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Public funding of political parties when campaigns are informative

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Abstract

The paper considers public funding of political parties when some voters are poorly informed about parties' candidates and campaigns are informative. For symmetric equilibria, it is shown that more public funding leads parties to chose more moderate candidates, and that an increase in the funding's dependence on vote shares induces further moderation and improves welfare. If parties are asymmetric, vote share dependent public funding benefits the large party and makes it moderate its candidate, while the smaller party reacts by choosing a more extremist candidate. On balance, however, if the parties are not too asymmetric, an increase in vote share dependent funding improves welfare and increases the likelihood that a moderate candidate wins the election.

Keywords: Political Economy, Parties, Public Funding, Informative Advertising, Campaign Finance.

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1 Introduction

Electoral campaigns and the funding of political parties are fundamental aspects of democracy. While private contributions and special interest groups have been the focus of much recent research most countries fund parties

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with significant amounts of public money and in many countries this funding is related to how parties fare at elections. In this paper we investigate the effect of such funding systems. We provide a model of rational behavior, where ideological parties receive public funding according to their vote share, which they can choose to spend on electoral campaigns. Voters are rational and ideologically motivated, and campaigns inform parts of the electorate about the ideological stance of the candidates. Uninformed voters try to infer whatever information is contained in the fact that they remain uninformed. We show that in symmetric equilibria a public funding system furthers candidate moderation and it does more so, the more funding depends on the vote share. Thus, the exact way the public funding is given out is crucial for how it works. There is a *campaign-multiplier effect*. We show that such a funding system enhances welfare, in fact in symmetric equilibria it is weakly Pareto improving.

These results are derived assuming that the parties are in a symmetric situation, i.e., they are equally large and have access to an equally efficient campaign technology. However, it has been argued that public funding depending on the parties' vote shares is unfair since large parties are advantaged by receiving larger monetary benefits[1]. The asymmetric case is therefore of interest. But the asymmetric case cannot be solved analytically and we must rely on numerical simulations. While public funding depending on vote shares indeed benefits the large party, more funding benefits voters at large, and total welfare increases as long as the asymmetry is not too pronounced. We have considered several kinds of asymmetries and report here on one where a party has access to a more efficient information technology. This could be because media are politically slanted, the party has better access to media or has a more efficient party organization. Whatever the reason, the examples reveal that increasing the share of public funding depending on vote shares benefits the advantaged party: its probability of winning the election increases. However, it also makes the party moderate its policy. Since it is more efficient at informing the electorate (and its larger share of the public funds makes it possible to inform even more), the benefits from moderation are larger. The disadvantaged party, on the other hand, reacts optimally by choosing a more extremist strategy. Its likelihood

[1] See for instance the Electoral Reform Green Paper of the Commonwealth of Australia, Commonwealth of Australia (2008), pp 33 ff

of winning the election becomes small, voters are poorly informed about its candidate, and then it is better to go with a more extremist strategy. This happens even though from an ideological point of view the parties are symmetric: the difference in moderation is not explained by differences in the moderation of parties' ideologies, but entirely by the asymmetry in the information technology.

Unless the parties are very asymmetric, total welfare for voters increase when funding becomes more dependent on vote shares as voters benefit from the moderation of the large party, who wins the election with a high probability. When, however, the parties are in a very asymmetric information, the welfare effect may be negative, as the large party is then already very moderate. Extra funds given to it then makes for almost no further moderation, while the small party becomes more extremist. On balance, this may worsen welfare.

When public funding depends on the vote share, the funding is given to parties after the vote share is realized, i.e. after the election. In reality, however, parties are often heavily indebted before elections, so they are able to spend (some of) the expected revenue from public funding before the election. We assume that the parties have access to a perfect credit market, so that they can spend the public funding on campaigns *before* the election, even though the funds are received *after the election.*

Public funding of political parties is widespread across the world: The International Institute for Democracy and Electoral Assistance (IDEA) database (Idea, 2008) lists 83 countries with some kind of public funding, in 69 it is related to how parties perform at elections. In 23 out of 25 Western European countries, parties receive public funding. The finer details vary, funding may depend on current representation in the legislature, or votes at current or previous elections. The purpose of the funding is election campaign activities (44 countries), general party administration (28 countries), while funds are not earmarked in 19 countries and in 8 countries the purpose is "other". In Denmark, for example, one vote gives 26.50 Danish kroner (2009 figure) (almost $4 EUR or 4.5 USD) per year. Bille (1997) notes that public funding has increased dramatically in the nineties. He finds that it accounts for 48-98% of major Danish parties' income. Mair (1994) finds that in many European countries public finance is at least as important for political parties as private finance is.

In our model, campaigns inform voters and this affects their voting. There is substantial empirical evidence to support the hypothesis that electoral campaigns affect voting, see e.g. Holbrook (1996). Starting with the famous panel study by Lazarsfeld, Berelston and Gaudet (1944) substantial evidence has also been collected showing that campaigns convey information to the voters. Alvarez (1998) finds (p 172) that "there are substantial reductions in voter uncertainty of the positions of the candidates during the campaign that are directly related to the flow of information". Ansolabehere and Iyengar (1995, p42) note that "exposure to advertising makes voters much more likely to refer to issues as reasons for supporting or opposing a candidate". Popkin (1991) stresses that campaigns convey information about candidate positions. Similarly, Brians and Wattenberg (1996, p 172) find that "citizens recalling political advertising have the most accurate knowledge of the candidates' issue positions". Just, Crigler and Wallach (1990) report experiments showing that political ad viewing convey more accurate candidate issue positions than televised debates. Although a lot of campaigning is done through advertisements, direct mail, telephone calls and canvassing, the ways campaigns work are manifold. Gelman and King (1993) stress the role of media. Holbrook (1996) argues that media's coverage of the campaigns is mostly about campaign events (conventions, debates etc.). Holbrok (1996) answers his title "Do campaigns matter?", with a resounding "yes", both for the electorates' information and voting.

The theoretical literature dealing with the effects of public funding is scarce but an interesting discussion is provided by the Electoral Reform Green Paper of the Commonwealth of Australia, Commonwealth of Australia (2008). The most closely related paper to our paper is Ortuno-Ortin and Schultz (2005) where we study public funding of political parties in a model where some voters are not rational and not interested in policies, but are impressionable in the sense of Baron (1994) (see also McKelvey and Ordeshok, 1987): Their vote is affected by campaigns with no informational content. In such an environment public funding of political parties leads to policy convergence and this convergence is more pronounced when funds depend on vote shares. However, the result depends on the presence of the irrational, impressionable voters who do maximize utility but just vote according to campaign expenditure and welfare analysis is therefore not possible. In the present paper all agents are described as rational policy motivated agents, which enables us to perform a proper welfare analysis.

The paper uses the framework of Coate (2004a) as a basis for analyzing public funding. Coate considers the case where parties receive contributions from partisan interest groups and shows that a limit on contributions reduces expected campaign spending, but it also decreases the likelihood that parties will select moderate candidates. Contribution limits therefore benefit extremist lobby groups and hurt moderate voters. Galeotti and Mattozzi (2011) introduces social networks in Coate's model and investigate the effect on political campaigning. Unlike us, Coate and Galeotti and Mattozzi only consider symmetric parties and are not concerned with public funding.

