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# Strategic Line Drawing between Debt and Equity

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**Abstract**

Corporate tax systems generally maintain a sharp distinction between debt and equity, however, the advent of hybrid instruments has transformed the universe of financial instruments into a debt-equity continuum and tax systems therefore need to draw lines that distinguish the set of debt instruments from the set of equity instruments. When countries draw these lines differently, there is a scope for international tax planning: A multinational firm financing a foreign investment with a hybrid instrument categorized as debt in the host country and equity in the home country combines the benefits of tax deductible interest payments in the host country and tax favored dividend payments in the home country. This paper develops a theoretical model of strategic line drawing between debt and equity in the presence of hybrid instruments. In the absence of international cooperation, lines are generally drawn in a globally suboptimal manner. The inefficiency typically derives from the endeavors of policymakers to draw lines in ways that facilitate hybrid financing by domestic multinational firms and impede hybrid financing by foreign multinational firms with a view to eroding foreign taxation of domestic firms and enforcing domestic taxation of foreign firms.

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# 1 Introduction

"*Although in a given case the bond and the stock features of an instrument seem to be hopelessly interwoven, the courts are called upon to untangle them and decide whether in the last analysis the bond or the stock character of the instrument prevails. For in the eyes of the law there are no 'hybrid securities'. In the law a person is either a creditor or a stockholder; he cannot be both.*" Rudolf E. Uhlman, 1937.

Corporate tax systems generally maintain a sharp distinction between debt and equity. While both debt and equity represent sources of financing from the perspective of the firm, payments to holders of debt, interest payments, are deductible from the corporate tax base whereas payments to holders of equity, dividends, are not. The distinction between debt and equity usually rests on the dichotomy between the shareholder, an active investor with a share in firm profits and losses, and the creditor, a passive investor entitled to a fixed return regardless of the business fortune.[1]

Legal scholars argue, however, that the distinction between debt and equity is one of degree rather than one of principle (Emmerich, 1985). Financial instruments differ in a large number of dimensions and firms frequently issue securities that resemble debt in some dimensions and equity in others. For instance, while pure debt instruments have a fixed maturity and a fixed return and pure equity instruments have no maturity and a return that is linked to firm profits, these characteristics can be combined differently to obtain two *hybrid instruments*: a perpetual loan with a fixed return and no maturity, and a profit sharing loan with a fixed maturity and a return that is linked to firm profits. Moreover, financial instruments may in any single dimension have characteristics that obscure the debt-equity distinction. For instance, a fixed maturity of 100 years is arguably an equity-like characteristic since it is economically almost equivalent to no maturity, however, it is less clear whether maturities of 10, 30 or 50 years should be considered more debt-like or equity-like characteristics. Rather than just two distinct financial instruments, debt and equity, the universe of financial instruments thus comprises a myriad of hybrid instruments that combine characteristics of standard debt and equity in different proportions and the legal literature therefore often refers to the universe of financial instruments as a debt-equity continuum (e.g. Hariton, 1994; Krahmal, 2005).

In the presence of hybrid instruments, any tax system that treats debt and equity differently needs *demarcation rules* that distinguish the set of debt instruments from the set of equity instruments. In the U.S., a long list of characteristics are considered when categorizing financial instruments as debt or equity for tax purposes, for instance *maturity* - whether the principal is reimbursed either at a fixed maturity or on demand; *seniority* - whether the claims of the holder are subordinate to the rights of general creditors

[1] The following is a typical formulation of the debt-equity dichotomy from classical case-law: "The essential difference between a stockholder and a creditor is that the stockholder's intention is to embark upon the corporate adventure, taking the risks of loss attendant upon it, so that he may enjoy the chances of profit. The creditor, on the other hand, does not intend to take such risks so far as they may be avoided, but merely to lend his capital to others who do intend to take them." (United States v. Title Guarantee & Trust Co., 123 F.2d 990, 993, 6th Cir. 1943; as cited in Hariton, 1994)

in the case of bankruptcy; *management* - whether the holder has voting rights; *return* - whether the return represents a legally enforceable claim and *label* - whether the instrument is labeled debt or equity.[2] Not surprisingly, however, demarcation rules vary considerably across countries. A recent comparative study of demarcation rules in the U.S., Canada, France and the Netherlands document material differences both in the legal principles underlying categorization of financial instruments for tax purposes and in the typical tax treatment of a number of specific hybrid financial instruments (Connors and Woll, 2001)

The variation in demarcation rules across countries introduces the possibility that the same financial instrument is categorized as debt in one country and equity in another country, which represents an important tax planning opportunity for multinational firms. To see this, consider a multinational firm investing in a foreign subsidiary. If the investment is financed with a hybrid instrument that is successfully categorized as debt in the host country and as equity in the home country, payments on the instrument are treated as tax deductible interest expenses at the level of the subsidiary and as tax favored dividends at the level of the parent company. The net result is a considerable tax saving compared to an investment in the form of pure debt or pure equity. In the former case, payments are consistently treated as interest and thus deductible at the level of the subsidiary but taxable at the level of the parent company. In the latter case, payments are consistently treated as dividends and thus tax favored at the level of the parent company but non-deductible at the level of the subsidiary.[3] Perpetual loans, profit sharing loans and convertible loans are examples of hybrid instruments that are often used in international tax planning because of the fact that they are typically treated as equity in the U.S. and as debt in many other countries (Krahmal, 2005).[4]

While there exists no quantitative evidence on the use of hybrid instruments in international tax planning, there are other types of evidence indicating that this is a matter of large empirical relevance: Some legal scholars provide detailed accounts of the important cross-country differences in demarcation rules and the specific tax planning opportunities created by these differences (Connors and Woll, 2001; Krahmal, 2005). Others have emphasized the impotence of general anti-abuse rules, which derives from the fact that hybrid instruments, by exploiting cross-country differences in tax rules, are not abusive under any single set of tax rules (Rosenbloom, 1999). Legal analysis thus demonstrates that hybrid instruments represent an effective and relatively low-risk tax planning device. Moreover, policymakers have shown an awakening interest in hybrid structures. For instance, the Internal Revenue Service recently announced that cross-border hybrid instruments were to be among its highest compliance priorities, which suggests that the adverse impact on collected corporate tax revenues is judged to be substantial (Internal Revenue Service, 2007). Finally, there is econometric evidence that recent declines in the effective tax burden on

[2] IRS guidelines in Notice 94/47

[3] A very related issue is that of hybrid entities, that is corporate entities, which for tax purposes are considered transparent by some countries and non-transparent by other countries.

[4] Hybrid financial instruments can also serve other purposes than international tax planning. By issuing securities that are categorized as debt for tax purposes and equity for financial reporting purposes, firms combine the tax advantages of debt financing with the advantages of equity financing in terms of favorable credit ratings (Engel, Erickson and Maydew, 1999). Similarly, banks and other financial institutions facing binding capital requirements may benefit from issuing securities that are categorized as debt for tax purposes and equity for regulatory purposes (Gergen and Schmitz, 1997).

foreign affiliates of US multinationals are due to more aggressive use of international tax planning rather than reductions in statutory tax rates (Altshuler and Grubert, 2005).[5] This empirical pattern is clearly consistent with widespread and increasing use of hybrid instruments for tax planning purposes.

All these considerations suggest that line drawing between debt and equity is associated with potentially important cross-border spill-over effects. For instance, if a country $i$ adopts a demarcation rule that categorizes almost all financial instruments as equity, it becomes relatively easy for firms in country $i$ to avoid taxation of foreign investments by means of hybrid instruments categorized as debt in the host country and equity in country $i$. Conversely, it becomes relatively difficult for foreign firms investing in country $i$ to obtain a similar tax advantage by means of hybrid instruments categorized as debt in country $i$ and equity in the home country. While the example illustrates that line drawing between debt and equity has consequences for government revenue and firm profits in foreign countries, it also points to a possible scope for *strategic line drawing*: Countries may draw lines between debt and equity with a view to influencing the financial decisions of domestic and foreign firms in ways that increase domestic welfare at the expense of foreign welfare.

In order to explore the implications of this hypothesis, we develop a model of line drawing between debt and equity in an international taxation setting. At the heart of the paper is a simple one-dimensional model of hybrid instruments and their classification for tax purposes. We assume that financial instruments are characterized by a value $z$ that corresponds to a location on the debt-equity continuum where lower values of $z$ reflect more debt-like characteristics and higher values of $z$ reflect more equity-like characteristics. Government choose the threshold value $\alpha$ that delineates debt and equity for tax purposes, however, to account for the judicial uncertainty often emphasized by legal scholars, we assume that tax assessments have a stochastic element such that financial instruments that are close enough to $\alpha$ face *ex ante* uncertainty about the classification for tax purposes.

We study the interaction between two countries, each of which is inhabited by a fixed number of firms endowed with a profitable investment project in the other country. Firms optimally decide whether to finance the foreign investment with standard financial instruments or with a hybrid financial instrument. In the latter case, firms choose the hybrid instrument that maximizes the probability of equity treatment in the home country and debt treatment in the host country. Clearly, the probability of obtaining the desired hybrid treatment is larger the higher the $\alpha$ of the host country since this facilitates debt treatment at the level of the borrowing subsidiary and the lower the $\alpha$ of the home country since this facilitates equity treatment at the level of the lending parent company.

In the spirit of much of the literature on international taxation, we compare the *optimal policies* prevailing under international cooperation to the *equilibrium policies* prevailing under non-cooperative policy making. In the cooperative setting, policies aim to provide protection against the use of hybrid instruments by firms in both countries, however, these objectives are conflicting since a higher value of $\alpha$ in one country, which makes hybrid instruments less attractive for firms in this country, at the

[5] Altshuler and Grubert (2005) explicitly mention hybrid structures as one of the tax planning tools most likely to be responsible for the decreasing effective tax rates but no hard evidence is presented in favor of this hypothesis.

same time makes hybrid instruments more attractive for firms in the other country. When countries are asymmetric in terms of the number of firms involved in foreign investment, the optimal policy involves a higher value of $\alpha$ in the net capital exporting country than in the net capital importing country so as to achieve a higher average level of protection against hybrid financing across all firms. In the non-cooperative setting, both governments typically desire a low value of $\alpha$ in their own country relative to the foreign country. By facilitating the use of hybrid instruments by domestic firms and impeding their use by foreign firms, a relatively low value of $\alpha$ erodes foreign taxation of domestic firms and enforces domestic taxation of foreign firms. This mechanism implies that equilibrium values of $\alpha$ are competed down to suboptimally low levels and that the difference between the equilibrium values of $\alpha$ chosen by the net capital exporting country and the net capital importing country is too small compared to the optimal policies.

Somewhat surprisingly, references to cross-border hybrid instruments in the finance and economics literatures are very scarce and no formal analysis exists of cross-border hybrid instruments, let alone of their implications for line drawing between debt and equity. In a companion paper, we study tax planning with cross-border hybrid financing in a framework with multi-dimensional financial instruments and multi-layered financial structures. The main insight is that the scope for tax planning with hybrid instruments derives from two types of cross-country differences in demarcation rules: differences in the relative 'weights' attributed to the various dimensions of financial instruments and differences in the threshold level of 'equityness' that triggers equity rather than debt treatment for tax purposes (Johannesen, 2011). The present paper draws on the more general framework developed in the companion paper, however, in order to analyze policy choices over demarcation rules and preserve tractability, the underlying model of hybrid financing is simplified by assuming one-dimensional financial instruments and single-layered financial structures.

The present paper relates to existing work on taxation in the presence of international tax planning. A host of papers study taxation of firms that have access to profit shifting techniques and analyze policy choices over tax rates, tax bases and tax enforcement (Haufler and Schjelderup, 2000; Peralta, Wauthy and Ypersele, 2006; Hong and Smart, 2010; Johannesen, 2010). Our analysis is clearly distinct from this literature in terms of both the tax planning technique and policy dimension considered and, by consequence, in terms of the mechanics of the model. The paper also relates to other work on line drawing in tax policy. The defining feature of line drawing problems is that policymakers can shift lines between the various categories used by the tax code but are unable to eliminate the categories themselves. In two seminal papers, Weisbach (1999, 2000) argues that line drawing problems are prevalent in the tax system and makes a case for line drawing based on efficiency arguments rather than doctrinal arguments. Both papers explicitly mention the distinction between debt and equity as one of the most prominent line drawing problems in current tax systems but does not present a formal analysis of the problem.

Finally, we draw the attention to two important limitations of the analysis. Firstly, we formulate the government problem as a line drawing problem where definitions of debt and equity can be changed but the categories themselves cannot be eliminated. The paper therefore does not contribute to the literature that compares the standard corporate tax system to alternative business tax systems, which abandon the discrimination between debt and equity either by granting a notional tax deduction for equity (the "ACE system") or by denying the tax deduction for debt (the "CBIT system").[6] Secondly, the paper does not aim to provide a general theory of line drawing between debt and equity but retains a sharp focus on how international tax planning with hybrid instruments affect optimal and equilibrium lines. Arguably, there are other important determinants of lines between debt and equity but to keep the analysis tractable these are modeled in a reduced-form fashion.

The paper is structured in the following way. Section 2 develops the model. Section 3 characterizes the economic equilibrium for a given set of policies. Section 4 derives globally optimal policies in a cooperative setting. Section 5 derives equilibrium policies in a non-cooperative setting. Section 6 provides some concluding remarks.

# 2 The model

This section develops a model of line drawing between debt and equity. The model comprises two countries, $A$ and $B$, the economies of which are tied together by firms residing in one country and investing in the other country. Both countries operate a standard corporate tax system that allows for deduction of interest payments from the taxable income and exempts foreign source income. To focus exclusively on hybrid financing and disregard financial strategies related to, for instance, profit shifting, we assume that both countries apply the same corporate tax rate $t$, hence government policy is only concerned with the rule delineating debt from equity. The first subsection presents a simple model of hybrid instruments and classification of such instruments for corporate tax purposes. The second and third subsections describe the objectives and constraints facing firms and governments respectively.

