accumulate machines, in spite of the lack of an ability to "feed" them with electricity.

The model can alternatively be expressed in terms of electricity consumption, which turns out to be useful empirically. Formally, insert (2) in (4) to obtain the law of motion of per capita electricity consumption.

$$\dot{e}(t) = \frac{a\epsilon^{1/a}}{\nu} \cdot e(t)^{2-1/a} - a\left(\frac{\mu}{\nu} + n\right) \cdot e(t). \tag{5}$$

Inspection shows that there exists a unique non-trivial steady-state of electricity consumption where $e = e^* \equiv \epsilon[\epsilon/(\mu + n\nu)]^{a/(1-a)}$. Since the second, negative term in (5) enters linearly, the steady-state is globally stable if the first, positive term enters less than linearly, i.e. for $2 - 1/a < 1$. To see this formally, stability requires that

$$\frac{\partial \dot{e}}{\partial e}_{|e=e^*} = \frac{a\epsilon^{1/a}}{\nu}\left(2 - \frac{1}{a}\right) \cdot (e^*)^{1-1/a} - a\left(\frac{\mu}{\nu} + n\right) < 0 \Leftrightarrow \left(2 - \frac{1}{a}\right) e^* < e^*.$$

Since $2 - 1/a < 1$ implies $a < 1$ and for any conceivable network $a \in [1/2, 3/4]$, the steady-state is globally stable.[12] For a sample of electricity-distributing systems the theory thus predicts β-convergence; electricity consumption per capita converges towards the same steady-state for networks sharing the same underlying parameters.

[12] For a three dimensional network, $a \in [1/2, 3/4]$. For a hypothetical two-dimensional network, $a \in [1/2, 2/3]$.

4. Discussion

The model holds various implications on issues of whether growth can be sustained, the plausibility of neoclassical growth theory and on technological change and economic growth. These implications are discussed in turn.

4.1. Neoclassical growth theory and its critics

The model allows some reconciliation between neoclassical growth theory and the work of its staunchest critics (Daly, 1977; Georgescu-Roegen, 1976). The central charge is that energy is introduced into the standard models in an unsatisfactory way (if not ignored altogether). That is, by including energy in the aggregate production function as a separate input, which can be substituted for by capital. This approach is fundamentally flawed, the argument goes, because it does not consider that any capital good is itself produced by means of energy. Here, we have explicitly taken into account that all capital is created, run, and maintained through energy use. Interestingly, we nevertheless arrive at a law of motion for capital which is structurally identical to that implied by the Solow model. Hence, the structure of the Solow (1956) model is not at variance with fundamental physical principles, like energy conservation.

4.2. Can growth be sustained?

In the last section we found that the maximum sustainable stock of capital per capita is bounded from above. For a proper assessment of the steady-state result note that it is not derived from the assumption of limited supply of energy. Instead, convergence towards a constant capital stock per capita is a consequence of energy demand, distribution, and the entailed decreasing returns from expanding networks. It is sometimes argued that economic growth is ultimately limited from above by energy availability (Daly, 1977). Interestingly, however, the present analysis demonstrates that – absent technological progress – economic growth is limited *even if* energy supply were unlimited. This brings us to the issue of how "technology" is said to be present in the model above.

4.3. Technological change

In the context of the model, technological change will come in the shape of changes in the key parameters: ν, μ and a. That is, innovations which lower electricity requirements or improve the efficiency of the network. These innovations are critically important in facilitating capital accumulation. Without them, the economy would eventually end up at e^* and capital accumulation comes to a halt at k^* on purely physical grounds.

Innovations that map into these parameters are easy to think of. The fact that technological advances make appliances less energy-hungry because, for example, smaller sizes of computers or microchips are fulfilling the same task that required large, energy consuming machines in the past, is captured by the model as a reduction of μ. A one time reduction of μ leads to temporarily higher growth according to (4) and convergence towards a permanently higher steady-state k^*. A perpetually declining μ could be conceptualized as convergence towards "the weightless economy" (Quah, 1999).

