The Effects of Entry in Bilateral Oligopoly*

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11th December 2013

Abstract

We show that a firm’s profits under Cournot oligopoly can be increasing in the number of firms in the industry if wages are determined by (decentralised) bargaining in unionised bilateral oligopoly. The intuition for the result is that increased product market competition following an increase in the number of firms is mirrored by increased labour market rivalry which induces (profit-enhancing) wage moderation, a result which does not occur if unions can coordinate their wage demands under centralised bargaining. Whether the product or labour market effect dominates depends both on the extent of union bargaining power and on the nature of union preferences. In an extension of the model, we also show that if a firm has a first-mover advantage in the Stackelberg sense, it is more likely to benefit from the positive effects of entry.

Running title: Effects of Entry in Bilateral Oligopoly.

Keywords: Unionised bilateral oligopoly, wage bargaining, firm profits.


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*The authors are grateful to Keith Cowling, Andrew Oswald, Mark Stewart, Mike Waterson, Huw Edwards and Arijit Mukherjee for helpful comments and discussion.

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1 Introduction

In the standard Cournot model of oligopoly, each firm’s profits decrease as the number of firms competing in the product market increases. This fundamental result in microeconomics was formally established by Seade (1980a). One important implication, for example, is that incumbent firms have an unambiguous incentive to deter entry by new firms. In this paper, we show that when firms’ costs (wages) are determined by decentralised bargaining between (downstream) firms and (upstream) labour unions in unionised bilateral oligopoly, then the relationship between profits-per-firm and the number of firms depends on relative bargaining power and on union preferences. If unions are relatively powerful and place sufficient weight on wages relative to employment, then an increase in the number of firms in the market can raise the profits of each firm, reversing the standard Cournot result. The intuition for the result is that increased product market competition following an increase in the number of firms is mirrored by increased labour market rivalry which induces (profit-enhancing) wage moderation, a result which does not occur if unions can coordinate their wage demands under centralised bargaining. The basic model we develop considers decentralised bargaining between a firm and an organised labour union. But as Booth (1995) and others have argued, the bargaining model is likely to be relevant wherever workers can exert bargaining power, whether or not they are formally organised into labour unions. For example, as Lindbeck and Snower (1988) have shown, ‘insider’ power is likely to prevail even in the absence of organised unions.

One implication of this result is that firms in unionised bilateral oligopoly do not necessarily have incentives to deter entry: a duopolist’s profits can exceed those of a monopolist, for example. A corollary of this is that the presence of unions might be associated with an increase rather than a decrease in product market competition. Thus, the model identifies a mechanism to counter that analysed in the classic model of Williamson (1968), according to which unions are associated with inhibiting product market competition. We also show that if a firm has a first-mover advantage, the result is strengthened: in fact, it is more likely that a Stackelberg leader can raise profits following the entry of other firms. A second corollary of our model is that when the bilateral oligopoly is characterised by upstream profit-maximising firms rather than by utility-maximising labour unions the profits of each downstream firm are necessarily falling in the number of firms, as in the standard model. This is because the firm-firm bilateral oligopoly can be characterised as a special case of the union-firm bilateral oligopoly, in which we can show that the upstream agent’s preferences are such that the implicit weight on the bargained price is not sufficient to cause profits to increase with entry.

Our finding that each Cournot firm’s profit can increase with the number of firms may also arise in

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1 Early work by Bain (1956) and Sylos-Labini (1962) sparked an interest in the broader topic of the effects of entry. The first formal attempts to directly study these effects are, to the best of our knowledge, found in Frank (1965), Okuguchi (1973) and Ruffin (1971).
different environments. Recently, Matsushima and Mizuno (2012) consider a simple Cournot model where downstream firms engage in process R&D. They find that the profits of upstream input suppliers may increase with entry since more upstream suppliers leads downstream firms to engage in more R&D, correctly anticipating a fall in the input price which comes about from entry in the upstream market. Mukherjee and Zhao (2008) have shown that profits of some incumbent firms may increase if they are relatively cost efficient relative to other incumbent firms and relative to Stackelberg followers.\(^2\) Their finding is, unlike ours, independent of the vertical structure.\(^3\) Naylor (2002) shows conditions under which industry profits are increasing with the number of firms in the market, but does not address the issue of the individual firm’s profit level. It is less surprising that industry profits can increase with the number of firms as such a result consistent with falling profits-per-firm. In the related literature on unionised oligopoly, Dowrick (1989) develops a framework in which unions act as the upstream agent and shows how the bargained wage varies with market size, but does not focus on the relationship between profits and the number of firms. Horn and Wolinsky (1988) examine a differentiated oligopoly with upstream agents (unions) and downstream firms, but assume a duopolistic market.\(^4\) In the literature on unions and entry deterrence, the usual approach builds on Williamson’s (1968) insight that incumbent firms might collude with unions to enforce industry-wide wage premia in order to deter entry. Unions are seen as an employer instrument to preserve product market power. In the model we outline below, it emerges that in the presence of unions firms might have reduced incentives to deter entry: in other words, in contrast to the Williamson insight, unions might have a pro-competitive impact within an imperfectly competitive product market. In a related literature, Bughin (1999) compares firms’ and unions’ preferences over bargaining scope and finds that entry deterrence is an influence on the choice of bargaining agenda.

