Exchange Rate Dynamics in a Small Open Economy

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Abstract: I study the behavior of the nominal exchange rate in a small open economy with wage rigidity. When there are two traded goods (imports and exports) a consequence of the small open economy assumption is an extreme form of consumption home-bias. Despite assuming the law of one price holds for each good, (consumption based) purchasing power parity does not hold and this frees the domestic real interest rate from the exogenous world rate. In this environment, as long as money demand is not unit interest elastic, a permanent unanticipated change in the supply of money generates additional dynamics in the exchange rate. If the substitution elasticity between goods is unity and money demand interest inelastic, a rise in the supply of money causes the short-run exchange rate to overshoot. When the two traded goods are close substitutes (empirically the more plausible case), the home country accumulates net foreign assets and exchange rate overshooting is magnified. Asset accumulation therefore amplifies the reaction of the nominal exchange rate to monetary shocks.

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I. Introduction

The idea that monetary shocks generate additional dynamics in the exchange rate has a long standing tradition in international macroeconomics. Dornbusch’s (1976) original contribution was to show that, in a Mundell-Fleming model with well functioning asset markets, these dynamics are a natural consequence of financial arbitrage.\(^1\) However, the problems with this type of analysis are almost as widely recognized - the structure of Dornbusch’s economy is one of directly postulated relationships that cannot account for the intertemporal nature of economic decision making. Nevertheless, the argument for exchange rate overshooting is persuasive.\(^2\) I investigate Dornbusch’s overshooting result in a new open economy macro setting. This approach stresses the use of intertemporal optimization and includes a rationale for the nominal rigidities (and demand determined output in the short-run) inherent the Mundell-Fleming approach through monopolistically competitive product markets.\(^3\)

Underpinning the analysis is the idea that small open economies exporting a specialized output have different consumption baskets than their larger neighbors, i.e. there is consumption home-bias. Although this is an implicit assumption in the Mundell-Fleming approach, analyzing small open economies in a micro-founded setting, it is more usual to adopt one of two other types of goods market structure. Either there is a common endowment or domestic production is introduced by means of a nontraded goods sector. The second formulation has the obvious advantage in that it becomes possible to trace out the effects of monetary policy on relative prices. In this more general case, even if the law of one price (LOP) holds for the traded good, purchasing power parity (PPP) does not hold so that the reaction of the real exchange rate can also be analyzed.\(^4\) However, breaking the PPP restriction also has important implications for the behavior of the nominal exchange rate, which is the focus of this paper.\(^5\)

One important feature of the traded-nontraded goods structure is that because trade only occurs in a common good it does not imply the domestic economy can influence world prices. However,

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\(^2\)Some of the empirical literature is not so positive. See, for example, Eichenbaum and Evans (1995) on ‘delayed overshooting’. In an interesting paper Bluedorn and Bowdler (2005) have recently questioned these results.

\(^3\)I take an explicitly macro approach and am interested in the initial reaction of the exchange rate. This contrasts with the new micro exchange rate economics approach which considers the role of ‘dispersed information’. See, for e.g., Evans and Lyons (2002).

\(^4\)The traded-nontraded structure has been used extensively; for example, in Jones (1974) a nontraded goods sector is introduced as a necessary component for models of international trade and in Dornbusch (1983) nontraded goods force a wedge between the domestic and foreign real interest rates in an analysis of the real exchange rate. It is also possible to assume both traded and nontraded goods are produced if one were interested in sector spillovers within the domestic economy.

\(^5\)In particular, an argument for nominal exchange rate overshooting appears in the Appendix of Obstfeld and Rogoff (1995). Hau (2002) focuses on real exchange rate overshooting. Lane (2001) also use a traded-nontraded structure in an analysis of the real exchange rate. All of these papers assume price rigidities so that real and nominal exchange rate overshooting amount to the same thing.
when a small open economy produces a specialized output, export demand depends explicitly on the relative price. When this assumption is modeled correctly, an implication must then be that the share of domestic goods in the foreign economy’s consumption basket is negligible, or rather, consumption baskets are ‘non-identical’. If consumption baskets were identical this would then have the counter-factual implication that the domestic economy exported all of it’s output whilst consuming only the foreign good. Therefore, when there are two traded goods, non-identical preferences are a consequence of the small open economy assumption and there is an extreme form of consumption home-bias, which also breaks the PPP assumption.6

This simple argument forms the basis for studying the nominal exchange rate in this paper. I assume an ‘import-export’ structure, focusing on the implications of consumption home-bias (and deviations from PPP) for the behavior of the nominal exchange rate when there is wage rigidity. Some previous work has commented on the implications of consumption home-bias for the behavior of the nominal exchange rate and is relevant. Most notably, Warnock (2003) introduces a form of consumption home-bias (named home-product bias) by altering the Dixit-Stiglitz (1977) preferences used in the original Redux model of Obstfeld and Rogoff (1995).7

In this case, consumption home-bias generates a nominal exchange rate overshooting result following a change in the money supply.8 As with much recent analysis, the model Warnock (2003) presents is of two interdependent economies (of equal size) so that the ‘world’ rate of interest is also endogenous. In this case it is usual to simplify the analysis by assuming preferences are identical.9 But this has the implication that consumption based PPP holds and in this case each country faces an identical real interest rate and consumption growth rate. Given an uncovered interest parity (UIP) condition, a direct implication is that additional exchange rate dynamics cannot be generated.10

Interestingly, the specialized outputs assumption has been adopted in the small open economy setting recently, but only in a limited number of papers. For example, Parrado and Velasco (2002), Monacelli (2004) and Gali and Monacelli (2005) all use the import-export structure.

6Although it is questionable whether all small open economies can alter their terms of trade there is an intuitive appeal to this setup. In countries such as the US imported goods form only a minor proportion of the overall consumption basket whereas in smaller economies the consumption basket is often comprised more heavily of imported goods. Economies with this import-export structure are sometimes referred to as semi-small open economies because the world interest rate is exogenous but the terms of trade are endogenized.

7As with Obstfeld and Rogoff (1995) the degree of substitution between domestic and foreign goods is equal to the monoplistic mark-up. This is somewhat restrictive as it is then necessary to assume the substitution elasticity between goods is larger than one.

8Another paper that reproduces the overshooting result but by introducing a pricing to market assumption (and therefore deviations from LOP) is Betts and Devereux (2000). Benigno and Theonissen (2002) calibrate a model that includes all three of the features that generate movements in the real exchange rate; that is, nontraded goods, consumption home-bias and pricing to market. Finally, Chari et al. (2002) use a more elaborated model which includes capital accumulation and nominal wage and price rigidities.

9Warnock’s (2003) preferences nest this as a special case.

10This is not to say that changes in the exchange rate are proportional to changes in the money supply when nominal rigidities are present, as I discuss below.
The first focuses on optimal monetary policy in the face of demand and supply shocks, similar
in spirit to Poole’s (1970) analysis, the second focuses on the behavior of the real and nominal
exchange rates in relation to the work of Mussa (1986) and Gali and Monacelli (2005) examine
the role of inflation targeting. But all of these papers limit money’s role to that of a unit
of account and thus the economy is cashless, in the sense of Woodford (2003). Money is not
modelled explicitly and therefore plays only a passive role. Here, I model money by assuming
it enters directly into the utility function of the domestic household. This introduces a role
for the interest elasticity of money demand, a parameter stressed in Dornbusch (1976) as being
important in generating exchange rate dynamics.

Consistent with previous work I find additional dynamics in the nominal exchange rate arise
from a permanent unanticipated increase in the money supply. For example, it is not neces-
sary to introduce a nontraded sector to generate the type of results found in Dornbusch (1976)
because when a small open economy exports a specialized output PPP fails and this produces
the desired result. In particular, when money demand is interest inelastic and the substitu-
tion elasticity between goods is unity there is an initial degree of overshooting in the nominal
exchange rate. Overshooting depends on the extent to which the real side of the economy
(and hence the nominal wage rigidity) interacts with the monetary side, and this is shown to
depend entirely on the interest elasticity of money demand. An increase in the money supply
depresses the real interest rate and stimulates domestic output. When the interest elasticity
of money demand is suitably low this creates a liquidity effect reducing the short-run nominal
interest rate. Overshooting is then a consequence of financial arbitrage.

