Abstract

Corporate tax systems generally maintain a sharp distinction between debt and equity. However, the advent of hybrid instruments has transformed the universe of financial instruments into a debt-equity continuum and tax systems thus need to draw lines that distinguish the set of debt instruments from the set of equity instruments. When countries draw these lines differently, there is a scope for international tax planning: A multinational firm financing a foreign investment with a hybrid instrument categorized as debt in the host country and equity in the home country combines the benefits of tax deductible interest payments in the host country and tax favored dividend payments in the home country. This paper develops a theoretical model of strategic line drawing between debt and equity in the presence of hybrid instruments. In the absence of international cooperation, lines are generally drawn in a globally suboptimal manner. The inefficiency typically derives from the endeavors of policymakers to draw lines in ways that facilitate hybrid financing by domestic multinational firms and impede hybrid financing by foreign multinational firms with a view to eroding foreign taxation of domestic firms and enforcing domestic taxation of foreign firms.
1 Introduction

"Although in a given case the bond and the stock features of an instrument seem to be hopelessly interwoven, the courts are called upon to untangle them and decide whether in the last analysis the bond or the stock character of the instrument prevails. For in the eyes of the law there are no 'hybrid securities'. In the law a person is either a creditor or a stockholder; he cannot be both." Rudolf E. Uhlman, 1937.

Corporate tax systems generally maintain a sharp distinction between debt and equity. While both debt and equity represent sources of financing from the perspective of the firm, payments to holders of debt, interest payments, are deductible from the corporate tax base whereas payments to holders of equity, dividends, are not. The distinction between debt and equity usually rests on the dichotomy between the shareholder, an active investor with a share in firm profits and losses, and the creditor, a passive investor entitled to a fixed return regardless of the business fortune.¹

Legal scholars argue, however, that the distinction between debt and equity is one of degree rather than one of principle (e.g. Emmerich, 1985). Financial instruments differ in a large number of dimensions and firms frequently issue securities that resemble debt in some dimensions and equity in others. For instance, while pure debt instruments have a fixed maturity and a fixed return and pure equity instruments have no maturity and a return that is linked to firm profits, these characteristics can be combined differently to obtain two hybrid instruments: a perpetual loan, which has a fixed return and no maturity, and a profit sharing loan, which has a fixed maturity and a return that is linked to firm profits. Moreover, financial instruments may in any single dimension have characteristics that obscure the debt-equity distinction. For instance, a fixed maturity of 10,000 years is arguably an equity-like characteristic since it is economically almost equivalent to no maturity, however, it is less clear whether maturities of 20, 50 or 100 years should be considered more debt-like or equity-like characteristics. Rather than just two distinct financial instruments, debt and equity, the universe of financial instruments thus comprises a myriad of hybrid instruments that combine characteristics of standard debt and equity in different proportions and the legal literature therefore often refers to the universe of financial instruments as a debt-equity continuum (e.g. Hariton, 1994; Krahmal, 2005).

In the presence of hybrid instruments, any tax system that treats debt and equity differently needs to draw lines that distinguish the set of debt instruments from the set of equity instruments. We refer to these lines as demarcation rules. Not surprisingly, there are large differences in demarcation rules across countries. In the U.S., for instance, tax authorities consider the following properties when determining whether a financial instrument should be treated as debt or equity for tax purposes:² (i) maturity -

¹The following is a typical formulation of the debt-equity dichotomy from classical case-law: "The essential difference between a stockholder and a creditor is that the stockholder’s intention is to embark upon the corporate adventure, taking the risks of loss attendant upon it, so that he may enjoy the chances of profit. The creditor, on the other hand, does not intend to take such risks so far as they may be avoided, but merely to lend his capital to others who do intend to take them." (United States v. Title Guarantee & Trust Co., 123 F.2d 990, 993, 6th Cir. 1943; as cited in Hariton, 1994)
²IRS guidelines in Notice 94/47
whether the principal is reimbursed either at a fixed maturity or on demand; (ii) seniority - whether the claims of the holder are subordinate to the rights of general creditors, for instance in the case of bankruptcy; (iii) management - whether the holder has voting rights; (iv) return - whether the return represents a legally enforceable claim; (v) label - whether the instrument is labelled debt or equity. Connors and Woll (2001) review the general principles underlying delineation of debt and equity and the typical tax treatment of a number of specific hybrid financial instruments in Canada, France and the Netherlands and document very material cross-country differences.

The variation in demarcation rules across countries introduces the possibility that the same financial instrument is categorized as debt in one country and equity in another country. This represents an important tax planning opportunity for multinational firms. To see this, consider a multinational firm investing in a foreign subsidiary. If the investment is financed with a hybrid instrument that is successfully categorized as debt in the host country and as equity in home country, payments on the instrument are treated as tax deductible interest expenses at the level of the subsidiary and as tax favored dividends at the level of the parent company. The net result is a considerable tax saving compared to an investment in the form of pure debt or pure equity. In the former case, payments are consistently treated as interest and thus deductible at the level of the subsidiary but taxable at the level of the parent company (home country taxation). In the latter case, payments are consistently treated as dividends and thus tax favored at the level of the parent company but non-deductible at the level of the subsidiary (host country taxation).

A question naturally arising is whether the use of hybrid instruments for tax planning purposes is a serious concern or a mere theoretical possibility. While there is no direct empirical evidence on the use of hybrid instruments in international tax planning, there are other types of evidence suggesting that this is a matter of large empirical relevance: Legal practitioners have provided detailed accounts of the cross-country differences in the rules delineating debt and equity and the specific tax planning opportunities created by these differences (Connors and Woll, 2001). Legal scholars have emphasized the impotence of general anti-abuse rules in combatting cross-border hybrid instruments precisely because this type of tax planning, by exploiting differences in tax rules, is not abusive under any single set of tax rules (Rosenbloom, 1999). These two type of contributions clearly demonstrate that cross-

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3 A very related issue is that of hybrid entities, that is corporate entities, which for tax purposes are considered transparent by some countries and non-transparent by other countries.

4 Hybrid financial instruments may serve other purposes than international tax planning. By issuing securities that are categorized as debt for tax purposes and equity for financial reporting purposes, firms may combine the tax advantages of debt financing with the advantages of equity financing in terms of favorable credit ratings (Engel et al., 1999; Mills and Newberry, 2005). Similarly, banks and other financial institutions facing binding capital requirements may benefit from issuing securities that are categorized as debt for tax purposes and equity for regulatory purposes (Gergen and Schmitz, 1997).

5 An empirical assessment of the extent to which firms employ cross-border hybrid instruments presents obvious difficulties. Since the classification of financial instruments for tax purposes need not coincide with the classification for financial reporting purposes, reliable estimates require matched firm-level tax data from several jurisdictions. We are not aware of any existing datasets that satisfy this requirement.

6 In the words of Rosenbloom (1999): "The beauty of international tax arbitrage, when practiced most skillfully, is that
border hybrid instruments represent an effective and relatively low-risk tax planning device. Moreover, policymakers have shown an awakening interest in hybrid structures. For instance, the Internal Revenue Service recently announced that cross-border hybrid instruments were to be among its highest compliance priorities, which suggests that this type of tax planning is judged to have a substantial adverse impact on collected corporate tax revenues (Internal Revenue Service, 2007). Finally, there is econometric evidence suggesting that recent declines in the effective tax rates facing foreign affiliates of US multinationals are due to more aggressive use of international tax planning rather than changes in statutory tax rates (Altshuler and Grubert, 2005). This is clearly consistent with widespread use of hybrid instruments for tax avoidance purposes.

These considerations suggest that line drawing between debt and equity is associated with potentially important cross-border spill-over effects. For instance, if a country $i$ adopts a demarcation rule that categorizes almost all financial instruments as equity, it becomes relatively easy for firms in country $i$ to avoid taxation of foreign investments by means of hybrid instruments categorized as debt in the host country and equity in country $i$. Conversely, it becomes relatively difficult for foreign firms investing in country $i$ to obtain a similar tax advantage by means of hybrid instruments categorized as debt in country $i$ and equity in the home country. While the example illustrates that line drawing between debt and equity has consequences for government revenue and firm profits in foreign countries, it also points to a possible scope for strategic line drawing: Countries may draw lines between debt and equity with a view to influencing the financial decisions of domestic and foreign firms in ways that increase domestic welfare at the expense of foreign welfare.

In order to explore this hypothesis, we develop a model of line drawing between debt and equity in an international taxation setting. At the heart of the paper is a simple one-dimensional model of hybrid instruments and their classification for tax purposes. We assume that financial instruments are characterized by a value $z$ that corresponds to a location on the debt-equity continuum. Lower values of $z$ reflect more debt-like characteristics whereas higher values of $z$ reflect more equity-like characteristics. Countries set a threshold value $\alpha$ that delineates debt and equity for tax purposes. To account for the uncertainty emphasized by legal scholars, we assume that tax assessments have a stochastic element. Thus, if $z$ is sufficiently close to $\alpha$, there is $ex ante$ uncertainty about the classification of the instrument for tax purposes, however, the probability that the instrument is classified as equity is increasing in $z - \alpha$.

The model includes two countries $A$ and $B$, each inhabited by firms that are endowed with profitable investment projects in the foreign country. Firms optimally decide whether to finance the foreign investment with standard debt and equity or with a hybrid instrument. In the latter case, firms choose the hybrid instrument that maximizes the probability of equity treatment in the home country and debt treatment in the host country. Clearly, the scope for hybrid financing is larger the higher the threshold value $\alpha$ of the host country since this facilitates debt treatment at the level of the borrowing subsidiary.

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7 Altshuler and Grubert (2005) explicitly mention hybrid structures as one of the tax planning tools most likely to be responsible for the decreasing effective tax rates but no hard evidence is presented in favor of this hypothesis.
and the lower the \( \alpha \) of the home country since this facilitates equity treatment at the level of the lending parent company.

We characterize the policies \( \alpha^A \) and \( \alpha^B \) prevailing under cooperative and non-cooperative policy making respectively. Assuming that country \( A \) has a larger multinational sector than country \( B \), the optimal distance \( \alpha^A - \alpha^B \) is positive. Intuitively, an international tax environment with \( \alpha^A - \alpha^B \) offers more protection against hybrid financing by the relatively large number of firms in \( A \) than by the relatively small number of firms in country \( B \). Moreover, the optimal location of \( \alpha^A \) and \( \alpha^B \) is symmetric around the policy parameter \( \bar{\alpha} \), which is preferred for non-tax purposes. In the non-cooperative setting, both governments typically desire a low value of \( \alpha \) in their own country relative to the foreign country. By facilitating the use of hybrid instruments by domestic firms and impeding their use by foreign firms, this erodes foreign taxation of domestic firms and enforces domestic taxation of foreign firms. This mechanism implies that the equilibrium distance \( \alpha^A - \alpha^B \) is too small and the equilibrium location of \( \alpha^A \) and \( \alpha^B \) given the distance is too low compared to the optimal set of policies.

Somewhat surprisingly, references to cross-border hybrid instruments in the finance and economics literatures are very scarce and no formal analysis exists of cross-border hybrid instruments, let alone of their implications for optimal line drawing between debt and equity. A companion paper, Johannesen (2010a), develops a model of cross-border hybrid instruments while taking international differences in demarcation rules for given. The latter paper characterizes the scope for cross-border hybrid financing as well as the ‘optimal cross-border hybrid instrument’ in a framework with multi-dimensional financial instruments and demarcation rules. The main insight is that the scope for tax planning with cross-border hybrid instruments derives from two types of cross-country variation in demarcation rules: (i) variation in the relative ‘weights’ attributed to the different dimensions of financial instruments, (ii) variation in the threshold level of ‘equity-ness’ that triggers equity rather than debt treatment for tax purposes. The present paper draws on Johannesen (2010a), however, in order to endogenize demarcation rules and preserve tractability, the underlying model of hybrid instruments is simplified. Essentially, we assume that countries employ the same relative weights in their demarcation rules, which implies that the universe of financial instruments collapses to a one-dimensional debt-equity continuum and demarcation rules reduce to the threshold value \( \alpha \).

The paper relates to existing work on capital taxation in the presence of international tax planning. A number of papers analyze taxation of firms that have access to profit shifting techniques (e.g. Haufler and Schjelderup, 2000; Hong and Smart, 2010; Johannesen, 2010b). Our paper is, however, clearly distinct from this literature both in terms of the policy dimension considered, existing papers generally analyze choices over capital tax rates, and, by consequence, in terms of the mechanics of the model.

The paper also relates to other work on optimal line drawing in tax policy. The defining feature of line drawing problems is that policymakers can shift lines between the various categories used by the tax code but are unable to eliminate the categories themselves. In two seminal papers, Weisbach (1999, 2000) argues that line drawing problems are prevalent in the tax system and makes a case for line drawing based on efficiency arguments rather than doctrinal arguments. Both papers explicitly mention
the distinction between debt and equity as one of the most prominent line drawing problems in current tax systems but does not present a formal analysis of the problem. In a more recent paper, Kleven and Slemrod (2009) characterize optimal commodity taxation in terms of the lines that optimally group the infinite number of possible commodities into a finite number of tax categories.