A permanently lower value of ν captures an innovation that admit new machines to be produced at lower energy costs. Such a parametric change seems to fit quite nicely with the sort of innovations growth theories usually refer to as "general purpose technologies" (GPT). GPT innovations are viewed as "fundamental" innovations which tend to "reset" the economy, and instigate (ultimately) a growth "spurt". The process, however, may involve a non-monotonous adjustment process, with an initial slump of productivity while the GPT forces a replacement of old machines with new ones that employ the new basic technology.

Bresnahan and Trajtenberg (1996) who where among those who initiated GPT research asked (p. 84): "Could it be that a handful of technologies had a dramatic impact on growth over extended periods of time? What is it in the nature of the steam engine, the electric motor, or the silicon wafer, that make them prime suspects of having played such a role?" They gave a very broad answer which is still used in the literature (see e.g., Jovanovich and Rousseau, 2005). The technology must be pervasive (spread to most sectors), there must be scope for improvement over time (lowering the costs of its use) and it must be innovation spawning, i.e. it enables the production of new products. The following "handful" of technologies are usually referred to as GPT's: the waterwheel, the steam engine, electricity, railways, motor vehicles, and IT.

Based on the theory developed above we can suggest a more precise answer to Bresnahan and Trajtenberg's question. A GPT must improve either the use of energy (waterwheel, steam), its delivery through a network (railways, cars) or both (electricity, IT). Interestingly, while not all proposals of GPT candidates available in the literature coincide perfectly, electricity and IT, the technologies that revolutionized both the use and distribution of energy, are always on the lists. Speculating about what could possibly be the next GPT experts usually come up with nano-technology; again a new system for distributing energy at a new (finer) level of network with less power loss. With our theory at hand it becomes intuitive why other seemingly equally fundamental innovations (e.g. the decoding of the DNA) are not GPT's: they do not (much) improve the distribution and use of non-human energy.

An *ad hoc* way to mimic the arrival of a GPT within the standard Solow model is to simultaneously vary total factor productivity and the depreciation rate. The first parameter change is meant to capture the long-run increase in productivity, and the second captures the initial slump, originating from obsolescence of machines embodying the old technology (see Aghion and Howitt, 1998, Ch. 8.4). The problem is that both measures move the steady-state in opposite directions and some fine-tuning is needed to create the desired transitional and long-run effects.

In the current model, we have a – while admittedly equally *ad hoc*– somewhat more elegant way to produce the desired growth trajectory: a decrease of ν. For the following discussion we assume – based on the empirical evidence presented below – that (4) has not only bearing on physically feasible growth but also on actual economic growth. A lower value for ν means that machines can be produced at lower energy costs, which, for example, could have been initiated through the transistor replacing the energy-intensive vacuum tube in electronic devices. From inspection of (4) we see that a lower ν has a double effect. It raises both the first term, "productivity", and the second term, "depreciation". It is easy to see from the steady-state solution for k that a lower value for ν unambiguously raises the steady-state level. Starting at the original steady-state (using tube technology) $\dot{k}$ equals zero initially. Evaluating the RHS of (4) after the fall of ν, we see (since $a < 1$) that initially the negative effect through the depreciation channel dominates. In conclusion, energy saving technological progress causes GPT-like adjustment dynamics with an initial slump, recovery, and convergence towards a higher steady-state level.

5. Empirical evidence

In this section we confront the model with cross-sectional data for the 50 US States, covering the period 1960–2000. In order to do so, we must deal with two major issues: one practical, one conceptual.

The practical problem is that data on our notion of capital per capita, k, does not exist as it encompasses both electrical appliances in homes, and in factories. Only the latter constitutes capital in the System of National Accounts. Hence, we cannot test the model's predictions regarding capital accumulation. However, we can also state the law of motion in terms of electricity consumption per capita, as demonstrated above. Electricity is measurable, which makes it feasible to perform empirical tests.

The conceptual issue relates to whether tests are meaningful *a priori*. As it stands the model is one that relates to the physically feasible amount of capital accumulation, which need not coincide with actual capital accumulation. The pragmatic way to resolve this issue is to try and see how well the model fits the data. If it proves to be a poor fit, this is evidence that the economy is *not* operating anywhere near the physical boundary.