However, the story changes slightly when the substitution elasticity between goods is not re-
stricted to be unity. In this case, despite the interest elasticity of money demand having to be
different from unity to generate exchange rate dynamics, the elasticity of substitution between
goods plays an important role in determining whether the exchange rate under or overshoots.
If goods are close substitutes (empirically the more plausible case) the exchange rate overshoots,
but by more than the unit substitution case. The reason for this result is shown to be asset
accumulation. With a non-unit substitution elasticity money shocks cause both current ac-
count and exchange rate dynamics. Asset accumulation causes long-run changes in output (i.e.
changes in output beyond the length of the assumed wage rigidity) which, in turn, affects the
reaction of the current period real interest rate, altering the size of the liquidity effect.12 That
asset accumulation magnifies the reaction of the exchange rate to monetary shocks is contrary
to the standard idea in Obstfeld and Rogoff (1995). As noted in Kollmann (2001), there, the
exchange rate reacts less than proportionately to the change in the money supply. Thus it

11Kollmann’s (2001) analysis is an exception to this rule being that it’s focus is on the role of the money
supply. But that analysis is a strictly quantitative exercise accounting for underlying micro elements found both
in Betts and Devereux (2000) and Chari et al. (2002).
12Overshooting is therefore driven by the interaction of two elasticities allowing for four possibilities. For
example, it is entirely consistent to have exchange rate overshooting when money demand is interest elastic, so
long as the traded goods are viewed as compliments.
seems home-bias has some important ramifications.

The rest of the paper is organized as follows. In Section 2 I describe the model. Section 3 works through the solution method for the case of unit elastic consumer demands and examines the reaction of the nominal exchange rate to permanent unanticipated increases in the money supply. Section 4 considers the more general constant elasticity of substitution (CES) case to help better understand the interaction between the current account and exchange rate when there are money shocks. Section 5 concludes.

II. Model Economy

The structure of the economy is closely related to the class of new open economic macroeconomics models that assume optimizing agents, (one period) nominal rigidities and monopolistic competition. There are two economies (domestic and foreign). Both economies consist of a continuum of \( j \in [0, 1] \) households which supply a differentiated labor type. Households hold real money balances, nominal bonds and consume domestic and foreign goods, which are imperfect substitutes in utility. Households own firms and firms choose amongst labor types producing a homogenous output, sold at home and exported abroad. A government controls the supply of money by making lump-sum transfers directly to households with revenues generated from seigniorage.

The foreign economy is assumed to be large relative to the domestic economy implying that the domestic economy takes financial conditions in the foreign economy as given. Here, smallness also implies that domestic exports form a negligible component of the foreign economy’s consumption basket. Consumption, output and the nominal price of domestic production are denoted with \( h \)-subscripts and for foreign consumption, output and prices I use \( f \)-subscripts. Asterisks denote foreign economy variables.

A. Firms

Each firm chooses between differentiated labor types and produces a single good. Firms solve a static profit maximization problem.

\[
\max_{\{l_t(j)\}} \theta_t(j) = P_{h,t}y_t - w_t(j)l_t(j)
\]  

subject to a homogeneous constant elasticity of substitution production function as in Blanchard and Kiyotaki (1987),

\[
y_t = \left( \int_0^1 l_t(j)^{\sigma-1}/\sigma \, dj \right)^{\sigma/\alpha(\sigma-1)}
\]  

where \( l_t(j) \) and \( w_t(j) \) are the \( j \)th individuals labor supply and nominal wage, \( P_{h,t} \) is the GDP deflator, \( y_t \) domestic output, \( \alpha > 1 \) implies decreasing returns to scale in production and \( \sigma > 1 \).
measures the elasticity of input substitution. The solution to this problem implies conditional labor demand is,

\[ l_t(j) = \left( \frac{w_t(j)}{w_t} \right)^{-\sigma} y_t^\alpha \]  

(3)

where \( w_t = \left( \int_0^1 w_t(j)^{1-\sigma} dj \right)^{1/(1-\sigma)} \) is the wage index and \( \sigma \) now measures the elasticity of demand with respect to the relative wage. Since all households set the same wage in equilibrium, \( w_t = w_t(j) \) \( \forall j \), and final labor demand is given by,

\[ y_{h,t} = (\alpha w_t / P_{h,t})^{1/(1-\alpha)} \]  

(4)

Supply of output here depends on the GDP deflator and this introduces a role for the terms of trade. The standard competitive labor demand condition is given by re-substituting (4) into the production function.

B. Households

Household \( j \) picks a sequence of nominal bond and money holdings, consumption and a desired wage rate. It also allocates expenditures between domestic and foreign produced goods. Suppressing the \( j \) index where possible the households intertemporal problem is,

\[ \max_{\{B_t, C_t, M_t, w_t(j)\}} U_0 = \sum_{t=0}^\infty \beta^t u(C_t, m_t, l_t) \]  

(5)

subject to the sequence of constraints,

\[ B_t - B_{t-1} \geq \vartheta_t + w_t(j)l_t(j) - P_t C_t - T_t + i_{t-1}B_{t-1} + M_{t-1} - M_t \]  

(6)

and conditional labor demand, (3). \( \beta \in (0, 1) \) is the discount factor (the exogenous rate of time preference), \( B_t \) are bonds denominated in domestic currency paying net nominal interest \( i_t \), \( C_t \) is total consumption (consumption index), \( m_t = M_t / P_t \) are real money balances, \( P_t \) is the consumer price index (CPI), \( T_t \) are lump-sum transfers and \( \vartheta_t \) are monopoly profits.\(^{14}\)

Period utility is assumed to have a semi-CRRA form, \( u = \ln C_t + am_t^{1-\epsilon} / (1 - \epsilon) - \phi l_t^\epsilon / \kappa \), where \( a > 0 \) is a measure of monetary frictions, \( \epsilon > 0 \) determines the substitutability of real balances with the consumption index and labor, \( \phi > 0 \) is the weight attached to the disutility of labor and \( \kappa > 1 \) is a measure of the elasticity of labor substitution. The first order conditions imply,

\[ P_{t+1}C_{t+1} = P_tC_t/\beta (1 + i_t) \]  

(7)

\(^{13}\)The assumption \( \sigma > 1 \) is required to generate a well defined equilibrium and is equivalent to the monopolist operating on the elastic portion of it’s demand curve.

\(^{14}\)Bond denomination is irrelevant in the analysis because Ricardian Equivalence holds in the infinite horizon representative agent set-up I use.
\[ am_t^{-\epsilon} P_t^{\epsilon - 1} = 1/P_t C_t - 1/\beta P_{t+1} C_{t+1} \quad (8) \]
\[ w_t = \sigma \phi C_t / (\sigma - 1) l_t^{\lambda - \kappa} \quad (9) \]

Equation (7) is the standard consumption Euler equation. Equation (8) is the money Euler equation which implies the demand for money takes the form, \[ m_t^\epsilon = a C_t (1 + i_t) / i_t, \] with \( \epsilon > 0 \) now also measuring the interest elasticity of money demand. The wage equation (9) demonstrates that the optimal wage is a function of the monopoly mark-up, \( \sigma / (\sigma - 1) \), i.e. the desired mark-up of households over marginal cost, and the marginal rate of substitution between consumption and leisure. This condition gives a natural way of incorporating nominal rigidity into the model, as the money wage is negotiated one period in advance. Finally, the usual arbitrage (UIP) condition connecting the exchange rate with domestic and foreign interest rates holds, \( s_{t+1} (1 + i_t^*) = s_t (1 + i_t) \), where \( s_t \) is the nominal exchange rate and \( i_t^* \) is the foreign net nominal interest rate.