The remainder of the paper is structured in the following way. Section 2 develops the model. Section 3 characterizes the economic equilibrium for a given set of policies. Section 4 derives globally optimal policies in a cooperative setting. Section 5 derives equilibrium policies in a non-cooperative setting. Section 6 provides some concluding remarks.

2 The model

This section develops a model of line drawing between debt and equity. The model comprises two countries A and B, the economies of which are tied together by firms residing in one country and investing in the other country. Both countries operate a standard corporate tax system that allows for deduction of interest payments from the taxable income and exempts foreign source income. To focus exclusively on hybrid financing and disregard financial strategies related to, for instance, profit shifting, we assume that both countries apply the same corporate tax rate \( t \), hence government policy is only concerned with the rule delineating debt from equity.

The first subsection presents a simple model of hybrid instruments and classification of such instruments for corporate tax purposes. The second and third subsections describe the objectives and constraints facing firms and governments respectively.

2.1 Hybrid instruments

Financial instruments are assumed to differ in a single, continuous dimension scaled to range the interval \([0; 1]\). A financial instrument is thus fully characterized by a value \( z \in [0; 1] \). Instruments with \( z \) closer to zero have properties closer to debt whereas instruments with \( z \) closer to one have properties closer to equity. The corporate tax base is characterized by a threshold value \( \alpha \in [0; 1] \) that delineates debt and equity for tax purposes.

In real-world tax systems, delineation rules are typically not truly deterministic but leave considerable discretion to individual tax administrators and judges. Arguably, this creates \( \textit{ex ante} \) uncertainty about the tax treatment of a given financial instrument.\(^8\) In order to capture this uncertainty in the model, we assume that tax authorities make individual assessments of each hybrid instrument, on the basis of which the instrument is categorized as either debt or equity. Specifically, we assume that the assessment of a hybrid instrument with characteristics \( z \) is given by

\[
Z = z + \varepsilon
\]

\(^8\) The lack of legal certainty is noted by several legal scholars. For instance, Emmerich (1985) writes: "But because of the wide variety of instruments and transactions that have required classification as debt or equity, the courts have spawned a bewildering variety of tests and standards requiring highly fact-bound and uncertain legal determinations."
where \( \varepsilon \) is random draw from a uniform distribution with mean zero and density \( \gamma \). The financial instrument is thus categorized as equity if \( Z \geq \alpha \) and debt if \( Z < \alpha \). We let \( p(z) \) denote the probability that a financial instrument with characteristics \( z \) is categorized as equity, which implies that \( 1 - p(z) \) is the probability that the same instrument is categorized as debt. Given the distributional assumptions about \( \varepsilon \), it is easy to show that [see Appendix B]:

\[
p(z) = \begin{cases} 
0 & \text{if } z < \alpha - \frac{1}{2\gamma} \\
\frac{1}{2} + \gamma(z - \alpha) & \text{if } \alpha - \frac{1}{2\gamma} < z < \alpha + \frac{1}{2\gamma} \\
1 & \text{if } \alpha + \frac{1}{2\gamma} < z 
\end{cases}
\]  

(1)

Intuitively, there is uncertainty about the tax treatment if \( z \) is sufficiently close to \( \alpha \) and the probability of equity treatment is increasing linearly in \( z \) within this intermediate range. If \( z \) is sufficiently much smaller than \( \alpha \) the instrument is treated as debt with certainty whereas if \( z \) is sufficiently much larger than \( \alpha \) the instrument is treated as equity with certainty. The parameter \( \gamma \) can be perceived as a measure of the precision of the tax assessment.

Firms in country \( A \) investing in country \( B \) obtain a tax advantage if the financial instrument is treated as equity in country \( A \) and debt in country \( B \). It is easy to see that there exists a set of hybrid instruments with a strictly positive probability of achieving this tax treatment provided that \( \alpha^A - \alpha^B < 1/\gamma \). Intuitively, if \( z \) is not so much smaller than \( \alpha^A \) so as to be categorized as debt with certainty in country \( A \) and not so much larger than \( \alpha^B \) so as to be categorized as equity with certainty in country \( B \), the financial instrument may possibly obtain the desired tax treatment. Similarly, firms in country \( B \) investing in country \( A \) obtain a tax advantage if the financial instrument is treated as equity in country \( B \) and debt in country \( A \). There exists a set of hybrid instruments with a strictly positive probability of achieving this treatment provided that \( \alpha^B - \alpha^A < 1/\gamma \). Consequently, it is generally possible for firms in one country to obtain their desired hybrid tax treatment and, moreover, it is possible for firms in both countries to obtain their desired hybrid tax treatment provided that \( |\alpha^A - \alpha^B| < 1/\gamma \).

The scope for hybrid instruments is illustrated in figure 1. In country \( A \), instruments with \( z \in [z_1; z_3] \) may be treated as either debt or equity depending on the stochastic outcome of the assessment made

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9In the present framework, the possibility that some firms in country \( A \) have financial instruments categorized as equity in country \( A \) and debt in country \( B \) whereas at the same time some firms in country \( B \) have financial instruments categorized as equity in country \( B \) and debt in country \( A \) hinges crucially on the stochastic formulation of the demarcation rule. It should be noted, however, that in the multi-dimensional model of hybrid financing developed in Johannesen (2010a), there is often a scope for hybrid financing by firms in both countries when countries assign different relative weights to the different dimensions of financial instruments in their demarcation rules. To see this, consider the two-dimensional case where financial instruments are characterized by a vector \( z = (z^1, z^2) \) with \( z^k \in (0, 1) \) for \( k = 1, 2 \). As an extreme example of differences in relative weights assume that the deterministic demarcation rule of country \( A \) only takes into account the first dimension of financial instruments so that \( z \) is treated as equity if and only if \( z^1 \geq \alpha^A \) whereas the demarcation rule of country \( B \) only considers the second dimension so that \( z \) is treated as equity if and only if \( z^2 \geq \alpha^B \). Regardless of the values of \( \alpha^A \) and \( \alpha^B \), the instrument \( z = (1, 0) \) allows firms in country \( A \) investing in country \( B \) to obtain the desired hybrid treatment with certainty whereas firms in country \( B \) investing in country \( A \) can obtain the opposite hybrid treatment by implementing the instrument \( z = (0, 1) \).
by the tax authorities. Instruments with $z < z_1$ are treated as debt with certainty whereas instruments with $z > z_3$ are treated as equity with certainty. Similarly, in country $B$, instruments with $z \in [z_2; z_4]$ may be treated as either debt or equity depending on the assessment outcome. Instruments with $z < z_2$ are treated as debt with certainty whereas instruments with $z > z_4$ are treated as equity with certainty. Any instrument with $z \in [z_1; z_4]$ may possibly be treated as equity in country $A$ and debt in country $B$ as desired by firms in country $A$ whereas instruments with $z \in [z_2; z_3]$ may possibly be treated as debt in country $A$ and equity in country $B$ as desired by firms in country $B$.

While hybrid instruments from the perspective of firms represent an opportunity for non-taxation, the stochastic tax environment described above, in principle, also implies a risk of double taxation. For instance, if a firm in country $A$ finances an investment in country $B$ with a hybrid instrument that is categorized as debt in country $A$ and equity in country $B$, the firm has no deductible interest payments in country $B$ but is nevertheless taxed on interest income in country $A$. We assume that such double taxation does not occur. Instead we assume that in cases where inconsistent categorization of a financial instrument would give rise to double taxation, the two countries agree to treat the instrument consistently either as equity (with probability $1/2$) or as debt (with probability $1/2$). This assumption has strong foundations in the prevailing legal institutions of international taxation. Most developed countries have extensive networks of bilateral double tax conventions with other developed countries with the aim of eliminating double taxation. Typically, double tax conventions are based on the OECD model convention which provides definitions of dividend and interest payments for the purposes of the convention. In cases where conflicting interpretations of a convention lead to double taxation, tax authorities are committed to endeavor to resolve the conflict by mutual agreement with a view to eliminating the double taxation.

2.2 Firms

Countries are inhabited by domestically owned firms, each of which is endowed with a single profitable investment project in the other country. Investment projects require $k$ units of capital and generate a gross revenue of $y$. Firms undertaking a foreign investment are composed of two entities: a parent company in the home country and a subsidiary in the foreign country. The parent company raises equity capital in external capital markets and transfers the funds to the subsidiary by means of a financial instrument. External investors require a fixed rate of return of $r$, hence the before-tax profits generated by an investment project amount to $\pi = y - rk$.

When parent companies invest in foreign subsidiaries, they choose between a hybrid financial structure and a standard financial structure composed of a pure debt and a pure equity instrument. Firms are risk-neutral and thus opt for the mode of finance that maximizes expected profits. As a tie-breaker, we

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10While we should therefore expect double tax conventions to eliminate double taxation of hybrid instruments, they do not by any means prevent non-taxation of such instruments. As discussed by Rosenbloom (1999), this asymmetry derives from the fact that double tax conventions are elective for tax payers who may always reject a treaty and invoke their rights under domestic law. In the words of Rosenbloom: "Since international tax arbitrage generally (though perhaps not invariably) builds upon differences in domestic laws, not treaties, an election to rely on domestic law would leave the tax payer with the same arbitrage opportunities as if the treaty did not exist at all" (p. 164).
assume that firms only opt for hybrid financing when this yields strictly larger expected after-tax profits than financing with standard debt and equity instruments.

Firms opting for hybrid finance choose the hybrid instrument \( z^* \) that maximizes expected profits. Depending on the stochastic outcomes of the tax assessments in the home and host countries, hybrid finance may give rise to a tax saving. If the hybrid instrument is successfully categorized as equity in the home country and debt in the host country, the tax liability in the host country is \( t(y - rk) \) while there is no taxation in the home country. In this case, payments on the instrument \( rk \) are treated as tax deductible interest expenses in the host country and as non-taxable dividend income in the home country. If the hybrid instrument is categorized as debt in both countries, the tax liability in the host country is \( t(y - rk) \) and the tax liability in the home country is \( trk \). Conversely, if the hybrid instrument is treated as equity in both countries, the tax liability in the host country is \( ty \) while there is no tax liability in the home country. In either case of unsuccessful hybrid financing, the total tax burden thus amounts to \( ty \). The tax saving generated by successful hybrid financing amounts to \( trk \).

While hybrid instruments may generate a tax saving, we also assume that they involve a fixed cost \( c \). The cost may reflect fees to tax advisors, lawyers, accountants and auditors for implementation and management of the hybrid finance structure or inefficiencies related to a capital structure that is distorted by tax considerations. We assume that firms are heterogeneous with respect to the fixed cost \( c \). Specifically, it is assumed that \( c \) is uniformly distributed over the interval \([0, \sigma]\) with density \( \delta \) and is uncorrelated with country of residence. We also assume that \( \sigma \) is sufficiently large to ensure that there in any policy environment are firms choosing not to implement a hybrid instrument.\(^{11}\)

Firms opting for standard financial instruments finance the foreign investment with a fixed fraction \( \beta \) of internal debt and a fraction \((1 - \beta)\) of equity. The tax liability in the host country is \( t(y - \beta rk) \) whereas the tax liability in the home country is \( t\beta rk \) with the tax base comprising interest income. The total tax liability is simply \( ty \). Intuitively, internal debt shifts taxable income from the host country to the home country, however, with symmetry in corporate tax rates, the level of internal debt has no net effect on the total tax liability. The significance of the parameter \( \beta \) is to determine the allocation of taxing rights of firms using standard financial instruments. For these firms, a fraction \( \beta \) of the normal return to capital is taxed by the home country whereas the remaining fraction \((1 - \beta)\) is taxed by the host country.\(^{12}\)

For notational simplicity and without loss of generality, we normalize the mass of firms with profitable foreign investment projects in the two countries to one, that is \( \delta \sigma = 1 \). We allow for a possible asymmetry in the size of the multinational sector of the two countries by assuming that a fraction \( \theta \geq 1/2 \) of the total mass of firms is located in country \( A \). We shall often simply refer to \( \theta \) and \((1 - \theta)\) as the size of

\(^{11}\)As will become clear, this assumption implies that \( \sigma > trk \) such that for firms with \( c = \sigma \) the fixed cost of hybrid financing exceeds the expected tax saving even in the most favorable tax environment where the tax saving occurs with certainty.

\(^{12}\)Johannesen (2010c) reviews the available empirical evidence on the capital structure of multinational firms and shows that internal loans on average account for around 25%-30% of the financing of foreign affiliates. See section 5.3 for a discussion of this point.
country $A$ and $B$ respectively.

For future reference, we define $\tau^N \equiv trk$ and $\tau^P \equiv t(y - rk)$ where $\tau^N$ is the tax bill associated with taxation of the normal return to capital $rk$ and $\tau^P$ is the tax bill associated with taxation of pure economic profits $(y - rk)$.

