SOLUTIONS

PROBLEM 1: VERBAL QUESTIONS, VARIOUS TOPICS

Question 1.1. Draws on material discussed in the context of Jones (1995)

The student may agree or disagree. The important part is the explanation.

Solow is referring to the fact that endogenous growth models of this variety require us to believe in an aggregate production function which can be written: Y=AK. That is, featuring constant returns to scale in capital input.

From a technical perspective the logic of the statement is very clear, since if one deviates from strict linearity the properties of the model changes radically (i.e. imagine embedding such a production function into, say, an otherwise standard Solow model). If we write $Y=AK^{\alpha}$ we end up concluding that growth will come to a halt, insofar as "A" is constant and α is smaller than 1. Conversely, if $\alpha>1$ we would be looking at a growth "explosion"; accelerating growth rates. The latter is undoubtedly unsustainable in a world of finite resources. Hence, if it were to materialize itself we would expect an ultimate collapse (even though Solow, in the paper mentioned above, seems to think of this path as "too-good-to-be-true", it is likely to be the other way around). In any case, it is disturbing that the model seemingly hinges on a very particular parameter restriction.

On the other hand one may argue that what endogenous growth requires is "only" that the above production function is linear in the "limit". That is, $F(K,L) \rightarrow AK$. The required condition for endogenous growth in the 1 sector case is that the marginal product of capital is bounded away from zero, at some (arbitrary) positive level, when the capital labor ratio tends to infinity. A standard production function which yields this result would be a CES production function, with elasticity of substitution above one. Not a very "un-robust" case. Moreover, empirically, this case (elasticity of substitution above 1) is hard to completely dismiss.

One may also observe that moving from $\alpha=1$, to say $\alpha<1$ (however close to one), is not a "hairs breath" change in assumptions. In terms of production possibility sets, it is huge change.

Finally, another angle on the discussion concerns the sources of constant returns. In the course it has been discussed that none of the existing "stories" will do. Learning effects do no seem to be large enough to motivate constant returns; government productive investments hold a similar difficulty. Still, it remains an open question whether these sorts of theories, taken together, could be sufficient.

Question 1.2. Draws on Kneller et al., 1999; B&S Ch. 4

The student may agree or disagree. The important part is the explanation.

If the student agrees with the statement, the argument should be that higher taxation will inhibit the individuals desire to accumulate capital, either physical or human. Consequently, if growth is fuelled by capital accumulation, growth will decline.

A more nuanced answer, however, will be that the statement is too strong in that the growth impact depends on how the revenue is spend. If the revenue is used for investments in, say, infrastructure, health or schooling, the net impact could in fact be positive. Theoretically this is motivated by the growth model with productive government investments; more investments may on net allow for an increase in the net return on investments, thus stimulating growth.

Empirically, the study by Kneller et al. (1999) highlights the importance of the *use* of the revenue when making assessments about the impact from taxation, which would be relevant to bring up in the discussion. (Kneller et al. distinguish between "productive" and "unproductive" spending, where the former includes infrastructure investments, spending on schooling, health etc and the latter is mainly transfers).

Question 1.3. Draws on Hendricks (2002).

We may write the production function

$$Y = AK^{\alpha}H^{1-\alpha} \tag{0.1}$$

where the human component is given by

$$H = e^{\psi s}qL \tag{0.2}$$

In H enters the number of people in the labor force, L, the productivity impact from the quantity of schooling (s), and finally the quality component, *q*.

It is possible to measure L, K and s. The parameters ψ and α can also be calibrated. The former using the "Mincer return" from the labor literature on the topic, and the latter by using national accounts data on capital's share in total income (which requires us to assume competitive factor markets). However, "A" and "q" are unobserved, for which reasons they seemingly are impossible to disentangle.

A recent study by Hendricks (2002) provides a clever way of moving forward. By invoking data on the wages of immigrants – i.e. by observing people educated in different countries but participating in the same labor market – it is possible to find the quality level as a residual.

In theory wages of agent i is given as

$$w = (1-\alpha)(K/Y)^{\frac{\alpha}{1-\alpha}} Ahq \tag{0.3}$$

Hence, the wages of worker i, educated in country j, but working in the US, *relative to an indigenous US worker* would give us

$$\frac{w^{i}{}_{j}}{w_{US}} = q \tag{0.4}$$

If we simply normalize the quality level q=1 for the US worker. In this manner one may measure quality of human capital for workers originating from a large number of countries.

In the end, Hendriks does not find that variation in *q* can explain very much of observed labor productivity differences across countries.