I adopt a simple constant elasticity of substitution consumption index, which defines the relation between the consumer price index and domestic price. In previous studies, such as Corsetti and Pesenti (2001), the slightly stronger restriction of Cobb-Douglas preferences (a unit substitution elasticity between the two goods), simplifies the behavior of the current account significantly and this special case is taken up in section III. The households intratemporal problem (between domestically produced goods, \( C_h \), and goods produced in the foreign economy, \( C_f \)) is,

\[ \max_{\{C_{h,t}, C_{f,t}\}} C_t = \left( n^{1/\lambda} C_{h,t}^{(\lambda-1)/\lambda} + (1-n)^{1/\lambda} C_{f,t}^{(\lambda-1)/\lambda} \right)^{\lambda/(\lambda-1)} \quad (10) \]

subject to the nominal constraint, \( P_t C_t = P_{h,t} C_{h,t} + P_{f,t} C_{f,t} \), with \( P_t C_t \) for now, taken as given. The demands for the domestic and foreign produced goods are then,

\[ C_{h,t} = n (P_t / P_{h,t})^\lambda C_t \quad (11) \]
\[ C_{f,t} = (1-n) (P_t / P_{f,t})^\lambda C_t \quad (12) \]

Given this, the CPI is defined as,

\[ P_t = \left( n P_{h,t}^{1-\lambda} + (1-n) P_{f,t}^{1-\lambda} \right)^{1/(1-\lambda)} \quad (13) \]

Here, \( n \) measures openness, so as \( n \to 1 \) the domestic economy is closed to trade. The elasticity of substitution between domestic and foreign goods is captured by \( \lambda > 0 \) and as \( \lambda \to 1 \) it is straightforward to show \( P_t = P_{h,t}^n P_{f,t}^{1-n} \). In (10), when \( \lambda > 1 \) the two goods are substitutes and when \( \lambda < 1 \) they are compliments.\(^{16}\)

\(^{15}\)Preferences in this model are such that \( \epsilon \) determines both the consumption elasticity and the interest elasticity of money demand. Empirical estimates put the consumption elasticity of money demand at or below 1 (Mankiw and Summers, 1986) and the interest elasticity of money demand at 0.1 (Koenig, 1990). In this case, a reasonable magnitude for the consumption elasticity of demand will imply an unrealistically high interest elasticity. One natural extension to allow the elasticities to differ is to make consumption of CRRA form and not of log form.

\(^{16}\)Empirical estimates used in Chari et al. (2002) suggest a range of 1-2 for the elasticity of substitution. Gali and Monacelli (2005) and Smets and Wouters (2002) set the elasticity parameter at 1.5.
Foreign households face the same choice over the two goods but the small open economy assumption implies \( n^* \to 0 \) so that \( C^*_t \approx C^*_f, t \) and \( P^*_t \approx P^*_f, t \), which is normalized to unity as it is completely exogenous.\(^{17}\) This assumption does not mean export demand is zero, since \( C^*_t \) itself is large. Using the assumption that the law of one price holds for the domestic good \((P_{h,t} = s_t P^*_{h,t})\) foreign consumption of domestic production is,

\[
C^*_{h,t} = g^*_t (s_t / P_{h,t})^\lambda
\]

so that \( g^*_t \equiv n^* C^*_t \) is a measure of export demand.\(^{18}\) The parameter \( g^*_t \) is restricted to be non-zero and finite and is exogenous from the viewpoint of the domestic economy.

As stressed above, an important point is that despite assuming the law of one price holds for both traded goods the small open economy assumption implies (consumption based) PPP does not hold and the real exchange rate is not constant in the face of exogenous shocks. Recall, the LOP assumption holds where \( s_t \) is the nominal exchange rate and \( P^*_{f,t} \) the exogenous foreign currency price. As \( P_t \neq s_t \), I define the consumption based real exchange rate as \( q_t \equiv s_t / P_t \), where an increase in \( q_t \) is a real depreciation in the domestic currency. It is important to keep in mind that the differential in PPP (equivalently that \( q_t \neq 1 \)) derives directly from the assumption that \( n \neq n^* \).

C. Government

The government is a consolidated fiscal and monetary authority, connected by a simple budget constraint. The monetary authority prints money and collects seigniorage revenues. These are transferred to the fiscal authority which makes a lump-sum transfer to each household. The fiscal authority does not issue bonds so the consolidated constraint is,

\[
T_t = (\mu_t - 1) M_t
\]

where \( \mu_t \equiv M_{t-1} / M_t \) is defined as the inverse money growth rate. As the counterpart of an increase (decrease) in the money stock is a lump-sum transfer (tax) of equal size to all households there exists no marginal redistribution associated with government transfers. With the representative agent taking nominal prices as given when choosing a desired path of nominal money holdings and the government rebating revenues lump-sum to the public inflation still discourages holding nominal balances as the money transfer is unrelated to the optimal money demand decision.

III. Exchange Rate Dynamics with Cobb-Douglas Preferences

Given the description of the economy I begin by looking at a special case in which consumer preferences are characterized by a Cobb-Douglas sub-utility function, i.e. \( \lambda = 1 \) in equation

\(^{17}\)One can think of the share of imports in the foreign economy’s CPI as being negligible and hence the foreign economy is effectively closed.

\(^{18}\)If \( C^*_t \equiv y_{f,t} \), where \( y_{f,t} \) is foreign GDP, then \( g^*_t \) (\( = n^* y_{f,t} \)) approximates export demand.
This assumption has been used by, for example, Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2002) to restrict current account dynamics and allow for a relatively simple solution method. In particular, this restriction allows for a closed-form (exact) solution to the model and an analysis of changes in monetary policy of an arbitrary size. As I show below, the zero current account result in these papers holds in the small open economy model I develop. In particular, the zero current account result is shown to be invariant to the money demand function but is dependent on the intertemporal substitution elasticity of consumption being unity, unlike Corsetti and Pesenti (2001).

This additional requirement is a direct result of relaxing the PPP assumption and also arises in Gali and Monacelli’s (2005) small open economy analysis for the same reason.

A. Model Solution and ADAS System

I derive a ‘final’ aggregate demand and supply (ADAS) system and use it to examine the effects of a permanent unanticipated change in the money supply. Because the Cobb-Douglas sub-utility function simplifies the solution method I am also able to appeal to a simple diagrammatic analysis to show the effects of a change in the money supply on the exchange rate. But there is a complication. Due to the specification of preferences over real balances (i.e. CRRA and not logarithmic) it is necessary to solve the real and the monetary sides of the economy jointly. Initially an explanation of consumption, the real interest rate, and the real exchange rate in the domestic economy is required, independent of monetary factors. Then, by characterizing the domestic monetary side using a nominal interest rate difference equation, the two sides of the economy can be solved jointly whilst accounting for the rigidity in the nominal wage. Although it is not possible to appeal to the exogenous foreign rate of interest in any solution to the monetary side the analysis of the real side is simplified by this exogeneity condition.

I. Real Side

As agents have perfect foresight they are only surprised in the initial period. Markets therefore clear in all future periods and for periods $t \geq 1$ the monopolistic supply of labor equals the demand for labor. Labor market clearing is given by the following expression.

$$y_t = \left(\alpha \phi \sigma C_t / (\sigma - 1) \left(\frac{P_{h,t}}{P_t}\right)\right)^{1/(1-\kappa)}$$

for $t \geq 1$ \hspace{1cm} (16)

This expression determines aggregate supply for periods $t \geq 1$ and it is clear that domestic output depends on total consumption and the ratio of the CPI to GDP deflator, all of which are endogenous.

The second condition required to determine the real side of the economy is a goods market clearing condition. This first requires the resource constraint of the domestic economy to be

\[\text{additionally separable in utility.}\]
satisfied, i.e. that world consumption of the domestic good is matched by domestic production levels.

\[ y_t = C_{h,t} + C^*_{h,t} \]  

Combining this with the CPI when \( \lambda = 1 \) (i.e. \( P_t = P_{h,t}^n P_{f,t}^{1-n} \)), the demand conditions (11) and (14) and assuming \( C^*_t = C^* \) for all \( t \) such that \( g_t^* = g^* \), produces the following goods market clearing expression.\(^{20}\)

\[ y_t = (nP_t C_t + g^* s_t) / P_{h,t} \]  

Now an increase in \( g^* \) can be viewed as an ‘output shifter’ and because output need not equal consumption, unlike a closed economy, a trade deficit (surplus) satisfying excess demand (supply) is possible. This is captured by the \( g^* s_t - \text{term} \) which also represents the value of exports of the domestic good.\(^{21}\)

It should also be clear that both the goods and labor market clearing equations make output a function of consumption and relative prices. However, due to the Cobb-Douglas sub-utility assumption, the inverse terms of trade for the domestic economy, denoted \( \tau_t = P_{f,t} / P_{h,t} \) (a reduction in \( \tau_t \) represents an improvement in a country’s terms of trade), relates to the real exchange rate through the degree of trade openness alone. Thus I have, \( q_t = \tau^n_t \).\(^ {22}\) In this case it is clear that (16) and (18) form an implicit sub-system for periods \( t \geq 1 \) in the variables \( y_t, q_t, \) and \( C_t \) only.