### 2.3 Governments

The corporate tax system is characterized by the tax rate $t$ and the tax base parameter $\alpha$ that determines the tax treatment of hybrid instruments. We assume throughout the paper that $t$ is fixed, hence $\alpha$ is the only available policy instrument.

The optimal policy balances three concerns: As usual in models of international taxation, the objective function of the government in country $i$ includes government revenue $R^i$ as well as disposable income of domestic residents, which in this model simply equals the sum of after-tax profits earned by domestic firms denoted by $\Pi^i$. For simplicity, government revenue carries a constant weight in the social welfare function that exceeds the weight of private disposable income. Finally and more unusually, we assume that the level of $\alpha^i$ has a direct effect on social welfare. Specifically, we take a reduced-form approach by assuming that deviations of $\alpha^i$ from some target level $\bar{\alpha}$ give rise to a welfare loss. A possible interpretation is that the economy besides the multinational sector that is described explicitly in the model comprises a domestic sector where deviations from $\bar{\alpha}$ create costly distortions. An important merit of this specification is that it allows us to identify two types of inefficiencies. Firstly, the distance between $\alpha^A$ and $\alpha^B$ may be suboptimal. In general, the distance matters for efficiency by determining how well the international tax environment protects global tax revenue against hybrid financing by firms in country $A$ relative to hybrid financing by firms in country $B$. Secondly, for a given distance $\alpha^A - \alpha^B$, the location of $\alpha^A$ and $\alpha^B$ on the debt-equity continuum may be suboptimal. If, for instance, both governments for strategic reasons desire a low value of $\alpha$ relative to the other country, the equilibrium values of $\alpha^A$ and $\alpha^B$ could both be below $\bar{\alpha}$, in which case increasing $\alpha^A$ and $\alpha^B$ by the same (small) amount would unambiguously raise welfare in both countries. Absent direct policy preferences over $\alpha^i$, welfare would only depend on the distance between $\alpha^A$ and $\alpha^B$ and the model would be unable to capture inefficiencies related to the location of $\alpha^A$ and $\alpha^B$. For the sake of completeness, however, we shall also discuss the properties of the policy outcomes that arise without a direct policy preference over $\alpha^i$.

Drawing on these considerations, we posit that country $i$ has a social welfare function of the following form:

$$ W^i = \Pi^i + \mu R^i - \frac{\phi}{2}(\alpha^i - \bar{\alpha})^2 $$

where $\mu > 1$ and $0 < \bar{\alpha} < 1$. The analysis generally assumes that $\phi > 0$ except when it is explicitly stated that $\phi = 0$. For tractability, we have chosen a standard quadratic loss function to represent policy preferences over $\alpha^i$. Under the partial equilibrium interpretation of the model alluded to above, it is natural to think of $\mu$ as the marginal cost of public funds from other sources than multinational firms.

Absent international cooperation, the government of country $i$ sets $\alpha^i$ so as to maximize $W^i$ while
taking $\alpha_j^j$ as given. Under international cooperation, the two governments set $\alpha^A$ and $\alpha^B$ cooperatively so as to maximize the sum of the welfare levels in the two countries $W = W^A + W^B$.

2.4 Game structure and solution

We analyze two-stage games with the following structure: (i) governments set policy parameters $\alpha^A$ and $\alpha^B$ either cooperatively or non-cooperatively depending on the institutional setting while correctly anticipating firm responses to policies; (ii) firms choose optimal financial policies and undertake foreign investment. We are particularly interested in the outcomes of the first-stage policy game. In the cooperative case, the solution to the policy game was defined in the previous section as the policy vector $(\alpha^{A*}, \alpha^{B*})$ that maximizes global welfare $W$. We shall refer to this vector as the optimal policy. In the non-cooperative case, we follow most of the literature on international taxation by identifying a Nash equilibrium. We shall refer to a policy vector $(\alpha^A, \alpha^B)$ that constitutes a Nash equilibrium in the non-cooperative policy game as the equilibrium policy.

As it turns out, it is convenient to present policy outcomes in terms of the difference between the policy parameters $\alpha^A - \alpha^B$ and the average of the policy parameters $(\alpha^A + \alpha^B)/2$. Clearly, the difference and the average together implicitly define the level of $\alpha^A$ and $\alpha^B$. We shall refer to the difference $\alpha^A - \alpha^B$ as the distance between policy parameters and to the average $(\alpha^A + \alpha^B)/2$ as the location of the policy parameters.

3 Economic equilibrium

This section takes a first step towards identifying the policy equilibrium by solving for the economic equilibrium for a given set of policies. The first subsection characterizes the optimal financial policies of firms given the policy environment. The second subsection derives expressions for the resulting equilibrium profits and government revenues.

3.1 Optimal financial policies

The financial policies of firms consist of two choices: (i) firms decide whether to finance the foreign investment with an hybrid instrument or with standard debt and equity instruments, (ii) conditional on using hybrid finance, firms decide on the specific characterics $z$ of the hybrid instrument to be implemented. We start analyzing (ii) and subsequently turn to (i).

Firms in country $i$ investing in country $j$ are seeking to have the financial instrument financing the investment characterized as equity in country $i$ and as debt in country $j$. Assume that $\alpha^i - \alpha^j < 1/\gamma$ so that this tax treatment is indeed possible. Conditional on using a hybrid instrument to finance the foreign investment, the firm chooses its characteristics $z$ to maximize the following profit function:

$$\pi - c - (\pi^P + \pi^N) + p^i(z)(1 - p^j(z))\gamma^N$$

(2)
subject to the constraints that $0 \leq z \leq 1$. The first three terms are deterministic and capture profits $\pi$ net of costs of setting up a hybrid structure $c$ and taxation of the normal return to capital and pure profits $(\tau^P + \tau^N)$. The last term is stochastic and capture the expected tax saving from the hybrid structure where $p^i(z)(1 - p^j(z))$ is the probability of obtaining the desired tax treatment in both countries and $\tau^N$ is the tax saving under this desired tax treatment. The following lemma gives the solution to this maximization problem.

**Lemma 1** In a policy environment $(\alpha^A, \alpha^B)$ satisfying that $-1/\gamma < \alpha^A - \alpha^B < 1/\gamma$, the optimal hybrid instrument is characterized by:

$$z^* = \frac{\alpha^A + \alpha^B}{2}$$

**Proof.** See Appendix □

Lemma 1 states that the $z^*$ characterizing the optimal hybrid instrument is exactly halfway between $\alpha^A$ and $\alpha^B$. The intuition for this result is straightforward. It is clear from (2) that the optimal financial instrument is the one that maximizes the probability of obtaining the desired hybrid tax treatment $p^i(z)(1 - p^j(z))$. The essential trade-off associated with the choice of hybrid is that while raising $z$ increases the probability of equity treatment in country $i$, that is $p^i(z)$, it also reduces the probability of debt treatment in country $j$, that is $1 - p^j(z)$. Raising the probability of equity treatment in country $i$ through an increase in $z$ is more (less) valuable in terms of expected tax savings when the probability of debt treatment in country $j$ is large (small), that is when $z$ is initially low (high). Similarly, reducing the probability of debt treatment in country $j$ through an increase in $z$ is less (more) costly in terms of expected tax savings when the probability of equity treatment in country $i$ is small (high), that is when $z$ is initially low (high). This mechanism implies that the probability of obtaining the desired hybrid tax treatment is maximized exactly when the probabilities $p^i(z)$ and $1 - p^j(z)$ are equalized and this is the case when $z$ is equidistant from $\alpha^A$ and $\alpha^B$.

Having defined the optimal hybrid instrument $z^*$, we introduce the following short-hand notation: We let $p^i = p^i(z^*)$ denote the probability that the optimal hybrid instrument is treated as equity in country $i$ and let $q^i = p^i(1 - p^j)$ denote the probability that firms in country $i$ financing an investment in country $j$ with the optimal hybrid instrument $z^*$ achieves the desired tax treatment.

We now turn to the dichotomous decision whether to finance foreign investment with a hybrid instrument or with a combination of standard debt and equity. Inserting $q^i$ on the place of $p^i(z)(1 - p^j(z))$ in (2) yields an expression for maximized after-tax profits under hybrid financing for a firm in country $i$. After-tax-profits under financing with standard debt and equity instruments amount to $\pi - \tau^P - \tau^N$. Comparing the two expressions, it is easy to see that there exists a threshold value $c^i = q^i\tau^N$ where firms with $c^i < c^i$ opt for hybrid financing and firms with $c^i \geq c^i$ opt for financing with standard debt and equity. The threshold value $q^i\tau^N$ captures the expected tax saving from hybrid financing.

Finally, we present some intermediate results relating to the behavioral responses of financial policies to policy changes, which will prove useful in the sections 4 and 5.

**Lemma 2** For country $i = A, B$ and country $j \neq i$, it holds that
\[(a) \frac{dp^i}{d\alpha^i} = -\frac{dp^j}{d\alpha^i} = -\frac{\gamma}{2} \text{ for } i \neq j\]

\[(b) \frac{dq^i}{d\alpha^i} = -\frac{dq^j}{d\alpha^i} = -\gamma p^i \text{ for } i \neq j\]

\[(c) q^i = (p^i)^2\]

**Proof.** See Appendix.

While these intermediate results are simple and intuitive, they constitute important building blocks for the further analysis. To see the intuition for (a), recall that firms adjust the characteristics of hybrid instruments \(z^*\) in response to policy changes so as to ensure that the optimal hybrid is always equidistant from \(\alpha^A\) and \(\alpha^B\). It follows that increasing \(\alpha^i\) by one unit while holding \(\alpha^j\) constant causes a half unit increase in \(z^*\) so that the total effect is a decrease in \(p^i\) and an increase in \(p^j\) of the same size. The result in (b) states that an increase in \(\alpha^i\) and a decrease in \(\alpha^j\) both reduce \(q^i\) by the same amount \(\gamma p^i\). This result is key to understanding the mechanics of the model. Firstly, it shows that probabilities \(q^i\) and \(q^j\) depend only on the distance \(\alpha^i - \alpha^j\). Secondly, it implies that increasing the distance \(\alpha^i - \alpha^j\) has a large negative effect on \(q^i\) when \(p^i\) is large, that is when \(\alpha^i - \alpha^j\) is initially small, and a small negative effect on \(q^i\) when \(p^i\) is small, that is when \(\alpha^i - \alpha^j\) is initially large. Intuitively, when the probability of equity treatment in country \(i\) is small (large), reducing the probability of debt treatment in country \(j\) has a small (large) impact on the total probability of successful implementation of a hybrid instrument by firms in country \(i\).

### 3.2 Profits and revenue

To shorten the expressions to be derived below, we define \(x^{i,f}\) as the mass of firms in country \(i\) opting for finance of type \(f\) where \(f = H\) indicates hybrid financing and \(f = S\) indicates financing with standard debt and equity instruments. It is easy to see that \(x^{HA} = \theta \delta \bar{c}^A, \ x^{SA} = \theta \delta (\bar{c} - \bar{c}^A); \ x^{HB} = (1 - \theta)\delta \bar{c}^B\) and \(x^{SB} = (1 - \theta)\delta (\bar{c} - \bar{c}^B)\).

With these definitions, we may state total after-tax profits earned by firms resident in country \(A\) and \(B\) respectively in the following way:

\[
\Pi^A = x^{HA} \left\{ q^A (\pi - \tau^P) + (1 - q^A)(\pi - \tau^P - \tau^N) - \psi^A \right\} + x^{SA} (\pi - \tau^P - \tau^N) \\
\Pi^B = x^{HB} \left\{ q^B (\pi - \tau^P) + (1 - q^B)(\pi - \tau^P - \tau^N) - \psi^B \right\} + x^{SB} (\pi - \tau^P - \tau^N)
\]

where \(\psi^A = \bar{c}^A / 2\) and \(\psi^B = \bar{c}^B / 2\) capture the average costs of implementing a hybrid financing structure taken over the firms that optimally use hybrid financing. Firms in country \(i\) financing foreign investment with a hybrid instrument earn after-tax profits \(\pi - \tau^P - c\) with probability \(q^i\) and after-tax profits \(\pi - \tau^P - \tau^N - c\) with probability \(1 - q^i\) whereas firm using standard debt and equity earn after-tax profits \(\pi - \tau^P - \tau^N\) with certainty.

Finally, define \(p^{ij}\) as the probability that a firm in country \(i\) implementing a hybrid instrument to finance its foreign investment end up paying taxes on the normal return to capital in country \(j\) for
i = A, B and j = A, B:

\[ \rho^{AA} \equiv (1 - p^A)(1 - p^B) + \frac{1}{2}(1 - p^A)p^B \]

\[ \rho^{AB} \equiv p^Ap^B + \frac{1}{2}(1 - p^A)p^B \]

\[ \rho^{BB} \equiv (1 - p^A)(1 - p^B) + \frac{1}{2}(1 - p^B)p^A \]

\[ \rho^{BA} \equiv p^Ap^B + \frac{1}{2}(1 - p^B)p^A \]

The first term of \( \rho^{ij} \) captures the probability that consistent tax assessments in the two countries give taxing rights to country \( j \), for instance in the case of \( \rho^{AA} \) that the hybrid instrument used by a firm in country A is categorized as debt in both countries A and B. The second term of \( \rho^{ij} \) captures the probability that inconsistent tax assessments cause both countries to claim the taxing rights and that the taxing rights are eventually ceded to country \( j \), for instance in the case of \( \rho^{AA} \) that the hybrid instrument used by a firm in country A is assessed as debt in country A and equity in country B and that the two countries agree to categorize it as debt.