In judging the fit of the model it is fortunate that the theory provides a set of very strong predictions. In particular, the network theory predicts that the estimate for the scaling parameter a should fall in an interval between 1/2 and 3/4. The model also holds structural predictions, reflected in required parameter restrictions, as discussed below. Finally, the model holds predictions about the

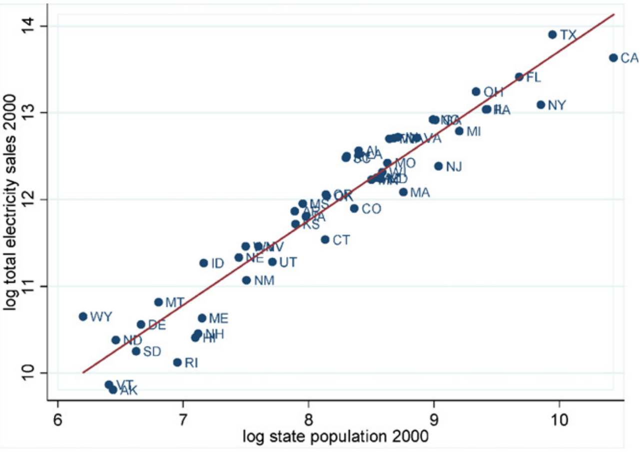

Fig. 1. Population size and electricity consumption across US states. Electricity consumption vs. population size: 50 US States in 2000. The line illustrated is estimated by OLS, and yields the following result: log(Electricity) = 3.95 + 0.98·log(Population), R^2 = 0.92, 95% CI for population coefficient is (0.87,1.08). A test of the hypothesis that the slope is equal to one returns a *p*-value of 0.63 and thus cannot be rejected.

nature of the growth process with respect to electricity consumption per capita: conditional β-convergence should prevail.[13]

5.1. Testing a key assumption

Before we test the model in it its entirety, we test the validity of the crucial underlying assumption of the model, using our cross-sectional data set for US States: the postulated linear association between total electricity consumption and total population (Eq. 1). This is the first place where the theory may fail.

Our test is based on the following specification:

$$\log(E_i) = a_0 + a_1 \cdot \log(P_i) + \varepsilon_i, \qquad (6)$$

where *i* denotes the unit of observation. The variable ε_i is a noise term. That is, structurally $L_i^D = a_0 + \varepsilon_i$, which allows the size of the network to vary across units of observation. Our assumption, for which we require validation, is that $a_1 = 1$. The identifying assumption for ordinary least squares (OLS) to deliver an unbiased estimate for a_1 is that $E(\varepsilon \cdot P) = 0$, where $E(\cdot)$ is the expectation operator. In terms of the model, this means that the size of population is uncorrelated with the capital-labor ratio, since the latter determines *L*. Under the null (the model is correct), this assumption is plausible.

We obtained data for electricity sales across US states from the US Energy Information Administration. Specifically, electricity sold to end users: the residential sector, the commercial sector, and the industrial sector.[14] From this source we also obtained data for state populations.

Fig. 1 provides a visual impression of the results from estimating Eq. (6) by OLS and reports the parameter estimates; the regression relates to the year 2000. As is visually obvious, we cannot reject the assumption of proportionality between total electricity consumption and population size. The coefficient for log(*P*) is close to 1, with a fairly narrow 95% confidence interval. Hence, Fig. 1 informs us that a potential empirical failure of the theory cannot be ascribed to a violation of Eq. (1). This evidence

[13] Hence, in this section we employ a basic approach to testing for convergence which goes back to Mankiw et al. (1992). Related convergence equations have been estimated in an effort to understand energy efficiency, the ratio of total energy consumption and GDP (e.g., Mulder and De Groot, 2007), as well as pollution (e.g., Brock and Taylor, 2004).

[14] Data was obtained through the web site http://www.eia.doe.gov/emeu/states/_seds.html.

corroborates previous findings for German cities and Chinese urban administrative units (Kühnert et al., 2006; Bettencourt et al., 2007).