Baron (1994) considers lobbying in a model where some voters are impressionable and parties get campaign finance from extreme lobby groups. Parties' policies do therefore not converge to the median voter's preferred policy. The introduction of public funding (as a lump-sum, and independent of vote-share), mitigates the power of interest groups and the parties' policies become less polarized. Contrary to Baron, our argument does not depend on public funding mitigating lobby groups' power and non-rational voters. Roemer (2006) presents a model with both informed and impressionable voters, where informed citizens endogenously choose party membership. Campaigns persuade impressionable voters. He compares several funding systems, among which are purely private funding and matching funds but does not consider public funding depending on vote shares. Ashworth (2006) studies the effect of incumbency advantages in fund-raising. He finds that public matching funds improve welfare in districts where parties campaign in the absence of matching funds and reduces welfare in other districts. Troumpounis (2012) compares how public funding based on performance at the election (funding per vote) and public funding based on the representation in the parliament (funding per seat) affect voter turnout. The model, however, assumes that the proposals of parties are given and voters know them. Public funding only plays the role of increasing parties' incentives to exert more effort to convince citizens to go to the polls.

Special interest groups donations to political parties when the funds are used for informative campaigns are considered in a number of papers, see e.g. Austen-Smith (1987), Prat (2002), Potters, Sloof and Van Winden (1997) and Coate (2004b). Schultz (2007) considers the effect of informative campaigns on redistribution among different groups in society.

The organization of the paper is as follows: Section 2 presents the basic model. Section 3 introduces the public funding system, and Section 4 discusses voters information and beliefs, Section 5 concerns parties' candidate selection. Section 6 analyses the effects of the public funding system. Asymmetries are considered in Section 7. A few concluding remarks are contained in Section 8. A few proofs are relegated to an Appendix.

2 Model

The basic set up follows Coate's (2004a) model of political competition among parties with private campaign contributions. If a candidate with ideology y is elected, a voter with ideology x gets utility

$$u(y;x) = -\left|y - x\right|. \tag{1}$$

We can interpret this as voters' utility directly depends on the candidate's ideology or alternatively that a candidate cannot commit to a particular policy before the election, so she will choose a policy equal to her bliss point if elected. There is a continuum of voters, divided into partisans and swing voters. Partisans vote for their party regardless of what the party does, leftists for party L and rightists for party R. Leftists have ideologies uniformly distributed on $[0, d]$ and rightists have ideologies uniformly distributed on $[1 - d, 1]$. There are equally many partisans supporting each party, so the election is decided by the swing voters. Swing voters are uniformly distributed on the interval $[\mu - \tau, \mu + \tau]$, which is in between d and $1 - d$. Swing voters ideologies are ex ante uncertain: μ is the realization of a random variable uniformly distributed on $[1/2 - \varepsilon, 1/2 + \varepsilon]$. We assume that

$$\varepsilon \leq 1/2 - \tau - d, \tag{2}$$

so that the swing voters' ideologies are in between those of the partisans. We also assume that

$$\tau \geq \varepsilon + d/4. \tag{3}$$

This latter assumption will ensure that the ideologies of swing voters are sufficiently different, so that for all realizations of uncertainty, there is a swing voter who is indifferent between the two parties. Hence, no party will receive all votes from the swing voters. Let η, $0 < \eta < 1$, denote the fraction of the population who are swing voters.

The parties' members are partisans and we assume that the distribution of members' ideologies is the same as the partisans'. Each party chooses among its members a candidate, who runs at the election. We imagine a party deciding on its candidate by a vote among the members, so the median member will be decisive. This member will never choose a more extreme candidate than herself, but may want to choose a more moderate candidate in order to increase the chance that the party wins the election. To simplify matters, we assume that a party can choose among two candidates only: The median member ($d/2$ for party L) and the most moderate member (d for party L)[2].

Swing voters are ex ante uninformed about the parties' choice of candidates but can learn through campaigns. If a party spends c on its campaign, a fraction $\phi(c)$ of the voters learn about its candidate. Parties can inform about the ideology of their own candidate only and they have to do so truthfully. An interpretation is that a candidate has a track record from his political life, e.g. a voting history in the legislature, and a party can advertise this. Parties cannot target their campaigns to particular groups of voters, so that the probability that a voter becomes informed is independent of her ideology[3]. One can imagine that a party advertises on television or in magazines, and that only those voters who happen to see its advertising will learn about its candidate. Some voters, however, might well become informed regardless of whether they see advertisements or not. They might read newspapers, listen to radio stations or the news on television. This could easily be included in the following, but in order to simplify the exposition we will disregard this and assume that the campaign technology is given by

$$\phi(c) = \frac{c}{c+\alpha}, \tag{4}$$

where $\alpha > 0$ reflects the costliness of the campaign technology. The smaller is α the more efficient are campaigns in reaching a large fraction of the voters.

Parties' campaign finance come from public funding. The funds depend on the vote-share obtained in the election and are therefore available *after* the election. If a party wishes to spend the funds on campaigns *before* the

[2] Coate (2004a) shows that parties will in fact end up choosing among two types of candidates only in equilibrium even though they can choose among all members.

[3] See Galeotti and Mattozzi (2011) for an interesting extension of Coate's model, where voters share information among likeminded and parties can target their campaigns.

election it must either take a loan or spend money received in the previous election. We assume that the parties have access to a perfect credit market, where they can raise loans to be repaid by the funds earned in the election. The market is risk neutral and has complete information about the parties' candidates and rational expectations regarding parties' vote share. We thus abstract from issues of credit-worthiness and the dynamic issues involved if parties' credit is limited to a fraction of the public funding they are expected to receive. In most countries parties are in fact heavily indebted before elections[4].

When a party's vote share is v, it receives

$$\Psi(v) = \psi_0 + \psi_1 \cdot v, \tag{5}$$

public funds. The parameter $\psi_1 > 0$ measures how much the public funding system depends on the vote share. In order for the model to be well-behaved, there has to be limits to how much public funding responds to an increase in the vote share. In particular, we will assume that

$$\psi_1 \leq 2\alpha/\eta. \tag{6}$$

An increase in party $L's$ vote share equal to $d\tilde{v}$ increases $L's$ public funding with $\psi_1 d\tilde{v}$. If this is spent on campaigning, the number of swing voters informed about party $L's$ policy increases with $\phi\prime\eta\psi_1 d\tilde{v}$. Since from (4) $\phi\prime < 1/\alpha$, it follows that $d\phi_L/d\tilde{v} \leq \psi_1\eta/\alpha$. Assumption (6) therefore bounds $d\phi_L/d\tilde{v}$ so that $d\phi_L/d\tilde{v} < 2$.

A party is free to use the public funding as it wishes; it may use it for campaign finance but it may also use it on matters internal to the party, such as the party organization or events for partisans etc. To make the model as simple as possible, we assume that whatever utility the parties' members derive from the latter form of spending it is lexicographically inferior to the utility derived from politics. Hence a party will spend the funding on campaigns if and only if it increases the chance that the party wins the election. Otherwise, the funding is spent on matters internal to the party.

The timing is as follows: First, the parties elect candidates to run at the election anticipating the public funds they will receive and the effects

[4] In fact the public funding system varies across countries, in some coutries the funding depends on the current election in some on the previous, see Idea (2008). With a perfect credit market, this distinction is not important .

of the campaigns on voter information and voter behavior. Then, they take loans based on their expected vote share. The loans are then spent either on campaigns or internal matters in the party. Some of the voters observe a party's campaign advertisement and learn the ideology of the party's candidate. Voters, who do not observe an advertisement from a party, take this into account and update their beliefs about the party's candidate. Then the election is held, partisans always vote for their party and swing voters vote sincerely given their knowledge or beliefs about the parties' candidates. Finally, public funds are distributed and the parties repay their loans. [5]

3 Funding and campaigns

At the candidate selection stage each party chooses a candidate. A party's selection rules are such that the median member decides. Party L can choose between its median member $d/2$ and its most moderate member d, party R can choose among its median member $1-d/2$ and its most moderate member $1-d$. Viewed from the whole spectrum of ideologies, the median member, $d/2$ (or $1-d/2$), is *extremist,* e, and the most moderate member, d (or $1-d$), *moderate,* m. To shorten the notation, we say that parties choose candidates among types $t \in \{e, m\}$.