The rest of this paper is organized as follows. In Section 2, we outline the basic model and in Section 3 we examine how firms’ profits vary with the number of firms. Section 4 augments the basic model to the case of a Stackelberg leader. Section 5 addresses the issue of firm-firm rather than union-firm bilateral oligopoly. Section 6 examines the sensitivity of the results to assumptions regarding the level at which wage bargaining takes place. Section 7 closes the paper with conclusions and further remarks.

\(^2\)The requirements for their result to hold are rather specific. In fact, all of the following must be satisfied: there must be a large number of cost inefficient incumbents relative to cost efficient incumbents and entrants, and the cost difference between the incumbents, and that between the efficient incumbents and entrants must also be large.

\(^3\)Similar papers which demonstrate a positive relationship between firm profits and entry due to a vertical structure include: Tyagi (1999) and Mukherjee et al. (2009).

\(^4\)Similarly, Naylor (1999) considers unionised oligopoly in the context of international trade and economic integration, but does not allow the number of firms to vary.
2 The Model

We follow Horn and Wolinsky (1988) in supposing that the upstream agents are firm-specific trade unions bargaining with firms over the wage rate. We analyse a non-cooperative two-stage game in which \( n \) identical firms produce a homogeneous good. In the first stage (the labour market game), each firm independently bargains over its wage with a local labour union: bargaining is decentralised. The outcome of the labour market game is described by the solution to the \( n \) union-firm pairs’ sub-game perfect best-reply functions in wages. In the second stage (the Cournot product market game), each firm sets its output - given pre-determined wage choices from stage 1 - to maximize profits. We proceed by backward induction.

(i) **Stage 2: the product market game**

Let linear product market demand be written as:

\[
p = a - bX, \tag{1}
\]

where \( X = \sum_{i=1}^{n} x_i \). Profit for the representative firm \( i \) can be written as:

\[
\pi_i = \left[ a - b \sum_{i=1}^{n} x_i - w_i \right] x_i \tag{2}
\]

where \( w_i \) is the outcome of the wage bargain for union-firm pair \( i \). In this short-run analysis, we exclude non-labour costs. We also assume a constant marginal product of labour, and set this as a numéraire.

Under the Cournot-Nash assumption, differentiation of (2) with respect to \( x_i \) yields the first-order condition for profit maximisation by firm \( i \), from which it is straightforward to derive firm \( i \)'s best-reply function in output space as:

\[
x_i = \frac{1}{2b} \left[ a - w_i - b \sum_{j=1}^{n} x_j \right]. \tag{3}
\]

Solving across the \( n \) first-order conditions, the \( n \) best-reply functions can be re-written as sub-game perfect labour demand equations. From Equation (3) for example, the expression for firm \( i \)'s labour demand is:

\[
x_i = \frac{1}{(n+1)b} \left[ a - nw_i + \sum_{j=1}^{n} w_j \right]. \tag{4}
\]

It is useful to express firm \( i \)'s profits in terms of the vector of all firms’ wages. Substituting (4) in (2), we obtain,

\[
\pi_i = \frac{1}{(n+1)^2 b} \left[ a - nw_i + \sum_{j=1}^{n} w_j \right]^2. \tag{5}
\]
From (5), it follows that in symmetric equilibrium, with $w_i = w$,

$$\pi_i = \frac{1}{(n + 1)^2 b} [a - w]^2, \quad \forall \ i,$$  \hspace{1cm} (6)

where $w$ is the outcome of the Stage 1 wage-bargaining game. We note that if $w$ is set exogenously at the competitive level, $\bar{w}$, or if unions have no bargaining power, then, with $w = \bar{w}$ in (6), the firm’s profits are falling in $n$, the number of firms in the industry, as

$$\frac{d\pi_i}{dn} = -\frac{2}{(n + 1)^3 b} [a - \bar{w}]^2 < 0,$$  \hspace{1cm} (7)

which is the standard Cournot oligopoly result.

(ii) **Stage 1: the labor market game**

We assume that the representative trade union $i$ bargaining with firm $i$, has the objective described by the Stone-Geary utility function:

$$U_i = [w_i - \bar{w}]^{2\alpha_i} x_i^{2(1 - \alpha_i)}$$  \hspace{1cm} (8)

where $w$ denotes the wage which would obtain in a competitive non-unionised labour market. We choose the quadratic form for the Stone-Geary utility as this captures the special case of rent maximisation if $\alpha = 1/2$.\footnote{This form will be convenient for comparison with the case of firm-firm bilateral oligopoly considered in Section 5 below.}

Under the assumption of a right-to-manage model of Nash-bargaining over wages, we write the maximand as:

$$B_i = U_i^{\beta_i} w_i^{1 - \beta_i},$$  \hspace{1cm} (9)

where we assume that disagreement payoffs are zero. $\beta$ represents the union’s Nash-bargaining power in the asymmetric wage bargain.