A problem lies with self-selection. However, Henriks argue that micro evidence on the topic does not lead one to believe that the effect of self-selection is strong enough to overturn the conclusion that "A" accounts for the lion's share of observed differences in labor productivity.

PROBLEM 2

Main readings include: B&S Ch. 4; Jones (1995), McGratten (1999); Irwin & Klenow (1994).

Question 2.1.

The problem faced by the individual firm is

$$\max_{K,L} Y_i - rK_i - wL_i$$

The two first order conditions are

$$r = \alpha \frac{Y_i}{K_i}, w = (1-\alpha)\frac{Y_i}{L_i}$$

By implication each firm chooses the same factor intensity: $k_i = k = \frac{\alpha}{1-\alpha}\frac{w}{r}$. As a result it follows that

$$\begin{aligned} Y &= \sum Y_i = \sum A(t)^{1-\alpha} k^{\alpha}{}_i L_i \\ &= A(t)^{1-\alpha} k^{\alpha} \sum L_i = A(t)^{1-\alpha} k^{\alpha} L. \end{aligned}$$

With $k := K/L$.

<u>Question 2.2.</u>

(a) The specification can be thought to capture learning by doing effects. With greater production knowledge is accumulated, which works so as to increase worker productivity. Specifically, the specification reflects external learning; knowledge obtained in individual firms diffuses to the rest of the economy. Hence, it is production in the economy at large which elevates productivity. Empirical evidence on this is found in Irwin and Klenow (1994).

b) Using the assumption implies

$$Y = K^{\alpha}\left(\bar{A}Y^{\mu}L\right)^{1-\alpha}$$

$$\Leftrightarrow Y = K^{\frac{\alpha}{1-\mu(1-\alpha)}}\left(\bar{A}L\right)^{\frac{1-\alpha}{1-\mu(1-\alpha)}}.$$

Consequently, equilibrium factor prices are

$$r = \alpha K^{\frac{\alpha}{1-\mu(1-\alpha)}-1}\left(\bar{A}L\right)^{\frac{1-\alpha}{1-\mu(1-\alpha)}}$$

$$w = (1-\alpha) K^{\frac{\alpha}{1-\mu(1-\alpha)}}\left(\bar{A}L\right)^{\frac{1-\alpha}{1-\mu(1-\alpha)}} L^{-1}$$

c) In order to ensure endogenous growth we need the marginal product of capital to be bounded away from zero. This requires us to impose (cf. the equilibrium real rate of the return):

$$\frac{\alpha}{1-\mu(1-\alpha)} = 1 \Leftrightarrow \mu = 1$$

Accordingly, we need to assume that when output doubles, productivity doubles through the learning-by-doing mechanism. This is too much to expect if the empirical literature on the topic is to be a guide. Irwin and Klenow (1994) examine external learning effects within the semi-conductor industry. While they do find evidence in favour of the existence of external learning across industries, and even countries, the impact is much more limited than what we have to impose to generate endogenous growth.

Question 2.3.

The household optimization problem can be written

$$\max_{\{c\}_0^\infty} U = \int_0^\infty \log\left(c(t)\right) L e^{-\rho t} dt$$

s.t. *c(t)* >0,

$$\dot{b}(t) = rb(t) + w(t) - c(t),$$

$$\lim_{t\to\infty} b(t)e^{-rt} \geq 0,$$

Standard computations lead to the Keynes-Ramsey Rule (the Hamilton should be written out, and the first order conditions stated plus the derivations of the rule).

We may also observe, that since the economy is closed per capita wealth must equal per capita capital: b(t)=k(t). Due to constant returns to scale to K and L in the production function (at the level of the individual firm), we know that rb +w=y.

Question 2.4.

a) Balanced growth implies that all endogenous variables (c, k, y etc) grow at one and the same rate. Accordingly, the growth rate of the economy is pinned down by the Keynes-Ramsey rule. Subsituting for the real rate of return we get

$$\gamma = \alpha\left(\bar{A}L\right)^{\frac{1-\alpha}{\alpha}} - \rho$$

b) There are two empirical difficulties which “AK” models face.

First, as can be seen the size of the labor force enters the growth rate. Accordingly, the model predicts that larger countries should grow faster. There is little empirical evidence to suggest that this is in fact true.