When party L has chosen a candidate of type t, and party R has chosen a candidate of type t', the expected vote share to party L is denoted $v_{tt'}$ and the expected vote share to party R is $1 - v_{tt'}$.

In this case the expected public funding to party L is

$$\Psi\left(v_{tt'}\right) = \psi_0 + \psi_1 \cdot v_{tt'},$$

while that to party R is

$$\Psi\left(1 - v_{tt'}\right) = \psi_0 + \psi_1 \cdot \left(1 - v_{tt'}\right).$$

As explained above, we assume that the parties have access to a perfect and risk neutral credit market, so they can obtain a credit contract stipulating that they get a loan equal to the expected public funding conditional on

[5] Notice that if parties have access to a perfect credit market, public funding will not prevent the entry of new parties to the political arena. For the time being, however, we shall disregard the issue of entry.

repaying the realized public funding after the election. If the credit market was not risk neutral, the parties would only be able to obtain some fraction of the expected public funding as a loan. The total amount of funding available to the parties at election time would therefore be smaller, but it would still depend on the expected vote share. This would change nothing qualitatively in the following but make formulas a bit longer.

When parties have chosen candidates and have received their loans, they decide on campaign spending. Swing voters are ideologically more moderate than any of the partisans, so ceteris paribus they prefer moderate candidates. A party therefore has an incentive to advertise, if indeed it has selected a moderate candidate, while it would like not to advertise an extremist candidate. If the party's candidate is extremist, the party uses the funding on events internal to the party. Parties can only advertise their own candidate and advertising has to be truthful. The idea is that the parties can advertise a candidate's track record and this record is verifiable. Since parties can only advertise their own candidate, negative campaigning is excluded. The party's campaign strategy therefore depends on the type of candidate it has chosen. If party L has chosen a moderate candidate and faces an expected vote share $v_{mt'}$ its expected public funding is $\Psi(v_{mt'})$ and it spends all this funding on campaign advertisements, $c(v_{mt'}) = \Psi(v_{mt'})$. Campaigns cannot be targeted specifically to any group of voters, so the fraction of swing voters informed about party $L's$ candidate is $\phi(c(v_{mt'}))$. To shorten notation, we write $\phi_L^{mt'} \equiv \phi(c(v_{mt'}))$. Notice that the fraction of voters informed about party $L's$ candidate's type depends on $R's$ candidate's type as well, since $R's$ candidate affects the expected vote shares and therefore the expected public funding which party L receives.

If the party has chosen an extremist candidate, it does not campaign, $c(v_{et'}) = 0$, and $\phi_L^{et'} \equiv \phi(0) = 0$.

4 Voters' information and beliefs

All swing voters are initially uninformed about the parties' candidates. If a voter sees a campaign advertisement from one of the parties she learns the party's candidate's type. A voter who does not see an advertisement revises her beliefs about the party's candidate in view of this (lack of) information. She will update her beliefs using the parties' strategies and Bayes' rule.

Following Coate (2004a) we first focus on the symmetric mixed strategy equilibrium. This is the only symmetric equilibrium in this environment. The reason is intuitive, suppose there were a pure strategy equilibrium where both parties chose moderate candidates. Then all voters would expect this, regardless of whether they were informed by campaigns or not and in fact parties would have no incentive to inform through campaigns, they would prefer to use the public funding on matters internal to the party. But then it would be beneficial for a party, L say, to deviate and choose an extremist candidate, who non-informed voters mistakenly would perceive as moderate. Party R may choose to campaign for its moderate candidate in face of the deviation of party L, thus trying to convince voters that party L has deviated to an extremist candidate. This would be an out of equilibrium move, and voters beliefs could in principle react such that they believe party L has chosen an extremist candidate. However, if party R can change change voter beliefs in this way, it would do it regardless of whether party L deviated or not. Hence the potential symmetric equilibrium with moderate candidates is undermined this way. Either way, the conclusion is that a symmetric equilibrium where both parties choose moderate candidates does not exist.

Similarly, if both parties choose extremist candidates in a potential equilibrium, it is worthwhile for a party to deviate to a moderate candidate, spend a little on campaigning and win the election with a higher probability. As also noted by Coate there must be uncertainty about the parties' choice of candidates for informative advertising to have an effect. One could alternatively have assumed that swing voters were uncertain about the identity of the median voter of each party[6]. As is well known the mixed strategy equilibrium, we focus on is close to the equilibrium of a game with such uncertainty if the uncertainty is small. One could therefore think of the uncertainty as stemming from uncertainty about the parties' median voters' ideologies.

Let e.g. σ_L denote the probability with which party L chooses an extremist candidate and similarly σ_R the probability with which party R chooses an extremist candidate.

After the parties have done their campaigning, a voter has four informational states. Let tt denote she that knows both candidates' types , $t\varnothing$, that

[6]This general route is pursued in Schultz (2007), where voters are uncertain about the parties' preferences.

she knows candidate $L's$ type but not candidate $R's$ and so forth. Notice that since a party will never campaign when it has an extremist candidate, information states $ee, e\varnothing$ and $\varnothing e$ will not occur.

An uninformed voter uses Bayes' rule to update beliefs. Let e.g. $\rho\,(em|\varnothing m)$ denote the probability a voter assigns to the event that party $L's$ candidate is of type e and party $R's$ is of type m, given the voter has not seen an advertisement from party L but have been informed that party $R's$ candidate is of type m. Bayes rule gives

$$\rho\,(em|\varnothing m) = \frac{\sigma_L \phi_R^{em}}{\sigma_L \phi_R^{em} + (1-\sigma_L)\,(1-\phi_L^{mm})\,\phi_R^{mm}},$$

i.e. the probability that a voter is informed by party R that its candidate is moderate when party $L's$ candidate is extremist times the probability that party L chooses an extremist candidate divided by the probability that party $R's$ candidate is moderate and the voter is uninformed about party $L's$ candidate's type. The latter event occurs in two cases, when $L's$ candidate is extremist (and consequently does not advertise) and when the candidate moderate and party L advertises, but the voter does not see the advertisement.