For future reference, it should be noted that \( \rho^{AA} + \rho^{AB} + q^A = 1 \) and \( \rho^{BA} + \rho^{BB} + q^B = 1 \). This simply reflects that for firms in country \( i \) engaged in hybrid financing of a foreign subsidiary, the normal return to capital is either taxed in country A (with probability \( \rho^A \)), taxed in country B (with probability \( \rho^B \)) or not taxed at all because the financial instrument obtains the desired hybrid treatment (with probability \( q^i \)). Also note that \( \rho^{AA} = \rho^{AB} \) and \( \rho^{BA} = \rho^{BB} \) This property derives from the fact that the optimal hybrid instrument is equidistant from \( \alpha^A \) and \( \alpha^B \), which implies that \( p^A = 1 - p^B \) and \( 1 - p^A = p^B \).

Using the short-hand notation introduced above, we may write the government revenue of countries A and B in the following way:

\[ R^A = x^{HA}T^N \rho^{AA} + x^{SA}T^N \beta + x^{HB} \left\{ \rho^{BA}T^N + \tau^P \right\} + x^{SB} \left\{ (1 - \beta)T^N + \tau^P \right\} \]

\[ R^B = x^{HA} \left\{ \rho^{AB}T^N + \tau^P \right\} + x^{SA} \left\{ (1 - \beta)T^N + \tau^P \right\} + x^{HB}T^N \rho^{BB} + x^{SB}T^N \beta \]

Firms in country A financing investment in country B with a hybrid instrument \((x^{HA})\) pay taxes on pure profits in country B with certainty and taxes on the normal return in country A or B with probabilities \( \rho^{AA} \) and \( \rho^{AB} \) respectively. Likewise for firms in country B financing investment in country A with a hybrid instrument \((x^{HB})\). Firms in country A financing investment in country B with standard debt and equity \((x^{SA})\) pay taxes on pure profits in country B whereas taxation of the normal return to capital is shared between country A and B in proportion to the shares of debt and equity in the capital structure, \( \beta \) and \( 1 - \beta \) respectively. Likewise for firms in country B financing investment in country A with standard debt and equity \((x^{SB})\).

It should be noted that since the endogenous variables in the expressions \( \Pi^i \) and \( R^i \) depend only on the probabilities \( q^i \) and \( q^j \), the economic equilibrium is fully determined by the distance \( \alpha^A - \alpha^B \). Intuitively, the optimal financial policies of firms as well as the distribution of revenue between the two countries only depend on the probabilities \( q^i \) and \( q^j \), which, in turn, only depend on the distance \( \alpha^A - \alpha^B \).
4 Optimal policies

Under international cooperation, the two governments set \((\alpha^A, \alpha^B)\) cooperatively so as to maximize aggregate welfare \(W\). Differentiating aggregate welfare with respect to \(\alpha_i\) yields the following first-order conditions for optimal cooperative policies [See Appendix B]:

\[
\frac{\partial W}{\partial \alpha^i} = (1 - \mu)\tau^N \left\{ x^H_i \frac{dq^i}{\partial \alpha^i} + x^H_j \frac{dq^j}{\partial \alpha^i} \right\} - \mu \tau^N \left\{ q^j \frac{dx^{Hj}}{\partial \alpha^i} + q^j \frac{dx^{Hj}}{\partial \alpha^i} \right\} - \phi(\alpha^i - \bar{\alpha}) = 0
\]

where \(j \neq i\). The first term reflects the 'mechanical' welfare effect of a small change in \(\alpha_i\), that is the welfare effect working through changes in \(q^i\) and \(q^j\) holding the number of firms using hybrid financing constant.\(^\text{13}\) Intuitively, holding the number of firms using hybrid financing constant, changes in \(q^i\) and \(q^j\) merely transfer rents between firms and governments. Increases in \(q^i\) and \(q^j\) imply a higher probability that firms using hybrid finance avoid taxation of the normal return to capital. This increases private profits but reduces government revenue by the same amount leaving the net welfare effect negative since we have assumed that the government spending is more valuable than private disposable income, i.e. \(\mu > 1\). The second term reflects the 'behavioral' welfare effect of a small change in \(\alpha_i\), that is the welfare effect working through changes in the number of firms using hybrid financing \(x^{Hi}\) and \(x^{Hj}\). Note that changes in \(x^{Hi}\) and \(x^{Hj}\) have no impact on private disposable income since the firms that respond to a small change in \(\alpha^i\) by changing the mode of finance are initially indifferent between the two modes of finance. This is an application of the envelope theorem. It should be noted that \(dq^i/\partial \alpha^i\) and \(dq^j/\partial \alpha^i\) generally have opposite signs as do \(dx^{Hi}/\partial \alpha^i\) and \(dx^{Hj}/\partial \alpha^i\). This points to the fundamental trade-off underlying cooperative policies that while increasing the distance \(\alpha^i - \alpha^j\) increases protection against tax planning by firms in country \(i\), it also reduces protection against tax planning by firms in country \(j\). Finally, the last term captures the welfare effect that is due to direct preferences over \(\alpha^i\). Clearly, if \(\alpha^i\) is initially smaller (larger) than \(\bar{\alpha}\) so that a marginal increase in \(\alpha^i\) brings it closer to (further away from) its target level, this welfare effect is positive (negative).

The following lemma restates (3) in terms of \(\alpha\), \(q\) and the primitive parameters of the model:

**Lemma 3** *The first-order conditions for optimal cooperative policies may be stated as:*

\[
\frac{\partial W}{\partial \alpha^A} = \Delta(2\mu - 1) \left\{ \theta(q^A)^\frac{3}{2} - (1 - \theta)(q^B)^\frac{3}{2} \right\} - \phi(\alpha^A - \bar{\alpha}) = 0
\]

\[
\frac{\partial W}{\partial \alpha^B} = \Delta(2\mu - 1) \left\{ (1 - \theta)(q^B)^\frac{3}{2} - \theta(q^A)^\frac{3}{2} \right\} - \phi(\alpha^B - \bar{\alpha}) = 0
\]

*where:*

\[
\Delta \equiv \gamma \delta (\tau^N)^2 > 0
\]

**Proof.** See Appendix \(\blacksquare\)

The first term of each expression summarizes mechanical and behavioral effects of changes in \(\alpha^A\) and \(\alpha^B\). Note that these terms are perfectly symmetric in the sense that a marginal increase in \(\alpha^A\) are

\(\text{13}\) Strictly speaking, the effect is not purely mechanical since \(dq^A/\partial \alpha^i\) and \(dq^B/\partial \alpha^i\) also capture changes in \(q^A\) and \(q^B\) that are due to behavioral changes in the properties of the hybrid instrument implemented by firms \(z^*\).
associated with exactly the same effects as a marginal reduction in $\alpha^B$. Essentially, this derives from the property discussed above that the economic equilibrium is fully determined by the distance $\alpha^A - \alpha^B$.

In the equation $\partial W/\partial \alpha^A = 0$, the first term in curly brackets represents the positive welfare effect of raising $\alpha^A$ working through a decrease in $q^A$ whereas the second term represents the negative welfare effect working through an increase in $q^B$. In the equation $\partial W/\partial \alpha^B = 0$, the first term in curly brackets represents the positive welfare effect of raising $\alpha^B$ working through a decrease in $q^B$ whereas the second term represents the negative welfare effect working through an increase in $q^A$. The factor $2\mu - 1$ reflects that, incidentally, the mechanical and behavioral effects are of exactly the same magnitude. Thus, a reduction in $q^i$ transfers rents from firms to governments creating a mechanical welfare gain proportional to $\mu - 1$. Moreover, a reduction in $q^i$ has a positive behavioral effect on government revenue of exactly the same size bringing the total welfare gain to $2\mu - 1$.

We are now prepared to present the first result, which pertains to optimal policy under symmetry.

**Proposition 1** If countries are symmetric ($\theta = 1/2$), the socially optimal policy $(\alpha^{A*}, \alpha^{B*})$ is given by:

$$\alpha^{A*} = \alpha^{B*} = \tilde{\alpha}$$

**Proof.** See Appendix.

The first important implication of Proposition 1 is that with symmetric countries, the optimal distance between $\alpha^A$ and $\alpha^B$ is zero. To see the intuition for this result, recall that raising $\alpha^i - \alpha^j$ increases welfare by reducing $q^i$ and thus limiting the scope for hybrid financing by firms in country $i$ while at the same time reducing welfare by increasing $q^j$ and thus enlarging the scope for hybrid financing by firms in country $j$. Moreover, recall the discussion of lemma 2 where we noted that raising $\alpha^i - \alpha^j$ has a large (small) effect on $q^i$ when $\alpha^i - \alpha^j$ is initially small (large). Hence, the common policy instrument $\alpha^i - \alpha^j$ exhibits *decreasing marginal effectiveness* in combatting hybrid finance by firms in country $i$. This amounts to a force that pulls the socially optimal levels of $\alpha^A$ and $\alpha^B$ together. Under symmetry, this force plays out fully so that $\alpha^{A*} = \alpha^{B*}$ and protection against hybrid finance in country $A$ and $B$ is optimally at exactly the same level.

The second implication of Proposition 1 is that in the baseline case with direct preferences over policy parameters ($\phi > 0$) the optimal location of $\alpha^A$ and $\alpha^B$ is uniquely determined at $\tilde{\alpha}$. Intuitively any symmetric policy gives rise to the same economic equilibrium, hence the optimal location is the one that coincides with the direct policy preference $\tilde{\alpha}$. It is easy to see that in the absence of a direct policy preference ($\phi = 0$), all policies with zero distance between $\alpha^A$ and $\alpha^B$ are equally good and thus constitute optimal policies.

We now characterize optimal policies in an asymmetric environment with more firms in country $A$ investing in country $B$ than *vice versa*.

**Proposition 2** If countries are asymmetric ($\theta > 1/2$), the socially optimal policy $(\alpha^{A*}, \alpha^{B*})$ can be characterized in the following way:
(a) **Distance.** The optimal distance $\alpha^A - \alpha^B$ is implicitly determined by:

$$\alpha^A - \alpha^B = \frac{2\Delta(2\mu - 1)}{\phi} \left\{ \theta(q^A)^{\frac{2}{3}} - (1 - \theta)(q^B)^{\frac{2}{3}} \right\}$$

This implies that $\alpha^A - \alpha^B$ is positive and increasing in the size of $\theta$ and that $q^B > q^A > 0$.

(b) **Location.** The optimal location is given by:

$$\frac{\alpha^A + \alpha^B}{2} = \bar{\alpha}$$

**Proof.** See Appendix

Part (b) of Proposition 2 states that $\alpha^A$ and $\alpha^B$ are optimally located symmetrically around $\bar{\alpha}$. We recall that the economic equilibrium is fully determined by the distance $\alpha^A - \alpha^B$. Hence, for a given distance the optimal location is simply the one that minimizes the quadratic loss functions, which clearly implies that $\alpha^A$ and $\alpha^B$ are equidistant from $\bar{\alpha}$. Part (a) states that the optimal distance $\alpha^A - \alpha^B$ is positive and increasing in the size of $\theta$, which suggests that size asymmetry represents a force that pull the socially optimal levels of $\alpha^A$ and $\alpha^B$ apart. Intuitively, starting from a symmetric policy, a marginal increase in $\alpha^A - \alpha^B$ reduces $q^A$ and raises $q^B$ by the same amount, however, the positive welfare effect deriving from the reduction in $q^A$ is larger than the negative welfare effect deriving from the increase in $q^B$ simply because the number of firms in country A exceeds the number of firms in country B. The decreasing marginal effectiveness of the policy instrument $\alpha^A - \alpha^B$ in combatting hybrid finance by firms in country A, however, is still at play. This implies that as $\alpha^A - \alpha^B$ increases to the point where $q^A$ approaches zero, the marginal benefit of additional increases in $\alpha^A - \alpha^B$ in terms of improved protection against hybrid financing by firms in country A also approaches zero, hence $q^A$ is strictly positive in the social optimum.

It is easy to see that in the absence of a direct policy preference ($\phi = 0$), the optimal distance must make the expression in curly brackets equal zero. It can be shown that in this case the socially optimal distance is simply the one that minimizes the expected global number of firms that succesfully use hybrid instruments to avoid taxes on the normal return to capital.