Substituting (4), (6) and (8) into (9) yields,

$$B_i = \frac{1}{(n + 1)^{2(1 - \alpha\beta)} b^{1 + \beta - 2\alpha\beta}} [w_i - \bar{w}]^{2\alpha\beta} \left[ a - n w_i + \sum_{j=1}^{n} w_j \right]^{2(1 - \alpha\beta)}$$  \hspace{1cm} (10)

The first order condition derived from the Nash maximand, (10), is

$$\frac{\partial B_i}{\partial w_i} = \frac{2 [w_i - \bar{w}]^{2\alpha\beta - 1}}{(n + 1)^{2(1 - \alpha\beta)} b^{1 + \beta - 2\alpha\beta}} \left[ a - n w_i + \sum_{j=1}^{n} w_j \right]^{1 - 2\alpha\beta}$$  \hspace{1cm} \times \left\{ \alpha\beta \left[ a - n w_i + \sum_{\substack{j=1 \atop j \neq i}}^{n} w_j \right] - (1 - \alpha\beta)n[w_i - \bar{w}] \right\} = 0,$$  \hspace{1cm} (11)
from which it follows that, in symmetric sub-game perfect equilibrium,
\[ w = w_i = \bar{w} + \frac{\alpha \beta}{\alpha \beta + n(1 - \alpha \beta)} [a - \bar{w}]. \] (12)

From substitution of (12) in (4), we can represent symmetric equilibrium output by
\[ x_i = x = \frac{n(1 - \alpha \beta)}{(n + 1)b[\alpha \beta + n(1 - \alpha \beta)][a - \bar{w}]} \] (13)

Substituting (12) in (6) gives equilibrium firm profits of
\[ \pi_i = \pi = \frac{n^2(1 - \alpha \beta)^2}{(n + 1)^2b[\alpha \beta + n(1 - \alpha \beta)]^2} [a - \bar{w}]^2. \] (14)

It follows from Seade (1980a, 1980b) that the Cournot product market equilibrium characterised in (13) and (14) satisfies sufficient conditions for stability. For the linear demand case considered here, the sufficient conditions are that \( b > 0 \) and \( n > 0 \). The difference between our model and that of Seade (1980a) is that in our model costs are not exogenous, but are the result of strategic bargaining in the Stage 1 game. In the next section of the paper, we consider comparative static properties of the model.

3 Firm profits and the number of firms

We now investigate how the profits of each firm in sub-game perfect Nash equilibrium vary with the number of firms in the market. We establish Proposition 1.

**Proposition 1.** Profits-per-firm increase in the number of firms if and only if unions care sufficiently about wages and have sufficient bargaining power.

**Proof.** Differentiating (14) with respect to \( n \), we obtain
\[ \frac{\partial \pi_i}{\partial n} = \frac{2(1 - \alpha \beta)^2n}{(n + 1)^2[\alpha \beta + n(1 - \alpha \beta)]^2} \left[ \alpha \beta - n^2(1 - \alpha \beta) \right] [a - \bar{w}]^2, \] (15)

which is non-negative – implying that firm profits are non-decreasing in the number of firms – if and only if the following condition is satisfied:
\[ \frac{\alpha \beta}{1 - \alpha \beta} \geq n^2 \] (16)

From (16), it is clear that firm profits are more likely to be increasing in the number of firms the larger are both \( \alpha \) and \( \beta \) and the smaller is \( n \). If the product of \( \alpha \) and \( \beta \) is close to unity – for example, if wages are set by monopoly unions (\( \beta = 1 \)) with an objective function close to wage rate maximisation – then the value of
where the firm profits are increasing in the number of firms is potentially large. We proceed by re-writing (14) as
\[ \pi_i = \frac{n^2(1 - \delta)^2}{(n + 1)^2b[\delta + n(1 - \delta)]^2}[a - \bar{w}]^2. \] (17)
where \( \delta \) denotes the product of \( \alpha \) and \( \beta \). Evaluating (17) for various values of \( n \) yields:

For \( n = 1 \), \( \pi_{i|n=1} = \frac{(1 - \delta)}{2} \frac{[a - \bar{w}]^2}{b}. \) (18)

For \( n = 2 \), \( \pi_{i|n=2} = \frac{2(1 - \delta)}{3} \frac{[a - \bar{w}]^2}{b}. \) (19)

For \( n = 3 \), \( \pi_{i|n=3} = \frac{3(1 - \delta)}{4[3 - 2\delta]} \frac{[a - \bar{w}]^2}{b}. \) (20)

For \( n = 4 \), \( \pi_{i|n=4} = \frac{4(1 - \delta)}{5[4 - 3\delta]} \frac{[a - \bar{w}]^2}{b}. \) (21)

From comparison of (18) and (19), it follows that the profits of each duopolist exceed that of a monopolist if \( \delta \geq 0.67 \). That is,
\[ \pi_{i|n=2} > \pi_{i|n=1} \text{ if and only if } \delta > \hat{\delta}_2 = 0.67, \] (22)

where \( \hat{\delta}_2 \) is the critical value of \( \delta \) such that the profit of each of two firms under \( n \)-firm Cournot oligopoly (with \( n = 2 \)) is just equal to the profit level associated with the case of monopoly, in which \( n = 1 \). Similarly, we can show by successive pair-wise comparisons of (19), (20) and (21) that

\[ \pi_{i|n=3} > \pi_{i|n=2} \text{ if and only if } \delta > \hat{\delta}_3 = 0.86, \] (23)

and that

\[ \pi_{i|n=4} > \pi_{i|n=3} \text{ if and only if } \delta > \hat{\delta}_4 = 0.92, \] (24)