5.2. Testing the full theory

The second test concerns the ultimate structure of the model. In order to facilitate estimation we log-linearize (5) around the steady-state where $e=e^* \equiv \epsilon[\epsilon/(\mu+n\nu)]^{a/(1-a)}$:

$$\ln[e(t)] = \ln[e(0)]\exp(-\lambda t) + (1-\exp(-\lambda t))\left[\frac{1}{1-a}\ln(\epsilon) - \frac{a}{1-a}\ln(\nu) - \frac{a}{1-a}\ln\left(\frac{n+\mu}{\nu}\right)\right],$$

where $\lambda \equiv (1-a)(n+\mu/\nu)$.

This equation is, from a mechanical perspective, similar to that implied by the Solow model. The conventional approach to taking this model to the data consists of estimating the convergence equation by OLS (e.g., Mankiw et al., 1992).

However, the equation is *non-linear* in the parameter of interest (a). As a result, the model should not be estimated by OLS. Instead we will have to resort to an iterative procedure. Below we therefore report the results from estimating a by non-linear least squares (NLS).[15]

Unfortunately, some of the variables entering the estimation equation are unobservable: the energy costs of running and maintaining capital (μ), the costs associated with creating new capital (ν), and the efficiency parameter ϵ. From the theoretical discussion above, however, we have a prior for the *ratio* μ/ν; it should reflect the rate of capital depreciation. Hence, a reasonable number for μ/ν can be imposed from the literature. A typical finding for the US is a rate of capital depreciation of 6% (Nadiri and Prucha, 1996; McQuinn and Whelan, 2007), which we therefore impose *a priori*.

In sum, the equation we estimate on the cross-section of US states is:

$$\ln[e_i(T)] - \ln[e_i(0)] = -[1-\exp(-b_1\cdot(n_i+0.06)\cdot T)][b_2 + b_3\ln(n_i+0.06) + \ln e_i(0)] + u_i, \quad (7)$$

As already noted, we test the model using cross-sectional data for the period 1960–2000; accordingly, $T=40$. The predictions of our theory are:

(1) $b_1 > 0$,
(2) $b_3 > 0$ and
(3) $1-b_1 = b_3/(1+b_3) = a \in [1/2, 3/4]$.

If any of these three requirements are left unsupported, the theory – as it stands – is rejected. The parameter b_2 captures the influence from ϵ and ν; the model provides no specific guide in terms of what to expect in terms of sign or size.

The identifying assumption needed for unbiased estimates is independence between the error term, u_i, and the right hand side variables. In particular, this requires that *if* ν, ϵ and μ varies across the US states that this variation is random. We find this identifying assumption plausible. The parameters ν, ϵ and μ are arguably of a technological nature, as discussed above, and the US states are undoubtedly fairly homogenous in this respect; innovations are likely to spread rapidly.

Before we turn to the NLS estimates it is worth showing the basic correlations in the data. Consider Fig. 2 which shows the unconditional association between growth in electricity consumption per capita 1960–2000 versus initial electricity consumption per capita. The partial correlation is highly significant, as is obvious. Fig. 3 shows the unconditional correlation between growth in electricity consumption per capita and the log of average state population growth 1960–2000 (plus 0.06). In keeping with the theory the association is negative, and significant. A simple OLS regression where the nonlinearities are ignored can account for 60 % of the variation growth of electricity sales per capita across the 50 states. Naturally, the OLS estimates will be biased (since the empirical model is

[15] To our knowledge Dowrick (2004) is the first to make this point in the context of cross-country growth empirics, and to estimate the (augmented) Solow model by NLS. We essentially follow his approach when estimating Eq. (6).