It should be noted that the capital structure of firms using standard finance has no bearing on the socially optimal policies. Intuitively, while $\beta$ matters for the distribution of taxing rights between the two countries with a large (small) $\beta$ implying a large degree of home (host) country taxation, it does not affect the size of the global tax base.

5 **Non-cooperative policy equilibrium**

Absent international cooperation, government $i$ sets $\alpha^i$ so as to maximize the welfare of country $i$. Differentiating $W^i$ with respect to $\alpha^i$ yields the following first-order condition for optimal policies from the perspective of country $i$ taking policies of country $j \neq i$ as given [see Appendix B]:

$$\frac{\partial W^i}{\partial \alpha^i} = \left\{ \tau^N \left\{ x^{Hi} \frac{d\rho^i}{d\alpha^i} \right\} + \mu \tau^N \left\{ x^{Hi} \frac{d\rho^ij}{d\alpha^i} + x^{Hj} \frac{d\rho^ij}{d\alpha^j} \right\} + \mu \tau^N \left\{ \frac{dx^{Hi}}{d\alpha^i} (\rho^i - \beta) + \frac{dx^{Hj}}{d\alpha^j} (\rho^j - (1 - \beta)) \right\} - \phi(\alpha^i - \bar{\alpha}) \right\}$$
The first term captures the effect of a small change in $\alpha^i$ on the profits of firms in country $i$. As argued above, behavioral changes in the mode of finance have no impact on firm profits, hence this term merely reflects the mechanical effect working through changes in the probability that firms in country $i$ with hybrid instruments obtain the desired tax treatment. The second and third term capture the impact on government revenue. The second term reflects that a change in $\alpha_i$ alters the probability that firms using hybrid finance are taxed on the normal return to capital in country $i$. The third term reflects that behavioral changes in the mode of finance affect government revenue. A firm in country $i$ changing from standard financing to hybrid financing increases expected revenue in country $i$ by $N^i(\rho^{ii} - \beta)$ where we recall that $\rho^{ii}$ is the probability that the normal return to capital is taxed in country $i$ under hybrid financing and $\beta$ is the part of the capital stock that is financed with debt and thus subject to taxation in country $A$ under standard financing. Similarly, a firm in country $j$ changing from standard financing to hybrid financing increases expected revenue in country $i$ by $N_j(\rho^{ji} - (1 - \beta))$ where $\rho^{ji}$ is the probability that the normal return to capital is taxed in country $i$ under hybrid financing and $1 - \beta$ is the part of the capital stock that is financed with equity and thus subject to taxation in country $A$ under standard financing. Finally, the last term reflects the welfare effect that is due to direct preferences over $\alpha^i$.

Equation (4) is a useful starting point for a discussion of the inefficiencies associated with non-cooperative policy making. The inefficiencies highlighted by this framework derive from cross-border externalities of taxation. Clearly, the policy parameter of each country affects effective taxation of firms in both countries. When setting $\alpha$ non-cooperatively, however, governments only consider the effect on after-tax profits of domestic firms and the effect on domestic tax revenue while ignoring the effect on after-tax profits of foreign firms as well as the effect on the foreign government revenue. This becomes clear when comparing (4) to (3): Firstly, the latter equation includes a term $dq^i/d\alpha^i$ capturing the effect of the policy instrument of country $i$ on profits in country $j$. Secondly, the latter equation includes the effect on the revenues in both countries, or equivalently the effect on the probabilities of successful avoidance $q^i$ and $q^j$, whereas the former equation only considers the effect on taxation in country $i$, that is $\rho^{ii}$ and $\rho^{ji}$.

For future reference, it is convenient to define $\sigma \equiv 1/2 - \beta$. We shall refer to $\sigma$ as 'equity bias' since it captures the degree to which the equity part of standard financing $(1 - \beta)$ exceeds $1/2$. We are now prepared to present the following lemma, which restates the first-order conditions for optimal policies from the perspective of the individual countries $A$ and $B$ in the general case where $\theta \geq 1/2$ and $0 \leq \beta \leq 1$.

**Lemma 4** The first-order conditions for optimal policy in country $A$ and $B$ respectively may be stated as:

$$\frac{\partial W^A}{\partial \alpha^A} = \Delta \left\{ (\theta(q^A)^\frac{1}{2} [(\mu - 1)q^A - \mu\sigma] - (1 - \theta)(q^B)^\frac{1}{2} \mu[q^B + \sigma] \right\} - \phi(\alpha^A - \tilde{\alpha}) = 0$$

$$\frac{\partial W^B}{\partial \alpha^B} = \Delta \left\{ (1 - \theta)(q^B)^\frac{1}{2} [(\mu - 1)q^B - \mu\sigma] - \theta(q^A)^\frac{1}{2} \mu[q^A + \sigma] \right\} - \phi(\alpha^B - \tilde{\alpha}) = 0$$

**Proof.** See Appendix □

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Lemma 4 is the non-cooperative analogue of Lemma 3. At this stage, we do not provide a detailed interpretation of the first-order conditions. Instead, we shall build intuition by considering some special cases and subsequently return to the general case.

5.1 Symmetric countries \((\theta = 1/2)\)

This subsection analyzes the case where countries are symmetric in terms of the size of the multinational sectors, that is \(\theta = 1/2\). We first study the special case where there is no equity bias, that is \(\beta = 1/2\). Under the parametric assumptions, \(\theta = 1/2\) and \(\beta = 1/2\), the general first-order conditions for optimal policy from the perspective of individual countries that were derived in Lemma 4 reduce to:

\[
\frac{\partial W^A}{\partial \alpha^A} = \frac{\Delta}{2} \left\{ (q^A)^2 (\mu - 1) - (q^B)^2 \mu \right\} - \phi (\alpha^A - \bar{\alpha}) = 0
\]

\[
\frac{\partial W^B}{\partial \alpha^B} = \frac{\Delta}{2} \left\{ (q^B)^2 (\mu - 1) - (q^A)^2 \mu \right\} - \phi (\alpha^B - \bar{\alpha}) = 0
\]

In \(\partial W^A/\partial \alpha^A\), the first term in the curly brackets represent the welfare gain from a small increase in \(\alpha^A\) working through a decrease in \(q^A\) whereas the second term in the curly brackets represents the welfare loss working through an increase in \(q^B\). As discussed above, the global welfare effect of a change in \(q^i\) is proportional to \(2\mu - 1\). The welfare gain associated with an increase in \(\alpha^A\) as perceived by the government in country \(A\) is proportional to \(\mu - 1\) since exactly one half of the increase in government revenue accrues to country \(B\). The welfare loss associated with an increase in \(\alpha^A\) as perceived by the government in country \(A\) is proportional to \(\mu\) since exactly one half of the loss of government revenue is borne by country \(B\) and all of the increase in private profits occurs in country \(B\). A symmetric interpretation applies to \(\partial W^B/\partial \alpha^B\). We are now prepared to present the first result in this subsection:

**Proposition 3** If countries are symmetric \((\theta = 1/2)\) and there is no equity bias \((\beta = 1/2)\), the unique equilibrium in the non-cooperative policy game \((\alpha^A, \alpha^B)\) is given by:

\[(\alpha^A, \alpha^B) = (\bar{\alpha} - \frac{\Delta}{2\phi}q^2)\]

where \(q = q^A = q^B = 1/4\). In the special case \(\phi = 0\), the unique equilibrium is given by \(\alpha^A = \alpha^B = 0\).

**Proof.** See Appendix. ■

Proposition 3 states that the unique equilibrium is symmetric and has \(\alpha^A = \alpha^B\) below the target level \(\bar{\alpha}\). This result should be compared to the result presented in Proposition 1 stating that the socially optimal policy is \(\alpha^A = \alpha^B = \bar{\alpha}\). Intuitively, under the special parametric assumptions underlying Proposition 3, global revenue gains and revenue losses associated with changes in \(q^i\) are shared equally between the two countries whereas private gains and losses associated with changes in \(q^i\) only accrue to firms in country \(i\). A reduction in \(\alpha^i\) increases \(q^i\) and reduces \(q^j\), which benefits firms in country \(i\) at the expense of firms in country \(j\). Starting from a symmetric policy vector where \(q^A = q^B\), a marginal change in \(\alpha^i\) has no impact on government revenue, hence a small reduction in \(\alpha^i\) transfers rents from foreign firms to domestic firms. Clearly, this triggers a downward pressure on \(\alpha\), which is, however, mitigated
by the direct preference over policy outcomes. In the unique symmetric equilibrium, the marginal direct welfare loss associated with a further reduction in \( \alpha^i \), that is \( \phi(\bar{\alpha} - \alpha^A) \), is exactly offset by the marginal welfare gain in terms of profits earned by domestic firms, that is \( (\Delta/2)q^A_q^2 \). It is straightforward to show that in the absence of a direct policy preference over \( \alpha \), that is \( \phi = 0 \), there is no force mitigating the downward pressure on \( \alpha \), hence the unique equilibrium is \( \alpha^A = \alpha^B = 0 \).

We now turn to the more general case where \( 0 \leq \beta \leq 1 \). Under the parametric assumption \( \theta = 1/2 \), the general first-order conditions for optimal policy in country \( A \) and \( B \) derived in Lemma 4 reduce to:

\[
\frac{\partial W^A}{\partial \alpha^A} = \frac{\Delta}{2} \left\{ (q^A)^{1/2} \left[ (\mu - 1)q^A - \mu\sigma \right] - (q^B)^{1/2} [\mu q^B + \mu\sigma] \right\} - \phi(\alpha^A - \bar{\alpha}) = 0
\]

\[
\frac{\partial W^B}{\partial \alpha^B} = \frac{\Delta}{2} \left\{ (q^B)^{1/2} \left[ (\mu - 1)q^B - \mu\sigma \right] - (q^A)^{1/2} [\mu q^A + \mu\sigma] \right\} - \phi(\alpha^B - \bar{\alpha}) = 0
\]

These first-order conditions are similar to the ones derived above for the special case \( \beta = 1/2 \), however, dropping the restriction on \( \beta \) adds a piece to each of the two terms in curly brackets. In \( \partial W^A / \partial \alpha^A \), the first term in the curly brackets, which represents the revenue gain associated with an increase in \( \alpha^A \) working through a decrease in \( q^A \), now includes the piece \( -(q^A)^{1/2} \mu\sigma \) whereas the last term, which represents the revenue loss associated with an increase in \( \alpha^A \) working through an increase in \( q^B \), includes the piece \( -(q^B)^{1/2} \mu\sigma \). Intuitively, from the perspective of the government in country \( A \), equity bias reduces the revenue gain associated with an increase in \( \alpha^A \) because the firms in country \( A \) that are induced to switch from hybrid finance to standard finance by the resulting decrease in \( q^A \) are to a larger extent equity financed and thus taxed in country \( B \). Similarly, from the perspective of the government in country \( A \), equity bias increases the revenue loss associated with an increase in \( \alpha^A \) because the firms in country \( B \) that are induced to switch from standard finance to hybrid finance by the resulting increase in \( q^B \) are to a larger extent equity financed and thus taxed in country \( A \). We now describe the equilibrium under the more general assumption \( 0 \leq \beta \leq 1 \):

**Proposition 4** If countries are symmetric (\( \theta = 1/2 \)) and there is positive or negative equity bias (\( \beta \neq 1/2 \)), the unique equilibrium in the non-cooperative policy game (\( \alpha^A, \alpha^B \)) is given by:

\[
\alpha^A = \alpha^B = \bar{\alpha} - \frac{\Delta}{2\phi} (q^A_q^2 + 2\mu\sigma q^A_q^2)
\]

where \( q = q^A = q^B = 1/4 \). In the special case \( \phi = 0 \), the unique equilibrium is given by \( \alpha^A = \alpha^B = 0 \) when \( \mu\sigma > -1/4 \) and \( \alpha^A = \alpha^B = 1 \) when \( \mu\sigma > -1/4 \).

**Proof.** See Appendix. ■

Proposition 4 states that the unique equilibrium is symmetric and has \( \alpha^A = \alpha^B \) below or above the target level \( \bar{\alpha} \). The equilibrium level of \( \alpha^A \) and \( \alpha^B \) is decreasing in the equity bias \( \sigma \). As discussed above, equity bias reinforces the downward pressure by subtracting from the benefit of increasing \( \alpha^A \) in terms of a lower \( q^A \) and adding to its cost in terms of a higher \( q^B \). This implies that when equity bias is below a critical (negative) value, the equilibrium has \( \alpha^A = \alpha^B \) above the target level \( \bar{\alpha} \). Intuitively, increasing \( \alpha^A \) still reduces \( q^A \) and increases \( q^B \), which transfers rents from firms in country \( A \) to firms in country \( B \), however, this effect is dominated by the behavioral effects: a lower \( q^A \) induces firms in country \( A \) to
switch from hybrid finance to standard finance and firms in country B to switch from standard finance to hybrid finance. When firms using standard financial instruments depend heavily on debt finance, these behavioral effects have strong positive revenue effects in country A because standard finance is largely associated with home country taxation.

