

Notes on Endogenous growth and evidence on Learning-by-Doing*

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Abstract

The note touches upon the main difficulty involved in making “A” endogenous, and outlines various strategies for accomplishing this task. In addition, some empirical studies on productivity gains from internal and external learning, one plausible reason why endogenous growth may arise, are summarized.

1 Making Growth Endogenous

We have made two important modelling choices up until now when developing growth models. First, we have assumed that an aggregate production function exists, and that it exhibits constant returns to scale (CRTS) to “rival” factors of production (capital and human input) while increasing returns to rival and non-rival factors (including “technology”, A):

$$Y = F(K, AL), \lambda Y = F(\lambda K, A\lambda L) \text{ for } \lambda > 0.$$

We based the assumption of CRTS to rival factors of production on the replication argument. Second, we have assumed that perfect competition in goods and factor markets prevail.

Together these two assumptions imply that

$$Y = F_L AL + F_K K = wL + RK, \tag{1}$$

where w and R represents the wage and the return on capital, respectively. The first equality follows from Eulers’ theorem on homogenous functions, while the second uses that firm’s maximize profits, and that perfect competition prevail.

Endogenous growth requires us essentially to make “A” endogenous. But, there is a sticky issue here: Who is going to pay for changes in A ? Under CRTS and perfect competition, compensating conventional factors of production exhausts output, as is clear

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from equation (1). Accordingly, firms will not be able to pay for it directly. How do we deal with this problem?

Conceptually, there are five approaches to "solving" this dilemma.

1. Deviate from perfect competition. Models of endogenous growth through (privately funded) research and development fall into this category. By allowing for imperfect competition we may be able to motivate firm into paying for the required R&S costs. This sort of an approach is discussed in B&S Ch. 6-8.

2. Households pay for it directly. In this case we may think of "A" as human capital. Households invest in it, so as to increase their earning capabilities. In this process endogenous growth may arise. Such theories are discussed in B&S Ch. 5.

3. Forget "A"! Employ $Y = F(K, L)$ and assume that capital is a sufficiently powerful growth engine: $\lim_{k \rightarrow \infty} f'(k) > 0$. As shown in B&S Ch. 4 this will work in the sense that perpetual growth in income per capita is possible.¹ Perfect competition is consistent with this approach. But there are a couple of major drawbacks. First, recall that $\lim_{k \rightarrow \infty} f'(k) > 0$ implies that $\lim_{k \rightarrow \infty} \frac{f'(k)k}{f(k)} = 1$. The latter limit tells us that, asymptotically, the production function exhibits CRTS in the reproducible production factor: Capital. However, since markets are competitive $\frac{f'(k)k}{f(k)} = \frac{rk}{y}$, capitals' share in total income. Hence, this formulation will lead to the problematic prediction that capitals' share should approach one in the limit.

4. The government is paying (thus households, indirectly). Using tax revenue for investments in infrastructure or to finance public research, the government could ensure the expansion of A. This possibility is examined in B&S Ch. 4.4.

5. A rises as a by-product of investment and production effort. That is, when perfectly competitive firm's maximize profits, invest and produce, productivity rises as a (welcome) "side effect". So the solution is: "no-one really pays for it." Nevertheless, the evolution of A will be endogenous in that preferences and policies will determine its path. For example, policies that affect the incentive to produce and invest, will through the externality affect "technological knowledge" as well. A plausible source of such external productivity gains is learning-by-doing (LBD). Models' featuring endogenous growth through LBD is examined in B&S Ch. 4.3. In the next section we look a some evidence which suggest that LBD is a significant source of productivity growth.

2 Evidence on Learning

A very famous case study on learning, comes from the ship building industry. Specifically, from the construction of the so-called Liberty ship during World War II. This cargo vessel was produced on a number of independent ship yards over the period 1941-45. Importantly for present purposes the specifications of the vessel was unaltered throughout the war.

¹Given certain additional assumptions on parameters and per period utility; as detailed at the lectures.