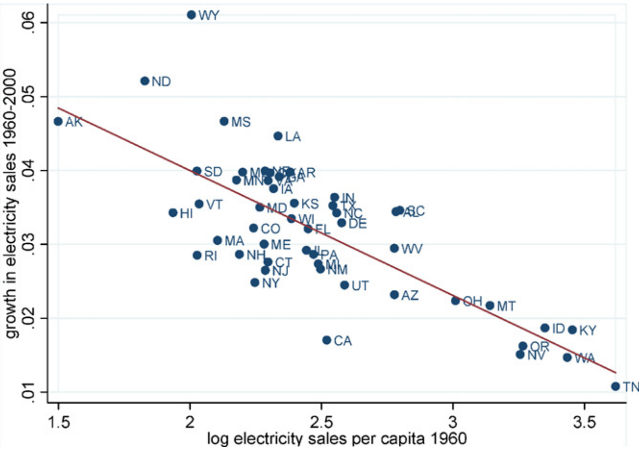

Fig. 2. Convergence. The figure shows the correlation between initial (log) electricity consumption per capita and subsequent growth in electricity consumption per capita: 50 US states, 1960–2000. The straight line is estimated by OLS.

misspecified) and cannot form the basis for statistical inference. The real test of the model requires us to invoke NLS.

Table 1 reports the results from estimating Eq. (7) by NLS. In panel A we report the results from estimating the model freely. That is, without imposing the restriction $1 - b_1 = b_3/(1 + b_3)$.

As can be seen from the R^2, the model accounts well for the cross-state variation in log changes in electricity consumption per capita over the 1960–2000 period. Moreover, the signs of the parameters are as predicted: $b_1 > 0$, $b_3 > 0$; both are significant at the 1% level of significance. Second, and more remarkably, the *structure* of the model is not rejected by the data either. That is, the implied equality of $1 - b_1$ and $b_3/(1 + b_3)$ is not rejected. Consequently, we can proceed to estimate the restricted model, where $a = 1 - b_1 = b_3/(1 + b_3)$ is imposed. This allows for a unique estimate for a, which we can compare with our prior.

The results are shown in Panel B. The point estimate of the scaling exponent a is 0.62; close to 2/3. Interestingly, the confidence interval for a coincides almost perfectly with the predicted range for a, as

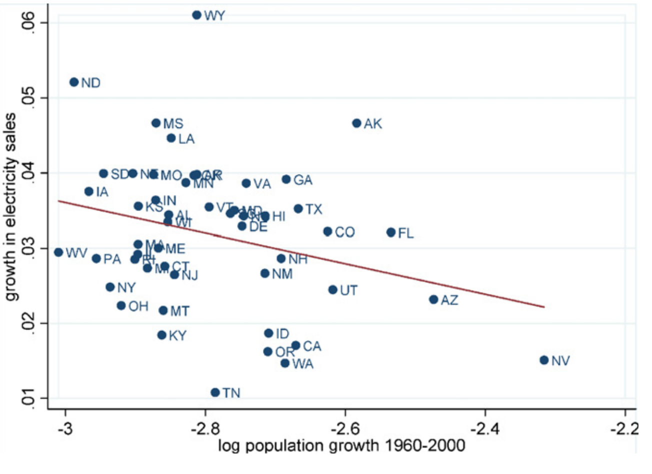

Fig. 3. Growth and state population growth. The figure shows the correlation between log state population growth (+0.06) and growth in electricity consumption per capita: 50 US.

Table 1
NLS estimates.

Dependent variable: log change in energy consumption per capita 1960–2000	
Panel A: unrestricted model	
b_1	0.36 (0.07)
b_2	−0.38 (0.89)
b_3	1.55 (0.37)
$1-b_1=(b_3/(1+b_3))$ (p-value)	0.60
R^2	0.96
Obs.	50
Panel B: restricted model; $1-b_1=(b_3/(1+b_3))$ *imposed*	
$1-b_1$	0.62 (0.05)
b_2	−0.05 (0.88)
95% CI for a	(0.52,0.73)
R^2	0.96
Obs.	50

Note: Heteroskedasticity robust standard deviations in parenthesis.

suggested by theory with $D=3$: $a\in[1/2, 3/4]$. In fact, we can reject a being smaller than the lowest admissible value under the theory: 1/2. At the same time, we can reject $a=3/4$ at the 5% level.