Indeed, it can be demonstrated that the critical level of \( \delta \) is always less than one: implying that for sufficiently high \( \delta \), an increase in \( n \) always leads to an increase in firm profits. We can show this by evaluating (17) at the value of \( n = N \) and at the value of \( n = N + 1 \) and comparing. It is straightforward to show that the value of the individual firm’s profits when \( n = N + 1 \) exceeds profits when \( n = N \) if and only if
\[ \delta > \hat{\delta}_{N+1} = \frac{N(N + 1)}{N(N + 1) + 1}, \] (25)

where \( \hat{\delta}_{N+1} \) is strictly less than unity \( \forall N \). In reality, however, \( \delta \) is unlikely ever to be sufficiently high that firm profits increase in \( n \) over and above the values considered explicitly in conditions (22) through (24). The implication of this is that profits-per-firm will be maximised when the oligopolistic industry consists of only a small number of firms. The novelty of our result is that this number is not necessarily equal to one.
Figure 1 plots (18) through (21) in \((\pi_i, \delta)\)-space and uses (22) through (24) to demonstrate the critical values of \(\delta\) at which the maximal values of profits-per-firm shift with the number of firms. Consider now Figure 2 which represents (22) through (24) in \((\alpha, \beta)\)-space to depict the combinations of \(\alpha\) and \(\beta\) which produce iso-profit contours for successive increments in the value of \(n\). In Region A, for example, all combinations of \(\alpha\) and \(\beta\) lie below the iso-profit schedule which satisfies (18) and (19) simultaneously. In this region, then, a monopolist’s profits always dominate the profits-per-firm associated with any alternative value of \(n\), \(n > 1\). Conversely, in Region B, each firm in a duopoly market earns profits which exceed those of the monopolist. Finally, Region C represents combinations of \(\alpha\) and \(\beta\) such that profits-per-firm are maximised when there are three firms competing in the Cournot oligopoly, and Region D when there are four or more firms.

What is the intuition for our result that profits-per-firm increase in the number of firms in the market
if $\delta = \alpha \beta$ is sufficiently high and $n$ sufficiently low? In the standard oligopoly model, an increase in the number of firms unambiguously reduces profits-per-firm through increased product market competition which reduces product price. We can see this mechanism working in the model of bilateral oligopoly developed in this paper. We substitute (4) in (1) and solve for the equilibrium price: this gives

$$p = \frac{1}{n+1} (a + nw),$$

(26)

where $w$ is given by (12). From (26), it follows that

$$\frac{dp}{dn} = \frac{n}{n+1} \frac{dw}{dn} - \left[ a - w \right] \left(\frac{n}{n+1}\right)^2.$$

(27)

Assuming that $\frac{dw}{dn} \leq 0$, as we demonstrate below, it follows from (27) that $\frac{dp}{dn}$ must be negative: an increase in $n$ leads to a fall in product price.

In addition to the profit-reducing effect of the fall in product price, however, the increase in the number of firms competing in the market also induces unions to moderate their wage demands. Dowrick (1989) established this effect in a model closely related to ours. We can see the result simply by differentiating (12)
with respect to \( n \), which yields

\[
\frac{dw}{dn} = -\frac{\delta(1-\delta)}{\delta + n(1-\delta)^2}(a-w) \leq 0. \tag{28}
\]

Furthermore, this wage moderation effect captured in (28) is increasing in the product \( \alpha\beta \). It is readily shown from (28) that \( \frac{d^2w}{dn^2} < 0 \). At one extreme, for example if \( \alpha\beta = 0 \), then there is no wage moderation effect associated with an increase in the number of firms: that is, there can be no wage moderation effect if unions exert no influence on the wage, as is implied by \( \alpha\beta = 0 \).

Thus, the presence of unions with influence over wages induces a (profit-enhancing) wage moderation effect to accompany the (profit-damaging) price-reducing effect of an increase in \( n \). Which effect dominates depends both on the product \( \alpha\beta \) - as shown in (28) - and on the size of \( n \) itself. To see this, consider (27) once more. As \( n \) becomes very large, the fraction \( \frac{n}{n+1} \) tends to one, implying that \( \frac{dp}{dn} \) tends to equal \( \frac{dw}{dn} \) minus the diminishing but positive term in square brackets. Hence, for sufficiently large \( n \) the absolute size of the price effect dominates that of the wage effect. For small enough \( n \), however, the fraction \( \frac{n}{n+1} \) in (27) is sufficiently less than one that \( \frac{dw}{dn} \) exceeds \( \frac{dp}{dn} \) and the wage moderation effect dominates, causing an increase in \( n \) to raise profits-per-firm for sufficiently high values of \( \alpha \) and \( \beta \). This result that profits-per-firm might be increasing in the number of firms is distinct from but closely related to the Horn and Wolinsky (1988) result that the merger of a downstream duopoly can lower profit through its effects on bargained input prices.

### 3.1 Entry deterrence incentives

Following Williamson (1968), unions have been characterised as a potential instrument with which incumbent firms can deter further market entry. In the standard Cournot oligopoly model, with profits-per-firm unambiguously decreasing in the number of firms in the market, there is an unambiguous incentive for firms to attempt to restrict entry. This was the explicit focus of the analysis of Seade (1980a) in establishing the nature of the relationship between the number of firms and profits-per-firm in the standard Cournot oligopoly model. But in the unionised bilateral oligopoly framework we have developed in the current paper, the very presence of unions with influence over wages leads to the possibility that, at least for small \( n \), profits-per-firm increase with \( n \). Thus, if (decentralised) unions have sufficient influence over wages, a single-firm monopolist might have incentives to encourage rather than to deter entry by one or more firms. If this were the case, a potential monopolist might accommodate that number of Cournot rivals which maximise profits per firm. Alternatively, the presence of influential unions might induce an incumbent monopolist toward a multi-divisional structure with distinct plants operating as if in competition with one another.