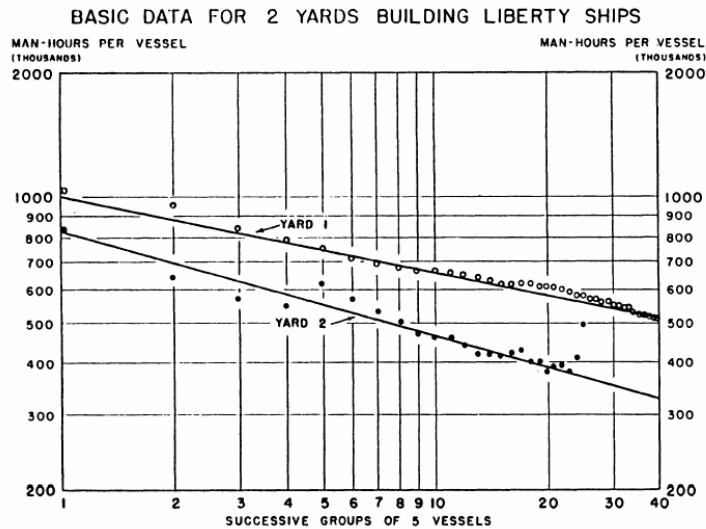


FIGURE 1.—Reductions in man-hours per vessel with increasing production. Merchant shipyards.

Figure 1: Source: Lucas (1993) “Making a Miracle”. *Econometrica*, 61, 251-72

Figure 1 shows the relationship between the number of hours used to produce a Liberty ship and the cumulated production for two separate yards. As is visually obvious, the number of man-hours used to produce a ship declined rather dramatically as production matured. Arguably, this decline can be ascribed to an improved ability of workers’ to assemble the ships – i.e., be a consequence of learning-by-doing.

Still, a scatter-plot is not altogether convincing. For example, suppose more capital (cranes and what not) were installed at the various yards during the war. This should make workers more productive, and reduce the amount of hours required to produce a ship. In addition, technological improvements – unrelated to learning – could be at work (better cranes). To look at the "Liberty Ship miracle" a bit more carefully one would have to resort to regression analysis.

2.1 Rapping’s study

Rapping (1965) attempts to quantify the gains from learning by estimating a Cobb-Douglas production function on a pooled data set encompassing 15 ship yards for the period 1941-45. Accordingly, he assume that total production at shipyard i , Y_i , is a function of "technology", A_i , capital input, K_i (approximated by the number of "shipbuilding ways" at each yard)² and labor input, L_i , (measured by hours worked):

$$Y_i = A_i K_i^\phi L_i^\delta,$$

²A "shipbuilding way" refers to the location where the hulls are being assembled. One would expect that other capital equipment might be correlated with the number of shipbuilding ways at any given yard.

where "technology" – or perhaps more fittingly : "productive knowledge" – depends on cumulated production *within each yard*, \tilde{Y} , exogenous technical change $T_t = T_0 e^{xt}$, and a stochastic component, ε_i :

$$A_i = \tilde{Y}_i^\beta T_t \varepsilon_i.$$

The error term $\ln \varepsilon_i$ is assumed to be noise with the usual properties needed to estimate the equation by OLS. Hence, taking logs and collecting terms yield the following equation which may form the basis for estimation:

$$\ln Y_i = \lambda_0 + \lambda_1 \cdot \text{time} + \lambda_2 \ln K_i + \lambda_3 \ln L_i + \lambda_4 \ln \tilde{Y}_i + u_i \quad (2)$$

where the parameters to be estimated are:

$$\lambda_0 \equiv \ln T_0; \lambda_1 \equiv x; \lambda_2 \equiv \phi; \lambda_3 \equiv \delta; \lambda_4 \equiv \beta,$$

while $u_i \equiv \ln \varepsilon_i$ is the error term. The particularly interesting point estimate is $\hat{\lambda}_4$, which measures how much learning contributed to output, i.e. the size of β .³ Before we review the results a few cautious remarks are in order.

First, one should expect that measurement error is an issue in the present context; both capital input and learning are undoubtedly measured with error. In isolation this will tend to bias both $\hat{\lambda}_2$ and $\hat{\lambda}_4$ towards zero.