That the real exchange rate and domestic output cannot be solved simultaneously demonstrates a key difference between micro-founded and ad-hoc approaches. In many respects the supply side of the model here is similar to the ad-hoc small open economy model of Sachs (1980). Solving the ad-hoc model is more straightforward because the aggregate supply and market clearing conditions can be solved simultaneously to determine the natural rate of output and the real exchange rate. But here an implicit system of the form \( q_t = q(C_t) \) and \( y_t = y(C_t) \) is generated, so this is not possible. Nevertheless, despite the complication, it is still possible, in this instance, to characterize the entire time path of the real interest rate.

To solve for the period \( t \geq 1 \) real interest rate I first combine the UIP condition with the Fisher equation, \( P_{t+1} (1 + r) = P_t (1 + i_t) \), which implies \( q_{t+1} (1 + r_t^*) = q_t (1 + r_t) \). As I assume that the foreign economy is in a steady state and as \( \beta = \beta^* \) is required to rule out the small economy growing large over time, then \( (1 + r_t^*) = 1/\beta \) for all \( t \). Given the implicit system

\(^{20}\)Thus the foreign economy is in a zero inflation steady-state.

\(^{21}\)If this term were to equal zero and \( n = 1 \) then it is clear that \( y_t = C_t \).

\(^{22}\)Note that the greater the degree of home-bias for the domestic economy (i.e. the larger \( n \)), then a given deterioration in the terms trade implies a greater depreciation in the real exchange rate. Benigno and Thoenissen (2002) refer to this as the home-bias channel.
above, \((1 + r_t) = q(C_{t+1}) / \beta q(C_t)\) and the real version of the consumption Euler equation now implies, 

\[
C_{t+1} / q(C_{t+1}) = C_t / q(C_t) \quad \text{for } t \geq 1
\]

(19)

Thus for periods \(t \geq 1\) the dynamics of the real side of the economy can be expressed as a self-contained difference equation which implies that the real interest rate is at its steady state level for periods \(t \geq 1\) and \(1 + r_t = 1 / \beta\). Similarly, \(C_t = C\) and \(q_t = q\) for \(t \geq 1\) and as \(y_t = y(C_t), y_{h,t} = y_h\) for \(t \geq 1\).

In the initial period, however, the money wage is fixed at \(w_0\). In this case, individuals are off their labor supply curves and labor is demand determined. But recall that labor demand is conditional on the GDP deflator, which is endogenous. From the definitions of the real exchange rate and consumer price index I can rewrite labor demand as a combination of the real and nominal exchange rates which will be determined below, thus temporarily solving the endogeneity problem.

\[
y_0 = \left( \alpha w_0 q_0^{1/n} / s_0 \right)^{1/(1-\alpha)}
\]

(20)

Goods market clearing is still applicable so that domestic output in the current period again depends implicitly on consumption and the real exchange rate. Specifically, combining clearing with (20) I can denote the second real side system (for the current period),

\[
q_0 = q(C_0, s_0; w_0)
\]

\[
y_0 = y_h(C_0, s_0; w_0)
\]

(21)

(22)

Note the difference between the future period and current period systems. The former is conditioned on consumption alone, whilst the latter is conditioned on consumption, the nominal exchange rate and the rigid money wage. Then, following the same steps as above, the current period domestic real interest rate is,

\[
1 + r_0 = q(C_1) / \beta q(C_0, s_0; w_0)
\]

(23)

Because the real interest rate is not invariant to shocks, it is not possible, in the current period, for the real side of the economy to be in a steady state. This result has important implications for the monetary side of the economy, which I now turn to.

II. Monetary Side

To describe the behavior of the monetary side of the economy I combine the consumption and money Euler equations to express the period \(t\) nominal interest rate as a function of the future nominal interest rate, the current period domestic real interest rate, the discount factor and the exogenous rate of money growth.

\[
i_t (1 + i_t)^{\xi -1} = (1 + r_t)^{\xi -1} i_{t+1} / (1 + i_{t+1}) \beta \mu_{t+1}
\]

(24)
As the nominal interest rate is a non-predetermined variable equation (24) needs to be unstable in it’s forward dynamics and satisfy a saddle path property. Since \(1 + r_t = 1/\beta\) for \(t \geq 1\) when \(\mu_t = \mu\) this property implies \(1 + i_t = 1/\beta\mu\) for \(t \geq 1\). From this result, the implications of home-bias in consumption for the monetary side of the economy become clearer. If there were no home-bias in consumption the domestic real interest rate would be pinned to the world real interest rate, which is exogenous; i.e. \(r_t = r^*_t = (1 - \beta) / \beta\) for all \(t\). With this restriction it would then be possible to solve for the level of the domestic nominal interest rate in all periods and not just all future periods. To solve for \(i_0\) in the current period I therefore need to solve (24) recursively. Doing this yields a key equation in the paper.

\[
i_0 (1 + i_0)^{\epsilon - 1} = (1 + r_0)^{\epsilon - 1} \frac{(1 - \beta \mu)}{\beta \mu^\epsilon}
\]  

(25)

Now, as a result of the rigid money wage, monetary policy has an additional influence over the nominal interest rate when \(\epsilon \neq 1\), giving rise to a liquidity effect. The regime for money growth is such that \(\mu_t = \mu\) for \(t \geq 1\) but \(\mu_0\) may differ from \(\mu\) so the complete time path is described by \((\mu_0, \mu)\). Note again that preferences over money balances force this particular solution method. If the interest elasticity of money demand was unity it would be possible to exploit a separability property between the real and monetary sides of the economy as it would then only be necessary to consider the monetary side when solving for the nominal interest rate. If there were no home-bias (i.e. preferences were such that \(n = n^*\)) the separability property would be redundant.

\section*{III. Current Account and Exchange Rate}

Given the behavior of nominal interest rate it is now possible to solve for the initial level of the nominal exchange rate. To do this I first write down the period national budget constraint (balance-of-payments condition). Using the household’s budget constraint, (6), and summing over all \(j\) households, along with the definition of firm profits, (1), and the government’s budget constraint, (15), I have,

\[
B_t - B_{t-1} (1 + i_{t-1}) = P_{h,t}y_t - P_tC_t
\]  

(26)

In equilibrium the end of period bond level is equal to domestic output minus the rate of absorption plus interest from claims on bonds.\(^{23}\) I then take the resource constraint and use the demand functions (10) and (11) to substitute out domestic output and the GDP deflator. With Cobb-Douglas preferences it is possible to then write the trade balance as a function of absorption and the value of exports alone. Finally, I solve the transformed condition forward, which implies,

\[
(1 + i_{-1})B_{-1} = \sum_{t=0}^{\infty} \frac{\Gamma_t}{(1 + i_0) \cdots (1 + i_{t-1})}
\]

\[
\Gamma_t = s_tg^* - P_tC_t (1 - n)
\]  

(27)

\(^{23}\)The timing on the interest rate is slightly different from the timing conventions used in, for example, Obstfeld and Rogoff (1995) but this does not alter the results.
where \((1 + i_0) \cdots (1 + i_{t-1}) \equiv 1\) when \(t = 0\) and the initial level of debt, \(B_{-1}\), is assumed equal to zero.\(^{24}\) Ponzi games are also ruled out so the following holds, \(\lim_{t\to\infty} B_t / (1 + i_t) \cdots (1 + i_0) = 0\).

Equation (27) is the national intertemporal budget constraint (NIBC).