## 2.1 Hybrid instruments

Financial instruments are assumed to differ in a single, continuous dimension scaled to range the interval $[0, 1]$. A financial instrument is thus fully characterized by a value $z \in [0, 1]$. Instruments with $z$ closer to zero have properties closer to debt whereas instruments with $z$ closer to one have properties closer to equity. The corporate tax base is characterized by a threshold value $\alpha \in [0, 1]$ that delineates debt and equity for tax purposes.

In real-world tax systems, demarcation rules are typically not truly deterministic but leave considerable discretion to individual tax administrators and judges. Arguably, this creates *ex ante* uncertainty about the tax treatment of a given financial instrument.[7] In order to capture this uncertainty in the model, we assume that tax authorities make individual assessments of each hybrid instrument, on the

[6] Mooij and Devereux (2009) discuss various aspects of the ACE and CBIT systems of business taxation including the limited practical experience with their implementation.

[7] The lack of legal certainty is noted by several legal scholars. For instance, Emmerich (1985) writes: "But because of the wide variety of instruments and transactions that have required classification as debt or equity, the courts have spawned a bewildering variety of tests and standards requiring highly fact-bound and uncertain legal determinations."

basis of which the instrument is categorized as either debt or equity. Specifically, we assume that the assessment of a hybrid instrument with characteristics $z$ is given by

$$Z = z + \varepsilon$$

where $\varepsilon$ is random draw from a uniform distribution with mean zero and density $\gamma$. The financial instrument is categorized as equity if $Z \geq \alpha$ and debt if $Z < \alpha$. We let $p(z)$ denote the probability that a financial instrument with characteristics $z$ is categorized as equity, which implies that $1 - p(z)$ is the probability that the instrument is categorized as debt. Given the distributional assumption about $\varepsilon$, it is easy to show that [see Appendix B1]:

$$p(z) = \begin{cases} 0 & & z < \alpha - \frac{1}{2\gamma} \\ \frac{1}{2} + \gamma(z - \alpha) & \text{if} & \alpha - \frac{1}{2\gamma} < z < \alpha + \frac{1}{2\gamma} \\ 1 & & \alpha + \frac{1}{2\gamma} < z \end{cases} \tag{1}$$

Intuitively, there is uncertainty about the tax treatment if $z$ is sufficiently close to $\alpha$ and the probability of equity treatment is increasing linearly in $z$ within this intermediate range. If $z$ is sufficiently much smaller than $\alpha$ the instrument is treated as debt with certainty whereas if $z$ is sufficiently much larger than $\alpha$ the instrument is treated as equity with certainty. The parameter $\gamma$ can be perceived as a measure of the precision of the tax assessment.

Firms in country $A$ investing in the other country $B$ obtain a tax advantage if the financial instrument is treated as equity in country $A$ and debt in country $B$. It is easy to see that there exists a set of hybrid instruments with a strictly positive probability of achieving this tax treatment provided that $\alpha^A - \alpha^B < 1/\gamma$. Intuitively, if $z$ is not so much smaller than $\alpha^A$ so as to be categorized as debt with certainty in country $A$ and not so much larger than $\alpha^B$ so as to be categorized as equity with certainty in country $B$, the financial instrument may possibly obtain the desired tax treatment. Similarly, firms in country $B$ investing in country $A$ obtain a tax advantage if the financial instrument is treated as equity in country $B$ and debt in country $A$. There exists a set of hybrid instruments with a strictly positive probability of achieving this treatment provided that $\alpha^B - \alpha^A < 1/\gamma$. Consequently, it is generally possible for firms in one country to obtain their desired hybrid tax treatment and, moreover, it is possible for firms in both countries to obtain their desired hybrid tax treatment provided that $|\alpha^A - \alpha^B| < 1/\gamma$.[8]

[8] In the present framework, the possibility that some firms in country $A$ have financial instruments categorized as equity in country $A$ and debt in country $B$ and, at the same time, some firms in country $B$ have financial instruments categorized as equity in country $B$ and debt in country $A$ hinges crucially on the stochastic formulation of the demarcation rule. It should be noted, however, that in the multi-dimensional and deterministic model of hybrid financing developed in Johannesen (2011), there is often a scope for hybrid financing by firms in both countries when countries assign different relative weights to the different dimensions of financial instruments in their demarcation rules. To see this, consider the two-dimensional case where financial instruments are characterized by a vector $\mathbf{z} = (z^1, z^2)$ with $z^k \in (0, 1)$ for $k = 1, 2$. As an extreme example of differences in relative weights assume that the deterministic demarcation rule of country $A$ only takes into account the first dimension of financial instruments so that $\mathbf{z}$ is treated as equity if and only if $z^1 \geq \alpha^A$ whereas the demarcation rule of country $B$ only considers the second dimension so that $\mathbf{z}$ is treated as equity if and only if $z^2 \geq \alpha^B$.

The scope for hybrid instruments is illustrated in Figure 1. In country $A$, instruments with $z \in [z_1; z_3]$ may be treated as either debt or equity depending on the stochastic outcome of the assessment made by the tax authorities. Instruments with $z < z_1$ are treated as debt with certainty whereas instruments with $z > z_3$ are treated as equity with certainty. Similarly, in country $B$, instruments with $z \in [z_2; z_4]$ may be treated as either debt or equity depending on the assessment outcome whereas instruments with $z < z_2$ and $z > z_4$ are treated as debt and equity respectively with certainty. Any instrument with $z \in [z_1; z_4]$ may possibly be treated as equity in country $A$ and debt in country $B$ as desired by firms in country $A$ whereas instruments with $z \in [z_2; z_3]$ may possibly be treated as debt in country $A$ and equity in country $B$ as desired by firms in country $B$.

While hybrid instruments from the perspective of firms represent an opportunity for non-taxation, the stochastic tax environment described above, in principle, also implies a risk of double taxation. For instance, if a firm in country $A$ finances an investment in country $B$ with a hybrid instrument that is categorized as debt in country $A$ and equity in country $B$, the firm has no deductible interest payments in country $B$ but is nevertheless taxed on interest income in country $A$. We assume that such double taxation does not occur. Instead we assume that in cases where inconsistent categorization of a financial instrument would give rise to double taxation, the two countries agree to treat the instrument consistently either as equity (with probability one half) or as debt (with probability one half). This assumption has strong foundations in the prevailing legal institutions of international taxation. Most developed countries have extensive networks of bilateral double tax conventions with other developed countries with the aim of eliminating double taxation. Typically, double tax conventions are based on the OECD model convention which provides definitions of dividend and interest payments for the purposes of the convention. In cases where conflicting interpretations of a convention lead to double taxation, tax authorities are committed to resolve the conflict by mutual agreement with a view to eliminating the double taxation.[9]

## 2.2 Firms

Countries are inhabited by domestically owned firms, each of which is endowed with a single profitable investment project in the other country. Investment projects require $k$ units of capital and generate a gross revenue of $y$. Firms undertaking a foreign investment are composed of two entities: a parent company in the home country and a subsidiary in the foreign country. The parent company raises

Regardless of the values of $\alpha^A$ and $\alpha^B$, the instrument $\mathbf{z} = (1, 0)$ allows firms in country $A$ investing in country $B$ to obtain the desired hybrid treatment whereas firms in country $B$ investing in country $A$ can obtain the opposite hybrid treatment by implementing the instrument $\mathbf{z} = (0, 1)$.

[9] While we should therefore expect double tax conventions to eliminate double taxation of hybrid instruments, they do not by any means prevent non-taxation of such instruments. As discussed by Rosenbloom (1999), this asymmetry derives from the fact that double tax conventions are *elective* for tax payers who may always reject a treaty and invoke their rights under domestic law. In the words of Rosenbloom: "Since international tax arbitrage generally (though perhaps not invariably) builds upon differences in domestic laws, not treaties, an election to rely on domestic law would leave the tax payer with the same arbitrage opportunities as if the treaty did not exist at all" (p. 164).

capital in external capital markets and transfers the funds to the subsidiary by means of a financial instrument. External investors require a fixed rate of return of $r$, hence the before-tax profits generated by an investment project amount to $\pi \equiv y - rk$.

When parent companies invest in foreign subsidiaries, they choose between a hybrid financial instrument and a standard financial structure composed of a pure debt and a pure equity instrument. Firms are risk-neutral and thus opt for the mode of finance that maximizes expected profits. As a tie-breaker, we assume that firms only opt for hybrid financing when this yields strictly larger expected after-tax profits than financing with standard debt and equity instruments.

Firms opting for a hybrid instrument choose the instrument $z^*$ that maximizes expected profits. Depending on the stochastic outcomes of the tax assessments in the home and host countries, hybrid finance may give rise to a tax saving. If the hybrid instrument is successfully categorized as equity in the home country and debt in the host country, the tax liability in the host country is $t(y - rk)$ while there is no taxation in the home country. In this case, payments on the instrument $rk$ are treated as tax deductible interest expenses in the host country and as non-taxable dividend income in the home country. If the hybrid instrument is categorized as debt in both countries, the tax liability in the host country is $t(y - rk)$ and the tax liability in the home country is $trk$. If the hybrid instrument is treated as equity in both countries, the tax liability in the host country is $ty$ while there is no tax liability in the home country. In either case of unsuccessful hybrid financing, the total tax burden thus amounts to $ty$. The tax saving generated by successful hybrid financing amounts to $trk$. The upper part of Table 1 summarizes possible tax outcomes under hybrid financing.

Firms opting for standard financial instruments finance the foreign investment with a fixed fraction $\beta^D$ of internal debt and a fixed fraction $\beta^E$ of equity where $\beta^D + \beta^E = 1$. The tax liability in the host country is $t(y - \beta^D rk)$ whereas the tax liability in the home country is $t\beta^D rk$ with the tax base comprising interest income. The total tax liability is simply $ty$. Intuitively, internal debt shifts taxable income from the host country to the home country, however, with symmetry in corporate tax rates, the level of internal debt has no net effect on the total tax liability. The significance of parameters $\beta^D$ and $\beta^E$ is to determine the allocation of taxing rights of firms using standard financial instruments. For these firms, a fraction $\beta^D$ of the normal return to capital is taxed by the home country whereas the remaining fraction $\beta^E$ is taxed by the host country. The lower part of Table 1 summarizes the tax outcome under standard financing.

While hybrid financing may generate a tax saving relative to standard financing, we also assume that it involves a fixed cost $c$. The cost may reflect fees to tax advisors, lawyers, accountants and auditors for implementation and management of the hybrid finance structure or inefficiencies related to a capital structure that is distorted by tax considerations. We assume that firms are heterogeneous with respect to the fixed cost $c$. Specifically, it is assumed that $c$ is uniformly distributed over the interval $[0; \bar{c}]$ with density $\delta$ and uncorrelated with country of residence. We also assume that $\bar{c}$ is sufficiently large to ensure that there, in any policy environment, are firms choosing not to implement a hybrid instrument.[10]

[10] As will become clear, this assumption implies that $\bar{c} > trk$ such that for firms with $c = \bar{c}$ the fixed cost of hybrid

For notational simplicity and without loss of generality, we normalize the mass of firms with profitable foreign investment projects in the two countries to one. We allow for a possible asymmetry in the size of the multinational sector of the two countries by assuming that a fraction $\theta^A \geq 1/2$ of the total mass of firms is located in country $A$ and the remaining fraction $\theta^B \leq 1/2$ is located in country $B$ where $\theta^A + \theta^B = 1$. We shall often simply refer to $\theta^A$ and $\theta^B$ as the sizes of country $A$ and $B$.

For future reference, we define the tax on the normal return to capital $\tau^N \equiv trk$ and the tax on pure economic profits $\tau^P \equiv t(y - rk)$. These definitions are convenient because they divide the total tax bill into one part $\tau^P$, which is payable in the source country with certainty, and another part $\tau^N$, which depending on the tax treatment of the financial instrument is payable in the home country (if debt treatment), in the source country (if equity treatment) or not at all (if desired hybrid treatment).

## 2.3 Governments

The corporate tax system is characterized by the tax rate $t$ and the demarcation rule $\alpha$ that determines the tax treatment of hybrid instruments. Under the maintained assumption that $t$ is fixed, $\alpha$ is the only available policy instrument. We posit that the government of country $i$ has the following objective function:

$$W^i = \Pi^i + \mu R^i - \frac{\phi}{2}(\alpha^i - \widetilde{\alpha})^2$$

As usual in models of international taxation, the revenue of the domestic government and the disposable income of domestic residents enter the objective function. Since our model is only concerned with cross-border investment, government revenue is simply the proceeds from taxing domestic firms investing in the foreign country and foreign firms investing in the home country, which we denote by $R^i$. Similarly, domestic disposable income is simply the after-tax profits derived by domestic firms from their foreign investment, which we denote by $\Pi^i$. It is natural to interpret $\mu > 1$ as the marginal cost of public funds from other sources than taxation of foreign investment.

The role of the final term is to allow for the possibility that governments use demarcation rules to pursue other policy objectives than the ones described in the underlying model of foreign investment. For instance, while the model explicitly describes the role of demarcation rules in shaping financing decisions related to cross-border investment, demarcation rules are also likely to affect financing decisions related to domestic investment. To see this, note that tax systems favoring debt over equity introduces a rationale for financial instruments that are sufficiently close to debt to be treated as such for tax purposes but retain key attributes of equity. It is well known that firms use hybrid instruments designed to serve as equity for financial reporting purposes and as debt for tax purposes, that banks issue hybrid securities at the same time serving to satisfy capital requirements and benefitting from debt treatment for tax purposes and that firms generally seek to achieve a capital structure that combines the flexibility of equity finance with the tax advantages of debt finance (Engel, Erickson and Maydew, 1999; Gergen and Schmitz, 1997). Demarcation rules are crucial in determining the scope for these uses of hybrid instruments and thus have

financing exceeds the expected tax saving even in the most favorable tax environment where the tax saving occurs with certainty.

a bearing on the effective taxation of domestic investment and the deadweight loss associated with the use of tax motivated hybrid instruments to finance domestic investment. In order to allow governments to use demarcation rules to affect these and other economic outcomes while keeping the model tractable, we take a reduced-form approach by assuming that for fixed levels of $R^i$ and $\Pi^i$, governments have a preferred demarcation rule $\widetilde{\alpha}$ and that deviations from $\widetilde{\alpha}$ are associated with a loss. The parameter $\phi$ measures the relative importance of these unspecified policy motives and except when explicitly stated otherwise the analysis assumes that $\phi$ is strictly positive and large enough to ensure that demarcation rules are chosen in the interior of the unit interval.