In theory, it is not entirely clear what the relevant dimensionality D of the network is. However, the estimate for a suggest $D=3$ is probably a good approximation. If, instead, $D=2$, we would expect a to fall in an interval between 0.5 and 2/3. Since the latter upper boundary can be rejected by the data (which support values nearly as high as 3/4), the notion that $D=2$ is inconsistent with our results.

With $D=3$ our estimates suggests the presence of inefficiencies in the electricity distributing network. As discussed in Section 2, this would be a consequence of the need for redundancy. The fact that our estimates are consistent with the fact that electricity networks, as compared with the biological counterparts, should be (mildly) inefficient is encouraging.

In sum, the above results support a model of electricity consumption which builds on two elements: (i) log–log scaling between capital per capita and electricity consumption per capita, and, (ii) an accounting equation relating electricity consumption to capital use, maintenance and accumulation. The state-level data suggests that a scaling coefficient around 2/3 is the best approximation.

The fact that the empirical model performs so well suggests that over the period in question the US economy must have been operating most of the time close to the feasibility constraint, as determined by our model. This is in itself an interesting, and perhaps surprising, finding. While this appears to be the viable conclusion, it should be stressed that our theory provides no explanation for this intriguing phenomenon. Understanding *how* this outcome may be brought about will require a fuller model detailing the intricate links between electricity consumption, investment, consumption and, naturally, GDP. Such a model has yet to be constructed.

6. Conclusion

The fundamental notion that economic growth originates from (and is limited by) energy has a long intellectual history, going back to Herbert Spencer's (1862) *First Principles*. According to Spencer the evolution of societies depends on their ability to harness increasing amounts of energy for the purpose of production. Differences in stages of development can be accounted for by energy: the more energy a society consumes the more advanced it is. Chemist and Nobel prize winner Wilhelm Ostwald (1907) developed the Spencerian ideas further. Ostwald emphasized that it is not the sheer use of energy, but the degree of efficiency by which raw energy is made available for human purposes that defines the stage of economic (and according to Ostwald also cultural) development of society.[16]

[16] Further refinements were made by several natural and social scientists, among them Frederick Soddy, Alfred Lottka, and Fred Cottrell.

The theory developed above demonstrates that this notion of development, when given a modern network interpretation, is compatible with neoclassical growth theory. Indeed, it coincides with the structural form of the economist's core model of economic growth, the Solow growth model. The central element of the theory, the network equation for electricity supply, receives support in US state-level data.

The theory has bearing on the fundamental "limits to growth" debate. In particular, while conceding the importance of energy for growth, the theory also highlights the crucial importance of human ingenuity. As shown above, absent technological change, growth will come to a halt even with unlimited supplies of energy, since energy dissipation increases as the economic network (appliances and machines connected) becomes larger. This result therefore implies that technology, associated with the harnessing and use of energy, is as important for growth prospects as the supply of energy itself; energy and technology are equal partners in development. Indeed, as argued above, "major" innovations (which usually are referred to as GPTs) can be seen as rare instances of progress, which in a profound way improve the harnessing, transformation, and/or distribution of energy. Integrating the literature on endogenous technological change, with the present model of capital accumulation, would therefore seem like a useful topic for future research.

The framework could also be adapted to the study of growth in the very long run. It seems widely conceded that human societies at large enjoy income and consumption levels of historically unprecedented magnitudes (e.g. Galor, 2005). A key implication of the model above is that such increases is inescapably linked to the ability of human societies to expand energy supply, which requires technological innovations. In particular then, such a long-run growth model would suggest that the recent harnessing of electricity during the 19th century should sow the seeds of a dramatic change in human societies. First, it is the period during which the modern day energy transport network is created. That is, this period represents the genesis of $e(t) = \epsilon k(t)^a$. Second, as a result, these innovations allowed for investment growth, and thus income growth, of unprecedented scale, by removing the constraint on capital accumulation previously imposed by energy supply in ways of the metabolism of humans and animals. Accordingly, integrating the framework above with the unified growth literature also seems like a fruitful avenue for future research.

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