I now split the NIBC into two parts; one where the nominal rigidity takes effect (short-run, \(t = 0\)), and one where it does not (long-run, \(t \geq 1\)). Using the fact that the domestic nominal interest rate is constant for periods \(t \geq 1\) it is straightforward to demonstrate from the consumption Euler equation that \(P_t C_t / P_{t+1} C_{t+1} = \mu\) for all \(t \geq 1\) and so as a consequence \(\Gamma_t / \Gamma_{t+1} = \mu\) for all \(t \geq 1\), or rather, that in all periods after the current period the trade balance grows at a constant rate; so now let \(\Gamma_t \equiv \Gamma\) for all \(t \geq 1\). From this I am able to re-write (27) as,

\[
0 = \Gamma_0 + (1 + i_0)^{-1} \Gamma / (1 - \beta \mu) \tag{28}
\]

where I have used \(\sum_{t=1}^{\infty} (\beta \mu)^t = \beta \mu / (1 - \beta \mu)\). Substituting out \((1 + i_0)\) using the Euler and UIP conditions \(\Gamma\) is a positive multiple of \(\Gamma_0\). Then, when the initial net foreign asset position is zero, it must be that \(\Gamma_0 = \Gamma = 0\) and more generally \(\Gamma_t = 0\) for all \(t\). Therefore despite the non-unit interest elasticity of money demand, in this small open economy, there is a zero current account condition for all periods. But this result holds only when both the intertemporal and intratemporal consumption elasticities are unity as in Gali and Monacelli (2005). If either departs from unity it is possible to generate current account surpluses or deficits from changes in the money supply depending on parameter restrictions (I examine a particular case in section IV). In Corsetti and Pesenti (2001) a zero current account condition holds even when the intertemporal elasticity differs from unity, but in that model PPP holds. Here, relaxing PPP requires a more stringent restriction on preferences to generate a similar result.

From this result and the definition of the trade balance the solution for the nominal exchange rate (the ‘simplified’ NIBC) is,

\[
s_t = P_t C_t (1 - n) / g^* \text{ for all } t \tag{29}
\]

Finally, given market clearing and the period \(t \geq 1\) money demand function I can now relate the nominal exchange rate to the monetary policy variable, \(M_t\),

\[
\begin{align*}
    s_t &= \eta M_t \text{ for } t \geq 1 \\
    \eta &= ((1 - \beta \mu) / \alpha)^{1/\epsilon} y^{n(\epsilon - 1)/\epsilon} ((1 - n) / g^*)^{n(\epsilon - 1) + 1)/\epsilon} \tag{30}
\end{align*}
\]

with \(\eta > 0\). Although it is not yet possible to show directly, this demonstrates how in period \(t = 0\) the reaction of the exchange rate to changes in the money supply is augmented through changes in output because instead \(y_0\) will appear in the current period version of (30). The changes in output derive from changes in the real interest rate which in turn also depend on the wage rigidity.

\(^{24}\)This final step is common in the literature and is crucial for generating a closed-form solution.
IV. ADAS System

To tie down $s_0$ and $y_0$ I first consider periods $t \geq 1$. To offer a full solution to the model the key insight is the behavior of the current account. Recalling $\Gamma_t = 0$ and noting this implies $C_t = y_tq_t^{(n-1)/n}$ when using the definition of the real exchange rate, the period $t \geq 1$ real side system can be written solely in terms of output and the real exchange rate.

\begin{align*}
q_t &= q(y_tq_t^{(n-1)/n}) \\
y_t &= y(y_tq_t^{(n-1)/n})
\end{align*}

This transformed system now pins down, $q_t$, $y_t$ and $C_t$ explicitly for periods $t \geq 1$. Although previously it was not possible to solve for domestic output and the real exchange rate simultaneously because each was implicitly dependent on consumption, in all periods after the current period it is possible to solve for domestic output, the real exchange rate and consumption simultaneously because the current account relation gives a third equation in all three variables. These equations therefore now provide solutions for the natural rate of output and the real exchange rate, which I denote $\overline{q}$ and $\overline{y}$.

It is also possible to show that this version of the model generates an explicit expression for the natural rate; that is, because the current account is zero in all periods, there is an explicit expression for future output that depends only on preference parameters and technology.

\[ \overline{y} = \left( (\sigma - 1) / \sigma \alpha \phi \right)^{1/\kappa} \alpha \]

As the monopoly distortion falls, that is as $\sigma \to \infty$, employment reaches the competitive level $l = (1/\phi\alpha)^{1/\kappa}$. As $q = q(C)$ and $y = y(C)$, $\overline{q} = \overline{C} (1 - n) / g^*$, and then $\overline{y}$, $\overline{q}$ and $\overline{C}$ are related to one another uniquely.

Tying down the equilibrium for periods $t \geq 1$ turns out to be the important step in tying down all remaining endogenous variables in the model because by substituting $\Gamma_0 = 0$ into the $t = 0$ implicit system it is now possible to solve for $y_0, q_0$ and $C_0$, conditional on $s_0$. Therefore output and the nominal exchange rate are related via the real sector relations and there is a closed-form (exact) solution which describes aggregate supply (AS).

\[ y_0^\alpha = (s_0g^*/(1 - n) \alpha \overline{w}_0) \]

A similar expression for aggregate demand is also obtainable. First note that from the UIP condition the current period real and nominal interest rates can be written respectively as,

\begin{align*}
1 + r_0 &= (\overline{y} / y(C_0, s_0; \overline{w}_0))^n / \beta \\
1 + i_0 &= \eta M_1 / \beta s_0
\end{align*}

where I have used the market clearing condition and the solution for the exchange rate (29) with $\eta$ given in (30) and $\overline{y}$ given by (33).\footnote{Using the demand equation and the zero current account condition, $\overline{y} = \overline{q}^{1/n} (g^*/(1 - n))$.} Substituting out the real and nominal interest rates
from the current period interest rate difference equation therefore produces a second closed-form (exact) relation between output and the nominal exchange rate which describes aggregate demand (AD).

\[ y_0^{1-\epsilon} = \frac{(\eta M_1 - s_0 \beta) (\eta M_1 / s_0 \beta)^{\epsilon-1}}{\beta s_0 \gamma} \]
\[ \gamma = \frac{(\sigma - 1) / \sigma \alpha \phi^{\sigma(\epsilon-1)/\kappa \alpha} (1 - \beta \mu)}{(\beta \mu)^\epsilon} \]  

with \( \gamma > 0 \). All remaining variables in equation (37) are exogenous since \( M_1 \) is the policy variable. The method I employ therefore transforms the current period implicit system for \( y_0 \), \( q_0 \) and \( C_0 \) into a simple pair of implicit equations for \( y_0 \) and \( s_0 \).

In this case appealing to a diagrammatic analysis is easiest because the simplification of Cobb-Douglas preferences allows me plot the loci in \((s_0, y_0)\) space. First, since aggregate supply is independent of the interest elasticity of money demand, \( \alpha > 1 \) and \( n \in (0, 1) \) the slope of (34) is unambiguously positive in \((s_0, y_0)\) space. But the slope of the aggregate demand locus (37) is not independent of the interest elasticity of money demand. When \( \epsilon = 1 \), preferences are logarithmic over real balances and aggregate demand is horizontal. But with \( \epsilon > 1 \) aggregate demand has a positive slope and \( \epsilon < 1 \) it is negative. Diagrammatically, I have the following representation of the model.

![Diagram of the Cobb-Douglas ADAS System](image)

Figure 1: The Cobb-Douglas ADAS System
As current period variables are tied down, and in all future periods variables are at their natural rate levels, it is straightforward to consider the effects exogenous changes in the money supply have on output and the exchange rate.

**B. Effects of a Monetary Expansion**

I now consider the effects of a monetary expansion. I restrict the analysis to a permanent unanticipated increase in the money supply focusing on domestic output and the nominal exchange rate. Because the zero current account condition is the result of the $\lambda = 1$ restriction this is the baseline case. To understand how output and the exchange rate behave in response to a monetary expansion I implicitly differentiate (34) and (37). \[26\]

**I. Monetary Policy without a Liquidity Effect**

From the proceeding discussion it should already be clear that the reaction of the exchange rate when money demand is interest unit-elastic is proportional to the change in the money supply. Setting the interest elasticity to unity in the difference equation implies the nominal interest rate jumps immediately to it’s steady state value. In this case a change in the money supply has no additional effect on the nominal interest rate and the nominal exchange rate increases each period by the money growth rate. Underlying this result is that money demand responds one-for-one with the change in consumption because $\epsilon$ determines the interest and consumption elasticities of money demand. This corresponds to the type of effect in models where (consumption based) PPP holds.