Second, the model would also suggest that if the investment share rises, growth should pick up. This is criticized by Jones (1995), who finds that over the last century growth has remained remarkably stable in the OECD area, whereas investment rates have trended upwards. This would seem to contradict a close association between growth and investments. On the other hand, McGratten argues that Jones tests’ a performed over too short of a period of time. She goes on to show that over very long periods of time (>100 years) there does appear to be a tendency for growth to accelerate when investment shares go up. Hence, it remains a somewhat controversial issue whether this prediction of the model is consistent with the evidence or not. The chief difficulty, as should be clear, lies with identifying the relevant time horizon over which models’ such as this should have bearing.

Question 2.5.

a) No, it will not be Pareto optimal since the model involves a positive externality from capital accumulation. As a result, investment will be too low, and growth too slow, from a social perspective. For a full score, however, the student should demonstrate this, by solving the planner’s problem.
b) A subsidy to capital investment will do. It should be financed in by way of non-distortionary taxes, such as a lump sum tax.

PROBLEM 3
Main readings: B&S Ch. 6, Jones (1995), Nonnemann and Vanhout (1995), Ha and Howitt (2005).

Question 3.1

From the equation we immediately see that the size of the labor enters. This captures a "scale effect", whereby increasing R&D input yields faster growth. Jones (1995) criticises this aspect of the model, and proposes a modification whereby the growth rate of per capita GDP is determined by the growth rate of R&D input ("semi-endogenous growth"); ultimately the growth rate of population. This model has also been criticized on empirical grounds however (Ha and Howitt, 2005). The key problem is that TFP growth has remained trend less over the last 50 years or so (in the US), whereas the growth rate of R&D input (whether measured by scientists and engineers or expenditures) have declined.

The student may also note that this model would suggest a positive association between total expenditures on R&D (fraction of GDP) and growth. This prediction is consistent with the evidence (Nonnemann and Vanhout, 1995).

Question 3.2

The social planner's problem is

$$\max_{\{c\}_0^\infty} U = \int_0^\infty \log\left(c(t)\right) L e^{-\rho t} dt$$

st. $c(t)>0$, $x>0$

$$\dot{N}(t) = \frac{1}{\eta}\left(Y(t) - c(t)L - N(t)x\right),$$

$$Y(t) = AL^{1-\alpha}x^{\alpha}N(t),$$

$$N(t) \geq 0 \text{ for all t}$$

The current value Hamiltonian

$$H(c, x, N, \lambda, t) = \log\left(c(t)\right) L e^{-\rho t} + \lambda\left(\frac{1}{\eta}\left(Y - cL - Nx\right)\right)$$

The first order conditions wrt c, x and N:

$$\frac{1}{c} L e^{-\rho t} = \frac{\lambda}{\eta} L$$

$$\lambda\left(\frac{1}{\eta}\left(\alpha\frac{Y}{x} - N\right)\right) = 0 \Rightarrow \frac{Y}{N} = \frac{x}{\alpha}$$

$$\lambda\left(\frac{1}{\eta}\left(\frac{Y}{N} - x\right)\right) = -\dot{\lambda}$$

If we differentiate the FOC wrt c, with respect to time, and substitute into the FOC wrt N

$$\frac{\dot{c}}{c} = \frac{1}{\eta}\left(\frac{Y}{N} - x\right) - \rho$$

Next insert the FOC wrt x:

$$\frac{\dot{c}}{c} = \frac{1}{\eta}\left(\frac{1-\alpha}{\alpha}\right)x - \rho.$$

To find the solution for x we combine the production function and the FOC wrt x

$$\frac{Y}{N} = \frac{x}{\alpha} = AL^{1-\alpha}x^{\alpha} \Rightarrow$$

$$x = \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L$$

In sum, we have shown that

$$\frac{\dot{c}}{c} = \gamma = \frac{1}{\eta}\left(\frac{1-\alpha}{\alpha}\right)x - \rho, \text{where } x = \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L$$

The first equality follows from the fact that we are dealing with an “AK production function” – balanced growth is therefore known to prevail.

The difference in the two solutions is due the presence of imperfect competition in the market scenario; the exponent for α differs. The implication of monopoly power is that too few varieties are produced in the market equilibrium. This implies that the return on R&D falls, compared with the first best solution, for which reason the growth rate falls short of the first best solution.

Another way to look at the difference between the market and the planner is to say that while the market sees the value of a new idea by the implied monopoly profits, the planner views the value by the size of the “consumer surplus” generated by a new intermediate good. Hence, from a social perspective there is too little R&D going on in the market economy.

Question 3.3

The optimal policy aims to ensure that the equilibrium production of x coincides with the planner’s solution. A subsidy to final goods production (or intermediate goods production) would work. The subsidy must be financed a non-distortionary tax. For example, a lump sum tax.