Second, we are not controlling for the quality of the labor force. Hence, we are probably faced with an omitted variable problem as well. This too will bias our estimates of, in particular, ϕ and δ .⁴ For example, if more capital were allocated to yards with a better educated labor force, ϕ will be biased upward.

Finally, K and L could be endogenously determined in their own right. Specifically, suppose that the yards were optimizing and imagine that a yard receives a positive productivity shock (a "high" realization of ε_i). This should work so as to increase labor demand and thus labor input. But this implies that the covariance between L_i and the error terms, $\ln \varepsilon_i$, is non-zero (positive, as it were). If indeed this is the case then $\hat{\delta}$ will tend to be overestimated, compared with its "true" value. A similar argument goes for ϕ of course.

Bearing these considerations in mind, Figure 2 shows Rappings results. Row (7) and (8) are comparable to the specification (2). The difference between (7) and (8) lies in the measure used for \tilde{Y} . Rapping measures cumulated output in three ways, corresponding to his variables C_1 , C_2 and C_3 . $C_{1t} \equiv \sum_{s=0}^{s=t} Y_s$, $C_{3t} \equiv \sum_{s=0}^{s=t-1} Y_s$, whereas C_2 is a convex combination of the two. Now if C_1 is used, then Y_t enters both the right hand side, and the left hand side of the regression, which is likely to produce a spurious relation between output and the learning variable. This is probably why Rapping focuses on C_2 and C_3 . In row (7) the point estimate for β is reported to be 0.34. Its highly significant

³A " ^ " signifies the OLS estimator for the variable.

⁴It doesn't seem terribly likely that the quality of the labor force (measured in terms of schooling) would change very much over the relatively short period under consideration. Still, if it did, this trend would affect the estimate for x and β .

TABLE 1. — MULTIPLE REGRESSION RESULTS
(BASED ON 48 OBSERVATIONS)

Regression No. and Dependent Variable	Independent Variable ^a									R ²
	M	K	C ₁	C ₂	C ₃	T ₂	T ₁	0		
(1) X	1.102	.573								.923
t	22.14	4.04								
(2) X	1.116	.634					.228			.957
t	.29.98	5.95					6.07			
(3) X	1.139	.285				.276				.960
t	31.41	2.57				6.55				
(4) X	.918	.229	.286							.980
t	30.54	2.92	11.38							
(5) X	.970	.251		.235						.979
t	33.80	3.15		10.97						
(6) X	1.110	.293			.110					.969
t	35.12	3.04			8.23					
(7) X	.901	.067		.342			-.147			.982
t	24.28	.66		7.69			-2.69			
(8) X	1.110	.260			.120		-.027			.969
t	34.66	2.01			4.07		-.38			
(9) X	.936	.259		.261				-.037		.980
t	29.64	3.37		10.98				-2.19		

^a M = man-hours; K = physical capital; C₁ = cumulated output 1; C₂ = cumulated output 2; C₃ = cumulated output 3; T₂ = yard time; T₁ = calendar time; 0 = output of types other than Liberties; and t = standard t statistic.

Figure 2: Source: Rapping, 1965.

and robust to the inclusion of a time trend. Indeed the time trend comes out negative. The parameter values for δ and ϕ are 0.9 and 0.07 respectively. Both seem implausible. As mentioned above, there might be several reasons why $\hat{\delta}$ and $\hat{\phi}$ should be biased in either direction. The impact from measurement error might be very dominant, forcing $\hat{\phi}$ downward, whereas if L were endogenously determined (exhibiting a positive correlation with ε), this could motivate the very high point estimate for its share.

From row (8) its clear that the point estimate is reduced by a factor of nearly three when the measure of learning is changed from C_2 to C_3 . It is still significant, however, whereas the trend now is insignificant. The estimate $\hat{\phi}$ rises to a somewhat more plausible level, while $\hat{\delta}$ remains essentially unchanged. In sum, Rappings results are clearly consistent with the notion that learning took place at the Liberty ship yards; even controlling for exogenous technical change and capital. Still, one may worry about the data used to measure capital, and moreover, about the risk of omitted variables of various kinds.