In conclusion, when symmetric countries engage in non-cooperative policymaking the equilibrium distance between $\alpha^A$ and $\alpha^B$ is at the optimal level ($\alpha^A = \alpha^B$). The equilibrium location of $\alpha^A$ and $\alpha^B$ is increasing in the extent to which firms opting for standard finance rely on internal debt rather than equity to finance foreign investment. There is a threshold value of $\beta$ below (above) which the equilibrium location is lower (higher) than the socially optimal location. This threshold value of $\beta$ is larger than $1/2$.

5.2 Asymmetric countries ($\theta > 1/2$)

We now turn to the asymmetric case where country A is larger than country B in terms of the multinational sector, that is $\theta > 1/2$. To build intuition, we first consider a special case where there is no equity bias, that is $\sigma = 0$. Under the parametric assumption, $\beta = 1/2$, the general first-order conditions for optimal policy in country A and B derived in Lemma 4 reduce to:

\[
\frac{\partial W^A}{\partial \alpha^A} = \Delta \left\{ \theta(q^A)^{2 \phi} (\mu - 1) - (1 - \theta)(q^B)^{2 \phi} \mu \right\} - \phi (\alpha^A - \bar{\alpha}) = 0
\]

\[
\frac{\partial W^B}{\partial \alpha^B} = \Delta \left\{ (1 - \theta)(q^B)^{2 \phi} (\mu - 1) - \theta(q^A)^{2 \phi} \mu \right\} - \phi (\alpha^B - \bar{\alpha}) = 0
\]

In $\partial W^A/\partial \alpha^A$, the first term in the curly brackets as usually represents the welfare gain from a small increase in $\alpha^A$ working through a decrease in $q^A$ whereas the second term in the curly brackets represents the welfare loss working through an increase in $q^B$. Compared to the symmetric case, the former effect is larger and the latter effect smaller. This is simply because the size asymmetry amplifies both mechanical and behavioral effects in country A and diminishes both effects in country B. We now characterize the equilibrium under the parametric assumptions $\theta > 1/2$ and $\beta = 1/2$.

Proposition 5 If countries are asymmetric ($\theta > 1/2$) and there is no equity bias ($\beta = 1/2$), the unique equilibrium in the non-cooperative policy game ($\alpha^A, \alpha^B$) can be characterized in the following way:

(a) Distance. The equilibrium distance is implicitly determined by:

\[
\alpha^A - \alpha^B = \frac{\Delta(2\mu - 1)}{\phi} \left\{ \theta(q^A)^{2 \phi} - (1 - \theta)(q^B)^{2 \phi} \right\}
\]

This implies that $\alpha^A - \alpha^B$ is positive and increasing in $\theta$.

(b) Location. The equilibrium location is given given by

\[
\frac{\alpha^A + \alpha^B}{2} = \bar{\alpha} - \frac{\Delta}{2\phi} \left\{ \theta(q^A)^{2 \phi} + (1 - \theta)(q^B)^{2 \phi} \right\}
\]

Proposition 5 implies that in this non-cooperative setting the location $(\alpha^A + \alpha^B)/2$ is too low whereas the distance $\alpha^A - \alpha^B$ is positive but too small compared to the optimal policy derived in Proposition 2. Intuitively, the globally optimal policy has $\alpha^A - \alpha^B > 0$, which implies that $q^A < q^B$ so that the international tax environment provides more protection against tax planning by firms in the large country.
A than by firms in the small country B. Evaluated at the globally optimal policy both countries have an incentive to lower the policy parameter \( \alpha \) for the same reason as in the symmetric case analyzed in Proposition 3: A lower value of \( \alpha \) increases the scope for hybrid finance by firms in country and reduces it for firms in country \( j \), which increases welfare in country \( i \) at the expense of country \( j \). This explains why the location is suboptimally low in the non-cooperative setting. As both countries lower the policy parameter \( \alpha \) from the globally optimal level, \( \alpha_A \) comes closer to \( \bar{\alpha} \), which reduces the marginal welfare cost associated with a further reduction in \( \alpha_A \), whereas \( \alpha_B \) moves away from \( \bar{\alpha} \), which increases the marginal welfare cost associated with a further reduction in \( \alpha_B \). This explains why \( \alpha_A \) is reduced more than \( \alpha_B \) as compared to their globally optimal levels so that the distance is suboptimally small in the non-cooperative setting.

Finally, we consider the general asymmetric case, which also allows for equity bias, that is \( 0 \leq \beta \leq 1 \). The first-order conditions for optimal policy in country \( A \) and \( B \) are the ones presented in Lemma 4. Intuitively, equity bias has the same effect on government incentives as in the symmetric case. From the perspective of the government in country \( i \), equity bias reduces the revenue gain associated with an increase in \( \alpha_i \) because the firms in country \( i \) that are induced to switch from hybrid finance to standard finance by the resulting decrease in \( q^i \) are to a larger extent equity financed and thus taxed in country \( j \). Similarly, from the perspective of the government in country \( j \), equity bias increases the revenue loss associated with an increase in \( \alpha_i \) because the firms in country \( j \) that are induced to switch from standard finance to hybrid finance by the resulting increase in \( q^j \) are to a larger extent equity financed and thus taxed in country \( i \). We now characterize the equilibrium in the general case:

**Proposition 6** If countries are asymmetric (\( \theta > 1/2 \)) and there is positive or negative equity bias (\( \beta \neq 1/2 \)), the unique equilibrium in the non-cooperative policy game \((\alpha_A, \alpha_B)\) can be characterized in the following way:

(a) **Distance.** The equilibrium distance \( \alpha_A - \alpha_B \) is implicitly determined by:

\[
\alpha_A - \alpha_B = \frac{\Delta(2\mu - 1)}{\phi} \left\{ \theta(q^A)^{1/2} - (1 - \theta)(q^B)^{1/2} \right\}
\]

This implies that \( \alpha_A - \alpha_B \) is positive and increasing in \( \theta \).

(b) **Location.** The equilibrium location is given by:

\[
\frac{\alpha_A + \alpha_B}{2} = \bar{\alpha} - \frac{\Delta}{2\phi} \left\{ \theta(q^A)^{1/2}[q^A + 2\mu\sigma] + (1 - \theta)(q^B)^{1/2}[q^B + 2\mu\sigma] \right\}
\]

This implies that the location \((\alpha_A + \alpha_B)/2\) is decreasing in the equity bias \( \sigma \).

Proposition 6 implies that the location \((\alpha_A + \alpha_B)/2\) may be either too low or to high and the distance \( \alpha_A - \alpha_B \) is positive but too small compared to the optimal policy derived in Proposition 2 above. As in the symmetric case, equity bias reinforces the downward pressure on \( \alpha \), however, does not affect the equilibrium distance \( \alpha_A - \alpha_B \). Intuitively, this is because a change in the debt-capital ratio \( \beta \) affects the incentives of country \( A \) and \( B \) to change \( \alpha_A \) and \( \alpha_B \) respectively in the same way thus leaving the equilibrium distance between the two unaltered.
In conclusion, when asymmetric countries engage in non-cooperative policymaking the equilibrium distance is positive ($\alpha^A > \alpha^B$) but smaller than the socially desirable level. The international tax environment thus provides more protection against tax planning by firms in the large country $A$ than by firms in the small country $B$ as efficiency requires but the difference is not sufficiently large. The equilibrium location of $\alpha^A$ and $\alpha^B$ is increasing in the extent to which firms opting for standard finance rely on internal debt rather than equity to finance foreign investment. There is a threshold value of $\beta$ below (above) which the equilibrium location is lower (higher) than the socially optimal location. This threshold value of $\beta$ is larger than $1/2$.

5.3 Empirical evidence on $\beta$

We have seen that the equilibrium location depends crucially on the value of $\beta$, which is an empirically observable parameter. The empirical evidence on the internal capital markets of multinational firms allows us to determine a range of empirically realistic values of $\beta$, on the basis of which we can ascertain whether an equilibrium location above or below the optimal location is the most likely empirical outcome.

The most detailed empirical evidence on the structure of intra-firm investment derives from a dataset on German inbound and outbound foreign direct investment collected by the German Central Bank. Buettner and Wamser (2009) summarize the financial structure of foreign affiliates of German firms and report average ‘internal debt-asset ratios’ of around 24%, ‘external debt-asset ratios’ of around 35% and ‘equity-asset ratios’ of around 41%. Ramb and Weichenreider (2005) conversely consider German affiliates of non-German firms and find average ‘internal debt-asset ratios’ of around 30%, ‘external debt-asset ratios’ of around 23% and ‘equity-asset ratios’ of around 47%. Two papers based on US data provide less detailed evidence on the capital structure of foreign affiliates of US multinational firms (Desai et al., 2004; Altshuler and Grubert, 2002). Importantly, the US data only distinguish between loans from the parent company and loans from other sources, which does not allow us to compute the ‘internal debt-asset ratio’ and ‘external debt-asset ratio’ since internal loans may derive from other affiliates than the parent company.

When computing an approximate value of $\beta$ in these two empirical samples, we need to take into account that while our theoretical model does not include external debt, this source of finance plays a prominent role empirically. Recalling that the role of $\beta$ and $1 - \beta$ in the theoretical model is to capture the fraction of the normal return that is subject to home country and host country taxation respectively and noting that external debt is untaxed at the firm level under a standard corporate tax system, we may compute an approximate empirical equivalent to $\beta$ as the fraction of internal debt in the part of the capital stock, which is subject to firm taxation, that is:

$$\beta^{EMP} = \frac{\text{internal debt}}{\text{internal debt} + \text{equity}}$$

Applying this formula yields $\beta^{EMP} = 0.37$ for the sample of Buettner and Wamser (2009) and $\beta^{EMP} = 0.39$ for the sample of Ramb and Weichenreider (2005). While we are unable to apply the formula directly to the samples of US multinational firms, the reported levels of equity and loans from the parent
company seem to suggest even lower values of $\beta^{EMP}$.\textsuperscript{14}

In sum, the lesson from the available empirical evidence on the capital structure of multinational firms is that empirically realistic values of $\beta$ are comfortably below 1/2. This implies that in a typical case of cross-border investment, less than one half of the taxable part of the normal return to capital is subject to home country taxation and more than one half is subject to host country taxation. In terms of the predictions of the model, this suggests that the most likely empirical outcome of non-cooperative line-drawing between debt and equity is threshold values $\alpha$ lower than the socially optimal level, or, in other words, that the equilibrium location is below the optimal location.

6 Concluding remarks

This paper has developed a theoretical model of strategic line drawing between debt and equity. Firms are endowed with profitable foreign investment projects that may be financed either with a hybrid instrument or a combination of pure debt and equity. Hybrid instruments are costly to implement but generate a tax saving in the event that they are success fully categorized as debt in the host country and equity in the home country. Governments are faced with a continuum of financial instruments ranging from pure debt to pure equity and need to draw lines that delineate debt instruments and equity instruments for tax purposes. Policy choices affect the optimal financial choices of the firms, in particular the choice between hybrid and standard financing and the properties of the optimal hybrid instrument.

The first set of results characterized the globally optimal policies prevailing under cooperative policymaking. In this case, the central trade-off is that policy changes that reduce the scope for hybrid financing by firms in one country at the same time enlarge the scope for hybrid financing by firms in the other country. We thus obtain the intuitive result that the optimal distance between the threshold values of the demarcation rules provides more protection against hybrid financing by firms in the relatively large country. The optimal location of the threshold values of the demarcation rules is always symmetric around the location that is preferred for non-tax purposes.

The second set of results characterizes the generally suboptimal policies prevailing under non-cooperative policymaking. The equilibrium distance between the threshold values of the demarcation rules is too small. This implies that the international tax environment provides too little protection against hybrid financing by firms in the relatively large country and too much protection against hybrid financing by firms in the relatively small country. The equilibrium location of the threshold values of the demarcation rules may be too low or too high depending on the capital structure of firms opting for standard financing. For realistic parameter values, the equilibrium location is suboptimally low. The intuition

\textsuperscript{14}Buettner and Wamser (2009) decompose the ‘internal debt-asset ratio’ of around 24% into a ‘parent debt-asset ratio’ of around 13% and a ‘other affiliate debt-asset ratio’ of around 11%. Altshuler and Grubert (2002) report a ‘total debt-asset ratio’ of around 54% and a ‘parent debt-asset ratio’ of around 11%. Assuming that US multinational firms apply the same split between ‘parent debt-asset ratio’ and ‘other affiliate debt-asset ratio’ as German multinational firms, the implied ‘internal debt-asset ratio’ in the sample of Altshuler and Grubert (2002) is around 20% and the resulting value of $\beta^{EST}$ is around 0.3. Similar computations generate a value of $\beta^{EST}$ around 0.25 for the sample of Desai et al. (2004).
for this result is that policymakers endeavor to draw lines in ways that facilitate hybrid financing by
domestic multinational firms and impedes hybrid financing by foreign multinational firms with a view to
eroding foreign taxation of domestic firms and enforcing domestic taxation of foreign firms. This causes
a competitive pressure to categorize a larger subset of financial instruments as equity or, equivalently, a
race-to-the-bottom in the threshold values $\alpha^A$ and $\alpha^B$.