For a voter who is not informed about $L's$ candidate but is informed that $R's$ candidate is moderate, the expected candidate from party L is

$$\begin{aligned} l_{\varnothing m}^E &= \rho\,(em|\varnothing m)\,\frac{d}{2} + \rho\,(mm|\varnothing m)\,d \\ &= \frac{\sigma_L \phi_R^{em} \frac{d}{2} + (1-\sigma_L)\,(1-\phi_L^{mm})\,\phi_R^{mm} d}{\sigma_L \phi_R^{em} + (1-\sigma_L)\,(1-\phi_L^{mm})\,\phi_R^{mm}}. \end{aligned}$$

In the sequel, we are going to focus on symmetric equilibria. In a symmetric equilibrium, $\phi_L^{mm} = \phi_R^{mm} = \phi^{mm}$, so we get

$$l_{\varnothing m}^E = \frac{\sigma_L \phi_R^{em} \frac{1}{2} + (1-\sigma_L)\,(1-\phi^{mm})\,\phi^{mm}}{\sigma_L \phi_R^{em} + (1-\sigma_L)\,(1-\phi^{mm})\,\phi^{mm}} d \tag{7}$$

5 Voting

If a voter expects party $L's$ candidate to be of type l^E and party $R's$ candidate to be of type r^E, we get from (1) that she is indifferent between the parties if her ideology is

$$x^* = \frac{l^E + r^E}{2}. \tag{8}$$

If a voter's ideology is $x < x^*$ she prefers party L, and if $x > x^*$ she prefers party R. We assume that swing voters vote sincerely and vote for the party they like the best. Swing voters are uniformly distributed on $[\mu - \tau, \mu + \tau]$, therefore the fraction of swing voters with ideology $x \leq x^*$ is[7]

$$\frac{1}{2} + \frac{x^* - \mu}{2\tau}.$$

The median swing voter's ideology, μ, is uniformly distributed on $[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$, so the expected share of swing voters with ideology $x \leq x^*$ is

$$F(x^*) = \int_{\frac{1}{2}-\varepsilon}^{\frac{1}{2}+\varepsilon} \left(\frac{1}{2} + \frac{x^* - \mu}{2\tau} \right) \frac{1}{2\varepsilon} d\mu = \frac{1}{2} + \frac{2x^* - 1}{4\tau}. \tag{9}$$

When parties have chosen candidates l and r and the fraction of voters learning about party $L's$ candidate is ϕ_L and the fraction learning about party $R's$ candidate is ϕ_R, party $L's$ expected vote share among the swing voters is

$$\begin{aligned} \tilde{v} = \phi_L \phi_R F\left(\frac{l+r}{2}\right) + \phi_L (1 - \phi_R) F\left(\frac{l + r^E_{l,\varnothing}}{2}\right) + \\ (1 - \phi_L) \phi_R F\left(\frac{l^E_{\varnothing,r} + r}{2}\right) + (1 - \phi_L)(1 - \phi_R) F\left(\frac{l^E_{\varnothing\varnothing} + r^E_{\varnothing\varnothing}}{2}\right). \end{aligned} \tag{10}$$

Consider the case when party L chooses an extremist candidate and party R a moderate candidate. Then party L does not advertise and $c_L = 0$ and $\phi_L = 0$, while party R does and $\phi_R = \phi_R^{em}$. Inserting this and (9) and using that in a symmetric equilibrium, $r^E_{\varnothing\varnothing} = 1 - l^E_{\varnothing\varnothing}$, gives that the expected vote share to L is

$$\tilde{v}_{em} = \frac{1}{2} + \frac{1}{4} \frac{\phi_R^{em} \left(l^E_{\varnothing,m} - d \right)}{\tau}. \tag{11}$$

As $l^E_{\varnothing,m} < d$, it follows that $\tilde{v}_{em} < \frac{1}{2}$.

The median swing voter's ideology, μ, is uniformly distributed on $[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$, so the probability that her ideology, μ, is less than x^*, is

$$\Pr\{\mu \leq x^*\} = \begin{cases} 0 & \text{if} \quad x^* < \frac{1}{2} - \varepsilon \\ \frac{x^* - \left(\frac{1}{2} - \varepsilon\right)}{2\varepsilon} = \frac{1}{2} + \frac{2x^* - 1}{4\varepsilon} & \text{if} \quad \frac{1}{2} - \varepsilon \leq x^* \leq \frac{1}{2} + \varepsilon \\ 1 & \text{if} \quad \frac{1}{2} + \varepsilon \leq x^*. \end{cases} \tag{12}$$

[7] The formula requires that $x^* \in \{\mu - \tau, \mu + \tau]$ for all realizations of μ. This is ensured by assumption (3). E.g. to see that $x^* \geq \mu - \tau$, recall that $\mu \leq \frac{1}{2} + \varepsilon$. Hence, $\mu - \tau \leq \frac{1}{2} + \varepsilon - \tau$. On the other hand $x^* \geq \frac{\frac{d}{2} + 1 - d}{2}$. Now use (3).

When candidate types are e and m, respectively, party L does not campaign and a voter observes $R's$ campaign with probability ϕ_R^{em}. In this case, she votes L if her ideology $x \leq \frac{l^E_{\varnothing,m}+1-d}{2}$. Similarly, with probability $(1-\phi_R^{em})$ she does not observe $R's$ campaign in which case she votes L if $x \leq \frac{1}{2}$. Using (12) we find that the probability that a majority of swing voters votes for party L, and party L therefore wins the election, becomes

$$\pi_{em} = \frac{1}{2} + \phi_R \frac{l^E_{\varnothing,m} - d}{4\varepsilon}. \tag{13}$$

Using (11) we then have

$$\pi_{em} = \frac{1}{2} + \frac{\tau}{\varepsilon}\left(\tilde{v}_{em} - \frac{1}{2}\right). \tag{14}$$

6 Parties' candidate selection

At the candidate selection stage, each party chooses a candidate, whom the median member finds best given the strategy of the other party. As stated above, we focus on the symmetric mixed strategy equilibrium.

Consider party L and let the probability of winning for party L, given types tt', be denoted $\pi_{tt'}$. Given that party R chooses an extremist candidate with probability σ_R, the expected utility for the median member of party L, Mr. $d/2$, from choosing a moderate candidate, d, is

$$\sigma_R\left(-\pi_{me}\left|d-\frac{d}{2}\right| - (1-\pi_{me})\left|1-\frac{d}{2}-\frac{d}{2}\right|\right)$$
$$+(1-\sigma_R)\left(-\pi_{mm}\left|d-\frac{d}{2}\right| - (1-\pi_{mm})\left|1-d-\frac{d}{2}\right|\right),$$

which reduces to

$$-\sigma_R\left(\pi_{me}\frac{d}{2} + (1-\pi_{me})(1-d)\right) - (1-\sigma_R)\left(\pi_{mm}\frac{d}{2} + (1-\pi_{mm})\left(1-\frac{3d}{2}\right)\right).$$

If party L chooses an extremist candidate, the expected utility for the median member becomes

$$-\sigma_R(1-\pi_{ee})(1-d) - (1-\sigma_R)(1-\pi_{em})\left(1-\frac{3d}{2}\right).$$

In a mixed strategy equilibrium, the median member is indifferent between choosing a moderate and an extremist candidate, which gives

$$\sigma_R = \frac{(2-3d)\,\pi_{em} - (2-4d)\,\pi_{mm}}{(2-3d)(\pi_{em}+\pi_{me}) - (2-4d)(\pi_{mm}+\pi_{ee})}$$

Using that in a symmetric equilibrium $\pi_{me} = 1 - \pi_{em}$ and $\pi_{mm} = \pi_{ee} = \frac{1}{2}$, we get

$$\sigma_R = \frac{1}{2} + \left(\frac{2}{d} - 3\right)\left(\pi_{em} - \frac{1}{2}\right). \tag{15}$$

Inserting (14) into (15) we obtain that in a symmetric equilibrium, there is the following relation between the probability for choosing an extremist candidate, σ_R, and the expected vote share, $\tilde{v}_{em}$, among the swing voters

$$\sigma_R(\tilde{v}_{em}) = \frac{1}{2} - \frac{\tau}{\varepsilon}\left(\frac{2}{d} - 3\right)\left(\frac{1}{2} - \tilde{v}_{em}\right). \tag{16}$$

Notice that $2/d - 3 > 0$ as $d < 1/2$. In a symmetric equilibrium, the parties put equal weight on an extremist candidate, so $\sigma_R = \sigma_L$. We therefore see that the higher is $\tilde{v}_{em}$ - the expected vote share among swing voters to a party with an extremist candidate when the other party has chosen a moderate candidate - the more parties put weight on extremist candidates. This is very intuitive. An extremist candidate is ideologically better for the decisive median member of a party. The cost of choosing such a candidate is that the expected vote share is low (recall that $\tilde{v}_{em} < 1/2$). The smaller is this cost, the more a party tends to choose an extremist candidate.