More formally, the reaction of the economy to a change in the money supply can here be summarized by the following two conditions.

\[
\begin{align*}
\partial y_0 / \partial M_1 &= (y_0^{1-\alpha} g^*/\alpha^2 (1-n) \bar{w}_0) (s_0/M_1) \\
\partial s_0 / \partial M_1 &= s_0/M_1
\end{align*}
\]

(38) \hspace{1cm} (39)

The obvious point is that as $(\partial s_0 / \partial M_1) M_1/s_0 = 1$ the short-run change in the exchange rate is proportional to the increase in the money supply. But it is also important that when there is a permanent increase in the money supply output increases. Intuitively, output rises because the temporary change in money growth leads to price rises, lower real wages (recall the nominal wage is predetermined) and an increase in labor demand. As labor is demand determined, in the short-run, there is an increase in output. When the economy is more open, i.e. when $n$ is lower, monetary policy is less effective at increasing output but when $\epsilon = 1$ this has no impact on the reaction of exchange rate. This is because setting $\epsilon = 1$ means that (34) and (37) do not form an interdependent system of equations as the term in domestic output no longer enter into the AD equation. As a result, there is no feedback effect on the exchange rate from a change

\[26\]Details are in the Appendix.
in output and, as stressed earlier, the real and monetary sides of the model are independent. This feedback effect is the key to the behavior of the exchange rate in more general cases.

II. Monetary Policy with a Liquidity Effect

I now consider the case that money demand is interest inelastic. I then have the following representation of the effects of a change in the money supply.

![Diagram showing exchange rate overshooting](image)

Figure 2: Exchange Rate Overshooting when $\lambda = 1$

To understand the effects of shocks on the system as a whole it is straightforward to see from figure two that an exogenous increase in the money supply shifts the monetary relation up/left (AD to AD'). Again output is higher as wage costs are lower. But the shift in the aggregate demand curve now implies there is an additional change in the exchange rate. In the short-run the response of the exchange rate is more than proportional to the change in the money supply and $s_0$ is greater than it’s long run level, given by $\bar{s}/\mu$. The overshooting of the exchange rate is greater the lower the sensitivity of money demand to changes in the interest rate (a higher $\epsilon$) because in this case the asset market compensates more and more for the distortion.

---

27 Again when $\epsilon = 1$ the AD curve is horizontal which implies $s_0 = \bar{s}/\mu$, i.e. no overshooting.
produced by the rigid money wage. As wages adjust and the economy reaches its new steady-state (long-run) the aggregate supply curve shifts to intersect with AD' (AS to AS') and output returns to its natural rate level, given by (33). The behavior of the exchange rate in this case is clearly seen in the following expression,

\[
\left(\frac{\partial s_0}{\partial M_1}\right) \frac{M_1}{s_0} = \frac{\alpha \left(\left(1 + i_0\right) \epsilon + (1 - \epsilon)\right)}{(1 - \epsilon) \left(\left(i_0\right) n + \alpha\right) + (1 + i_0) \alpha \epsilon}
\]  

(40)

When \(\epsilon > 1\), \(\left(\frac{\partial s_0}{\partial M_1}\right) \frac{M_1}{s_0} > 1\). As should also be clear from the discussion above, the reaction of the exchange rate is a result of consumption home-bias arising from the import-export structure of the goods market. When there are two traded goods \((1 + r_t) \neq (1 + r_t^*)\) and when \(\epsilon \neq 1\), the domestic real interest rate influences the nominal interest rate altering the behavior of the exchange rate when there is a change in the money supply.

This result can also be understood a little more directly by simply looking at the aggregate demand curve. As current output always rises with a monetary expansion (this is shown formally in the Appendix for the \(\epsilon \neq 1\) case) and is uniquely related to the real exchange rate the real interest rate falls. When \(\epsilon > 1\), the nominal interest rate must then fall causing the exchange rate to overshoot via the UIP condition. Taking the initial rise in output as given it is sufficient to differentiate both sides of (25) with respect to \(y_0\). This produces,

\[
\frac{\partial i_0}{\partial y_0} = \varrho n \left(1 - \epsilon\right) / i_0^{1/(\epsilon - 1)} \left(\epsilon + i_0^{-1}\right)
\]  

(41)

where \(\varrho \equiv \left(\frac{\gamma n}{\beta}\right) \left((1 - \beta \mu) / (\beta \mu^*)\right)^{1/(\epsilon - 1)} > 0\). The sign of the derivative then only depends on the magnitude of the interest elasticity of money demand. As a check, if this is unity there should be no overshooting. Setting \(\epsilon = 1\), implies \(\partial i_0/\partial y_0 = 0\) but if \(\epsilon > 1\), then \(\partial i_0/\partial y_0 < 0\), and so that the nominal interest rate falls through an induced liquidity effect and the exchange rate overshoots.\(^{28}\)

It is also clear both from (40) and (41) how overshooting depends on the degree of trade openness. Recall that as \(n\) lowers the economy is more open to trade; therefore, the more open the economy, the lower the degree of exchange rate overshooting. This result is also comparable to much of the literature that has employed simple linear approximations when solving this class of model. In particular, Hau (2002) demonstrates with a traded-nontraded goods structure, that in a more open economy, money shocks (the same in form as I consider here) produce smaller changes in the exchange rate. Note that this is also consistent with the idea that changes in output have a feedback effect on the exchange rate. When \(\epsilon = 1\), monetary policy is less effective at raising output the more open the economy. But this implies

\(^{28}\)Note also that an undershooting result cannot be ruled out. This is also true in the original overshooting model as long as money demand is sensitive to changes in output. As Rogoff (2002) notes this is quite unrealistic, but the overriding difference between the model I present (and this class of models in general) and the original analysis of Dornbusch (1976) is that the interest elasticity determines the relationship between money demand and consumption, not money demand and output. Thus a quicker response may be less unrealistic.
that in the more general case here there should be less overshooting because the feedback effect is weaker - this is exactly what happens.

**IV. A More General Case - CES Preferences**

I now consider the more general case. In particular, I allow the substitution elasticity between domestic and foreign goods to be equal to $\lambda > 0$. This has a number of important implications. First, recall that $\epsilon$ governs the consumption and interest elasticities of money demand. As the consumption basket is comprised of two goods and there is now a non-unit degree of substitution between them this can, in principle, affect both the sign and the magnitude of the change in the interest/exchange rate. Of course, $\epsilon$ is still the dominant factor, as when $\epsilon = 1$ the exchange rate will not have any additional dynamics, regardless of the value of $\lambda$.

Second, the zero current account result will no longer hold. In the analysis of Obstfeld and Rogoff (1995) money shocks result in a current account surplus because the intratemporal substitution elasticity is pinned to the parameter governing the monopoly mark-up, which is assumed be larger than one. In Tille (2001), once these two parameters are dealt with separately, it is the intratemporal elasticity alone that determines the reaction of the current account to monetary shocks. There, when goods are substitutes (equivalent to $\lambda > 1$ above), a change in the money supply again causes a current account surplus. But this is a model in which PPP holds. Gali and Monacelli (2005) consider a small open economy with the same import-export structure to that developed above and argue that both the intra and intertemporal consumption substitution elasticities determine the reaction of the current account to monetary shocks - a similar result was shown to hold in the previous section. Since the intertemporal substitution elasticity is assumed to be unity, $\lambda$ is now the key parameter determining the reaction of the current account to monetary policy.\(^{29}\)

Finally, allowing for $\lambda \neq 1$ makes the model non-linear and, in particular, the demand functions for the specialized outputs. The previous solution method is then no longer possible and to solve I take a first-order approximation around the steady-state of the model (I choose to do this around a zero inflation and zero trade balance steady-state where $P_h = P = P_f = s$ as in Monacelli (2004)). Note from here on that a circumflex denotes the log deviation of a variable from its steady-state, i.e. for any variable $x_t$, $\hat{x}_t \equiv (x_t - x) / x$, where $x$ is the steady-state value of $x_t$.\(^{30}\)

**A. Real and Monetary Sides of the Economy**

The period $t \geq 1$ goods and labor market clearing equations can now be rewritten up to a first-order approximation as an implicit system in the same endogenous variables as before, i.e. consumption, output and the real exchange rate. However, despite the approximation, it is

\(^{29}\)Arguably, this is empirically more relevant than departing from a unit intertemporal substitution elasticity.