References


Appendix A

Proof of Lemma 1:
Maximization of (2) with respect to \( z \) subject to \( 0 \leq z \leq 1 \) yields the first-order condition:

\[
\frac{\partial p^i(z)}{\partial z} (1 - p^i(z)) - \frac{\partial p^i(z)}{\partial z} p^i(z) + \lambda_0 - \lambda_1 = 0 \tag{5}
\]

where \( \lambda_0 \) and \( \lambda_1 \) are the lagrange multipliers associated with the constraints \( z \geq 0 \) and \( z \leq 1 \) respectively. Note that since \(-1/\gamma < \alpha^A - \alpha^B < 1/\gamma\), there exists a hybrid \( z' \) satisfying \( p^i(z') > 0 \) and \( 1 - p^i(z') > 0 \). The optimal hybrid \( z^* \) must therefore at least satisfy that \( p^i(z^*) > 0 \) and \( 1 - p^i(z^*) > 0 \) in order not to be strongly dominated by \( z' \). Also, note from (1) that for \( k = A, B \) it holds that \( \partial p^k(z)/\partial z = \gamma \)
for \( \alpha^k - 1/2\gamma < z < \alpha^k + 1/2\gamma \) whereas \( \partial p^k(z)/\partial z = 0 \) for values of \( z \) outside these bounds. Assume that \( \lambda_0 > 0 \). This requires that the optimal hybrid is \( z = 0 \) and implies that \( \lambda_1 = 0 \). Since \( z \leq \alpha^i \) and \( z \leq \alpha^j \), it must hold that \( 1 - p^i(z) \leq 1/2 \leq p^i(z) \). Moreover, since \( 0 < p^i(z) \leq 1/2 \), it must hold that \( \partial p^i(z)/\partial z = \gamma \geq \partial p^i(z)/\partial z \). It follows that the left-hand side of (5) is positive. Hence, \( \lambda_0 = 0 \). Similarly, assume that \( \lambda_1 > 0 \). This requires that the optimal hybrid is \( z = 1 \) and implies that \( \lambda_0 = 0 \). Since \( z \geq \alpha^i \) and \( z \geq \alpha^j \), it must hold that \( 1 - p^i(z) \leq 1/2 \leq p^i(z) \). Moreover, since \( 0 < 1 - p^i(z) \leq 1/2 \), it must hold that \( \partial p^i(z)/\partial z = \gamma \geq \partial p^i(z)/\partial z \). It follows that the left-hand side of (5) is negative. Hence, \( \lambda_1 = 0 \). Note that since, by assumption, \(-1/\gamma < \alpha^A - \alpha^B < 1/\gamma\), there exists no hybrid satisfying \( p^i(z) = 1 - p^i(z) = 1 \). This implies that the optimal hybrid \( z^* \) satisfies \( \partial p^i(z^*)/\partial z > 0 \) or \( \partial p^i(z^*)/\partial z < 0 \). However, it follows from (5) that if \( \partial p^i(z^*)/\partial z > 0 \) then \( \partial p^i(z^*)/\partial z > 0 \) and vice versa. Hence, \( \partial p^i(z^*)/\partial z = \partial p^i(z^*)/\partial z = \gamma \). This implies that \( p^i(z^*) = 1 - p^i(z^*) \). It follows from (1) that \( \alpha^A - z^* = z^* - \alpha^B \) and, consequently, that \( z^* = (\alpha^A + \alpha^B)/2 \).

Proof of Lemma 2:
(a) Differentiating \( p^i \) with respect to \( \alpha^i \) yields:

\[
\frac{dp^i}{d\alpha^i} = \frac{\partial p^i}{\partial \alpha^i} + \frac{\partial p^i}{\partial z^*} \frac{dz^*}{d\alpha^i}
\]

From the expression for \( p(z) \) derived in section 2.2, it follows that \( \partial p^i/\partial \alpha^i = -\gamma \) and \( \partial p^i/\partial z = \gamma \). Moreover, it follows from the definition of \( z^* \) that \( \partial z^*/\partial \alpha^i = 1/2 \). Putting together these pieces, we obtain \( dp^i/d\alpha^i = -\gamma/2 \). Differentiating \( p^i \) with respect to \( \alpha^i \) yields:

\[
\frac{dp^i}{d\alpha^i} = \frac{\partial p^i}{\partial \alpha^i} + \frac{\partial p^i}{\partial z^*} \frac{dz^*}{d\alpha^i}
\]

From the expression for \( p(z) \) derived in section 2.2, it follows that \( \partial p^i/\partial \alpha^i = 0 \) and \( \partial p^i/\partial z = \gamma \). Hence, the result \( dp^i/d\alpha^i = \gamma/2 \).

(b) Differentiating \( q^i \) with respect to \( \alpha^i \) yields:

\[
\frac{dq^i}{d\alpha^i} = \frac{dp^i}{d\alpha^i} (1 - p^i) - \frac{dp^i}{d\alpha^i} p^i
\]

Use \( dp^i/d\alpha^i = -dp^i/d\alpha^i = -\gamma/2 \) from (a) and \( p^i = 1 - p^i \) from (c) to obtain \( dq^i/d\alpha^i = -\gamma p^i \). Differentiating \( q^i \) with respect to \( \alpha^j \) yields:

\[
\frac{dq^i}{d\alpha^j} = \frac{dp^i}{d\alpha^j} (1 - p^j) - \frac{dp^i}{d\alpha^j} p^i
\]
Following the same procedure that we used to derive \( dq^i / d\alpha^i \), we obtain \( dq^i / d\alpha^j = \gamma p^i \).

(c) It follows from Lemma (1) that \( z^* \) is equidistant from \( \alpha^A \) and \( \alpha^B \), that is \( z^* - \alpha^A = \alpha^B - z^* \). Using the expression for \( p(z) \) derived in section 2.2, it is easy to see that \( p^i = 1 - p^i \). Using the definition \( q^i = p^i (1 - p^i) \), it follows that \( q^i = (p^i)^2 \).

**Proof of Lemma 3:**

Use the results from Lemma 2 that \( dq^A / d\alpha^A = -\gamma p^A \) and \( dq^B / d\alpha^A = \gamma p^B \); definitions \( x^{HA} \equiv \theta \delta z^A \) and \( x^{HB} \equiv (1 - \theta) \delta z^B \) where \( \delta z^A \equiv q^A x^{N} \) and \( \delta z^B \equiv q^B x^{N} \); \( dx^{HA} / d\alpha^A = -\theta \delta x^{N} \gamma p^A \) and \( dx^{HB} / d\alpha^A = (1 - \theta) \delta x^{N} \gamma p^B \) to restate \( \partial W / \partial \alpha^A \) in the following way:

\[
\frac{\partial W}{\partial \alpha^A} = \begin{cases} 
(1 - \mu) \tau^N \{ -\theta \delta \gamma \tau^N q^A p^A + (1 - \theta) \delta \gamma \tau^N q^B p^B \} - \\
\mu \tau^N \{ -\theta \delta \gamma \tau^N p^A q^A + (1 - \theta) \delta \gamma \tau^N p^B q^B \} - \phi(\alpha^A - \bar{\alpha})
\end{cases}
\]

Use the definition of \( \Delta \) and the result from Lemma 2 that \( q^A = (p^A)^2 \) and \( q^B = (p^B)^2 \) to simplify:

\[
\frac{\partial W}{\partial \alpha^A} = (2\mu - 1) \Delta \left\{ \theta (q^A)^2 - (1 - \theta)(q^B)^2 \right\} - \phi(\alpha^A - \bar{\alpha})
\]

Follow the same procedure to derive \( \partial W / \partial \alpha^B \).

**Proof of Proposition 1:**

Evaluate the first-order conditions derived in Lemma 3 at \( \theta = 1/2 \) and compute \( \partial W / \partial \alpha^A - \partial W / \partial \alpha^B \) to obtain the equilibrium condition:

\[
\alpha^A - \alpha^B = \frac{\Delta (2\mu - 1)}{\phi} \left\{ (q^A)^2 - (q^B)^2 \right\}
\]

At \( \alpha^A = \alpha^B \), it holds that \( q^A = q^B \), hence the equilibrium condition is satisfied. At any, \( \alpha^A > \alpha^B \), it holds that \( q^A < q^B \), hence the left-hand side is positive and the right-hand side is negative so that the equilibrium condition cannot be satisfied. At any, \( \alpha^A < \alpha^B \), it holds that \( q^A > q^B \), hence the left-hand side is negative and the right-hand side is positive so that the equilibrium condition cannot be satisfied. It follows that \( \alpha^A = \alpha^B \). Now compute \( \partial W / \partial \alpha^A + \partial W / \partial \alpha^B \) and rearrange to obtain \( (\alpha^A + \alpha^B) / 2 = \bar{\alpha} \). Insert \( \alpha^A = \alpha^B \) to obtain \( \alpha^A = \alpha^B = \bar{\alpha} \).

**Proof of Proposition 2:**

(a) Compute \( \partial W / \partial \alpha^A + \partial W / \partial \alpha^B \) and rearrange to obtain \( (\alpha^A + \alpha^B) / 2 = \bar{\alpha} \).

(b) Compute \( \partial W / \partial \alpha^A - \partial W / \partial \alpha^B \) and rearrange to obtain:

\[
\alpha^A - \alpha^B = \frac{2\Delta (2\mu - 1)}{\phi} \left\{ \theta (q^A)^2 - (1 - \theta)(q^B)^2 \right\}
\]

Note that \( q^A \) is a continuous and decreasing function of \( \alpha^A - \alpha^B \) whereas \( q^B \) is continuous and increasing function of \( \alpha^A - \alpha^B \). The right-hand side is therefore a continuous and increasing function of \( (\alpha^A - \alpha^B) \) and a continuous and increasing function of \( \theta \). It follows that starting from values of \( \theta, q^A \) and \( q^B \).
that satisfy the equilibrium condition, an increase in \( \theta \) requires an increase in \( (\alpha^A - \alpha^B) \) to reestablish equilibrium or, in other words, \((\alpha^{A*} - \alpha^{B*})\) is increasing in \( \theta \). Moreover, at \( \alpha^A = \alpha^B \), it holds that \( q^A = q^B \). Hence, at \( \alpha^A = \alpha^B \), the right-hand side is positive while the left-hand side is zero and the equilibrium condition is not satisfied. Clearly, \( \alpha^A > \alpha^B \) is required to satisfy the equilibrium condition and it follows that \( q^A < q^B \) in equilibrium. Finally, when \( \alpha^A - \alpha^B \) is so large that \( q^A = 0 \), the right-hand side is negative and the equilibrium condition is not satisfied. Hence, \( q^A > 0 \) in equilibrium.

**Proof of Lemma 4:**

Use definitions of \( x^{HA}, x^{HB}, \tilde{c}^A, \tilde{c}^B \) and \( \sigma; dq^A/d\alpha^A = -\gamma p^A; dq^B/d\alpha^A = \gamma p^B; d\alpha^A/d\alpha^A = -\theta\delta T N \gamma p^A \) and \( dx^{HB}/d\alpha^A = (1-\theta)\delta T N \gamma p^B; dp^{A}/d\alpha^A = -(dq^A/d\alpha^A)/2 \) and \( dp^{B}/d\alpha^A = -(dq^B/d\alpha^A)/2 \) to restate \( \partial W^A/\partial \alpha^A \) in the following way:

\[
\frac{\partial W^A}{\partial \alpha^A} = \left\{ \begin{array}{l}
\tau^N \left\{ -\theta \gamma \delta q^A \tau N p^A \right\} + \mu \tau^N \left\{ (\theta \delta q^A \tau N)(\gamma p^A) \frac{1}{2} \right\} - ((1-\theta)\delta T N \gamma p^B)(\gamma p^B) \frac{1}{2} \\
\mu \tau^N \left\{ -\theta \delta T N \gamma p^A (-q^A/2 + \sigma) + (1-\theta)\delta T N \gamma p^B (-q^B/2 - \sigma) \right\} - \phi(\alpha^A - \bar{\alpha})
\end{array} \right.
\]

Use the definition of \( \Delta; q^A = (p^A)^2 \) and \( q^B = (p^B)^2 \) to obtain:

\[
\frac{\partial W^A}{\partial \alpha^A} = \Delta \left\{ \theta(q^A)^{\frac{1}{2}} [(\mu - 1)q^A - \mu \sigma] - (1-\theta)(q^B)^{\frac{1}{2}} [q^B + \sigma] \right\} - \phi(\alpha^A - \bar{\alpha}) = 0
\]

Follow the same procedure to derive \( \partial W^B/\partial \alpha^B \).