We consider two institutional settings. Firstly, government may act non-cooperatively in which case each government sets its demarcation rule so as to maximize its own objective function. Secondly, governments may engage in international cooperation, in which case the two governments set their demarcation rules so as to maximize the sum of the objective functions $W \equiv W^A + W^B$.

### 2.4 Game structure and solution

We analyze two-stage games with the following structure: (i) governments set demarcation rules $\alpha^A$ and $\alpha^B$ either cooperatively or non-cooperatively depending on the institutional setting while correctly anticipating firm responses to policies; (ii) firms choose optimal financial policies and undertake foreign investment. We are particularly interested in the outcomes of the first-stage policy game. In the co-operative case, the solution to the policy game was defined in the previous section as the policy vector $(\alpha^{A*}, \alpha^{B*})$ that maximizes $W$. We shall refer to this vector as the *optimal policy.* In the non-cooperative case, we follow most of the literature on international taxation by identifying a Nash equilibrium in the game where country $i$ sets its demarcation rule so as to maximize $W^i$ while taking the demarcation rule of the other country $-i$ as given. We shall refer to the policy vector $(\alpha^{Ao}, \alpha^{Bo})$ that constitutes a Nash equilibrium in the non-cooperative policy game as the *equilibrium policy.*

As it turns out, it is convenient to describe policy outcomes in terms of the difference between the demarcation rules $\alpha^A - \alpha^B$ and the average of the demarcation rule $(\alpha^A + \alpha^B)/2$. Together, the difference and the average implicitly define the individual levels of $\alpha^A$ and $\alpha^B$. We shall refer to the difference $\alpha^A - \alpha^B$ as the *distance* between the demarcation rules and to the average $(\alpha^A + \alpha^B)/2$ as the *location* of the demarcation rule on the debt-equity continuum.

## 3 Economic equilibrium

This section takes a first step towards identifying the policy equilibrium by solving for the economic equilibrium for a given set of policies. The first subsection characterizes the optimal financial policies of firms given the policy environment. The second subsection derives expressions for the resulting equilibrium profits and government revenues.

### 3.1 Optimal financial policies

The financial policies of firms consist of two choices: (i) firms decide whether to finance the foreign investment with a hybrid instrument or with standard debt and equity instruments, (ii) conditional on using hybrid finance, firms decide on the specific characteristics $z$ of the hybrid instrument to be implemented. We start analyzing the latter of these decisions and subsequently turn to the former one.

Firms in country $i$ investing in the other country $-i$ are seeking to have the financial instrument financing the investment characterized as equity in country $i$ and as debt in country $-i$. Assume that $\alpha^i - \alpha^{-i} < 1/\gamma$ so that this tax treatment is indeed possible. Conditional on using a hybrid instrument to finance the foreign investment, the firm chooses its characteristics $z$ to maximize expected after-tax profits:

$$\pi - c - \tau^P - \tau^N + p^i(z)(1 - p^{-i}(z))\tau^N \tag{2}$$

subject to the constraints that $0 \leq z \leq 1$. The first four terms are deterministic and capture profits $\pi$ net of three cost components: the cost of setting up a hybrid structure $c$, the tax on pure profits $\tau^P$ and the tax on the normal return to capital $\tau^N$. The last term is stochastic and capture the expected tax saving from the hybrid structure where $p^i(z)(1 - p^{-i}(z))$ is the probability of obtaining the desired tax treatment in both countries and $\tau^N$ is the tax saving under this desired tax treatment. The following lemma gives the solution to this maximization problem.

**Lemma 1** *Assuming that demarcation rules satisfy $-1/\gamma < \alpha^A - \alpha^B < 1/\gamma$, the optimal hybrid instrument is characterized by:*

$$z^* = \frac{\alpha^A + \alpha^B}{2}$$

**Proof.** See Appendix A ■

Lemma 1 states that the $z^*$ characterizing the optimal hybrid instrument is exactly halfway between $\alpha^A$ and $\alpha^B$. The intuition for this result is straightforward. It is clear from (2) that the optimal financial instrument is the one that maximizes the probability of obtaining the desired hybrid tax treatment $p^i(z)(1 - p^{-i}(z))$. The essential trade-off associated with the choice of hybrid is that while raising $z$ increases the probability of equity treatment in country $i$, that is $p^i(z)$, it also reduces the probability of debt treatment in country $-i$, that is $1 - p^{-i}(z)$. Raising the probability of equity treatment in country $i$ through an increase in $z$ is more (less) valuable in terms of expected tax savings when the probability of debt treatment in country $-i$ is large (small), that is when $z$ is initially low (high). Similarly, reducing the probability of debt treatment in country $-i$ through an increase in $z$ is less (more) costly in terms of expected tax savings when the probability of equity treatment in country $i$ is small (high), that is when $z$ is initially low (high). This mechanism implies that the probability of obtaining the desired hybrid tax treatment is maximized exactly when the probabilities $p^i(z)$ and $1 - p^{-i}(z)$ are equalized and this is the case when $z$ is equidistant from $\alpha^A$ and $\alpha^B$.

Having defined the optimal hybrid instrument $z^*$, we introduce the following short-hand notation: We let $p^i \equiv p^i(z^*)$ denote the probability that the optimal hybrid instrument is treated as equity in country $i$ and let $q^i \equiv p^i(1-p^{-i})$ denote the probability that firms in country $i$ financing an investment in the other country $-i$ with the optimal hybrid instrument $z^*$ achieves the desired tax treatment.

Turning to the choice between hybrid financing and standard financing, it is easy to see from (2) that expected after-tax profits under hybrid financing amount to $\pi - \tau^P - (1-q^i)\tau^N - c$. By comparison, after-tax-profits under standard financing amount to $\pi - \tau^P - \tau^N$. It follows directly that there is a threshold value $\widetilde{c}^i \equiv q^i \tau^N$ for which firms with $c^i < \widetilde{c}^i$ optimally choose hybrid financing and firms with $c^i \geq \widetilde{c}^i$ optimally choose standard financing. The threshold value $q^i \tau^N$ captures the expected tax saving from hybrid financing.

Finally, we present some intermediate results, which will prove useful in sections 4 and 5.

**Lemma 2** *For country $i = A, B$ it holds that*

$$\text{(a) } \frac{dp^i}{d\alpha^i} = \frac{d(1-p^{-i})}{d\alpha^i} = -\frac{\gamma}{2} \text{ for } i = A, B$$

$$\text{(b) } \frac{dq^i}{d\alpha^i} = -\frac{dq^i}{d\alpha^{-i}} = -\gamma p^i \text{ for } i = A, B$$

$$\text{(c) } q^i = (p^i)^2$$

**Proof.** See Appendix A ■

These intermediate results constitute important building blocks for the further analysis and are key to understanding the mechanics of the model. Subresult (a) states that an increase in $\alpha^i$ reduces the probability that the optimal hybrid receives equity treatment in country $i$ and debt treatment in country $-i$ by the same amount. Intuitively, the characteristics of the optimal hybrid instruments are adjusted in response to policy changes so as to ensure that the optimal hybrid is equidistant from the two demarcation rules, which implies that the probability of equity treatment in country $i$ optimally equals the probability of debt treatment in country $-i$. Subresult (b) has several important implications. Firstly, changes in $\alpha^i$ and $\alpha^{-i}$ of equal size leave the probability of successful hybrid treatment $q^i$ unchanged. Intuitively, $q^i$ depends only on the distance between demarcation rules $\alpha^i - \alpha^{-i}$ and not on their location. Secondly, increasing the distance $\alpha^i - \alpha^{-i}$ reduces the probability of successful hybrid treatment $q^i$. Intuitively, raising $\alpha^i$ lowers the probability of equity treatment in country $i$ whereas reducing $\alpha^{-i}$ lowers the probability of debt treatment in country $-i$ for *any* given financial instrument. Finally, increasing the distance $\alpha^i - \alpha^{-i}$ has a more negative effect on $q^i$ when $\alpha^i - \alpha^{-i}$ is initially small and a less negative effect on $q^i$ when $\alpha^i - \alpha^{-i}$ is initially large. In other words, increasing the distance $\alpha^i - \alpha^{-i}$ reduces $q_i$ with decreasing marginal effectiveness. Intuitively, recall the definition $q^i = p^i(1-p^{-i})$ and note that increasing the distance $\alpha^i - \alpha^{-i}$ reduces $p^i$, which has a larger impact on $q^i$ when $(1-p^{-i})$ is initially larger (that is when $\alpha^i - \alpha^{-i}$ is initially smaller), and reduces $(1-p^{-i})$, which has a larger impact on $q^i$ when $p^i$ is initially larger (that is when $\alpha_i - \alpha^{-i}$ is initially smaller). Subresult (c) holds because the optimal hybrid instrument is exactly halfway between $\alpha^i$ and $\alpha^{-i}$ so that $p^i = (1-p^{-i})$.

### 3.2 Profits and revenue

To shorten the expressions to be derived below, we define $x^{fi}$ as the mass of firms in country $i$ opting for finance of type $f$ where $f = H$ indicates hybrid financing and $f = S$ indicates financing with standard debt and equity instruments. It is easy to see that $x^{Hi} = \theta^i \delta \widetilde{c}^i$ and $x^{Si} = \theta^i \delta(\overline{c} - \widetilde{c}^i)$. With these definitions, we may state total after-tax profits earned by firms resident in country $i$ in the following way:

$$\Pi^i = x^{Hi}\left\{q^i(\pi - \tau^P) + (1-q^i)(\pi - \tau^P - \tau^N)\right\} - \psi^i + x^{Si}(\pi - \tau^P - \tau^N)$$

where

$$\psi^i \equiv \int_0^{\widetilde{c}^i} c\delta\theta^i dc$$

measures total implemntation costs incurred by firms in country $i$ with hybrid financing structures. Firms in country $i$ financing foreign investment with a hybrid instrument earn after-tax profits $\pi - \tau^P - c$ with probability $q^i$ and after-tax profits $\pi - \tau^P - \tau^N - c$ with probability $1 - q^i$ whereas firm using standard debt and equity earn after-tax profits $\pi - \tau^P - \tau^N$ with certainty.

Finally, define $\rho^{ii}$ as the probability that a firm in country $i$ implementing a hybrid instrument to finance its foreign investment pay taxes on the normal return in the home country $i$ and define $\rho^{i-i}$ as the probability that it pays taxes on the normal return in the source country $-i$:

$$\rho^{ii} \equiv (1-p^i)(1-p^{-i}) + \frac{1}{2}(1-p^i)p^{-i}$$

$$\rho^{i-i} \equiv p^i p^{-i} + \frac{1}{2}(1-p^i)p^{-i}$$

The first term of $\rho^{ii}$ is the probability that the tax assessments in both countries categorize the instrument as debt. The second term of $\rho^{ii}$ is the probability that conflicting tax assessments in the two countries give rise a double taxation dispute, which is resolved by the two countries agreeing to treat the instrument as debt. In both cases, the normal return is taxed in the home country. Similarly, the first term of $\rho^{i-i}$ is the probability that the tax assessments in both countries categorize the instrument as equity whereas the second term is the probability that a double taxation dispute is resolved by the two countries agreeing to treat the instrument as equity. In both cases, the normal return is taxed in the host country. For future reference, it should be noted that $\rho^{ii} + \rho^{i-i} + q^i = 1$. This simply reflects that for firms in country $i$ engaged in hybrid financing of a foreign subsidiary, the normal return is either taxed in the home country $i$ (with probability $\rho^{ii}$), taxed in the source country $-i$ (with probability $\rho^{i-i}$) or not taxed at all because the financial instrument obtains the desired hybrid treatment (with probability $q^i$). Also note that $\rho^{ii} = \rho^{i-i}$, which implies that the home country and the host country have the same probability $(1-q^i)/2$ of getting to tax the normal return under hybrid financing.

Using this short-hand notation, we may write the government revenue of countries $i$ in the following way:

$$R^i = x^{Hi}\tau^N\rho^{ii} + x^{Si}\tau^N\beta^D + x^{H-i}\left\{\rho^{-ii}\tau^N + \tau^P\right\} + x^{S-i}\left\{\beta^E\tau^N + \tau^P\right\}$$

The first two terms represent domestic revenue from taxing the cross-border investments of domestic firms. Domestic firms with hybrid financing ($x^{Hi}$) pay domestic taxes on the normal return with probability $\rho^{ii}$. Domestic firms with standard financing ($x^{Si}$) pay domestic taxes on the debt share $\beta^D$ of the normal return with certainty. The last two terms represent domestic revenue from taxing the cross-border investments of foreign firms. Foreign firms with a hybrid instrument ($x^{H-i}$) pay domestic taxes on pure profits with certainty and on the normal return with probability $\rho^{-ii}$. Foreign firms with standard financing ($x^{S-i}$) pay domestic taxes on pure profits and on the equity share $\beta^E$ of the normal return with certainty.