\(^{30}\)More details of the the following conditions are presented in the Appendix.
still possible to characterize the behavior of the real interest rate \textit{exactly}. Combine the UIP condition with the Fisher equation, the real consumption Euler equation and impose $\beta = \beta^*$. The resulting equation, when combined with the implicit system, can be written as,

\[
\hat{C}_{t+1} - q \left( \hat{C}_{t+1} \right) = \hat{C}_t - q \left( \hat{C}_t \right) \quad \text{for } t \geq 1
\]

(42)

This is the CES analog of (19), above. This difference equation has the property such that, $\hat{C}_{t+1} = \hat{C}_t \equiv \hat{C}$ for $t \geq 1$, where $\hat{C}$ denotes the long-run change in real consumption. But it must then be that the condition also holds exactly, so, $C_{t+1} = C_t \equiv C$ for $t \geq 1$. Therefore I can conclude that, for periods $t \geq 1$, $1 + r_t = 1 / \beta$, even in the CES case.

As the real interest rate result holds exactly it is immediate that the main result for the monetary side of the economy presented above also holds. Thus, the current period nominal interest rate is affected by changes in the current period real interest rate, but in all future periods is at it’s steady-state level, given that I assume there is a permanent unanticipated rise in the money supply. This also implies the following characterization of the monetary side of the economy holds.

\[
\hat{i}_0 = \frac{(\epsilon - 1) (1 - \beta)}{\epsilon - \beta (\epsilon - 1)} \hat{r}_0
\]

(43)

with, $\hat{i}_t = 0$ for $t \geq 1$ and when $\epsilon > 1$, $\partial \hat{i}_0 / \partial \hat{r}_0 > 0$, consistent with the liquidity effect described above.

In linear terms the policy regime now is such that in period $t = 0$ the money stock, $\hat{M}_t$, unexpectedly jumps permanently from zero to $\hat{M}$ for all $t$. And in terms of long-run reactions, I must now also have that nominal exchange rate jumps from zero to $\hat{s}_0$ but $\hat{s}_0 \neq \hat{s}_t$ for $t \geq 1$ and $\hat{s}_t = \hat{s}$ for all $t \geq 1$. Also note, nominal consumption has the property that $\hat{\Omega}_t = \hat{\Omega}$ for $t \geq 1$. From the definition of nominal consumption the jump to the steady-state in period $t = 1$ must also be true of the CPI and the GDP deflator. It is worth restating that variables with a circumflex and without a sub-script denote the long-run deviation from steady-state.

B. Current Account and Exchange Rate Revisited

I now turn to solve for the nominal exchange rate using the national intertemporal budget constraint, as above. With the more general CES preferences the behavior of the trade balance is no longer independent of the relative price (specifically a $P_h$ term enters). Using the CPI, the linear version of the CES-NIBC can be expressed as,

\[
\hat{a}_{-1} = \sum_{t=0}^{\infty} \beta^t \hat{\Gamma}_t
\]

\[
\hat{\Gamma}_t = (1 - n) \left[ (1 - n) (1 - \lambda) \hat{P}_{h,t} + (\lambda + n (\lambda - 1)) \hat{s}_t - \hat{\Omega}_t \right]
\]

(44)

where again I can split the constraint into two periods; one corresponding to the short-run ($t = 0$) and the other the long-run ($t \geq 1$). Here, assuming a zero inflation and zero trade
balance steady-state simplifies the expression considerably because it eliminates interest rate differentials.\textsuperscript{31}

Solving for the exchange rate first requires the elimination of the GDP deflator (for both the long and short-run) now using an ‘intermediate’ ADAS system, given by the implicit real side system or labor demand, which would be required even in money demand was unit interest elastic. But as a result of this, and when $\epsilon \neq 1$, it is not possible to then directly apply the Euler and UIP conditions to solve out for the nominal exchange rate because doing so introduces the current period nominal interest rate. However, it is possible to solve for the current period nominal exchange rate by using the money demand function. That the money demand function is necessary to solve for the nominal exchange rate gives rise to an interaction between the current account and interest rate, or rather, a second real-monetary interaction.\textsuperscript{32}

From (44), and using linear approximations for the Euler and UIP conditions, it is possible to show $\hat{s}_0 = s \left( \hat{\Omega}_0, \hat{i}_0 \right)$. The change in the interest rate is then captured by appealing to the current period money demand function such that,

\[
\hat{s}_0 = s \left( \hat{\Omega}_0; \hat{M} \right) \tag{45}
\]

There are some key differences between this expression and it’s Cobb-Douglas counterpart, given by (29).\textsuperscript{33} In that case it was possible to solve for the current period nominal exchange rate as a function of current period nominal consumption alone. Now, because of the second interaction between the monetary and the real sides of the economy an additional term in nominal money balances enters. This does not matter in terms of the solution because the policy regime is specified as an exogenous sequence of money supplies, but it is a direct consequence of $(\epsilon, \lambda) \neq 1$. It is possible to eliminate the extra term, as when $\lambda = 1$, $\hat{M}$ plays no additional role in the NIBC, for any value of $\epsilon$. Likewise, if $\epsilon = 1$, $\hat{M}$ would also be eliminated, leaving only the GDP deflator to be determined through the intermediate ADAS system.

Now consider the future period exchange rate, that is, $\hat{s}_t = \hat{s}$ for periods $t \geq 1$. I take the same approach, using the UIP and Euler equations, except substituting out $\hat{s}_0$ and $\hat{\Omega}_0$; this set of conditions imply, $\hat{s} = s \left( \hat{\Omega}, \hat{i}_0 \right)$. Again I use the current period money demand function to account for the change in the interest rate but now I also need use the future (i.e. $t \geq 1$) demand for money, which means I can write,

\[
\hat{s} = s \left( \hat{M} \right) \tag{46}
\]

\textsuperscript{31}Note that because the zero inflation steady-state is evaluated at a zero trade balance $\hat{\Gamma}_i$ is such the trade balance is not represented by it’s log deviation but by it’s absolute deviation deflated by the initial steady-state value of nominal consumption, $\Omega_{-1}$, i.e., $\hat{\Gamma}_i = (\Gamma_i - \Gamma_{-1}) / \Omega_{-1}$. Likewise, I define, $\hat{a}_{-1} \equiv (1 + i_{-1}) B_{-1}/\Omega_{-1}$.

\textsuperscript{32}Recall that the first real-monetary side interaction was present when $\lambda = 1$. It manifested itself in a non-separability property between the real and monetary sides of the economy when characterizing the interest rate dynamics.

\textsuperscript{33}Again, see the Appendix for more details.
The long-run expression for the nominal exchange rate is therefore only a function of the change in the money supply despite the real-monetary interaction introduced when the interest elasticity of money demand and the intratemporal consumption elasticity are not unity. Equation (46) should also be thought of as the analog of (30).

Some other points are also worth making. First, the current account reacts to changes in the money supply and, as is common in this type of open economy model the net foreign asset position is non-stationary. That is, there is a unit root in the net asset position and any change in the money supply causes a permanent accumulation/decumulation of assets. Here, the main implication of this is that, for a given money shock, the effect on output lasts longer than the length of the rigidity. Because agents accumulate assets over their entire lifetime, wealthier agents, for example, wish to supply less labor and therefore output is affected in future periods because labor is not determined by the demand side.

C. The CES ADAS System

Deriving the final ADAS system for the more general case requires the interest rate equation (43). Using the UIP and Fisher conditions, this implies,

\[ \hat{s}_0 = \hat{s} - \frac{(\epsilon - 1)(1 - \beta)}{\epsilon - \beta(\epsilon - 1)} (\hat{q} - \hat{q}_0) \]  

(47)

To solve fully, I now only need eliminate terms in the real exchange rate because the long-run nominal exchange rate, \( \hat{s} \), has already been expressed as a function of the policy variable, \( \hat{M} \). Doing this introduces changes in the current and future levels of output providing the key difference between the CD and the CES versions of the model. CD preferences imply \( \hat{q} = 0 \) but when the current account moves \( \hat{q} \neq 0 \).