In the current paper, we analyse the effects of entry in the presence of labour unions following the standard assumption that firms are identical. As Seade (1980a) observes, with a non-homogeneous industry, entry cannot be interpreted simply as an increase in firm numbers: it becomes necessary to model the
nature of the marginal firm and its entry/exit decision. In the next section, we model one type of industry non-homogeneity, one that gives an incumbent firm a first-mover advantage in the industry.

4 Firm profits with a Stackelberg leader

Thus far we have studied the effects of entry when all firms compete à la Cournot, without consideration of any structural advantages due to incumbency. In this section, we examine how the Stackelberg first-mover advantage of an incumbent firm affects the basic results in the previous section. This analysis is somewhat more complicated due to the asymmetric nature of the game played by, respectively, the Stackelberg leader and its union, and that of the followers and their unions. In equilibrium, unlike in the Cournot case, labour costs will differ across bargaining pairs in a way which depends on the firm’s order of moves. Denote by \( x_l \) the output of the Stackelberg leader, by \( x_k \) the output of follower \( k = 1, \ldots, m \), and by \( X_{-l} = \sum_{k=1}^{m} x_k \) the aggregate output of the \( m \) followers. Profits of the leader and that of a typical follower, respectively, can be written as:

\[
\pi_l = [a - bX - w_l] x_l \quad \text{and} \quad \pi_k = [a - bX - w_k] x_k,
\]

where \( X = x_l + X_{-l} \) denotes the aggregate output of all firms. The game now has the following stages: in the first, each firm independently bargains over its wage with a local labour union; in the second, the Stackelberg leader decides how much to produce; and in the final stage, \( m \) Stackelberg followers make their output decisions. The Stone-Geary utility functions representing the preferences of the union in the Stackelberg firm, and that representing workers in all of the \( m \) symmetric followers, respectively, are:

\[
U_l = [w_l - \overline{w}]^{2\alpha} x_l^{2(1-\alpha)} \quad \text{and} \quad U_k = [w_k - \overline{w}]^{2\alpha} x_k^{2(1-\alpha)} \quad \forall \ k.
\]

The maximands for the two types of firms, respectively, are:

\[
B_l = U_l^\beta x_l^{1-\beta} \quad \text{and} \quad B_k = U_k^\beta x_k^{1-\beta} \quad \forall \ k.
\]

Since the solution procedure for this game is similar to the Cournot game described in the previous section, we restrict the formal derivation to the Appendix, where we show that the output produced by the Stackelberg leader with \( m \) followers is:

\[
x_l = \frac{(1 - \delta)(\delta + 2m + 1)}{2m + 1 + \delta[1 - m(1 + \delta)]}(a - \overline{w}).
\]

The expression for the leader’s profits can also be solved as:

\[
\pi_l = \frac{1}{(m + 1)4b} \left( \frac{(1 - \delta)^2(\delta + 2m + 1)^2}{2m + 1 + \delta[1 - m(1 + \delta)]} \right)^2 (a - \overline{w})^2.
\]
Under what circumstances will the entry of a Stackelberg follower raise profits of a leader? As in the previous section, we proceed by evaluating (33) for various $m$:

For $m = 0$, \( \pi_{l|m=0} = \left( \frac{1-\delta}{2} \right)^2 \frac{(a-w)^2}{b}. \) \hfill (34)

For $m = 1$, \( \pi_{l|m=1} = \left( \frac{(1-\delta)(3+\delta)}{2(3-\delta^2)} \right)^2 \frac{(a-w)^2}{2b}. \) \hfill (35)

For $m = 2$, \( \pi_{l|m=2} = \left( \frac{(1-\delta)(5+\delta)}{2[5-\delta(1+2\delta)]} \right)^2 \frac{(a-w)^2}{3b}. \) \hfill (36)

For $m = 3$, \( \pi_{l|m=3} = \left( \frac{(1-\delta)(7+\delta)}{4[7-\delta(2+3\delta)]} \right)^2 \frac{(a-w)^2}{b}. \) \hfill (37)

From comparison of (34) and (35), it follows that the profits of a Stackelberg leader with one follower exceed that of a monopolist if and only if $\delta \geq 0.65$. That is,

\[ \pi_{l|m=1} > \pi_{l|m=0} \text{ if and only if } \delta > \hat{\delta}_1 = 0.65, \]

where $\hat{\delta}_1$ is the critical value of $\delta$ above which a first-moving firm can raise its profits by accommodating one Stackelberg follower, rather than by maintaining a monopoly position in its market.