It should be noted that the economic equilibrium $\left\{\Pi^A; R^A; \Pi^B; R^B\right\}$ is fully determined by the distance $\alpha^A - \alpha^B$. Intuitively, the optimal financial policies of firms as well as the distribution of government revenue between the two countries only depend on the probabilities $q^A$ and $q^B$, which, as we showed in Lemma 2, only depend on the distance $\alpha^A - \alpha^B$.

# 4 Optimal policies

Under international cooperation, the two governments set $(\alpha^A, \alpha^B)$ cooperatively so as to maximize aggregate welfare $W$. Differentiating aggregate welfare with respect to $\alpha_i$ yields the following first-order conditions for optimal cooperative policies [See Appendix B2]:

$$\frac{\partial W}{\partial \alpha^i} = (1-\mu)\tau^N \left\{ x^{Hi}\frac{dq^i}{d\alpha^i} + x^{H-i}\frac{dq^{-i}}{d\alpha^i} \right\} - \mu\tau^N \left\{ q^i \frac{dx^{Hi}}{d\alpha^i} + q^{-i}\frac{dx^{H-i}}{d\alpha^i} \right\} - \phi(\alpha^i - \widetilde{\alpha}) = 0 \qquad (3)$$

for $i = A, B$. The first term reflects the 'mechanical' welfare effect of a small change in $\alpha_i$, that is the welfare effect working through changes in $q^i$ and $q^{-i}$ holding the number of firms using hybrid financing constant.[11] Intuitively, holding the number of firms using hybrid financing constant, changes in $q^i$ and $q^{-i}$ merely transfer rents between the firms engaging in hybrid financing and governments. Increases in $q^i$ and $q^{-i}$ imply a higher probability that firms using hybrid finance avoid taxation of the normal return. This increases private profits but reduces government revenue by the same amount implying a net decrease in welfare under the maintained assumption that the marginal cost of funds exceeds unity. The second term reflects the 'behavioral' welfare effect of a small change in $\alpha_i$, that is the welfare effect working through changes in the number of firms using hybrid financing. Changes in $x^{Hi}$ and $x^{H-i}$ have no impact on private disposable income since the firms that respond to a small change in $\alpha^i$ by switching between hybrid financing and standard financing are initially indifferent between the two modes of finance. This is an application of the envelope theorem. Finally, the last term captures the effects of the demarcation rule outside the realm of cross-border investment. Clearly, if $\alpha^i$ is initially smaller (larger) than $\widetilde{\alpha}$ so that a marginal increase in $\alpha^i$ brings it closer to (further away from) $\widetilde{\alpha}$, this effect is positive (negative).

[11] Strictly speaking, the effect is not purely mechanical since $dq^A/d\alpha^i$ and $dq^B/d\alpha^i$ also capture changes in $q^A$ and $q^B$ that are due to adjustments of the properties of the optimal hybrid instrument $z^*$.

It should be noted that $dq^i/d\alpha^i$ and $dq^{-i}/d\alpha^i$ generally have opposite signs as do $dx^{Hi}/d\alpha^i$ and $dx^{H-i}/d\alpha^i$. This points to the fundamental trade-off that while increasing the $\alpha^i$ increases protection against hybrid financing by firms in country $i$, it also reduces protection against hybrid financing by firms in country $-i$.

The following lemma restates (3) in terms of demarcation rules, probabilities of successfully implementing hybrid structures and the primitive parameters of the model:

**Lemma 3** *The first-order conditions for optimal cooperative policies may be stated as:*

$$\frac{\partial W}{\partial \alpha^i} = \Delta \left\{ \begin{array}{c} (2\mu - 1)\theta^i (q^i)^{\frac{3}{2}} \\ -(2\mu - 1)\theta^{-i} (q^{-i})^{\frac{3}{2}} \end{array} \right\} - \phi(\alpha^i - \widetilde{\alpha}) = 0 \textit{ for } i = A, B$$

*where* $\Delta \equiv \gamma\delta(\tau^N)^2 > 0$.

**Proof.** See Appendix A ■

The first line in the curly brackets represents the positive welfare effect of raising $\alpha^i$ working through a decrease in $q^i$ whereas the second line represents the negative welfare effect working through an increase in $q^{-i}$. The factor $2\mu - 1$ reflects that in this model, incidentally, the mechanical and behavioral effects are of exactly the same magnitude. Thus, a reduction in $q^i$ causes a mechanical transfer of rents from firms in country $i$ using hybrid financing to governments thus creating a net mechanical gain proportional to $\mu - 1$. Moreover, a reduction in $q^i$ causes some firms in country $i$ to shift from hybrid financing to standard financing, which gives rise to a behavioral revenue gain of exactly the same size as the mechanical revenue gain thus bringing the total net gain to $2\mu - 1$.

We are now prepared to present the first result, which pertains to optimal demarcation rules under symmetry.

**Proposition 1** *If countries are symmetric ($\theta^A = \theta^B$) the socially optimal demarcation rules ($\alpha^{A*}, \alpha^{B*}$) are given by:*

$$\alpha^{A*} = \alpha^{B*} = \widetilde{\alpha}$$

**Proof.** See Appendix A ■

The first important implication of Proposition 1 is that with symmetric countries, the optimal distance between the demarcation rules is zero. To see the intuition for this result, recall that raising the distance $\alpha^i - \alpha^{-i}$ increases welfare by reducing $q^i$ and thus limiting the scope for hybrid financing by firms in country $i$ while at the same time reducing welfare by increasing $q^{-i}$ and thus enlarging the scope for hybrid financing by firms in country $-i$. As discussed above, the distance $\alpha^i - \alpha^{-i}$ exhibits decreasing marginal effectiveness in combating hybrid finance by firms in country $i$ such that an increase in $\alpha^i - \alpha^{-i}$ reduces $q^i$ by a larger amount and raises $q^{-i}$ by a smaller amount when $\alpha^i - \alpha^{-i}$ is initially small. This force, which pulls optimal demarcation rules together, plays out fully under symmetry so that the two countries optimally apply the same demarcation rule and, consequently, protection against hybrid finance is at the same level in the two countries.

The second implication of Proposition 1 is that the optimal location of demarcation rules is uniquely determined at $\widetilde{\alpha}$. Intuitively, any symmetric policy gives rise to the same equilibrium values $\{\Pi^A; R^A; \Pi^B; R^B\}$, hence the location of the demarcation rules is optimally chosen to coincide with the value that is optimal for other policy purposes. Under the alternative assumption that demarcation rules have no other effects than shaping the financing of foreign investment ($\phi = 0$), any pair of demarcation rules $\alpha^A = \alpha^B$ is as good as $(\alpha^{A*}, \alpha^{B*})$ and the optimal location becomes indeterminate.

We now proceed to characterize optimal demarcation rules in an asymmetric environment where more firms in country $A$ invest in country $B$ than *vice versa*.

**Proposition 2** *If countries are asymmetric ($\theta^A > \theta^B$) the socially optimal demarcation rules ($\alpha^{A*}, \alpha^{B*}$) can be characterized in the following way:*

*The* ***optimal distance*** $\alpha^{A*} - \alpha^{B*}$ *is implicitly determined by:*

$$\alpha^{A*} - \alpha^{B*} = \frac{2\Delta(2\mu - 1)}{\phi} \left\{ \theta^A (q^A)^{\frac{3}{2}} - \theta^B (q^B)^{\frac{3}{2}} \right\}$$

*which implies that (i) the optimal distance is positive and increasing in the size asymmetry $\theta^A - \theta^B$ and (ii) the optimum satisfies $q^B > q^A > 0$.*

*The* ***optimal location*** *is given by:*

$$\frac{\alpha^{A*} + \alpha^{B*}}{2} = \widetilde{\alpha}$$

**Proof.** See Appendix A ■

The first part of Proposition 2 states that optimally the demarcation rule of the larger country exceeds the demarcation rule of the smaller country and that the optimal distance between the two is increasing in the size asymmetry. Intuitively, starting from a symmetric policy, a marginal increase in the distance $\alpha^A - \alpha^B$ reduces $q^A$ and raises $q^B$ by the same amount, however, the positive welfare effect deriving from the reduction in $q^A$ is larger than the negative welfare effect deriving from the increase in $q^B$ simply because the number of firms in country $A$ exceeds the number of firms in country $B$. The decreasing marginal effectiveness of the distance $\alpha^i - \alpha^{-i}$ in combating hybrid finance by firms in country $i$, however, is still at play: As the distance $\alpha^A - \alpha^B$ increases to the point where $q^A$ approaches zero, the marginal benefit of additional increases in $\alpha^A - \alpha^B$ in terms of improved protection against hybrid financing by firms in country $A$ also approaches zero. Hence, $q^A$ is strictly positive in the social optimum even when $\theta^A$ is very large relative to $\theta^B$. The second part of Proposition 2 states that the socially optimal demarcation rules are located symmetrically around $\widetilde{\alpha}$. We recall that the economic equilibrium is fully determined by the distance between the demarcation rules. For a given distance, the optimal location is therefore simply the one that minimizes the sum of the quadratic loss functions, which requires that the two demarcation rules are equidistant from $\widetilde{\alpha}$.

Under the alternative assumption that demarcation rules have no other economic effects than shaping the financing of foreign investment ($\phi = 0$), it can be shown that the optimal distance is the one that minimizes the expected global number of firms that successfully use hybrid financing [See Appendix B3]. As in the symmetric case, the optimal location is indeterminate.

It should be noted that the capital structure of firms using standard finance has no bearing on the socially optimal policies. Intuitively, while $\beta^D$ and $\beta^E$ matter for the distribution of taxing rights between the two countries with a large (small) $\beta^D$ and small (large) $\beta^E$ implying a large (small) degree of home country taxation and a small (large) degree of host country taxation, it does not affect the size of the global tax base.

# 5 Non-cooperative policy equilibrium

Absent international cooperation, government $i$ sets $\alpha^i$ so as to maximize the welfare of country $i$. Partial differentiation of $W^i$ with respect to $\alpha^i$ yields the following first-order condition that characterizes the optimal demarcation rule from the perspective of country $i$ given the demarcation rule of the other country $-i$ [see Appendix B4]:

$$\frac{\partial W^i}{\partial \alpha^i} = \left\{ \begin{array}{c} \tau^N \left\{ x^{Hi} \frac{dq^i}{d\alpha^i} \right\} + \mu\tau^N \left\{ x^{Hi} \frac{d\rho^{ii}}{d\alpha^i} + x^{H-i} \frac{d\rho^{-ii}}{d\alpha^i} \right\} \\ +\mu\tau^N \left\{ \frac{dx^{Hi}}{d\alpha^i}(\rho^{ii} - \beta^D) + \frac{dx^{H-i}}{d\alpha^i}(\rho^{-ii} - \beta^E) \right\} \end{array} \right\} - \phi(\alpha^i - \widetilde{\alpha}) \tag{4}$$

The first term in the curly brackets captures the effect of a small change in the demarcation rule on private disposable income: a change in $\alpha_i$ alters the probability that hybrid instruments obtain the desired tax treatment and thus mechanically affect the expected profits of firms using hybrid financing. The second term in the curly brackets is the mechanical revenue effect: the mirror image of the change in the probability that hybrid instruments obtain the desired tax treatment is the change in the probability that hybrid instruments are taxed in country $i$. The third term in the curly brackets is the behavioral revenue effect: a change in $\alpha_i$ alters the financing decision of firms that are initially indifferent between the two modes of finance. A firm in country $i$ switching from standard financing to hybrid financing increases expected revenue in country $i$ by $\tau^N(\rho^{ii} - \beta^D)$ where we recall that $\rho^{ii}$ is the probability that the normal return is taxed in country $i$ under hybrid financing and $\beta^D$ is the part of the normal return that is subject to taxation in country $i$ under standard financing. Similarly, a firm in country $-i$ switching from standard financing to hybrid financing increases expected revenue in country $i$ by $\tau^N(\rho^{-ii} - \beta^E)$ where $\rho^{-ii}$ is the probability that the normal return is taxed in country $i$ under hybrid financing and $\beta^E$ is the part of the normal return that is subject to taxation in country $i$ under standard financing. Finally, the last term reflects the welfare effect that is due to other policy motives for using $\alpha^i$.

We are now prepared to present the following lemma, which restates the first-order conditions characterizing optimal demarcation rules from the perspective of individual countries.

**Lemma 4** *The first-order condition for the optimal non-cooperative policy in country $i$ may be stated as:*

$$\frac{\partial W^i}{\partial \alpha^i} = \Delta \left\{ \begin{array}{c} (\mu - 1)\theta^i (q^i)^{\frac{3}{2}} + \frac{\mu}{2} \left( \beta^D - \beta^E \right) \theta^i (q^i)^{\frac{1}{2}} \\ -\mu\theta^{-i} (q^{-i})^{\frac{3}{2}} + \frac{\mu}{2} \left( \beta^D - \beta^E \right) \theta^{-i} (q^{-i})^{\frac{1}{2}} \end{array} \right\} - \phi(\alpha^i - \widetilde{\alpha}) = 0$$

**Proof.** See Appendix A ■

Lemma 4 is the non-cooperative analogue of Lemma 3. The first line thus reflects the *gain* to country $i$ from a small increase in $\alpha^i$ working through a decrease in $q^i$ that makes it less favorable for domestic firms to use hybrid financing whereas the second line represents the *loss* to country $i$ working through an increase in $q^{-i}$ that makes it more favorable for foreign firms to use hybrid financing. It is highly instructive to compare the first-order conditions in the non-cooperative settings to the corresponding first-order conditions in the cooperative setting.