7 The effect of the public funding system

In this section we focus on the effect of the public funding system. Party $L's$ expected votes consist of votes from the left partisans and the fraction $\tilde{v}_{tt'}$ of the swing voters. The total vote for party L is therefore $v_{tt'} = (1-\eta)/2 + \eta\tilde{v}_{tt'}$ and the expected public funding to party L is

$$\Psi\left(\frac{1-\eta}{2} + \eta\tilde{v}_{tt'}\right) = \psi_0 + \psi_1 \cdot \left(\frac{1-\eta}{2} + \eta\tilde{v}_{tt'}\right).$$

Inserting (7) into (11) gives

$$\tilde{v}_{em} = \frac{1}{2} - \frac{d}{8\tau}\phi_R^{em} \cdot \frac{\sigma_L \phi_R^{em}}{\sigma_L \phi_R^{em} + (1-\sigma_L)(1-\phi^{mm})\phi^{mm}}. \tag{17}$$

When the expected vote share to party L is $\tilde{v}_{em}$, party $R's$ expected vote share is $1 - \tilde{v}_{em}$ and we get

$$\phi_R^{em} = \frac{\psi_0 + \psi_1 \cdot \left(\frac{1-\eta}{2} + \eta(1-\tilde{v}_{em})\right)}{\psi_0 + \psi_1 \cdot \left(\frac{1-\eta}{2} + \eta(1-\tilde{v}_{em})\right) + \alpha}. \tag{18}$$

In a symmetric equilibrium the vote shares to the parties are 1/2 each when they both choose a moderate or they both choose an extremist candidate. In order to determine the equilibrium, we therefore just need to determine the vote share to party L when its candidate is extremist and $R's$ is moderate. (By symmetry we then also have the vote share to L when its candidate is moderate and $R's$ is extremist). Inserting into (17) we obtain that in a symmetric equilibrium, $\tilde{v}_{em}$ is determined as the solution to the equation

$$\begin{aligned}\tilde{v}_{em} &= \frac{1}{2} - \frac{d}{8\tau}\left(\frac{\psi_0 + \psi_1 \cdot \left(\frac{1-\eta}{2} + \eta\left(1 - \tilde{v}_{em}\right)\right)}{\psi_0 + \psi_1 \cdot \left(\frac{1-\eta}{2} + \eta\left(1 - \tilde{v}_{em}\right)\right) + \alpha}\right) \cdot \qquad (19)\\ &\left(\frac{\sigma_L\left(\frac{\psi_0+\psi_1\cdot\left(\frac{1-\eta}{2}+\eta(1-\tilde{v}_{em})\right)}{\psi_0+\psi_1\cdot\left(\frac{1-\eta}{2}+\eta(1-\tilde{v}_{em})\right)+\alpha}\right)}{\sigma_L\left(\frac{\psi_0+\psi_1\cdot\left(\frac{1-\eta}{2}+\eta(1-\tilde{v}_{em})\right)}{\psi_0+\psi_1\cdot\left(\frac{1-\eta}{2}+\eta(1-\tilde{v}_{em})\right)+\alpha}\right) + \left(1 - \sigma_L\right)\left(1 - \phi^{mm}\right)\phi^{mm}}\right).\end{aligned}$$

First we consider the case where public funding does not depend on the vote share. We then have

Proposition 1 *Assume that public funding is given independent of the vote share so that $\psi_1 = 0$. Consider a change in the public funding system, so that more funding is given, i.e. ψ_0 increases. This change moderates the parties choice of candidates: the probability that an extremist candidate is chosen, σ_L $=\sigma_R$ decreases*

Proof. See the Appendix.

When there is more public funding - or in fact any kind of funding, which is independent of the vote share - parties become more moderate in their choice of candidates. The reason is that a party with a moderate candidate facing an extremist opponent gets an advantage. With more funding at hand, the party can inform a larger share of the electorate, that it has in fact a moderate candidate. This increases the party's vote share and makes it more attractive to choose a moderate candidate. In equilibrium therefore, parties behave more moderately. This effect is not peculiar to public funding, it follows from the fact that a larger fraction of the electorate becomes informed, when more campaign finance is at hand. It is also present in Coate's (2004a) analysis.

In order to investigate the effect stemming from the fact that public funding depends on vote shares and are not given lump sum, we now consider a change of the public funding system, so that it becomes more dependent on the vote share, while the amount of money given out is constant. Both parties receive ψ_0 and the total number of votes is normalized to one, so the total amount given out by the funding system is

$$\Psi_T = 2\psi_0 + \psi_1 \cdot 1.$$

A change of the funding system $d\psi_0, d\psi_1$ fulfilling $d\psi_1 > 0$ and $2d\psi_0 + d\psi_1 = 0$, puts more weight on the vote share and does not change the amount of total funds given out - so the effect of more funding, investigated in Proposition 1 is neutralized.

Proposition 2 *The more public funding depends on vote shares, the more moderate is the parties' choice of candidates: Consider a change in the public funding system, so that the total amount of public funding is constant but the share of funding that depends on the vote share increases, i.e. a change* $d\psi_1 > 0$ *such that* $2d\psi_0 + d\psi_1 = 0$*. This change moderates the parties choice of candidates; the probability that an extremist candidate is chosen,* $\sigma_L = \sigma_R$*, decreases.*

Proof. See the Appendix.

Proposition 2 shows that the way public funding is given is central for how it works. When funding depends more on vote shares, the parties get an extra incentive to moderate the candidate choice. The reason is that the parties realize that the choice of candidate now has funding consequences and they take this into account. With vote dependent public funding, the gains from choosing a moderate over an extremist candidate are higher. A moderate candidate will get a higher expected vote share, and therefore obtains a higher level of expected public funding. This expected public funding is used for raising money on the credit market which is used for advertising the moderate candidate, increasing his expected vote share even more. There is a *campaign multiplier effect*. Hence, from the point of view of the parties the trade off governing the choice of candidate has been twisted. A more moderate candidate, whom the party members dislike, since he is too moderate in their eyes, becomes more attractive, since he is able to raise more money to the party and thus has a larger chance of winning. Both parties therefore, in equilibrium, moderate their choice of candidate: they put more weight on the moderate candidate.

As is clear from the proof given in Appendix, assumption (6) is a sufficient and far from necessary condition, ensuring that the Proposition holds. The important feature is that the funding effects of policy moderation do not become too large, if they do the comparative statics are not sensible. In this case more money to the party with a moderate candidate would in fact give it a lower vote share. If funding responds too much to the vote share the multiplier-effect becomes excessively strong, in which case the model is not well-behaved[8].