A more recent study of the Liberty Ship yards is conducted by Thompson (2001). The key innovations of the Thompson study are two fold. First, better data for capital is collected, and he moreover attempts to correct for the utilization of capital over time. Secondly, Thompson uses panel data techniques, which allow A_i to vary across yards. This would in theory mediate the problem of omitted – slow moving – variables. Thompson obtains $\hat{\delta} = 0.253$ while $\hat{\phi} = 0.78$; both of which seem much more plausible than Rapping's original estimates, and roughly consistent with constant returns to capital and labor.

Interestingly, his point estimate for β is close to being an average of Rappings: 0.26.

Taken together Rapping's study, and the more recent work of Thompson do seem to imply that learning mattered for productivity. But how much? A common way of expressing the size of the learning gain is in terms of *learning rates*.

2.2 Learning rates

Suppose we use the production function above, imposing constant returns $\delta = 1 - \phi$. Then we know that the cost of producing one unit of output is given by the unit cost function:⁵

$$MC = \frac{1}{A} \left(\frac{w}{\delta}\right)^\delta \left(\frac{R}{1-\delta}\right)^{1-\delta} = \frac{\left(\frac{w}{\delta}\right)^\delta \left(\frac{R}{1-\delta}\right)^{1-\delta}}{\tilde{Y}_i^\beta T_t},$$

where the second equality follows from substituting for A , while ignoring the error term, ε_i .

Now, suppose you fix $\left(\frac{w}{\delta}\right)^\delta \left(\frac{R}{1-\delta}\right)^{1-\delta}$ and T_t . Then you can answer the following question: "how much does marginal cost decline if cumulated output, \tilde{Y} , doubles?" This number represents the learning rate. In formal terms

$$\frac{MC_{\tilde{Y}=2}}{MC_{\tilde{Y}=1}} = 2^{-\beta}$$

or

$$1 - \frac{MC_{\tilde{Y}=2}}{MC_{\tilde{Y}=1}} = \frac{MC_{\tilde{Y}=1} - MC_{\tilde{Y}=2}}{MC_{\tilde{Y}=1}} = 1 - 2^{-\beta} \equiv LR.$$

The estimated learning rate, LR , based on Thompson's estimate of β is:

$$1 - 2^{-0.26} \approx 0.16.$$

Hence, every time output doubles, marginal costs decline by approximately 16 percent.

2.3 Dimensions of Learning, and the need for more evidence

Ultimately we would like to argue that LBD is of macroeconomic importance. Moreover, if we are to be able to maintain our assumption of perfect competition, firm's should not internalize the gains from learning.⁶ If the productivity of a firm depends on own learning (i.e. own cumulated output), then one speaks of *internal learning*. The work of Thompson and Rapping suggest, that this form of learning was indeed significant in the Liberty shipyards. Their studies have bearing solely on internal learning, since the measure of learning used in both is the cumulated output *within* each shipyard. Now, Liberty shipyards may or may not have internalized these gains. But it seems plausible that most firm's operating under less special circumstances will (ultimately at least) pick up on this mechanism. Hence, albeit suggestive, the two studies above do not document learning effects of a nature that can motivate our theoretical approach.

⁵C.f. the Appendix.

⁶Remember that we have increasing returns to K, L and A ; where A will capture learning. Perfect competition and increasing returns are hard to reconcile.

In stead we need to find evidence in favor of *external learning*, i.e. production experience of one firm spills over to other firms, operating in the same industry within the same country, or even in another country. Its much harder to argue convincingly that firms systematically will internalize external learning. Consequently, if external learning can be documented then this would provide us with some empirical foundations for arguing that firms' take LBD gains as given, and ultimately, that the market solution may be inefficient.

The next section summarizes the basic approach and (some of the) main results from an empirical study aimed at gauging the size of such external learning gains. This entails a change of scenery, however, from the ship yards of the second world war, to the high tech semiconductor industry of the present day.