First, consider the special case where standard finance uses debt and equity in equal proportions $\left(\beta^D = \beta^E\right)$ such that the last term in both lines disappears. In this special case, the two sets of first-order conditions are identical except that the gain is proportional to $(\mu - 1)$ in the non-cooperative case as opposed to $(2\mu - 1)$ in the cooperative case and the loss is proportional to $\mu$ in the non-cooperative case as opposed to $(2\mu - 1)$ in the cooperative case. As for the gain from a small increase in $\alpha^i$, the difference is due to the fact that exactly one half of the increase in government revenue caused by a decrease in $q^i$ accrues to country $-i$. This externality is taken into account by country $i$ in the cooperative setting but not in the non-cooperative setting. As for the loss from a small increase in $\alpha^i$, the difference is due to the fact that exactly one half of the loss of government revenue caused by an increase in $q^{-i}$ is incurred by country $-i$ and the entire increase in after-tax profits accrues to firms in country $-i$. Again, this externality is only taken into account by country $i$ in the cooperative setting.

Next, consider the general case where standard finance uses debt and equity in different proportions $\left(\beta^D \neq \beta^E\right)$. If $\beta^D > \beta^E$, this adds to the gain and reduces the loss associated with an increase in $\alpha^i$. Intuitively, lowering $q^i$ is more beneficial to country $i$ if standard finance involves a relatively large share of debt such that the marginal domestic firms responding to the decrease in $q^i$ by adopting standard finance are mostly subject to taxation in the home country $i$. Similarly, raising $q^{-i}$ is less costly to country $i$ if standard finance involves a relatively large share of debt such that the marginal foreign firms responding to the increase in $q^{-i}$ by abandoning standard finance are mostly subject to taxation in the home country $-i$.

We first analyze the case where countries are symmetric in terms of the size of the multinational sectors and derive the following proposition:

**Proposition 3** *When countries are symmetric ($\theta^A = \theta^B$), the unique equilibrium in demarcation rules ($\alpha^{Ao}, \alpha^{Bo}$) is given by:*

$$\alpha^{Ao} = \alpha^{Bo} = \widetilde{\alpha} - \frac{\Delta}{8\phi}\left(\frac{1}{4} + \mu\left(\beta^E - \beta^D\right)\right)$$

**Proof.** See Appendix A. ∎

Proposition 3 states that the unique equilibrium is symmetric and has demarcation rules either below or above the social optimum $\widetilde{\alpha}$ depending on the parameters. Specifically, equilibrium demarcation rules are decreasing linearly in $\beta^E - \beta^D$ and take a value below the social optimum when $\beta^E - \beta^D$ is above a threshold level $-1/4\mu$ and above the social optimum when $\beta^E - \beta^D$ is below this threshold level.

To see the intuition for these results, first consider the special case $\beta^E = \beta^D$. Starting from symmetric demarcation rules, a marginal reduction in $\alpha^i$ transfers rents from the domestic government to domestic firms through an increase in $q^i$ but the domestic government exactly recuperates the lost revenue from foreign firms through a decrease in $q^{-i}$. The net effect of undercutting the demarcation rule of the other country is therefore a desirable transfer of rents from foreign firms to domestic firms, which drives demarcation rules *below* the social optimum. In the equilibrium, the marginal gain of undercutting in terms of increased private profits equals the marginal cost in terms of distortions in other policy dimensions.

Next, consider the general case $\beta^E \neq \beta^D$. As noted in the discussion of Lemma 4, a larger share of debt in standard financing makes it relatively more attractive for a country $i$ to raise its demarcation rule. This is because the marginal domestic firms that adopt standard financing as a result of the increase in $\alpha^i$ are subject to more taxation in their home country $i$ when $\beta^D$ is large relative to $\beta^E$ whereas the marginal foreign firms that abandon standard financing as a result of the increase in $\alpha^i$ are subject to less taxation in the host country $i$. Hence, a larger equity share intensifies the downward pressure on demarcation rules whereas a smaller equity share attenuates the downward pressure on demarcation rules. When the equity share is sufficiently low, equilibrium demarcation rules are *above* the social optimum.

In the special case $\phi = 0$, there is generally no interior equilibrium. Rather, when $\beta^E - \beta^D$ is above the threshold level $-1/4\mu$, the unique equilibrium is $\alpha^{Ao} = \alpha^{Bo} = 0$ and otherwise the unique equilibrium is $\alpha^{Ao} = \alpha^{Bo} = 1$. Intuitively, in the absence of other policy motives for using the demarcation rule, governments are only concerned with the value of their own demarcation rule relative to the demarcation rule of the other country. When $\beta^E - \beta^D$ is above the threshold, each country prefers to have a lower demarcation rule than the other country, which leads to an equilibrium where demarcation rules are at their minimum value so that all financial instruments except pure debt are classified as equity. When $\beta^E - \beta^D$ is below the threshold, each country prefers to have a higher demarcation rule than the other country, which leads to an equilibrium where demarcation rules are at their maximum value so that all financial instruments except pure equity are classified as debt [see Appendix B5].

We now turn to the asymmetric case where country $A$ is larger than country $B$ in terms of the size of the multinational sector.

**Proposition 4** *If countries are asymmetric ($\theta^A > \theta^B$), an equilibrium exists if the size asymmetry $|\theta^A - \theta^B|$ is not too large or the difference between the shares of debt and equity $|\beta^E - \beta^D|$ is not too large. Conditional on equilibrium existence, the unique equilibrium demarcation rules $(\alpha^{Ao}, \alpha^{Bo})$ can be characterized in the following way:*

*The* ***equilibrium distance*** *is implicitly determined by:*

$$\alpha^{Ao} - \alpha^{Bo} = \frac{\Delta(2\mu - 1)}{\phi} \left\{ \theta^A (q^A)^{\frac{3}{2}} - \theta^B (q^B)^{\frac{3}{2}} \right\}$$

*which implies that (i) the equilibrium distance is positive and increasing in the size asymmetry; (ii) the equilibrium distance is smaller than socially optimal distance; (iii) the equilibrium satisfies $q^B > q^A > 0$.*

*The* ***equilibrium location*** *is given by:*

$$\frac{\alpha^{Ao} + \alpha^{Bo}}{2} = \widetilde{\alpha} - \frac{\Delta}{2\phi} \left\{ \theta^A [(q^A)^{\frac{3}{2}} + \mu(\beta^E - \beta^D)(q^A)^{\frac{1}{2}}] + \theta^B [(q^B)^{\frac{3}{2}} + \mu(\beta^E - \beta^D)(q^B)^{\frac{1}{2}}] \right\}$$

*which implies that the equilibrium location is increasing linearly in the difference* $\beta^E - \beta^D$.

**Proof.** See Appendix A. ■

When asymmetric countries engage in non-cooperative policymaking the *equilibrium distance* is positive but smaller than the socially optimal distance. The international tax environment thus provides more protection against hybrid financing by firms in the large country $A$ than by firms in the small country $B$ as optimality requires but the difference is not sufficiently large. The *equilibrium location* of $\alpha^A$ and $\alpha^B$ is decreasing in the extent to which firms opting for standard finance rely on equity rather than internal debt to finance foreign investment and there is a (negative) threshold value of $\beta^E - \beta^D$ below (above) which the equilibrium location is lower (higher) than the socially optimal location.

Intuitively, size asymmetry represents a force that pulls equilibrium demarcation rules a part. More size asymmetry *ceteris paribus* makes it attractive for the large country $A$ to raise its demarcation rule because the benefit of lowering $q^A$ is larger with many firms in country $A$ and the cost of increasing $q^B$ is smaller with fewer firms in country $B$. For the same reason, more size asymmetry *ceteris paribus* makes it attractive for the small country $B$ to lower its demarcation rule. Once again, however, the decreasing marginal effectiveness of the distance $\alpha^A - \alpha^B$ in combating hybrid financing in country $A$ imposes an upper bound on the equilibrium distance in the sense that $\alpha^{Ao} - \alpha^{Bo}$ is sufficiently small to ensure that $q^A$ is strictly positive even when the size asymmetry is large. The equilibrium location is shaped by the capital structure parameters in much the same way as in the symmetric case. In the special case $\beta^E = \beta^D$, the equilibrium location is *below* the optimal location $\widetilde{\alpha}$. A positive value of $\beta^E - \beta^D$ adds to the benefit of lowering demarcation rules and reinforces the downward pressure on demarcation rules whereas a negative value of $\beta^E - \beta^D$ reduces the benefit of lowering demarcation rules and attenuates the downward pressure on demarcation rules.

In the special case $\phi = 0$, there are three types of equilibrium. One possibility is $\alpha^{Ao} = \alpha^{Bo} = 1$. This equilibrium can only prevail when debt has a larger share in standard financing than equity ($\beta^D > \beta^E$) and is more likely to prevail when debt is much more important than equity ($\beta^D >> \beta^E$), when the country size asymmetry is small ($\theta^A \simeq \theta^B$) and when the shadow value of public funds is large ($\mu >> 1$). In this case, both countries prefer to have a higher demarcation rule than the other country, which leads to a race-to-the-top in demarcation rules. Another possibility is $\alpha^{Ao} = \alpha^{Bo} = 0$. This equilibrium is more likely to prevail when the country size asymmetry is small ($\theta^A \simeq \theta^B$), when equity is much more important than debt ($\beta^E >> \beta^D$) and when the shadow value of public funds is small ($\mu \simeq 1$). In this case, both countries prefer to have a lower demarcation rule than the other country, which leads to a race-to-the-bottom in demarcation rules. Otherwise, the equilibrium satisfies either $0 = \alpha^{Bo} < \alpha^{Ao}$ or $\alpha^{Bo} < \alpha^{Ao} = 1$. This type of equilibrium is more likely to prevail when the country size asymmetry is large ($\theta^A >> \theta^B$). In this case, both countries prefer that the larger country has a higher demarcation rule than the smaller country due to the shared incentive to protect government revenues from hybrid financing by firms in the larger country, however, the distance preferred by the two countries differs. If parameters are such that the smaller country $B$ prefers a greater distance than the larger country $A$, the equilibrium is $\alpha^{Ao} > \alpha^{Bo} = 0$. Conversely, if the larger country $A$ prefers a greater distance than the smaller country $B$, the equilibrium is $\alpha^{Bo} < \alpha^{Ao} = 1$ [see Appendix B6].

## 6 Empirical evidence on capital structure

We have seen that the equilibrium location depends crucially on the relative importance of home country and host country taxation in cross-border investment, which, in turn, is determined by the relative size of parameters $\beta^E$ and $\beta^D$. The aim of this section is to draw on evidence on the capital structure of multinational firms to determine whether equilibrium demarcation rules above or below the social optimum is the most likely empirical outcome.

The most detailed empirical evidence on the financing of foreign investment derives from a dataset on German inbound and outbound foreign direct investment collected by the German Central Bank. Two papers report summary statistics from this dataset. Buettner and Wamser (2009) consider foreign affiliates of German firms and report an average *equity-asset ratio* of around 41%, an average *internal debt-asset ratio* of around 24% and an average *external debt-asset ratio* of around 35%. Ramb and Weichenreider (2005) conversely consider German affiliates of non-German firms and report an average *equity-asset ratio* of around 47%, an average *internal debt-asset ratio* of around 30% and an average *external debt-asset ratio* of around 23%. Empirical evidence on the financing of US foreign investment based on survey data from the BEA and tax return data from the IRS is less detailed but generally consistent with these patterns (Desai et al., 2004; Altshuler and Grubert, 2002).

The simplifying assumptions of the theoretical model complicates a direct translation of descriptive capital structure statistics into ranges of plausible values of parameters $\beta^D$ and $\beta^E$. Firstly, the theoretical model assumes that foreign investment is fully financed with internal funds in the form of either debt or equity whereas the empirical evidence shows that foreign investment is often financed with a large fraction of external debt. It would be relatively straightforward to modify the model to let the capital structure under standard financing include a fraction $\beta^W$ of external debt. In this extended model, a fraction $\beta^W$ of the normal return would be untaxed, which would make standard financing more attractive relative to hybrid financing and raise the share of firms opting for standard financing. We conjecture, however, that the key determinant of policy outcomes would still be the share of the normal return taxed in the host country relative to the share taxed in the home country, that is $\beta^E - \beta^D$. In both studies referred to above, the reported share of equity $\beta^E$ is considerably higher than the reported share of internal debt $\beta^D$. Secondly, the theoretical model holds tax rates constant and treats $\beta^D$ and $\beta^E$ as fixed. By contrast, actual tax rates vary between countries and observed capital structures are endogenous outcomes shaped by these tax rate differences. For the purposes of interpreting the results from the theoretical model with symmetric tax rates, we should ideally consider capital structure parameters that are undistorted by tax incentives but such parameters are not directly observable. It is useful to note, however, that the German corporate tax rate has consistently been among the highest in the world in recent decades. The tax environment has thus provided non-German firms with an incentive to finance investment in Germany with internal debt rather than equity and even in this sample the reported value of $\beta^E$ is considerably larger than the reported value of $\beta^D$, which suggests that the empirical regularity $\beta^E > \beta^D$ holds rather generally.

In sum, the empirical evidence seems to suggest that in an average case of cross-border investment, the share of the normal return that is subject to host country taxation is comfortably above the share that is subject to home country taxation. In terms of the predictions of the model, this implies that the most likely empirical outcome of non-cooperative line drawing between debt and equity is demarcation rules below the socially optimal level.

## 7 Concluding remarks

This paper has developed a theoretical model of strategic line drawing between debt and equity. In the model, firms are endowed with profitable foreign investment projects that may be financed either with a hybrid instrument or a combination of pure debt and equity. Hybrid instruments are costly to implement but generate a tax saving in the event that they are successfully categorized as debt in the host country and equity in the home country. Governments are faced with a continuum of financial instruments ranging from pure debt to pure equity and need to draw lines that delineate debt instruments and equity instruments for tax purposes. Policy choices affect the optimal financial choices of the firms, in particular the choice between hybrid and standard financing and the properties of the optimal hybrid instrument.

The first set of results characterized the globally optimal policies prevailing under cooperative policymaking. In this case, the central trade-off is that policy changes that reduce the scope for hybrid financing by firms in one country at the same time enlarge the scope for hybrid financing by firms in the other country. We thus obtain the intuitive result that the *optimal distance* between the demarcation rules of the two countries provides more protection against hybrid financing by firms in the relatively large country. The *optimal location* of the threshold values of the demarcation rules is always symmetric around the location that is preferred for other policy purposes.