Voters have single peaked partwise linear utility functions, see (1). In equilibrium, each party wins with probability one half and each party's choice of candidate is random, so from the point of view of the voters, the election is a lottery. When public funding depends more on the vote share, parties put more weight on moderate candidates, which strictly increases the expected utility of swing voters. For partisans with ideologies more moderate than the median partisan ($x > d/2$ in party L and $x < 1 - d/2$ in party R) the same logic applies. For partisans with ideologies more extreme than the median partisan, utility is not affected by parties choosing more moderate policies. They dislike that their own party chooses the moderate candidate, but they like that the other party chooses its moderate candidate. A change in the equilibrium such that both parties increase the weight on the moderate candidate with the same amount leave their expected utility unchanged. This feature is due to the linearity of the utility function. Had utility been concave, the positive effect from the other party choosing a more moderate candidate would have dominated the negative effect from their own party choosing a more moderate candidate, so that the expected utility would increase. Summing up this discussion

Proposition 3 *Any change in the public funding system which leads parties to moderate their choice of candidates, i.e.* $\sigma_L = \sigma_R$ *decreases, is weakly Pareto improving: Such a change strictly increases the expected utility of all voters whose ideology is more moderate than the median partisans, i.e.* $x \in (d/2, 1 - d/2)$. *It does not affect the expected utility of voters who are more extreme than the median partisans, i.e. voters with* $x \in [0, d/2] \cup [1-d/2, 1]$.

[8] To get some intuition for how this could happen think of a 45 degree Keynesian model where the marginal propensity to consume exceeds one. In such a model a change in the autonomous demand would in fact decrease GDP, since consumption "reacts to much" to changes in GDP. The same phenomenon occurs in our model when public funding "reacts too much" to vote shares.

Proposition 3 does not take into account the cost of public funds. In so far as the change in the funding system involves more funds, there is of course a cost which has to be taken into account. To account for this effect one could include a term $-\gamma\left(\Psi_T\right)^2$ in the utility function. Clearly, an increase in Ψ_T will then only be Pareto improving for voters in $x \in (d/2, 1-d/2)$ if γ is sufficiently small. For voters $x \in [0, d/2] \ \cup \ [1-d/2, 1]$, any increase in Ψ_T would decrease their utility. This comment does, however, not affect changes in the public funding system such that the total amount of funding is constant, such as those considered in Proposition 2. For the sake of completeness we state

Corollary 4 *A change in the public funding system so that the total funding remains constant but depends more on vote shares (as in Proposition 2) is weakly Pareto improving.*

8 The asymmetric case

In the discussion of the public funding systems it has been argued that funding related to vote shares is unfair because it benefits large parties and puts smaller parties at a disadvantage. If the playing field is not level, public funding may tilt it even more (see for instance the Electoral Reform Green Paper of the Commonwealth of Australia, Commonwealth of Australia, 2008, pp 33). It is therefore important to consider the case where the parties are asymmetric. This complicates matters substantially so analytic results are not available, and we rely on a number of simulations [9].

There are several ways in which parties can be asymmetric in our model. One party may be advantaged by having a larger group of partisans or it may have access to a larger lump sum of money (perhaps from private funds). The partisans of one party may be more extremist than those of the other party. It may also be that it has an advantage in informing voters. This may be because it has better access to media, media may be slanted, or because it has a better party organization. In short, it may have a more efficient information technology.

We have considered all three cases, and qualitatively the results are similar. Here we focus on the case where one party has a more efficient information technology than the other party has. Recall that this technology

[9] The Mathematica notebooks used for the simulations are avaliable at request.

is given by equation (4) and it is more efficient the lower is α. We now assume that the parties have different $\alpha' s$ and in particular that party L is advantaged so that $\alpha_L < \alpha_R$. We have considered a series of numerical examples, which fulfill assumptions (2), (3) and (6), and we present one here, which illustrates our main findings. The total amount of public funds is 0.8, other parameters of the model are $d = 0.1;\ \varepsilon = 0.05;\ \tau = 0.2; \eta = 0.5$. As in Proposition 2, we focus on the effect of changes in ψ_1 keeping the total amount of public funding constant: We let ψ_1 increase from 0 to 0.8, simultaneously adjusting the lump sum part ψ_0, so that total public funding is constant. Notice that when $\psi_1 = 0$, all public funds are lump sum, while when $\psi_1 = 0.8$ they are entirely a function of vote shares. We consider different degrees of asymmetry in the information technology: In particular, we fix $\alpha_R = 1$, and we let α_L vary from 0.2, corresponding to a case where party L has a very large informational advantage, to $\alpha_L = 1$, where the situation is symmetric. To illustrate, if the public funds are divided evenly and $\alpha_L = 0.2$, then party L is able to inform 58% of the voters while party R is only able to inform 26%.

Table 1: Asymmetry in the Information Technology

	ψ_1	**0**	**0.266667**	**0.533333**	**0.8**
$\alpha_L = 0.2$	σ_L	0.0154	0.0148	0.0142	0.0135
	σ_R	0.293	0.295	0.296	0.298
	Prob. ext. wins	0.1304	0.1306	0.1307	0.1308
	Prob. L wins	0.5664	0.5668	0.5673	0.5679
	Welfare	-0.429076	-0.429081	-0.429085	-0.429090
α_L=0.5	σ_L	0.064	0.063	0.062	0.061
	σ_R	0.272	0.273	0.275	0.276
	Prob. ext. wins	0.1523	0.1521	0.1519	0.1517
	Prob. L wins	0.5489	0.5496	0.5501	0.5508
	Welfare	-0.429758	-0.429753	-0.429748	-0.429743
α_L=0.8	σ_L	0.133	0.132	0.131	0.130
	σ_R	0.2331	0.2338	0.2346	0.2353
	Prob. ext. wins	0.1749	0.1747	0.1744	0.1742
	Prob. L wins	0.5235	0.5239	0.5244	0.5248
	Welfare	-0.430467	-0.430459	-0.430451	-0.430443
α_L=1	σ_L	0.1940	0.1937	0.1935	0.1933
	σ_R	0.1940	0.1937	0.1935	0.1933
	Prob. ext. wins	0.1884	0.1881	0.1879	0.1877
	Prob. L wins	0.5	0.5	0.5	0.5
	Welfare	-0.430886	-0.430879	-0.430871	-0.430864

Total amount of public funds= 0.8, $d = 0.1$; $\varepsilon = 0.05$; $\tau = 0.2; \eta = 0.5$.

In Table 1 we present the results for four different values of α_L. In each case, the table contains each party's probability of choosing an extremist candidate, σ_L and σ_R, the probability that an extremist candidate wins, the probability of winning for party L, and total welfare for different values of ψ_1.

As can be seen from Table 1, a higher ψ_1 increases the probability of victory for party L, when it has an informational advantage, i.e. when $\alpha_L < 1$. Party L reacts choosing an extremist candidate with a lower probability σ_L, i.e. by moderating its policy. The reason is twofold; when ψ_1 increases party L receives more funds (in absolute as well as relative terms) making it possible for the party to inform even more voters. The high level of information among voters implies that a more moderate policy gives a larger vote response, and this makes a moderate policy more attractive. Furthermore, voters understand that the party receives large funds. If, therefore, a voter is not informed about the party's policy, she rationally updates her beliefs and puts relatively much weight on the probability that the party has an extremist policy, making it less likely that she is going to vote for party L. This response by the uninformed voters in itself makes an extremist policy less attractive and in the end party L becomes very moderate and its probability of victory increases.

On the other hand, the disadvantaged party R responds by choosing a more extremist strategy. Its relatively poor information technology implies that, for the same amount of campaign funds, it can inform fewer voters than party L. Hence, the vote response from a more moderate policy is smaller for party R than for party L. This makes a moderate policy less attractive for party R. Furthermore, since voters realize that party R cannot inform so many voters, they do not change beliefs very much when they are not informed by party R. Hence, the belief response to an extremist policy (where a party does not campaign) does not hurt party R so much. In the end, the higher ψ_1, the less funds party R receives and its response is to choose a more extremist policy.