2.4 Irwin and Klenow's study

Irwin and Klenow (1994) work with a panel data set comprising 32 firms, situated in various countries, and producing various kinds of chips during the period 1974-92. Focusing on the production of semiconductor chips has the advantage that, like the liberty ship, the nature of the product is observable and can be held fixed in the analysis. Whereas Rapping and Thompson has a production function as point of departure for their analysis, Irwin and Klenow goes through prices, and costs directly.

To fix ideas, suppose we are working with the same production function that Rapping introduced. Then, marginal costs are

$$MC_t = \frac{1}{A_t} \left(\frac{w_t}{\delta} \right)^\delta \left(\frac{R_t}{1-\delta} \right)^{1-\delta} \equiv \frac{\Psi_t}{A_t}.$$

If learning increases A , then this should work to reduce marginal costs – given Ψ . The problem is that marginal costs are not observable. But Irwin and Klenow argue that they can be inferred, under certain assumptions. Specifically, suppose all semiconductor firms are maximizing profits, but that imperfect competition prevail, and that firms engage in Cournot competition. Then the following relationship between marginal costs and the price should hold⁷

$$MC_i = p \left(1 + \frac{s_i}{\eta} \right),$$

where p is the market price of any given chip, s_i is the market share of firm i and η is the elasticity of demand. Now, for empirical purposes, p and s_i are both observables, and estimates for η can be obtained. In most of their analysis Irwin and Klenow work with $\eta = -1.8$ (but they argue that their results are not sensitive to plausible variation in this parameter). In sum, all the variables on the right hand side of the equation above are observables (or obtainable). Combining them allow one to infer MC. So putting the two above equations together yield

$$p \left(1 + \frac{s_i}{\eta} \right) = \frac{\Psi}{A}.$$

⁷See e.g. Tirole, 1993. "Industrial Organization", MIT press, Ch. 5.4.

Irwin and Klenow do not have data on Ψ , i.e factor prices. In order to approximate the level of costs associated with conventional inputs they use the consumer price index (CPI).

Now we need to specify A ; total factor productivity. Irwin and Klenow assume that

$$A_{it} = v_i E_{it}^\beta e^{u_{it}},$$

where v_i is a (firm specific) constant, E_i refers to production experience while u_{it} is specified as a stochastic process exhibiting a trend, meant to capture exogenous technical changes.⁸ As for E_i they assume

$$E_{it} = \tilde{Y}_i + \alpha (\tilde{Y}_c - \tilde{Y}_i) + \gamma (\tilde{Y}_w - \tilde{Y}_c).$$

\tilde{Y}_i represents cumulated output within firm i , whereas \tilde{Y}_c and \tilde{Y}_w refers to cumulated output within the same industry and country, and to cumulated output world wide, respectively. Accordingly, α and γ refers to *external learning* across firms in any given country (α), and across countries (γ). Putting all of this together leads to the following specification:

$$\ln \left(\frac{p_t \left(1 + \frac{s_{it}}{\eta} \right)}{CPI_t} \right) = \lambda_0 + \lambda_1 \ln \left(\tilde{Y}_{it} + \lambda_2 (\tilde{Y}_{ct} - \tilde{Y}_{it}) + \lambda_3 (\tilde{Y}_{wt} - \tilde{Y}_{ct}) \right) + u_{it}$$

where the parameters to be estimated relate to the underlying model in the following way

$$\lambda_0 = -\ln v_i; \lambda_1 = -\beta; \lambda_2 = \alpha; \lambda_3 = \gamma,$$

while u_{it} includes a time trend and the error term. This is not a linear expression, so Irwin and Klenow invoke an iterative estimation approach known as nonlinear least squares. Their estimates for β, α and γ are presented in the Table below (standard errors in parenthesis). First, it is interesting to note that the estimated elasticity of production experience wrt to costs (β) tend to be in the ballpark of what the estimates obtained by Thompson and Rapping for the Liberty Ship yards: between 0.2 and 0.4. Note also that external learning tend to be a significant source of cost reduction in the semi-conductor industry. Indeed, even world cumulated output seems to impact on individual firm costs – the marginal impact is about as high as the spillover from firms within the same country. Consequently, the Irwin and Klenow study supports the view that learning spillovers are in fact a significant source of increased productivity.