The profits of the Stackelberg leader will increase in the number of rivals it accommodates, depending on the values of $\delta$. Equations (36) and (37) show the Stackelberg leader’s profits when the number of followers is two and three, respectively. Comparison of (35) and (36) shows that the Stackelberg leader would prefer two followers rather than one if and only if $\delta > 0.79$. \hfill (39)

That is,

\[ \pi_{l|m=2} > \pi_{l|m=1} \text{ if and only if } \delta > \hat{\delta}_2 = 0.79, \]

Similarly, the Stackelberg leader would prefer three followers to two if and only if $\delta > 0.84$. That is,

\[ \pi_{l|m=3} > \pi_{l|m=2} \text{ if and only if } \delta > \hat{\delta}_3 = 0.84, \]

It is interesting to compare (38) with the equivalent equation for the Cournot case, that is (22). The critical values of $\delta$ are very similar, though slightly lower in the case of the Stackelberg firm, indicating that a preference by a monopoly firm to accommodate the entry of a competitor is marginally more likely when the firm is, post-entry, able to act as a Stackelberg leader rather than compete `a la Cournot. Likewise, we have $\hat{\delta}_3 = 0.79 < 0.86 = \hat{\delta}_3$ and $\hat{\delta}_4 = 0.84 < 0.92 = \hat{\delta}_4$. The fact that the critical values of $\delta$ are lower in the Stackelberg case does not depend on the number of entrants. Denote by $\hat{\delta}_M$ the critical value of $\delta$ above which the Stackelberg leader increases its profits by accommodating the $M$th entrant. We establish Proposition 2:

\hfill (40)

\[ \text{Note that } m = 0 \text{ describes the case of monopoly, a leader with no followers, which corresponds to the case of } n = 1 \text{ in Section 3 on Cournot oligopoly. Similarly, } m = 1 \text{ corresponds to } n = 2 \text{ etc.} \]

\hfill (38)

\[ \text{The values of } \hat{\delta}_2 \text{ and } \hat{\delta}_3 \text{ were found by simulation in Maple 16.} \]
Proposition 2. The critical level of $\delta$ above which a Cournot firm increases its profits following the entry of the $N$th competitor is strictly greater than the equivalent level of $\delta$ for a Stackelberg firm accommodating the $M$th follower, that is $\delta_{N+1} > \delta_M$ for $N = M$.

Proof. In Appendix B

The extent to which the Stackelberg leader benefits from entry exceeds the equivalent marginal benefit accruing to firms under Cournot competition. The intuition for this is that since the Stackelberg leader is able to capture a larger market share, it will also capture a greater share of the benefits of entry when $\delta$ is high.

5 Firms as upstream agents

Suppose that the upstream agent is not a utility-maximising trade union but is a profit-maximising firm with the objective function

$$\pi_{U_i} = (w_i - \bar{w})x_i, \tag{41}$$

where $\bar{w}$ represents the upstream firm’s fixed input price and $w_i$ now denotes the price of the intermediate product sold by upstream firms to their downstream firm pair. Bargaining is still assumed to be locally decentralised with an equal number of upstream and downstream agents. Then the firm-firm Nash bargain over the intermediate product price solves

$$B_{i}^E = \pi_{U_i}^{\beta} \pi_i^{1-\beta}. \tag{42}$$

Formally, this problem is equivalent to that described in equations (10) through (14) above, with the implicit value of $\alpha$ set at one-half. Hence, even in the extreme case in which upstream firms have all the bargaining power, so that $\beta = 1$, the implicit value of the product $\delta = \alpha \beta$ is never greater than one-half. Hence, it is always less than the critical value, $\hat{\delta}_2$, above which profits-per-duopolist exceed those of a monopolist. Proposition 3 follows.

Proposition 3. Profits-per-firm in the downstream industry are never higher than in the case of monopoly when upstream and downstream agents are both characterised as profit-maximising firms.

Proof. From condition (22) - see also the graphic representations in Figures 1 and 2 - it follows that the critical threshold value of $\hat{\delta}_2$, exceeds the maximum of $\delta$ associated with the case in which upstream agents are profit-maximising firms.

8This assumption is more plausible in the union-firm case where the existence of the union can be thought of as arising as an institutional response to the existence of the firm. A similar story to explain a one-to-one matching between the number of upstream and downstream agents in the case of firm-firm bargaining is less convincing
Centralisation of wage bargaining

In the basic union-firm model outlined in Section 3, we assumed explicitly that wage bargaining occurs at the decentralised level of the individual union-firm pair. The extent to which wage bargaining is decentralised or is centralised at either the industry or economy-wide level varies across countries and over time. The classic macroeconomic work of Calmfors et al. (1988) has exploited variation across countries in the level at which wage bargaining takes place in order to infer the nature of a relationship between the level of centralisation and a country’s macroeconomic performance. It has been argued that industry-level wage bargaining produces the worst possible outcome because it fails to internalise potential adverse externalities associated with union-firm wage bargaining. In contrast, it is argued that both fully decentralised bargaining and fully centralised bargaining force bargaining agents to internalise wage externalities and hence yield efficient outcomes. Consider the basic model of Section 3, but incorporating the assumption that all unions and firms negotiate jointly over the level of wages. Then the Nash maximand defined in (9) becomes

\[ B_C = \left( \sum U \right)^\beta \left( \sum \pi \right)^{1-\beta}, \]  

(43)

where \( \sum \pi \) is the sum of the individual firms’ profits - given by (6) - and \( \sum U \) is the sum over the unions’ utility functions - given by (8). In the Nash maximand, it is assumed that all bargained wages will be equal: thus, \( w_i = w \) by assumption. Substituting this and the sum over (4), (6)) and (8) in (43) yields the Nash centralised wage-bargaining maximand:

\[ B_C = \frac{n}{(n+1)^2(1-\alpha \beta) \beta \beta - 2 \alpha \beta} \left[ w - \bar{w} \right]^{2 \alpha \beta} [a - w]^{2(1-\alpha \beta)}. \]  