The second set of results characterized the generally suboptimal policies prevailing under non-cooperative policymaking. The *equilibrium distance* between the threshold values of the demarcation rules is too small. This implies that the international tax environment provides too little protection against hybrid financing by firms in the relatively large country and too much protection against hybrid financing by firms in the relatively small country. The *equilibrium location* of the threshold values of the demarcation rules generally depends on the parameterization of the capital structure chosen by firms opting for standard financing. Assuming that foreign investment is financed by at least as much equity as internal debt, which is the empirically most relevant case, the equilibrium location is suboptimally low and too many financial instruments are characterized as equity for tax purposes. The intuition for this result is that policymakers endeavor to draw lines in ways that facilitate hybrid financing by domestic multinational firms and impedes hybrid financing by foreign multinational firms with a view to eroding foreign taxation of domestic firms and enforcing domestic taxation of foreign firms. This causes a competitive pressure to categorize a larger subset of financial instruments as equity or, equivalently, a race-to-the-bottom in the threshold values $\alpha^A$ and $\alpha^B$.

At a more general level, the results contribute to the emerging understanding in the international taxation literature that individual countries generally have mixed policy incentives when facing multinational firms engaged in international tax planning. For instance, Peralta, Wauthy and Ypersele (2006) show that it may be optimal for individual countries not to enforce transfer price regulation since this makes them more competitive in attracting mobile multinational firms. Similarly, Johannesen (2010) demonstrates that it may be optimal for individual countries to apply low or zero barriers to profit shifting to tax havens since this deters domestic firms from setting up conduit financing structures with foreign entities and induces foreign firms to set up conduit financing with domestic entities. These insights relate to the present paper where individual countries generally have an incentive to deviate from the policy that optimally combats tax planning because the successful use of hybrid instruments by domestic firms transfers rents from foreign governments to domestic firms, which increases domestic welfare at the expense of foreign welfare.

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# Appendix A

**Proof of Lemma 1:**

Maximization of (2) with respect to $z$ subject to $0 \leq z \leq 1$ yields the first-order condition:

$$\frac{\partial p^i(z)}{\partial z}(1 - p^{-i}(z)) - \frac{\partial p^{-i}(z)}{\partial z}p^i(z) + \lambda_0 - \lambda_1 = 0 \tag{5}$$

where $\lambda_0$ and $\lambda_1$ are the Lagrange multipliers associated with the constraints $z \geq 0$ and $z \leq 1$ respectively. Note that since $-1/\gamma < \alpha^A - \alpha^B < 1/\gamma$, there exists a hybrid $z'$ satisfying $p^i(z') > 0$ and $1 - p^{-i}(z') > 0$. The optimal hybrid $z^*$ must therefore satisfy that $p^i(z^*) > 0$ and $1 - p^{-i}(z^*) > 0$ in order not to be strongly dominated by $z'$. Also, note from (1) that for $j = i, -i$ it holds that $\partial p^j(z)/\partial z = \gamma$ for $\alpha^j - 1/2\gamma < z < \alpha^j + 1/2\gamma$ whereas $\partial p^j(z)/\partial z = 0$ for values of $z$ outside these bounds. Assume that $\lambda_0 > 0$. This requires that the optimal hybrid is $z = 0$ and implies that $\lambda_1 = 0$. Since $z \leq \alpha^i$ and $z \leq \alpha^{-i}$, it must hold that $1 - p^{-i}(z) \geq 1/2 \geq p^i(z)$. Moreover, since $0 < p^i(z) \leq 1/2$, it must hold that $\partial p^i(z)/\partial z = \gamma \geq \partial p^{-i}(z)/\partial z$. It follows that the left-hand side of (5) is positive. This contradiction implies that $\lambda_0 = 0$. Similarly, assume that $\lambda_1 > 0$. This requires that the optimal hybrid is $z = 1$ and implies that $\lambda_0 = 0$. Since $z \geq \alpha^i$ and $z \geq \alpha^{-i}$, it must hold that $1 - p^{-i}(z) \leq 1/2 \leq p^i$. Moreover, since $0 < 1 - p^{-i}(z) \leq 1/2$, it must hold that $\partial p^{-i}(z)/\partial z = \gamma \geq \partial p^i(z)/\partial z$. It follows that the left-hand side of (5) is negative. This contradiction implies that $\lambda_1 = 0$. Note that since, by assumption, $-1/\gamma < \alpha^A - \alpha^B < 1/\gamma$, there exists no hybrid satisfying $p^i(z) = 1 - p^{-i}(z) = 1$. This implies that either the optimal hybrid $z^*$ satisfies $p^i(z^*) < 1$ in which case $\partial p^i(z^*)/\partial z = \gamma$ or it satisfies $1 - p^{-i}(z^*) < 1$ in which case $\partial p^{-i}(z^*)/\partial z = \gamma$. It also follows from (5) that under our previous finding that $p^i(z^*) > 0$ and $1 - p^i(z^*) > 0$, if $\partial p^i(z^*)/\partial z > 0$ then $\partial p^{-i}(z^*)/\partial z > 0$ and *vice versa*. Hence, $\partial p^i(z^*)/\partial z = \partial p^{-i}(z^*)/\partial z = \gamma$. This, in turn, implies that $p^i(z^*) = 1 - p^{-i}(z^*)$. It now follows from (1) that $\alpha^A - z^* = z^* - \alpha^B$ and, consequently, that $z^* = (\alpha^A + \alpha^B)/2$.

**Proof of Lemma 2:**

(a) Differentiating $p^i$ with respect to $\alpha^i$ yields:

$$\frac{dp^i}{d\alpha^i} = \frac{\partial p^i}{\partial \alpha^i} + \frac{\partial p^i}{\partial z^*}\frac{\partial z^*}{\partial \alpha^i}$$

From (1) it follows that $\partial p^i/\partial \alpha^i = -\gamma$ and $\partial p^i/\partial z = \gamma$. Moreover, it follows from the expression for $z^*$ that $\partial z^*/\partial \alpha^i = 1/2$. Putting together these pieces, we obtain $dp^i/d\alpha^i = -\gamma/2$. Differentiating $p^{-i}$ with respect to $\alpha^i$ yields:

$$\frac{dp^{-i}}{d\alpha^i} = \frac{\partial p^{-i}}{\partial \alpha^i} + \frac{\partial p^{-i}}{\partial z^*}\frac{\partial z^*}{\partial \alpha^i}$$

From (1) it follows that $\partial p^{-i}/\partial \alpha^i = 0$ and $\partial p^{-i}/\partial z = \gamma$. Hence, the result $dp^{-i}/d\alpha^i = \gamma/2$ and $d(1 - p^{-i})/d\alpha^i = -\gamma/2$.

(b) Differentiating $q^i$ with respect to $\alpha^i$ yields:

$$\frac{dq^i}{d\alpha^i} = \frac{dp^i}{d\alpha^i}(1 - p^{-i}) - \frac{dp^{-i}}{d\alpha^i}p^i$$

Use $dp^i/d\alpha^i = -dp^{-i}/d\alpha^i = -\gamma/2$ from (a) and $p^i = 1 - p^{-i}$ from (c) to obtain $dq^i/d\alpha^i = -\gamma p^i$. Differentiating $q^i$ with respect to $\alpha^{-i}$ yields:

$$\frac{dq^i}{d\alpha^{-i}} = \frac{dp^i}{d\alpha^{-i}}(1 - p^{-i}) - \frac{dp^{-i}}{d\alpha^{-i}}p^i$$

Following the same procedure that we used to derive $dq^i/d\alpha^i$, we obtain $dq^i/d\alpha^{-i} = \gamma p^i$.
(c) It follows from Lemma 1 that $z^*$ is equidistant from $\alpha^A$ and $\alpha^B$, that is $z^* - \alpha^A = \alpha^B - z^*$. Using the expression for $p(z)$ derived in section 2.2, it is easy to see that $p^i = 1 - p^{-i}$. Using the definition $q^i = p^i(1 - p^{-i})$, it follows that $q^i = (p^i)^2$.

**Proof of Lemma 3:**

Use subresult (b) of Lemma 2, definitions of $x^{Hi}$ and $\widetilde{c}^i$ and the easily derivable result that $dx^{Hi}/d\alpha^j = -\theta^i \delta \tau^N \gamma p^i$ for $j = i, -i$ to restate $\partial W/\partial \alpha^i$ in the following way:

$$\frac{\partial W}{\partial \alpha^i} = \begin{cases} (1-\mu)\tau^N \left\{-\theta^i \delta\gamma\tau^N q^i p^i - \theta^{-i}\delta\gamma\tau^N q^{-i} p^{-i}\right\} \\ -\mu\tau^N \left\{-\theta^i \delta\gamma\tau^N q^i p^i - \theta^{-i}\delta\gamma\tau^N q^{-i} p^{-i}\right\} - \phi(\alpha^i - \widetilde{\alpha}) \end{cases}$$

Introduce the definition of $\Delta$ and subresult (c) of Lemma 2 to obtain the expression in Lemma **3**.

**Proof of Proposition 1:**

Evaluate the first-order conditions derived in Lemma **3** at $\theta = 1/2$. Compute $\partial W/\partial \alpha^A - \partial W/\partial \alpha^B = 0$ and rearrange to obtain the first optimality condition:

$$\alpha^{A*} - \alpha^{B*} = \frac{\Delta(2\mu - 1)}{\phi}\left\{(q^A)^{\frac{3}{2}} - (q^B)^{\frac{3}{2}}\right\}$$

At $\alpha^A = \alpha^B$, it holds that $q^A = q^B$, hence the optimality condition is satisfied. At any, $\alpha^A > \alpha^B$, it holds that $q^A < q^B$, hence the left-hand side is positive and the right-hand side is negative so that the optimality condition cannot be satisfied. At any, $\alpha^A < \alpha^B$, it holds that $q^A > q^B$, hence the left-hand side is negative and the right-hand side is positive so that the optimality condition cannot be satisfied. It follows that an optimal pair of demarcation rules must satisfy $\alpha^{A*} = \alpha^{B*}$. Now compute $\partial W/\partial \alpha^A + \partial W/\partial \alpha^B = 0$ and rearrange to obtain $(\alpha^{A*} + \alpha^{B*})/2 = \widetilde{\alpha}$. Insert $\alpha^{A*} = \alpha^{B*}$. It follows that an optimal pair of demarcation rules must satisfy $\alpha^{A*} = \alpha^{B*} = \widetilde{\alpha}$. Finally, differentiate the first-order condition derived in Lemma 3 and use subresult (b) of Lemma 2 to obtain:

$$\frac{\partial^2 W}{\partial(\alpha^i)^2} = -\Delta(2\mu - 1)\frac{3}{4}\gamma\left[q^i + q^{-i}\right] - \phi < 0$$

This establishes that the objective function is strictly concave and, hence, that the solutions to the first-order conditions $\alpha^{A*} = \alpha^{B*} = \widetilde{\alpha}$ constitute a global optimum.

**Proof of Proposition 2:**

Let $\alpha^{A*}$ and $\alpha^{B*}$ denote the demarcation rules that satisfy the first-order conditions $\partial W/\partial\alpha^A = 0$ and $\partial W/\partial\alpha^B = 0$ derived in Lemma **3.** Compute $\partial W/\partial\alpha^A - \partial W/\partial\alpha^B = 0$ and rearrange to obtain the first optimality condition:

$$\alpha^{A*} - \alpha^{B*} = \Omega(\alpha^{A*} - \alpha^{B*}) \tag{6}$$

where the function $\Omega(\alpha^{A*} - \alpha^{B*})$ is defined in the following way:

$$\Omega(\alpha^{A*} - \alpha^{B*}) \equiv \frac{2\Delta(2\mu-1)}{\phi}\left\{\theta^A(q^A)^{\frac{3}{2}} - \theta^B(q^B)^{\frac{3}{2}}\right\}$$

Note that $\Omega(0) > 0$ (since $q^A = q^B$) and that $\Omega(\infty) < 0$ (since $q^A = 0$ and $q^B > 0$). Also note that $\Omega(\alpha^{A*} - \alpha^{B*})$ is contiunously decreasing in $\alpha^{A*} - \alpha^{B*}$:

$$\Omega'(\alpha^{A*} - \alpha^{B*}) = \frac{-3\Delta\gamma(2\mu-1)}{\phi}\left\{\theta^A q^A + \theta^B q^B\right\} < 0$$

Hence, (6) uniquely defines the optimal distance $\alpha^{A*} - \alpha^{B*} > 0$. It follows directly that $q^B > q^A$ in the optimum. It must also hold that $q^A > 0$ since assuming that $\alpha^{A*} - \alpha^{B*}$ is sufficiently large to yield $q^A = 0$ gives rise to the contradiction that the left-hand side of (6) is positive and the right-hand side is negative.

Now, compute $\partial W/\partial\alpha^A + \partial W/\partial\alpha^B = 0$ and rearrange to obtain the second optimality condition**:**

$$\frac{\alpha^{A*} + \alpha^{B*}}{2} = \widetilde{\alpha} \tag{7}$$

Together, the two optimality conditions uniquely define $\alpha^{A*}$ and $\alpha^{B*}$.

Differentiate $\partial W/\partial\alpha^i = 0$ with respect to $\alpha_i$ and use subresult (b) of Lemma 2 to obtain:

$$\frac{\partial^2 W}{\partial(\alpha^i)^2} = -\Delta(2\mu-1)\frac{3}{2}\gamma\left[\theta^i q^i + \theta^{-i}q^{-i}\right] - \phi < 0 \tag{8}$$

This establishes that the objective function is strictly concave and, hence, that $\alpha^{A*}$ and $\alpha^{B*}$ indeed constitute a global optimum.