Hence, our results point out that if the playing field is not level, so that one party has an advantage, the public funding system amplifies this asymmetry: it increases the probability of victory of the advantaged party.

Notice that from an ideological point of view the parties are symmetric. Still in equilibrium, the informational asymmetry implies that one party chooses a very moderate strategy and wins the election with a high probability, while the disadvantaged party reacts by choosing an extremist strategy and wins with a low probability. Hence, the difference in moderation is not explained by differences in the moderation of parties' ideologies, but entirely by the asymmetry in the information technology. The more public funding depends on vote shares the more pronounced this phenomenon becomes.

Our results support the often made claim that public funding benefits the advantaged party and this may be seen as unfair. However, from a welfare perspective the relevant question is not whether a party is more likely to win or not but whether welfare is positively or negatively affected. From Proposition 2 we know that in the symmetric case, a higher ψ_1 is welfare improving since both parties react by moderating their policies. In the asymmetric case, the moderation of the large party L is beneficial for total welfare, on the other hand the extremism of the smaller party R is not. The net effect is the sum of the two. As is clear from Table 1, the former effect dominates for values α_L equal to $0,5$ and above, i.e. as long as the information technologies of the two parties are not "too asymmetric". In fact numerous simulations, not reported here, show that the former effect dominates for α_L above $0,35$. In this case, a higher ψ_1 improves total welfare even though the advantaged party L benefits at the expense of party R. However, for small α_L, such as $\alpha_L = 0.2$, i.e. when the informational asymmetry is large, total welfare decreases. As can be seen in Table 1, in this case party L is already very moderate for low values of ψ_1 and it is hardly possible to further moderate its policy. Increasing ψ_1 transfers resources from party R to party L, but this does not make L much more moderate. On the other hand, party R reacts by becoming more extremist and this effect now dominates. In fact, we see that the probability that a party (be it L or R) wins with an extremist proposal increases. This makes for the overall negative total welfare effect.

Still, when the parties are not too different increasing the part of public funding which depends on vote shares improves welfare, even though it benefits the large and advantaged party at the expense at the smaller and disadvantaged party.

9 Concluding Remarks

We have considered public funding of political parties, where public funds depend on the vote share a party receives in the election. For symmetric parties, it was shown that this funding of campaign finance has the attractive feature that it leads to policy convergence. The convergence is increased when the funding depends on vote share compared to a situation when it is given in a lump sum fashion. Such a change induces a weak Pareto improvement, it strictly increases utility of all non-extreme votes, while voters at the extremes are unaffected by the change.

For asymmetric parties, the large party gains by increasing the public funding's dependence on vote shares and it moderates its policy. The smaller party looses and reacts by choosing a more extremist policy. As long as the asymmetry is not too large, this is welfare improving as the moderation of the large party and the greater likelihood it wins the election outweighs the extremism of the smaller party. However, when the parties are very asymmetric, the large party is already very moderate and a further moderation has to be marginal. Therefore the increased extremism of the smaller party dominates in total welfare and further dependence on vote shares may be detrimental to welfare.

In the paper it was assumed that the parties have access to a perfect credit market, so that they can spend the public funds before the election takes place. An interesting extension of the model would be to a case, where the credit market, albeit important, does not function as well as in the present paper. In this case parties would to some extent have to rely on funds received from the previous election's vote share. This would create a dynamic link between elections, where a party which performs well in an election will be in an advantageous position in future elections. In this way public funding may induce cycles in the support for parties. The investigation of this link will be the subject of further research.

10 Appendix

Proof of Proposition 1.

When public funding does not depend on vote shares, $\Psi = \psi_0$, and it follows that $\phi_R^{em} = \phi^{mm} = \phi$. Furthermore

$$\phi = \frac{\psi_0}{\psi_0 + \alpha}. \tag{20}$$

Hence (17) reduces to

$$\tilde{v}_{em} = \frac{1}{2} - \frac{d}{8\tau}\phi\left(\frac{\sigma_L}{\sigma_L + (1-\sigma_L)(1-\phi)}\right). \tag{21}$$

From (16)

$$\sigma_R = \frac{1}{2} - \frac{\tau}{\varepsilon}\left(\frac{2}{d} - 3\right)\left(\frac{1}{2} - \tilde{v}_{em}\right),$$

which gives

$$\tilde{v}_{em} = \frac{1}{\frac{\tau}{\varepsilon}\left(\frac{2}{d} - 3\right)}\left(\sigma_L - \frac{1}{2}\right) + \frac{1}{2},$$

so that (21) becomes

$$\frac{1}{\frac{\tau}{\varepsilon}\left(\frac{2}{d} - 3\right)}\left(\sigma_L - \frac{1}{2}\right) + \frac{1}{2} = \frac{1}{2} - \frac{d}{8\tau}\phi\left(\frac{\sigma_L\phi}{\sigma_L\phi + (1-\sigma_L)(1-\phi)\phi}\right),$$

giving

$$\left(\sigma_L - \frac{1}{2}\right) = b\phi\left(\frac{\sigma_L}{\sigma_L + (1-\sigma_L)(1-\phi)}\right),$$

where

$$b = -\frac{d}{8\tau\frac{1}{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)}} = \frac{1}{8}\frac{3d-2}{\varepsilon} < 0.$$

Solving for ϕ gives

$$\phi = \frac{2\sigma_L - 1}{3\sigma_L + 2b\sigma_L - 2\sigma_L^2 - 1}.$$

Hence, we have that σ_L is decreasing in ϕ if the right hand side is decreasing in σ_L. Differentiating

$$\frac{\partial\left(\frac{2\sigma_L-1}{3\sigma_L+2b\sigma_L-2\sigma_L^2-1}\right)}{\partial\sigma_L} = \frac{4\sigma_L^2 - 4\sigma_L + 2b + 1}{\left(3\sigma_L + 2b\sigma_L - 2\sigma_L^2 - 1\right)^2},$$

which is less than zero if

$$4\sigma_L^2 - 4\sigma_L + 2b + 1 < 0,$$

this if fulfilled if

$$2b + 1 = 2\frac{1}{8}\frac{3d-2}{\varepsilon} + 1 < 0,$$

$$\varepsilon < \frac{1}{2} - \frac{3}{4}d.$$

which is fulfilled under our maintained assumption $\varepsilon \leq 1/2 - \mu - d$.

It therefore follows that σ_L is decreasing in ϕ. Since ϕ is increasing in ψ_0 (cf (20) it follows that σ_L is decreasing in ψ_0.This proves the Proposition. □

Proof of Proposition 2

The assumption that total public funding is constant, $2d\psi_0 + d\psi_1 = 0$, implies

$$\frac{d\psi_0}{d\psi_1} = -\frac{1}{2}. \tag{22}$$

Notice that as $\phi^{mm} = \frac{\psi_0+\psi_1\cdot\left(\frac{1}{2}\right)}{\psi_0+\psi_1\cdot\left(\frac{1}{2}\right)+\alpha}$

$$\frac{d\phi^{mm}}{d\psi_1} = 0.$$

Let

$$A = \left(\frac{\phi_R^{em}}{\phi_R^{em} + \frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}}\right),$$

then (17) can be written

$$\tilde{v}_{em} = \frac{1}{2} - \frac{d}{8\tau}\phi_R^{em} A,$$

so

$$\frac{d\tilde{v}_{em}}{d\psi_1} = -\frac{d}{8\tau}\left(\frac{d\phi_R^{em}}{d\psi_1}A + \phi_R^{em}\frac{dA}{d\psi_1}\right). \tag{23}$$

Using (18) and (22) we have

$$\frac{d\phi_R^{em}}{d\psi_1} = \phi\prime \cdot \left(-\frac{1}{2} + \left(\frac{1-\eta}{2} + \eta\left(1-\tilde{v}_{em}\right)\right) - \psi_1\eta\frac{d\tilde{v}_{em}}{d\psi_1}\right), \tag{24}$$

where (cf. (4)), $\phi\prime > 0$.