**Proof of Proposition 3:**

Evaluate \( \partial W^A/\partial \alpha^A \) and \( \partial W^B/\partial \alpha^B \) at \( \theta = 1/2 \) and \( \sigma = 0 \). Compute \( \partial W^A/\partial \alpha^A - \partial W^B/\partial \alpha^B \) and rearrange to obtain:

\[
\alpha^A - \alpha^B = \frac{\Delta(2\mu - 1)}{2\phi} \left\{ (q^A)^{\frac{1}{2}} - (q^B)^{\frac{1}{2}} \right\} \tag{6a}
\]

At \( \alpha^A = \alpha^B \), it holds that \( q^A = q^B \), hence the equilibrium condition is satisfied. At any, \( \alpha^A > \alpha^B \), it holds that \( q^A < q^B \), hence the left-hand side is positive and the right-hand side is negative so that the equilibrium condition cannot be satisfied. At any, \( \alpha^A < \alpha^B \), it holds that \( q^A > q^B \), hence the left-hand side is negative and the right-hand side is positive so that the equilibrium condition cannot be satisfied. It follows that \( \alpha^A = \alpha^B \) in equilibrium. Now, compute \( \partial W/\partial \alpha^A + \partial W/\partial \alpha^B \) and rearrange to obtain:

\[
\frac{\alpha^A + \alpha^B}{2} = \bar{\alpha} - \frac{\Delta}{4\phi} \left\{ (q^A)^{\frac{1}{2}} + (q^B)^{\frac{1}{2}} \right\} 
\]

Finally, use \( \alpha^A = \alpha^B \) and \( q^A = q^B \) to restate as:

\[
\alpha^A = \alpha^B = \bar{\alpha} - \frac{\Delta}{2\phi} q^{\frac{1}{2}}
\]

where \( q = q^A = q^B = 1/4 \). To analyze the special case \( \phi = 0 \), evaluate \( \partial W^A/\partial \alpha^A \) and \( \partial W^B/\partial \alpha^B \) at \( \theta = 1/2 \) and \( \sigma = \phi = 0 \). Compute \( \partial W^A/\partial \alpha^A - \partial W^B/\partial \alpha^B \) and rearrange to obtain:

\[
\frac{\Delta}{2} \left( 2\mu - 1 \right) \left\{ (q^A)^{\frac{1}{2}} - (q^B)^{\frac{1}{2}} \right\} = 0
\]
This equation requires that \( q^A = q^B \) and, thus, that \( \alpha^A = \alpha^B \). It is easy to show that at \( q^A = q^B \), it holds that \( \partial W^A / \partial \alpha^A < 0 \) and \( \partial W^B / \partial \alpha^B < 0 \). Hence, \( \alpha^A = \alpha^B = 0 \) where neither country can reduce their policy parameter any further is the unique Nash equilibrium.

**Proof of Proposition 4:**
Evaluate \( \partial W^A / \partial \alpha^A \) and \( \partial W^B / \partial \alpha^B \) at \( \theta = 1/2 \). Compute \( \partial W^A / \partial \alpha^A - \partial W^B / \partial \alpha^B \) and rearrange to obtain (6a). By the argument invoked in the proof of proposition (3), \( \alpha^A = \alpha^B \) in equilibrium. Now, compute \( \partial W / \partial \alpha^A + \partial W / \partial \alpha^B \) and rearrange to obtain:

\[
\frac{\alpha^A + \alpha^B}{2} = \bar{\alpha} - \frac{\Delta}{4} \left\{ (q^A)^{\frac{1}{2}} [q^A + 2\mu \sigma] + (q^B)^{\frac{1}{2}} [q^B + 2\mu \sigma] \right\}
\]

Finally, use \( \alpha^A = \alpha^B \) and \( q^A = q^B \) to restate as:

\[
\frac{\alpha^A + \alpha^B}{2} = \bar{\alpha} - \frac{\Delta}{2} \left\{ q^\frac{1}{2} + 2\mu \sigma q^{\frac{1}{2}} \right\}
\]

where \( q = q^A = q^B = 1/4 \).

**Proof of Proposition 5:**
(a) Evaluate \( \partial W^A / \partial \alpha^A \) and \( \partial W^B / \partial \alpha^B \) at \( \sigma = 0 \). Compute \( \partial W^A / \partial \alpha^A + \partial W^B / \partial \alpha^B \) and rearrange to obtain:

\[
\frac{\alpha^A + \alpha^B}{2} = \bar{\alpha} - \frac{\Delta}{2\phi} \left\{ \theta (q^A)^{\frac{1}{2}} + (1 - \theta) (q^B)^{\frac{1}{2}} \right\}
\]

(b) Compute \( \partial W^A / \partial \alpha^A - \partial W^B / \partial \alpha^B \) and rearrange to obtain:

\[
\alpha^A - \alpha^B = \frac{\Delta (2\mu - 1)}{\phi} \left\{ \theta (q^A)^{\frac{1}{2}} - (1 - \theta) (q^B)^{\frac{1}{2}} \right\}
\]

This equilibrium equation implicitly defines the distance \( \alpha^A - \alpha^B \). Note that evaluated at \( \alpha^A = \alpha^B \), the right-hand side is positive since \( q^A = q^B \) and \( \theta > 1/2 \). Also note that the right-hand side is a continuous decreasing function of \( \alpha^A - \alpha^B \). Hence, there must be unique value of \( \alpha^A - \alpha^B < 0 \) satisfying the equilibrium equation. Now, starting from an initial equilibrium, consider an increase in the parameter \( \theta \).
An increase in \( \theta \) increases the value of the right-hand side for a given value of \( \alpha^A - \alpha^B \), hence it requires an increase in \( \alpha^A - \alpha^B \) to reestablish equilibrium.

**Proof of Proposition 6:**
(a) Consider the general first-order conditions derived in Lemma 4. Compute \( \partial W^A / \partial \alpha^A + \partial W^B / \partial \alpha^B \) and rearrange to obtain:

\[
\frac{\alpha^A + \alpha^B}{2} = \bar{\alpha} - \frac{\Delta}{2\phi} \left\{ \theta (q^A)^{\frac{1}{2}} [q^A + 2\mu \sigma] + (1 - \theta) (q^B)^{\frac{1}{2}} [q^B + 2\mu \sigma] \right\}
\]

(b) Compute \( \partial W^A / \partial \alpha^A - \partial W^B / \partial \alpha^B \) and rearrange to obtain (8). By the argument invoked in the proof of Proposition 5, the equilibrium distance \( \alpha^A - \alpha^B \) is positive and increasing in \( \theta \).
Appendix B

Derive $p(z)$ [section 2.1]:
By definition of $p(z)$, we have:

$$p(z) = \text{prob}(Z > \alpha)$$

Use the definition of $Z$ and rearrange to obtain:

$$p(z) = \text{prob}(\varepsilon > \alpha - z)$$

By assumption, $\varepsilon$ is uniformly distributed over the interval $[-1/2\gamma, 1/2\gamma]$ with density $\gamma$. It follows that:

$$\text{prob}(\varepsilon < \alpha - z) = \gamma(\alpha - z - (-1/2\gamma)) = \frac{1}{2} + \gamma(\alpha - z)$$

and, consequently, that:

$$p(z) = \text{prob}(\varepsilon > \alpha - z) = 1 - \text{prob}(\varepsilon < \alpha - z) = \frac{1}{2} + \gamma(z - \alpha)$$

Derive $\partial W/\partial \alpha^i$ [section 4]:
As indicated in the main text, aggregate welfare is given by:

$$W = \sum_{i=A,B} \Pi^i + \mu R^i - \frac{\phi}{2}(\alpha^i - \bar{\alpha})^2$$

Differentiating with respect to $\alpha^i$ yields:

$$\frac{dW}{d\alpha^i} = \left\{ \frac{d\mu}{d\alpha^i} \{ (q^i\tau^N - \psi^j) + \mu \tau^N (\rho^{ij} + \rho^{ij} - \beta - (1 - \beta) \} + \frac{d\mu}{d\alpha^i} \{ (q^i\tau^N - \psi^j) + \mu \tau^N (\rho^{ij} + \rho^{ij} - \beta - (1 - \beta) \} + \frac{d\mu}{d\alpha^i} \{ \mu x^{H_i \tau^N} + \frac{d\mu}{d\alpha^i} x^{H_j \tau^N} + \frac{d\mu}{d\alpha^i} x^{H_i \tau^N \phi} \} \{ d\mu^{ij} \frac{d\psi^i}{d\alpha^i} + \frac{d\mu^{ij}}{d\alpha^i} \} \left\{ \mu x^{H_i \tau^N} + \frac{d\mu^{ij}}{d\alpha^i} x^{H_j \tau^N - \phi(\alpha^i - \bar{\alpha})} \right\} \right\}$$

where we have used $dx^{H_k}/d\alpha^i = - dx^{S_k}/d\alpha^i$ for $k = i, j$. From the identity $\rho^{KA} + \rho^{KB} + q^k = 1$, it follows that $dp^{KA}/d\alpha^i + dp^{KB}/d\alpha^i = -dq^k/d\alpha^i$ for $k = A, B$. Use these expressions to simplify:

$$\frac{dW}{d\alpha^i} = \left\{ \frac{d\mu^{ih}}{d\alpha^i} \{ (1 - \mu) q^i \tau^N - \psi^j \} + \frac{d\mu^{ij}}{d\alpha^i} \{ (1 - \mu) q^i \tau^N - \psi^j \} + \frac{d\mu}{d\alpha^i} (1 - \mu) x^{H_i \tau^N} + \frac{d\mu}{d\alpha^i} (1 - \mu) x^{H_j \tau^N} + \frac{d\mu}{d\alpha^i} x^{H_i \tau^N} + \frac{d\mu}{d\alpha^i} x^{H_j \tau^N - \phi(\alpha^i - \bar{\alpha})} \right\}$$

Finally, use $dx^{HA}/d\alpha^i = \theta (d\bar{C}^A/d\alpha^i)$; $dx^{HB}/d\alpha^i = (1 - \theta) (d\bar{C}^B/d\alpha^i)$; $d\psi^i/d\alpha^i = (d\bar{C}^i/d\alpha^i)/2$ and definitions $\bar{C}^i \equiv q^i \tau^N$ and $\psi^i \equiv \bar{C}^i/2$ to obtain:

$$\frac{dW}{d\alpha^i} = (1 - \mu) \tau^N \left\{ \frac{dq^i}{d\alpha^i} x^{H_i \tau^N} + \frac{dq^j}{d\alpha^i} x^{H_j \tau^N} \right\} - \mu \tau^N \left\{ \frac{dx^{H_i}}{d\alpha^i} q^i + \frac{dx^{H_j}}{d\alpha^i} q^j \right\} - \phi(\alpha^i - \bar{\alpha})$$

Derive $\partial W^i/\partial \alpha^i$ [section 5]:
As indicated in the main text, the welfare of country $i$ is given by:
\[ W^i = \Pi^i + \mu R^i - \frac{\rho}{2}(\alpha^i - \bar{\alpha})^2 \]

Differentiating with respect to \( \alpha^i \) yields:

\[
\frac{dW^i}{d\alpha^i} = \begin{cases} 
\frac{dx^{H_i}}{d\alpha^i} \{ (q^i \tau^N - \psi^i) + \mu \tau^N (\rho^{ii} - \beta) \} + \frac{dx^{H_j}}{d\alpha^i} \{ \mu \tau^N (\rho^{jj} - (1 - \beta)) \} + \\
\frac{d\phi^j}{d\alpha^i} x^{H_i} \tau^N - \frac{d\phi^i}{d\alpha^i} x^{H_i} + \frac{d\phi^j}{d\alpha^i} \mu x^{H_i} \tau^N + \frac{d\rho^{ji}}{d\alpha^i} \mu x^{H_j} \tau^N - \phi(\alpha^i - \bar{\alpha})
\end{cases}
\]

where we have used that \( dx^{HA}/d\alpha^i = - dx^{SA}/d\alpha^i \). Use \( dx^{HA}/d\alpha^i = \theta \delta(d\bar{c}^A/d\alpha^i) \); \( d\psi^i/d\alpha^i = (d\bar{c}^i/d\alpha^i)/2 \) and definitions \( \bar{c}^i \equiv q^i \tau^N \) and \( \psi^i \equiv \bar{c}^i/2 \) to obtain:

\[
\frac{dW^i}{d\alpha^i} = \begin{cases} 
\tau^N \left\{ x^{H_i} \frac{dx^{A}}{d\alpha^i} \right\} - \mu \tau^N \left\{ x^{H_j} \frac{d\rho^{ji}}{d\alpha^i} + x^{H_j} \frac{d\rho^{ji}}{d\alpha^i} \right\} + \\
\mu \tau^N \left\{ \frac{dx^{H_i}}{d\alpha^i} (\rho^{ii} - \beta) + \frac{dx^{H_j}}{d\alpha^i} (\rho^{jj} - (1 - \beta)) \right\} - \phi(\alpha^i - \bar{\alpha})
\end{cases}
\]
Range with uncertainty about tax treatment in country A

Range with uncertainty about tax treatment in country B

Figure 1