Finally, differentiate (6) with respect to $\alpha^{A*} - \alpha^{B*}$ and $\theta^A$ and use subresult (b) of Lemma 2 to obtain:

$$\frac{d(\alpha^{A*} - \alpha^{B*})}{d\theta^A} = \frac{2\Delta(2\mu-1)\left\{(q^A)^{\frac{3}{2}} + (q^B)^{\frac{3}{2}}\right\}}{\phi + 3\Delta\gamma(2\mu-1)\left\{\theta^A q^A + \theta^B q^B\right\}} > 0$$

Hence, the optimal distance $\alpha^{A*} - \alpha^{B*}$ is increasing in the size asymmetry.

**Proof of Lemma 4**:

Use definitions of $x^{Hi}$ and $\widetilde{c}^i$; the results derived in Lemma 2; the result that $\rho^{ii} = \rho^{i-i}$; and the identities $\beta^E + \beta^D = 1$ and $1 = \rho^{ii} + \rho^{i-i} + q^i$ to restate $\partial W^i/\partial\alpha^i$.

**Proof of Proposition 3:**

Let $\alpha^{Ao}$ and $\alpha^{Bo}$ denote the demarcation rules that satisfy the first-order conditions $\partial W^A/\partial\alpha^A = 0$ and $\partial W^B/\partial\alpha^B = 0$ derived in Lemma **4.** Evaluate $\partial W^A/\partial\alpha^A$ and $\partial W^B/\partial\alpha^B$ at $\theta = 1/2$, compute $\partial W^A/\partial\alpha^A - \partial W^B/\partial\alpha^B = 0$ and rearrange to obtain the first equilibrium condition:

$$\alpha^{Ao} - \alpha^{Bo} = \frac{\Delta(2\mu-1)}{2\phi}\left\{(q^A)^{\frac{3}{2}} - (q^B)^{\frac{3}{2}}\right\}$$

Assuming that $\alpha^{Ao} = \alpha^{Bo}$, it holds that $q^A = q^B$, hence the equilibrium condition is satisfied. Assuming that $\alpha^{Ao} > \alpha^{Bo}$, it holds that $q^A < q^B$, hence the contradiction that the left-hand side is positive and the right-hand side is negative. At any, $\alpha^{Ao} < \alpha^{Bo}$, it holds that $q^A > q^B$, hence the contradiction that the left-hand side is negative and the right-hand side is positive. It follows that $\alpha^{Ao} = \alpha^{Bo}$. This implies that $q^A = q^B = 0.25$.

Now, compute $\partial W^A/\partial\alpha^A + \partial W^B/\partial\alpha^B = 0$ and rearrange to obtain the second equilibrium condition:

$$\frac{\Delta}{2}\left\{-(q^A)^{\frac{3}{2}} - (q^B)^{\frac{3}{2}} + \mu\left(\beta^D - \beta^E\right)\left[(q^A)^{\frac{1}{2}} + (q^B)^{\frac{1}{2}}\right]\right\} = 2\phi\left\{\frac{\alpha^A + \alpha^B}{2} - \widetilde{\alpha}\right\}$$

Finally, define $q \equiv q^A = q^B$ and rearrange to restate as:

$$\frac{\alpha^A + \alpha^B}{2} = \widetilde{\alpha} - \frac{\Delta}{2\phi}\left\{q^{\frac{3}{2}} + \mu\left(\beta^E - \beta^D\right)q^{\frac{1}{2}}\right\}$$

which may easily be rearranged to yield the expression stated in the proposition.

Differentiate $\partial W^i/\partial\alpha^i = 0$ with respect to $\alpha^i$ and use subresult (b) of Lemma 2 to obtain:

$$\frac{\partial^2 W^i}{\partial(\alpha^i)^2} = -\frac{3}{4}\gamma\left\{(\mu-1)q^i + \mu q^{-i}\right\} - \phi < 0$$

This establishes that the objective function is strictly concave and, hence, that $\alpha^{Ao}$ and $\alpha^{Bo}$ indeed constitute a Nash equilibrium.

**Proof of Proposition 4:**

Let $\alpha^{Ao}$ and $\alpha^{Bo}$ denote the demarcation rules that satisfy the first-order conditions $\partial W^A/\partial\alpha^A = 0$ and $\partial W^B/\partial\alpha^B = 0$ derived in Lemma **4.** Compute $\partial W^A/\partial\alpha^A - \partial W^B/\partial\alpha^B = 0$ and rearrange to obtain the first equilibrium condition:

$$\alpha^{Ao} - \alpha^{Bo} = \Gamma(\alpha^{Ao} - \alpha^{Bo}) \tag{9}$$

where the function $\Gamma(\alpha^{Ao} - \alpha^{Bo})$ is defined in the following way:

$$\Gamma(\alpha^{Ao} - \alpha^{Bo}) = \frac{\Delta(2\mu-1)}{\phi}\left\{\theta^A(q^A)^{\frac{3}{2}} - \theta^B(q^B)^{\frac{3}{2}}\right\}$$

Note that $\Gamma(0) > 0$ (since $q^A = q^B$) and that $\Gamma(\infty) < 0$ (since $q^A = 0$ and $q^B > 0$). Also note that $\Gamma(\alpha^{A*} - \alpha^{B*})$ is continuously decreasing in $\alpha^{Ao} - \alpha^{Bo}$:

$$\Gamma'(\alpha^{Ao} - \alpha^{Bo}) = \frac{-3\Delta\gamma(2\mu-1)}{2\phi}\left\{\theta^A q^A + \theta^B q^B\right\} < 0$$

Hence, (9) uniquely defines the optimal distance $\alpha^{Ao} - \alpha^{Bo} > 0$. It follows directly that $q^B > q^A$ in the equilibrium. It must also hold that $q^A > 0$ since assuming that $\alpha^{Ao} - \alpha^{Bo}$ is such that $q^A = 0$ gives rise to the contradiction that the left-hand side of (6) is positive and the right-hand side is negative. Also, note that since $\Gamma(\cdot)$ is larger than the function $\Omega(\cdot)$ defined in the proof of Proposition 2 for any distance $\alpha^A - \alpha^B$, it holds that $\alpha^{Ao} - \alpha^{Bo} < \alpha^{A*} - \alpha^{B*}$.

Now, compute $\partial W^A/\partial\alpha^A + \partial W^B/\partial\alpha^B = 0$ and rearrange to obtain the second equilibrium location:

$$\frac{\alpha^{Ao} + \alpha^{Bo}}{2} = \widetilde{\alpha} - \frac{\Delta}{2\phi}\left\{\theta^A[(q^A)^{\frac{3}{2}} + \mu(\beta^E - \beta^D)(q^A)^{\frac{1}{2}}] + \theta^B[(q^B)^{\frac{3}{2}} + \mu(\beta^E - \beta^D)(q^B)^{\frac{1}{2}}]\right\}$$

Since the distance is determined in (9) and the distance determines $q^A$ and $q^B$, it is clear that the equilibrium location decreases linearly with $\beta^E - \beta^D$. Together, the two equilibrium conditions define $\alpha^{Ao}$ and $\alpha^{Bo}$.

Differentiate $\partial W^i/\partial\alpha^i = 0$ with respect to $\alpha_i$ and use subresult (b) of Lemma 2 to obtain:

$$\frac{\partial^2 W^i}{\partial(\alpha^i)^2} = -\frac{3}{2}\gamma\left\{(\mu-1)\theta^i q^i + \mu\theta^{-i}q^{-i}\right\} - \gamma\frac{\mu}{4}(\beta^D - \beta^E)(\theta^i - \theta^{-i}) - \phi \lesseqgtr 0$$

The first term is unambiguously negative whereas the sign of the second term depends on parameters $\beta^D$, $\beta^E$, $\theta^i$ and $\theta^{-i}$. It is easy to see that when $|\beta^D - \beta^E|$ is not too large or when $|\theta^i - \theta^{-i}|$ is not too large, the first term dominates the second term and the second derivative is negative. In this case, the objective function is strictly concave and it follows that $\alpha^{Ao}$ and $\alpha^{Bo}$ constitute a Nash equilibrium. Finally, differentiate (9) with respect to $\alpha^{Ao} - \alpha^{Bo}$ and $\theta^A$ and use subresult (b) of Lemma 2 to obtain:

$$\frac{d(\alpha^{Ao} - \alpha^{Bo})}{d\theta^A} = \frac{2\Delta(2\mu-1)\left\{(q^A)^{\frac{3}{2}} + (q^B)^{\frac{3}{2}}\right\}}{2\phi + 3\Delta\gamma(2\mu-1)\left\{\theta^A q^A + \theta^B q^B\right\}} > 0$$

Hence, the equilibrium distance $\alpha^{Ao} - \alpha^{Bo}$ is increasing in the size asymmetry.

# Appendix B

**[B1] Derive $p(z)$ [section 2.1]:**

By definition of $p(z)$, we have:

$$p(z) = prob(Z > \alpha)$$

Use that $Z \equiv z + \varepsilon$ and rearrange to obtain:

$$p(z) = prob(\varepsilon < z - \alpha)$$

Use that $\varepsilon$ is uniformly distributed over the inteval $[-1/2\gamma, 1/2\gamma]$ to derive (1).

**[B2] Derive $\partial W/\partial\alpha^i$ [section 4]:**

Aggregate welfare is given by:

$$W = \sum_{i=A,B} \Pi^i + \mu R^i - \frac{\phi}{2}(\alpha^i - \widetilde{\alpha})^2$$

Using the expressions for $\Pi_i$ and $R_i$ indicated in the main text, this may be restated as:

$$W = \begin{cases} \left\{x^{HA} + x^{SA} + x^{HB} + x^{SB}\right\}\pi \\ +\left\{x^{HA} + x^{SA} + x^{HB} + x^{SB}\right\}(\mu - 1)\tau^P \\ +\left\{(1-q^A)x^{HA} + x^{SA} + (1-q^B)x^{HB} + x^{SB}\right\}(\mu - 1)\tau^N \\ -\psi^A - \psi^B - \frac{\phi}{2}(\alpha^A - \widetilde{\alpha})^2 - \frac{\phi}{2}(\alpha^B - \widetilde{\alpha})^2 \end{cases}$$

where we have used the identity $\rho^{ii} + \rho^{i-i} + q^i = 1$. The first line captures that all firms earn pre-tax profits $\pi$, the second line captures that all firms pay taxes on the pure profits generated by their foreign investment and the third line captures that firms using hybrid financing pay taxes on the normal return to capital with probability $(1-q^i)$ whereas firms using standard financing pay taxes on the normal return to capital with certainty. Differentiating with respect to $\alpha^i$ yields:

$$\frac{dW}{d\alpha^i} = \begin{cases} (1-\mu)\tau^N\left\{\frac{dq^i}{d\alpha^i}x^{Hi} + \frac{dq^{-i}}{d\alpha^i}x^{H-i}\right\} \\ +(1-\mu)\tau^N\left\{\frac{dx^{Hi}}{d\alpha^i}q^i + \frac{dx^{H-i}}{d\alpha^i}q^{-i}\right\} - \frac{d\psi^i}{d\alpha^i} - \frac{d\psi^j}{d\alpha^i} \\ -\phi(\alpha^i - \widetilde{\alpha}) \end{cases}$$

where we have used $dx^{Hj}/d\alpha^i = -\ dx^{Sj}/d\alpha^i$ for $j = i, -i$. The first line captures the mechanical effects of a change in $\alpha_i$ holding the number of firms using hybrid financing and standard financing fixed and the second line captures the 'behavioral effects' of a change in $\alpha_i$ through changes in the mode of finance. Finally, use that $dx^{Hj}/d\alpha^i = \theta^j\delta(d\widetilde{c}^j/d\alpha^i)$; that $d\psi^j/d\alpha^i = \theta^j\delta\widetilde{c}^j(d\widetilde{c}^j/d\alpha^i)$ and that $\widetilde{c}^j = q^j\tau^N$ for $j = i, -i$ to simplify and obtain (3).

**[B3] Derive optimal policy under asymmetry when $\phi = 0$ [section 4]:**

Evaluating $\partial W/\partial\alpha^i = 0$ at $\phi = 0$ and simplifying yields:

$$\theta^A(q^A)^{\frac{3}{2}} - \theta^B(q^B)^{\frac{3}{2}} = 0 \tag{10}$$

By the argument in the proof of Proposition **2**, a solution to this equation is indeed an optimum.

Minimization of the expected number of firms successfully implementing a hybrid instrument may formally be written as:

$$\min_{\alpha^A,\alpha^B} x^{HA}q^A + x^{HB}q^B$$

Using definitions of $x^{HA}$ and $x^{HB}$ the problem may be restated as

$$\min_{\alpha^A,\alpha^B} \theta^A \delta\tau^N (q^A)^2 + \theta^B \delta\tau^N (q^B)^2$$

The first-order conditions to this problem are:

$$2\delta\tau^N \left\{ \theta^A q^A \frac{\partial q^A}{\partial\alpha^i} + \theta^B q^B \frac{\partial q^B}{\partial\alpha^i} \right\} = 0 \tag{11}$$

for $i = A, B$. Using the auxiliary results derived in Lemma 2, it is easy to show that the two first-order conditions (11) are identical and equivalent to (10).