(44)

The first order condition derived from the centralised-bargaining Nash maximand is then

\[ \frac{\partial B_C}{\partial w} = \frac{2n[w - \bar{w}]^{2\alpha \beta}[a - w]^{-2\alpha \beta}}{(n+1)^2(1-\alpha \beta) \beta \beta - 2 \alpha \beta} \times \left[ \alpha \beta \left[ \frac{a - w}{w - \bar{w}} \right] - (1 - \alpha \beta) \right]. \]  

(45)

Setting this first order condition equal to zero and solving for \( w \) yields:

\[ w = w_i = \bar{w} + \alpha \beta (a - \bar{w}). \]  

(46)

This establishes Proposition 4.

**Proposition 4.** Under centralised bargaining, the wage is independent of \( n \), the number of firms in the industry.

It follows from Proposition 4 that in the case of centralised bargaining, there is no wage moderation effect associated with an increase in \( n \). This lies behind Calmfors et al. (1988) and related analyses (see also Moene et al., 1993) . It also follows from (46) that with perfect competition and decentralised bargaining, unions have no effect on wages: all wage externality effects are internalised. To see this within our model,
let $n$ become very large: then the bargained wage given by (12) tends to the competitive non-union level, $w$. With centralised (industry-level) bargaining, in contrast, even with large $n$, the wage will be set above the competitive level, as shown in (46). Under decentralised bargaining, a wage externality arises only with the introduction of imperfect competition, represented by a falling and finite value of $n$. Increasing $n$ is associated with increasingly internalising the negative wage externality: which is just an alternative interpretation of what we have previously referred to as the wage moderation effect of increasing the number of firms in competition.

7 Conclusions

In this paper, we have developed a simple model of a unionised oligopoly in order to demonstrate that the standard cornerstone Cournot result that profits-per-firm are falling in the number of firms in the product market is not necessarily valid when firms’ input prices are determined endogenously through bargaining with upstream agents (labour unions). We have shown that if wage bargaining is decentralised (that is, firm-specific), then profits-per-firm will increase with the number of competing firms if unions care sufficiently about wages, relative to employment, and possess sufficient bargaining power. One corollary of this result is that if unions do possess sufficient influence over wages, it is no longer clear that incumbent firms will have an incentive to deter market entry. Wage bargaining in the model is interpreted as firm-specific bargaining with labour unions. To the extent that non-union labour also possesses bargaining power, the model is likely to be of wider significance.

The intuition for our result is that when wages are determined endogenously through bargaining, an expansion in the number of firms has a wage moderation effect which offsets the detrimental effect on firm profits associated with competitive reductions in product price. The more workers care about wages and the more powerful they are in bargaining, the greater is this wage moderation effect. We have shown that the conditions necessary for unions (as the upstream agent) to have the (unintended) effect of translating an increase in firm numbers into an increase in firm profits are not satisfied when the upstream agents are profit-maximising firms. We have also shown that the result holds only if union-firm bargaining is decentralised. Under centralised (industry-wide) bargaining, there is no wage moderation effect associated with an increase in the number of firms: this is because the bargained wage is independent of firm numbers under industry bargaining. Finally, we extended the basic Cournot case to a setting where one firm was a leader in the Stackelberg sense. We found that a firm with a Stackelberg first-mover advantage is more likely to gain from entry than a firm competing à la Cournot.
Appendix A

As usual, the Stackelberg game is solved backwards so we begin by determining the output of the followers. Maximising the profits of follower $k$ in (29) yields:

$$x_k = \frac{1}{2b} \left[ a - w_k - b \sum_{j=1 \atop j \neq k}^m x_j - bx_l \right].$$

(47)

Solving these $m$ reaction functions for $x_k$ and $X_l$ yields:

$$x_k = \frac{1}{(m+1)b} \left[ a - mw_k + \sum_{j=1 \atop j \neq k}^m w_j - bx_l \right].$$

(48)

$$X_l = \frac{1}{(m+1)b} \left[ ma - \sum_{k=1}^m w_k - mbx_l \right].$$

(49)

The output of the Stackelberg leader can now be found by maximising its profits in (29) using (49). This yields:

$$x_l = \frac{1}{2b} \left[ a - w_l(m+1) + \sum_{k=1}^m w_k \right].$$

(50)

Plugging (50) back into (48) and (49) yields the following expression for output of follower $k$ and aggregate output of all followers:

$$x_k = \frac{1}{(m+1)2b} \left[ a - (2m+1)w_k + (m+1)w_l + \sum_{j=1 \atop j \neq k}^m w_j \right];$$

(51)

$$X_l = \frac{1}{(m+1)2b} \left[ ma - (m+2)\sum_{k=1}^m w_k + n(m+1)w_l \right].$$

(52)

Now plugging (50) and (52) into (1) gives an expression for the common equilibrium price:

$$p = \frac{1}{2(m+1)} \left[ a + (m+1)w_l + \sum_{k=1}^m w_k \right].$$

(53)

Using this expression for the price we can now find profits of the Stackelberg leader and a follower, respectively, as:

$$\pi_l = \frac{1}{(m+1)4b} \left[ a - (m+1)w_l + \sum_{k=1}^m w_k \right]^2;$$

(54)

$$\pi_i = \frac{1}{(m+1)^24b} \left[ a - (2m+1)w_k + (m+1)w_l + \sum_{j=1 \atop j \neq k}^m w_j \right]^2.$$

