Written Exam for the M.Sc. in Economics 2007-II

Economic Growth

Master's Course

12.06.2007

(4-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Problem 1
Verbal Discussion: Various Topics
Weight in final grade: 30 %

Question 1.1. The Robustness debate concerning "AK" models

In a much-cited paper, Robert Solow (1994) discusses the "AK-model" of endogenous growth.[1] In the end he concludes that (p. 51): "[…] this version of the endogenous-growth model is very un-robust. It can not survive without *exactly* constant returns to capital. But you would have to believe in the tooth fairy to expect that kind of luck." (Emphasis in original)

Do you agree with Solow? Explain.

Question 1.2. Growth and Taxes

Consider the following statement: "A lower tax on capital will always increase the long-run growth rate in models of endogenous growth featuring infinitely lived households"

Do you agree with the statement? Explain.

Question 1.3. Human capital quality and productivity differences

Explain how one may try to assess the importance of human capital *quality* differences in accounting for differences in labor productivity across countries.

Problem 2
Analytical Discussion: AK models of endogenous growth
Weight in final grade: 40 %

Consider the following model for a closed economy in the process of development. Output is produced by combining capital, K, labor, L, input, and productivity, A. The latter is taken as given by the individual firm. Consequently, for firm *i* we have:

$$Y_i(t) = K_i(t)^\alpha \left(A(t) L_i(t) \right)^{1-\alpha}$$

The price of output is normalized to 1, and all markets are competitive. The rate of capital depreciation is assumed to be zero: $\delta=0$. Time is continuous. The real rate of return is denoted by $r(t)$, and wages are denoted $w(t)$. The total labor force is constant over time.

Households are infinitely lived and derive utility from consumption, which is discounted at the constant rate ρ. They derive income from the return to accumulated wealth and from supplying 1

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[1] Robert Solow, 1994. "Perspectives on Growth Theory," *Journal of Economic Perspectives,* Vol. 8, (Winter), pp. 45–54.

unit of labor each period inelastically. Per capita wealth is denoted b(t). The per period utility function is given by $u(c) = \log(c(t))$, where c(t) is consumption per capita.

Question 2.1. Firm's problem and the aggregate production function

Derive the firm's inverse factor demand for capital and labor, and use these first order conditions from the firm's maximization problem to show that the aggregate production function can be written

$$Y(t) = K(t)^\alpha \left(A(t)L(t)\right)^{1-\alpha},$$

where $Y(t)$ is total output, $K(t)$ is the aggregate supply of capital, and $L(t)$ is total labor supply.

Question 2.2. Equilibrium factor prices

Assume

$$A(t) = \bar{A}Y(t)^\mu, \bar{A} > 0, 0 < \mu \leq 1 \qquad (*)$$

(a) Provide an economic interpretation of this assumption about productivity, $A(t)$.
(b) Derive the equilibrium factor prices, employing equation (*).
(c) What do we need to assume with respect to the size of μ in order to ensure endogenous growth? Is this assumption empirically reasonable?

Question 2.3. Consumer's problem

Write down the problem faced by the representative consumer and proceed to solve it. In particular, show that the optimal consumption trajectory fulfils
$$\dot{c}/c = r - \rho.$$

Question 2.4. The balanced growth path

Balanced growth prevails in this economy *given* the condition on μ from question 2.2.c. In this case the model is of the "AK" type.

(a) Use this information to derive the growth rate of the economy assuming the parameter restriction is fulfilled.
(b) Comment on the empirical predictions of this model, and discuss their validity.

Question 2.5. Welfare

(a) Is the decentralized solution Pareto Optimal? Explain.
(b) If not, what policy would ensure that the market solution becomes Pareto Optimal?

Problem 3
Analytical Discussion: R&D driven endogenous growth
Weight in final grade: 30 %

Consider an R&D driven growth model, where productivity expands due to increasing product variety. At any given instant in time *N(t)* differentiated goods are in existence. The equilibrium is symmetrical in the sense that the quantity of each variety produced is the same across varieties and given by *x*. In this symmetrical equilibrium we may write production of final output as

$$Y(t) = AL^{1-\alpha} x^{\alpha} N(t),$$ where A>0 is a parameter.

Hence, from a technical perspective the model is of the "AK type", since the labor force, L, is constant through time.

On the expenditure side, final output can be used for three purposes: Consumption, *C(t)=c(t)L*, production of intermediate goods, *N(t)x*, and production of ideas, $\eta(\frac{dN}{dt}) = \eta \dot{N}$. Hence, the output cost of a new idea is η.

Households are infinitely lived and derive utility from consumption, which is discounted at the constant rate ρ. The per period utility function is given by $u(c(t)) = \log(c(t))$, where *c*(t) is consumption per capita.

In an unregulated market economy, the growth rate of GDP per capita (γ) is given by

$$\gamma = \frac{1}{\eta}\frac{1-\alpha}{\alpha} x - \rho, \text{where } x = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L$$

Question 3.1. Empirical relevance

Consider the growth rate of the market economy stated above. Comment on the empirical predictions of this model, and discuss briefly their validity.

Question 3.2. Social Planner Solution
Show that the socially optimal growth rate of the economy is given by

$$\gamma = \frac{1}{\eta}\frac{1-\alpha}{\alpha} x - \rho, \text{ where } x = \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L,$$

and comment on the difference to the market solution.

Question 3.3 Optimal Policy
What policy could align the market solution with the social optimum?