**[B4] Derive $\partial W^i/\partial\alpha^i$ [section 5]:**

The objective function of country $i$ is given by:

$$W^i = \Pi^i + \mu R^i - \frac{\phi}{2}(\alpha^i - \widetilde{\alpha})^2$$

Using the expressions for $\Pi_i$ and $R_i$ indicated in the main text, this may be restated as

$$W^i = \left\{ \begin{array}{c} x^{Hi}\left\{ q^i(\pi - \tau^P) + (1-q^i)(\pi - \tau^P - \tau^N) \right\} - \psi^i + x^{Si}(\pi - \tau^P - \tau^N) \\ +\mu \left[ x^{Hi}\tau^N \rho^{ii} + x^{Si}\tau^N\beta^D + x^{H-i}\left\{ \rho^{-ii}\tau^N + \tau^P \right\} + x^{S-i}\left\{ \beta^E\tau^N + \tau^P \right\} \right] \\ -\frac{\phi}{2}(\alpha^i - \widetilde{\alpha})^2 \end{array} \right.$$

Differentiating with respect to $\alpha^i$ yields:

$$\frac{dW^i}{d\alpha^i} = \left\{ \begin{array}{l} \frac{dx^{Hi}}{d\alpha^i}\left\{ q^i\tau^N + \mu\tau^N(\rho^{ii} - \beta^D) \right\} + \frac{dx^{H-i}}{d\alpha^i}\left\{ \mu\tau^N(\rho^{-ii} - \beta^E) \right\} + \\ \frac{dq^i}{d\alpha^i}x^{Hi}\tau^N + \frac{d\rho^{ii}}{d\alpha^i}\mu x^{Hi}\tau^N + \frac{d\rho^{-ii}}{d\alpha^i}\mu x^{H-i}\tau^N - \frac{d\psi^i}{d\alpha^i} - \phi(\alpha^i - \widetilde{\alpha}) \end{array} \right.$$

where we have used that $dx^{Hj}/d\alpha^i = -\, dx^{Sj}/d\alpha^i$ for $j = i, -i$. Finally, use that $d\psi^j/d\alpha^i = \theta^j\delta\widetilde{c}^j(d\widetilde{c}^j/d\alpha^i)$; that $\widetilde{c}^j = \tau^N q^j$; and that $dx^j/d\alpha^i = \theta^j\delta(d\widetilde{c}^j/d\alpha^i)$ for $j = i, -i$ to simplify and obtain 4.

**[B5] Derive equilibrium policy under symmetry when $\phi = 0$ [section 5]:**

Setting $\phi = 0$ in the first-order condition derived in Lemma 4 and imposing size symmetry $\theta^A = \theta^B$ yields the following expression:

$$\frac{\partial W^i}{\partial\alpha^i} = \frac{\Delta}{2}\left\{ \begin{array}{c} (\mu-1)(q^i)^{\frac{3}{2}} + \frac{\mu}{2}\left(\beta^D - \beta^E\right)(q^i)^{\frac{1}{2}} \\ -\mu(q^{-i})^{\frac{3}{2}} + \frac{\mu}{2}\left(\beta^D - \beta^E\right)(q^{-i})^{\frac{1}{2}} \end{array} \right\} \tag{12}$$

Differentiate this expression with respect to $\alpha^i$ to obtain

$$\frac{\partial^2 W^i}{\partial(\alpha^i)^2} = -\frac{3}{4}\Delta\gamma\left\{ (\mu-1)q_i + \mu q_{-i} \right\} < 0$$

These results imply that there is a unique distance $\alpha^i - \alpha^{-i}$ that maximizes $W^i$. This preferred distance depends on $\mu$ and on $\beta^D - \beta^E$. Evaluate (12) at $\alpha^i = \alpha^{-i}$ where $q^i = q^{-i} = 0.25$ and set equal to zero to show that preferred distance is zero exactly when $\beta^D - \beta^E = 1/4\mu$. In this case, any pair of demarcation rules $\alpha^i = \alpha^{-i}$ constitute an equilibrium. When $\beta^D - \beta^E > 1/4\mu$, it holds that $W^i/\partial\alpha^i > 0$ at $\alpha^i = \alpha^{-i}$, hence the preferred distance $\alpha^i - \alpha^{-i}$ must be positive. Since both countries prefer to have a larger demarcation rule than the other country, the unique equilibrium is at $\alpha^i = \alpha^{-i} = 1$. Similarly, when $\beta^D - \beta^E < 1/4\mu$, it holds that $W^i/\partial\alpha^i < 0$ at $\alpha^i = \alpha^{-i}$, hence the preferred distance $\alpha^i - \alpha^{-i}$ must be negative. Since both countries prefer to have a lower demarcation rule than the other country, the unique equilibrium is at $\alpha^i = \alpha^{-i} = 0$.

**[B6] Derive equilibrium policy under asymmetry when $\phi = 0$ [section 5]:**

Setting $\phi = 0$ in the first-order condition derived in Lemma 4 yields the following expression:

$$\frac{\partial W^i}{\partial \alpha^i} = \Delta \left\{ \begin{array}{c} (\mu-1)\theta^i(q^i)^{\frac{3}{2}} + \frac{\mu}{2}\left(\beta^D - \beta^E\right)\theta^i(q^i)^{\frac{1}{2}} \\ -\mu\theta^{-i}(q^{-i})^{\frac{3}{2}} + \frac{\mu}{2}\left(\beta^D - \beta^E\right)\theta^{-i}(q^{-i})^{\frac{1}{2}} \end{array} \right\} \tag{13}$$

Differentiate this expression with respect to $\alpha^i$ to obtain

$$\frac{\partial^2 W^i}{\partial(\alpha^i)^2} = -\frac{3}{2}\gamma\left\{(\mu-1)\theta^i q^i + \mu\theta^{-i}q^{-i}\right\} - \gamma\frac{\mu}{4}(\beta^D - \beta^E)(\theta^i - \theta^{-i}) \tag{14}$$

In the following we assume that the second derivative is strictly negative for all relevant values of $\left\{\alpha^A, \alpha^B\right\}$ such that (13) uniquely defines a distance $\alpha^i - \alpha^{-i}$ that maximizes the objective of country $i$. First, note that if $\partial W^i/\partial\alpha^i = 0$ it is only coincidentally the case that $\partial W^{-i}/\partial\alpha^{-i} = 0$, hence there is generally no equilibrium where both demarcation rules are interior. In other words, it is generally not the case that the distance preferred by country $i$ coincides with the distance preferred by country $-i$. Next note that:

$$\left\{\frac{\partial W^A}{\partial \alpha^A}|\alpha^A = \alpha^B\right\} - \left\{\frac{\partial W^B}{\partial \alpha^B}|\alpha^A = \alpha^B\right\} = (2\mu - 1)(\theta^A - \theta^B)q^{\frac{3}{2}} > 0 \tag{15}$$

where we have used that $q \equiv q^A = q^B$.

Consider the case $\alpha^A = \alpha^B = 0$. It is easy to see that this is the unique Nash equilibrium if and only if:

$$\left\{\frac{\partial W^A}{\partial \alpha^A}|\alpha^A = \alpha^B\right\} < 0 \tag{16}$$

Under this condition, starting from symmetry both countries would prefer to undercut the demarcation rule of the other country, however, at $\alpha^A = \alpha^B = 0$ demarcation rules cannot be reduced any further. Clearly, symmetric policies $\alpha^A = \alpha^B > 0$ cannot be an equilibrium since both countries have an incentive to undercut. Similarly, asymmetric policies $\alpha^i > \alpha^{-i}$ cannot be an equilibrium since, under the assumption that second derivatives are negative, it holds that $\partial W^i/\partial\alpha^i < 0$ so that country $i$ can increase the value of the objective function by lowering $\alpha_i$. Rearranging (16) while using that $q^A = q^B = 0.25$ under symmetric policies yields:

$$\left(\beta^E - \beta^D\right) > \frac{1}{2}\left\{\frac{(\mu-1)}{\mu}\theta^A - \theta^B\right\}$$

It is easy to show that this condition is more likely to be satisfied when $\beta^E - \beta^D$ is large, when $\theta^A$ is small and when $\mu$ is small.

Consider the case $\alpha^A = \alpha^B = 1$. By a similar argument, it is easy to see that this is an equilibrium if and only if

$$\left\{ \frac{\partial W^B}{\partial \alpha^B} | \alpha^A = \alpha^B \right\} > 0 \tag{17}$$

Rewriting in a similar fashion yields:

$$\left( \beta^E - \beta^D \right) < \frac{1}{2} \left\{ \frac{(\mu - 1)}{\mu} \theta^B - \theta^A \right\}$$

Note that the right-hand side is unambiguously negative, hence this equilibrium is only possible when $\beta^D > \beta^E$. Moreover, the condition is more likely to be satisfied when $\beta^E - \beta^D$ is small, when $\theta^A$ is small and when $\mu$ is large.

When neither (16) nor (17) holds, it must hold that:

$$\left\{ \frac{\partial W^A}{\partial \alpha^A} | \alpha^A = \alpha^B \right\} > 0 > \left\{ \frac{\partial W^B}{\partial \alpha^B} | \alpha^A = \alpha^B \right\}$$

There are three possible cases (i)-(ii), which we address in turn. In case (i), it holds that:

$$\frac{\partial W^B}{\partial \alpha^B} < 0 \text{ for } \frac{\partial W^A}{\partial \alpha^A} = 0 \Leftrightarrow \frac{\partial W^A}{\partial \alpha^A} > 0 \text{ for } \frac{\partial W^B}{\partial \alpha^B} = 0$$

The implication sign follows from the assumption that second derivatives are negative. Both countries would optimally choose a positive distance $\alpha^A - \alpha^B > 0$, however, the distance preferred by country $A$ is smaller than the distance preferred by country $B$. The unique equilibrium in this case is $\alpha^B = 0$ and $\alpha^A$ at the positive level that satisfies $\partial W^A / \partial \alpha^A = 0$ given $\alpha^B = 0$. At these policies, country $A$ is at its optimum whereas country $B$ would prefer a larger distance but cannot increase the distance by reducing $\alpha^B$ further. In case (ii), it holds that:

$$\frac{\partial W^B}{\partial \alpha^B} > 0 \text{ for } \frac{\partial W^A}{\partial \alpha^A} = 0 \Leftrightarrow \frac{\partial W^A}{\partial \alpha^A} < 0 \text{ for } \frac{\partial W^B}{\partial \alpha^B} = 0$$

By a similar argument, this implies that the unique equilibrium is $\alpha^A = 1$ and $\alpha^B$ at the level that satisfies $\partial W^B / \partial \alpha^B = 0$ given $\alpha^A = 1$. Finally, there is case (iii), which is the knife-edge case where:

$$\frac{\partial W^B}{\partial \alpha^B} = 0 \text{ for } \frac{\partial W^A}{\partial \alpha^A} = 0 \Leftrightarrow \frac{\partial W^A}{\partial \alpha^A} = 0 \text{ for } \frac{\partial W^B}{\partial \alpha^B} = 0$$

This implies that the preferred distance of country $A$ coincides with the preferred distance of country $B$. In this case, any pair of demarcation rules located with this preferred distance constitutes a Nash equilibrium.

**Figure 1**

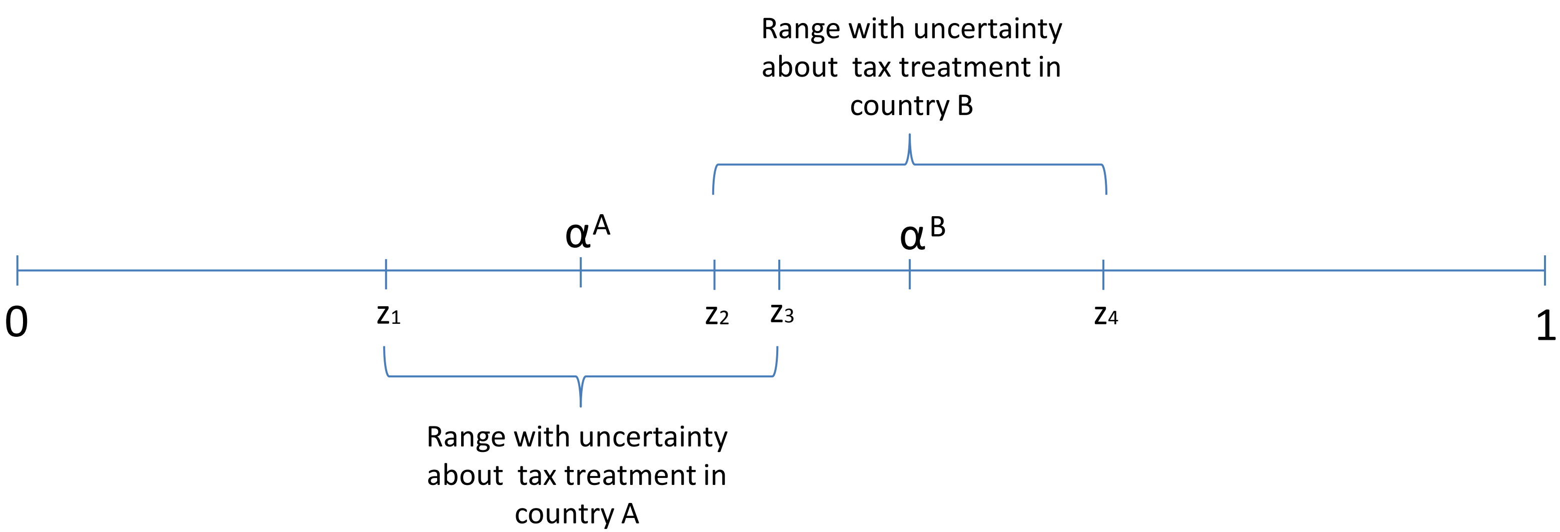
Range with uncertainty
about tax treatment in
country B
α^A
α^B
0
z1
z2
z3
z4
1
Range with uncertainty
about tax treatment in
country A

**Table 1: Tax treatment and tax bills**

| Type of financing | Tax environment | Possible outcomes | | Tax bills | | |
|---|---|---|---|---|---|---|
| | | home | host | home | host | total |
| hybrid | stochastic | equity | debt | 0 | t(y-rk) | t(y-rk) |
| | | equity | equity | 0 | ty | ty |
| | | debt | debt | trk | t(y-rk) | ty |
| standard | deterministic | $\beta^D$ debt / $\beta^E$ equity | | $t\beta^D rk$ | $t(y-\beta^D rk)$ | ty |