APPENDIX: THE COST FUNCTION

The objective is to minimize total costs, subject to the production technology:

$$\min TC = RK + wL$$

⁸For exact details on this term see the paper which is downloadable from the course web-site.

TABLE 3
LEARNING-BY-DOING SPILLOVERS: NONLINEAR LEAST SQUARES ESTIMATES

	β	α	γ	\bar{R}^2	Observations	$(\alpha - \gamma)^*$	Learning Rate
4K	-.350 (.006)	.300 (.048)	.310 (.047)	.90	374	.037	21.5
16K	-.488 (.008)	.176 (.038)	.238 (.039)	.90	489	.002	28.7
16K-5	-.217 (.009)	.263 (.074)	.241 (.096)	.85	118	.201	14.0
64K	-.375 (.005)	.335 (.096)	.425 (.105)	.91	623	.036	22.9
256K	-.314 (.006)	.328 (.105)	.369 (.099)	.82	593	.044	19.6
1M	-.293 (.009)	.130 (.048)	.247 (.072)	.76	399	-.033	18.4
4M	-.299 (.006)	.450 (.152)	.465 (.145)	.93	199	.102	18.7
16M	-.251 (.006)	.233 (.041)	.274 (.042)	.98	47	.010	16.0

NOTE.—See table 2.

* The largest value of $(\alpha - \gamma)$ that, as a null hypothesis, cannot be rejected at the 5 percent level.

Figure 3: Source: Irwin and Klenow (1994)

$$st$$

$$Y = AK^\alpha L^{1-\alpha}.$$

The lagrangian:

$$\mathcal{L} = RK + wL + \lambda(Y - AK^\alpha L^{1-\alpha})$$

We have three first order conditions:

$$\frac{\partial \mathcal{L}}{\partial K} : \lambda \alpha K^{\alpha-1} L^{1-\alpha} = \alpha \frac{Y}{K} = R$$

$$\Downarrow$$

$$K = \lambda \alpha \frac{Y}{r}$$

$$\frac{\partial \mathcal{L}}{\partial L} : \lambda (1 - \alpha) K^\alpha L^{-\alpha} = \lambda (1 - \alpha) \frac{Y}{L} = w$$

$$\Downarrow$$

$$L = \lambda (1 - \alpha) \frac{Y}{w}$$

and

$$Y = AK^\alpha L^{1-\alpha}.$$

Substituting $\frac{\partial \mathcal{L}}{\partial K}$ and $\frac{\partial \mathcal{L}}{\partial L}$ into the production function

$$Y = A \left(\lambda \frac{\alpha Y}{R} \right)^\alpha \left(\lambda \frac{(1 - \alpha) Y}{w} \right)^{1-\alpha}$$

allows us to solve for λ :

$$\lambda = \frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A}.$$

substituting this back into $\frac{\partial \mathcal{L}}{\partial K}$ and $\frac{\partial \mathcal{L}}{\partial L}$

$$\frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A} \alpha \frac{Y}{K} = r$$

$$\frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A} (1-\alpha) \frac{Y}{L} = w$$

and finally using that

$$\begin{aligned} TC &= wL + rK = \left(\frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A} (1-\alpha) \frac{Y}{L} \right) L + \left(\frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A} \alpha \frac{Y}{K} \right) K \\ &= \frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A} (1-\alpha) Y + \frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A} \alpha Y \end{aligned}$$

We get

$$TC = \frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A} Y.$$

Marginal cost are

$$MC = \frac{\partial TC}{\partial Y} = \frac{\left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w}{(1-\alpha)}\right)^{1-\alpha}}{A},$$

and independent of the level of production, by virtue of CRTS.

References

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- [3] Rapping, L., 1965. "Learning and World War II Production Functions". *Review of Economics and Statistics*, 47, 81-86.