(55)
The wage rate earned by supplying labour for a particular firm is derived from the labour game described in the previous section. To solve for the wage of the leader and a typical follower, respectively, we plug in (50) and (51) into the expressions for $U_l$ and $U_k$, respectively, in (30), and then substitute the resulting expressions into the maximands in (31). This yields, respectively,

$$B_l = \frac{1}{b^{1+\beta-2\alpha\beta}2^{\beta(1-\alpha)}[4(m+1)]^{1-\beta}}[w_l - \bar{w}]^{2\alpha\beta}$$

$$\times \left[ a - w_l(m+1) + \sum_{k=1}^{m} w_k \right]^{2(1-\alpha\beta)},$$

(56)

and,

$$B_i = \frac{1}{b^{1+\beta-2\alpha\beta}2^{\beta(1-\alpha)}2^{\beta(1-\alpha)}[4^{1-\beta}]}[w_l - \bar{w}]^{2\alpha\beta}$$

$$\times \left[ a - (2m+1)w_k + (m+1)w_l + \sum_{j=1, j \neq k}^{m} w_j \right]^{2(1-\alpha\beta)}.$$

(57)

Maximising, respectively, (56) and (57) with respect to $w_l$ and $w_k$ yield the expression for wages of a leader and a follower. The leader’s wage is in (56) and the interested reader may note that the solution for the follower is.

$$w = w_i = \frac{a\delta(1+\delta) + \{2n + 1 - \delta[\delta + n(1+\delta)]\}\bar{w}}{2n + 1 + \delta[1 - \delta(n+1)]},$$

(58)

where symmetry of followers, $i = 1, \ldots, n$ has been imposed. Equilibrium output and profit for the Stackelberg leader, respectively, can now be obtained by plugging (58) into (50) and (54). This yields the expressions in (32) and (33).

**Appendix B**

*Proof of Proposition 2.* We evaluate (33) at the value $m = M$ and at the value $m = M - 1$:

$$\pi_l|_{m=M} - \pi_l|_{m=M-1} = \frac{(1 - \delta)^2(a - \bar{w})^2}{4bM(M+1)[2M + 1 + \delta(1 - M(1+\delta))]^2[2M - 1 + \delta(1 - (M-1)(1+\delta))]^2} \times$$

$$\{ M(\delta + 2M + 1)^2(2M - 1 + \delta[1 - (M-1)(1+\delta)])^2 - (M + 1)(\delta + 2M - 1)^2(2M + 1 + \delta[1 - M(1+\delta)])^2 \}.$$  

(59)

This expression implicitly defines the value of $\tilde{\delta}_M$. Entry of the $M$th follower increases profit of the Stackelberg leader if the term in curly brackets $\{ \cdot \}$ in (59) is positive. That is, if and only if:

$$\{ M(\delta + 2M + 1)^2(2M - 1 + \delta[1 - (M-1)(1+\delta)])^2 -$$

$$(M + 1)(\delta + 2M - 1)^2(2M + 1 + \delta[1 - M(1+\delta)])^2 \} > 0.$$  

(60)
Or, alternatively,

\[
\{ \sqrt{M}(\delta + 2M + 1)(2M - 1 + \delta[1 - (M - 1)(1 + \delta)]) - \\
\sqrt{(M + 1)(\delta + 2M - 1)(2M + 1 + \delta[1 - M(1 + \delta)])} \} > 0. \tag{61}
\]

Evaluating this term at \( \delta = 0 \) yields:

\[- \left( \sqrt{M + 1} - \sqrt{M} \right)(2M + 1)(2M - 1) < 0, \tag{62}\]

which is strictly negative for \( M \geq 1 \). Evaluating (61) at \( \delta = 1 \) yields:

\[4 \left( \sqrt{M}(M + 1) - M\sqrt{M + 1} \right) > 0, \tag{63}\]

which is strictly positive for \( M \geq 1 \). It remains to be shown that (61) is monotonically increasing in \( \delta \in [0; 1] \).

Differentiating (61) wrt. \( \delta \) yields:

\[- 2 \left( \sqrt{M + 1} - \sqrt{M} \right)(\delta + 2M) + M \left( \sqrt{M + 1} - \sqrt{M} \right)(\delta(1 + \delta) \\
+ (1 + 2\delta)(\delta + 2M - 1)) + \sqrt{M}(1 + \delta)(1 + 3\delta), \tag{64}\]

which is strictly positive for \( M \geq 2 \) [NOTE TO ROBIN: explanation follows ]. This proofs there exists a unique intersection of \( \delta_M \in [0; 1] \). Next we show that \( \bar{\delta} > \check{\delta} \). Thus, evaluate (64) at \( \hat{\delta}_{N+1} \) from (25). This yields:

\[-1 - 6M - 448M^8 + 332M^{12} - 32M^9 + 428M^{10} + 536M^{11} + 112M^{13} + 16M^{14} + 51M^3 + 155M^5 + 150M^4 - 3M^2 - 9 \]

\[= \frac{(M^2 + M + 1)^6}{(M^2 + M + 1)^6} \tag{65}\]

which when I have written it more neatly can be shown to be positive.

\[\square\]

**References**