In a symmetric equilibrium, $\sigma_L = \sigma_R$. From (16), we therefore have

$$\frac{d\sigma_L}{d\tilde{v}_{em}} = \frac{d\sigma_R}{d\tilde{v}_{em}} = \frac{\tau}{\varepsilon}\left(\frac{2}{d} - 3\right).$$

Using this and (16), we get

$$\begin{aligned}\frac{d\left(\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)}{dv_{em}} &= \frac{d\left(\left(\frac{1}{\frac{1}{2}-\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(\frac{1}{2}-\tilde{v}_{em}\right)} - 1\right)\left(1-\phi^{mm}\right)\phi^{mm}\right)}{dv_{em}} \\ &= -\frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\left(\frac{1}{2}-\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(\frac{1}{2}-\tilde{v}_{em}\right)\right)^2},\end{aligned}$$

and thus

$$\begin{aligned}\frac{dA}{d\psi_1} &= \\ &= \frac{\frac{d\phi_R^{em}}{d\psi_1}\left(\phi_R^{em} + \frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right) - \phi_R^{em}\left(\frac{d\phi_R^{em}}{d\psi_1} - \frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\left(\frac{1}{2}-\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(\frac{1}{2}-\tilde{v}_{em}\right)\right)^2}\frac{d\tilde{v}_{em}}{d\psi_1}\right)}{\left(\phi_R^{em} + \frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)^2} \\ &= \frac{\frac{d\phi_R^{em}}{d\psi_1}\left(\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right) + \phi_R^{em}\left(\frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\sigma_L^2}\frac{d\tilde{v}_{em}}{d\psi_1}\right)}{\left(\phi_R^{em} + \frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)^2}.\end{aligned}$$

Inserting into (23) gives

$$\frac{d\tilde{v}_{em}}{d\psi_1} = -\frac{d}{8\tau}\cdot\left(\begin{array}{c}\frac{d\phi_R^{em}}{d\psi_1}\left(\frac{\phi_R^{em}}{\phi_R^{em}+\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}}\right) \\ +\phi_R^{em}\frac{\frac{d\phi_R^{em}}{d\psi_1}\left(\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)+\phi_R^{em}\left(\frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\sigma_L^2}\frac{d\tilde{v}_{em}}{d\psi_1}\right)}{\left(\phi_R^{em}+\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)^2}.\end{array}\right)$$

Collecting terms

$$\frac{d\tilde{v}_{em}}{d\psi_1} = -\frac{d}{8\tau}\frac{\frac{d\phi_R^{em}}{d\psi_1}\phi_R^{em}\left(\phi_R^{em} + 2\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right) + \left(\phi_R^{em}\right)^2\left(\frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\sigma_L^2}\frac{d\tilde{v}_{em}}{d\psi_1}\right)}{\left(\phi_R^{em} + \frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)^2}.$$

Using (24) gives

$$\begin{aligned}\frac{d\tilde{v}_{em}}{d\psi_1} &= -\frac{d}{8\tau}\cdot \\ &\frac{\left(\begin{array}{c}\left(\phi\prime\cdot\left(-\frac{1}{2}+\left(\frac{1-\eta}{2}+\eta\left(1-\tilde{v}_{em}\right)\right)-\psi_1\eta\frac{d\tilde{v}_{em}}{d\psi_1}\right)\right)\phi_R^{em}\left(\phi_R^{em}+2\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right) \\ +\left(\phi_R^{em}\right)^2\left(\frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\sigma_L^2}\frac{d\tilde{v}_{em}}{d\psi_1}\right)\end{array}\right)}{\left(\phi_R^{em}+\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)^2}\end{aligned}$$

Solving for $\frac{d\tilde{v}_{em}}{d\psi_1}$ gives

$$\begin{aligned}&\frac{d\tilde{v}_{em}}{d\psi_1} \\ =\ &\frac{-\left(\left(\phi\prime\cdot\left(-\frac{1}{2}+\left(\frac{1-\eta}{2}+\eta\left(1-\tilde{v}_{em}\right)\right)\right)\right)\phi_R^{em}\left(\phi_R^{em}+2\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)\right)}{\left(\begin{array}{c}\left(\phi_R^{em}+\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)^2\frac{8\tau}{d}-\phi\prime\psi_1\eta\phi_R^{em}\left(\phi_R^{em}+2\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right) \\ +\phi_R^{em}\phi_R^{em}\left(\frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\sigma_L^2}\right)\end{array}\right)}.\end{aligned}$$

The numerator is negative as $\tilde{v}_{em} < 1/2$, hence the sign of $\frac{d\tilde{v}_{em}}{d\psi_1}$ is opposite of the sign of the denominator.

Now the denominator is *larger than*

$$\begin{aligned}&\left(\phi_R^{em}+\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)^2\frac{8\tau}{d}-\phi\prime\psi_1\eta\phi_R^{em}\left(\phi_R^{em}+\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right) \\ &+\phi_R^{em}\phi_R^{em}\left(\frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\sigma_L^2}\right),\end{aligned}$$

which we can rewrite

$$\begin{aligned}&\left(\left(\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right)\frac{8\tau}{d}+\left(\frac{8\tau}{d}-\phi\prime\psi_1\eta\right)\phi_R^{em}\right)\left(\phi_R^{em}+\frac{1-\sigma_L}{\sigma_L}\left(1-\phi^{mm}\right)\phi^{mm}\right) \\ &+\phi_R^{em}\phi_R^{em}\left(\frac{\frac{\tau}{\varepsilon}\left(\frac{2}{d}-3\right)\left(1-\phi^{mm}\right)\phi^{mm}}{\sigma_L^2}\right). \qquad (25)\end{aligned}$$

All terms are positive if $\frac{8\tau}{d}-\phi\prime\psi_1\eta > 0$. From (4) $\phi\prime = \frac{\alpha}{(c+\alpha)^2} > 0$ and $\phi\prime\prime < 0$. Hence $\phi\prime < \frac{\alpha}{(0+\alpha)^2} = \frac{1}{\alpha}$ and $\frac{8\tau}{d}-\phi\prime\psi_1\eta > \frac{8\tau}{d}-\frac{\psi_1\eta}{\alpha}$

Using assumption (6) we have that

$$\frac{8\tau}{d} - \frac{\psi_1\eta}{\alpha} > \frac{8\tau}{d} - 2.$$

Using (3), $\tau \geq \varepsilon + d/4$, and the fact that $d < 1/2$,

$$\frac{8\tau}{d} > \frac{8\left(\varepsilon + d/4\right)}{d} > 2.$$

This implies that (25) is positive and therefore that $\frac{d\tilde{v}_{em}}{d\psi_1} < 0$.

From (16) we then get that $\frac{d\sigma_R}{d\psi_1} < 0$. This proves the Proposition $\square$.

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