Recall that the definition of the real exchange rate is, \( \hat{q}_t = \hat{s}_t - \hat{P}_t \) for all \( t \). To express the current real exchange rate in terms of current domestic output and the monetary policy variable, use the goods market clearing expression, labor demand and the CPI, with the solution for the current period nominal exchange rate. Schematically,

\[ \hat{q}_0 = q \left( \hat{y}_0, \hat{M} \right) \]  

(48)

In the future period again use the definition of the real exchange rate in the solution to the long-run nominal exchange rate, equation (46). By labor and goods market clearing and the period \( t \geq 1 \) money demand function (the point here is that nominal consumption enters the solution for the current period nominal exchange rate and is substituted out through the \( t = 0 \)

\[ \text{There are ways to circumvent this problem and they are covered in detail in Schmitt-Grohe and Uribe (2003). Although it is beyond the scope of the paper; Uzawa preferences, the introduction of overlapping generations and a debt elastic interest rate all deliver a stationary steady-state. For my purposes, however, this is not critical because, as I will show, it is that asset accumulation occurs that is important.} \]

\[ \text{This also implies there is no natural rate expression under CES preferences.} \]
implicit system. However, in the future period consumption does not appear in the solution for the nominal exchange rate and therefore using the implicit system reintroduces this variable. But the future money demand function provides a link between consumption, the exchange rate and the exogenous policy variable, solving the problem) the long-run real exchange rate can then be expressed only as a function of the change in the money supply.

\[ \hat{q} = q(\hat{M}) \]  \hspace{1cm} (49)

All three conditions now imply aggregate demand (AD) takes the following form.

\[ \hat{s}_0 = s(\hat{M}) - \frac{(\epsilon - 1)}{\epsilon - \beta(\epsilon - 1)} \left\{ q(\hat{M}) - q(\hat{y}_0; \hat{M}) \right\} \]  \hspace{1cm} (50)

To tie the model down then only requires a relationship between the nominal exchange rate and the level of output in the current period. In the current period labor is demand determined and so labor demand provides this expression. Using the goods clearing equation to express output as a function of the nominal exchange rate, the GDP deflator and nominal consumption and then using the solution for the current period exchange rate to express nominal consumption as a function of the nominal exchange rate and the money supply implies the following aggregate supply (AS) relationship holds.

\[ \hat{y}_0 = y(\hat{s}_0; \hat{M}) \]  \hspace{1cm} (51)

Equations (50) and (51) provide a solution to the model because together they provide an expression for the current period nominal exchange rate as a function of the policy variable. They are also equivalent to (34) and (37), respectively.

**D. Exchange Rate Dynamics Revisited**

The economy is complicated significantly by relaxing the assumption of a unit substitution elasticity between goods. To fix ideas it is therefore useful to first consider the Cobb-Douglas case (i.e. \( \lambda = 1 \), that has an exact solution) in linearized form. After doing this I then move on to allow for current account reactions to changes in the money supply to study the effect that changes in the current account have on the behavior of the exchange rate.

**I. Benchmark Case \( (\lambda = 1, \epsilon \neq 1) \)**

When preferences are Cobb-Douglas, \( \hat{s}_0 = \hat{\Omega}_0, \hat{s} = \hat{M}, \hat{q}_0 = n\hat{y}_0 \) and \( \hat{q} = 0 \). These simplifications occur as a result of the zero current account condition, which I can obtain when \( \lambda = 1 \),

\[ \text{Again note the difference between the CES and CD versions of the economy. In the CES case, monetary policy has two clear effects on the real-side of the economy; one via the exchange rate itself as before and a second effect through the money supply - it is this second effect that drops out when } \lambda = 1. \text{ This is again a result of second the interaction effect described above because to solve for the final AS curve I have directly accounted for change in the trade balance as a result of changes in the money supply.} \]
regardless of the value of $\epsilon$. Together these imply that the AD curve can be written from (50) as,

$$\hat{y}_0 = \frac{\epsilon - \beta (\epsilon - 1)}{(1 - \beta) n (1 - \epsilon)} \left\{ \frac{\hat{M}}{\hat{s}_0} \right\}$$

(52)

As before, I can draw this in ($\hat{s}_0, \hat{y}_0$) space and since $\beta \in (0, 1)$ it has a (linear) positive slope only when $\epsilon > 1$. The corresponding AS curve is,

$$\hat{y}_{h0} = \hat{s}_0 / \alpha$$

(53)

In ($\hat{s}_0, \hat{y}_0$) space it has a positive and steeper slope since the multiplier on the AD curve falls between zero and one whereas the slope of aggregate supply is governed by the returns to scale in production, which here are always less than one (as $\alpha > 1$). The combination of these two conditions then implies, when $\epsilon = 1$, there is no overshooting of the current exchange rate in response to a permanent increase in the money supply, i.e. $\hat{s}_0 = \hat{M}$. Analogously, when $\epsilon > 1$ there is overshooting and when $\epsilon < 1$ the exchange rate undershoots. Overshooting is also increasing in $\epsilon$ and decreasing in $n$, openness, as in the non-linear case. Here, because the solution method transforms the variables into deviations from their steady-state value it is straightforward to look at the equilibrium level of the exchange rate directly.

II. Exchange Rate Dynamics with Asset Accumulation ($\lambda > 1$)

Now I allow for asset accumulation and assume $\hat{M} = 1$.\textsuperscript{37} To make things clearer consider the following expressions,

$$\hat{s}_0 - s (1) = -\frac{(1 - \beta) (\epsilon - 1)}{\epsilon - \beta (\epsilon - 1)} \{ q (1) - q (y (\hat{s}_0; 1); 1) \}; \lambda > 1$$

$$\hat{s}_0 - 1 = \left( \frac{\alpha \{ \epsilon - \beta (\epsilon - 1) \}}{n (\epsilon - 1) (1 - \beta)} - 1 \right)^{-1}; \lambda = 1$$

(54)

where the right hand side represents the overshooting of the nominal exchange rate in response to monetary policy shock. First, it should be clear that in the $\lambda = 1$ case the analytical expression for overshooting is trivial, i.e. $\hat{s}_0 - 1$. However, the relative magnitude of the long-run and short-run change in the exchange rate for the $\lambda > 1$ case, is dependent on all the underlying structural parameters of the model and not just $\lambda$. Second, when $\lambda = 1$, the short-run change in the real exchange rate is $q (y (\hat{s}_0; 1); 1) = n\hat{y}_{h0} > 0$ whereas the long-run change is $q (1) = 0$, and these have been subsumed in the second expression. But, when $\lambda > 1$, $q (1) < 0$, i.e. the long-run real exchange rate falls even if in the short run it rises (a real depreciation of the domestic currency). As $q (1)$ enters negatively on the right-hand side of (54) it is immediate that this serves to increase the magnitude of exchange rate overshooting.\textsuperscript{38}

\textsuperscript{37}That the change in the money supply has been assumed equal to unity, i.e. $\hat{M} = 1$, does not hold well with the idea that the shock is ‘small’. But the figure gives an approximate idea of the initial reaction. It is possible to multiply the expressions by $10^{-2}$ to consider a one percentage change.

\textsuperscript{38}The exact analytical expression for $q (1) < 0$ is very cumbersome. In the next section I use a simple parameterized example to show the implications of this more clearly.
So what is the intuition for this result? As should be clear, the parameter $\lambda$, which magnifies the reaction of the exchange rate, also determines the reaction of the trade balance of the economy to a monetary shock. When $\lambda > 1$, the economy runs a current account surplus, and when $\lambda < 1$ the change in the money supply causes a current account deficit. On impact, the rise in $\hat{M}$ unambiguously raises the real exchange rate ($\hat{q}_0 > 0$) and the increased competitiveness of domestic goods leads to an increase in domestic output ($\hat{y}_0 > 0$) through the usual expenditure switching channel. When $\lambda > 1$, this also has a positive impact on the trade balance. But a key implication of changes in the net foreign asset position of the economy is that money shocks affect output beyond the current period; that is, beyond the length of the wage rigidity. In this case, there is also a fall in the long-run trade balance and an associated rise in the long run level of net foreign assets as, from (44), it must be $\hat{a}_{-1} = \hat{\Gamma}_0 + (\beta / (1 - \beta)) \hat{\Gamma}$. With $\hat{a}_{-1} = 0$, when $\hat{\Gamma}_0 > 0$, and long-run change in the trade balance is negative. This results in wealthier domestic households that supply less labor whilst consuming more goods and it is the fall in labor supply that reduces long run output with interest receipts on net foreign assets allowing for a permanent trade deficit. For a given interest elasticity of money demand, the overall impact of these changes in the behavior of the economy is to increase the liquidity effect.

This result may initially seem surprising when considering a main implication from the seminal work in this area, Obstfeld and Rogoff (1995). There the dynamics of the exchange rate are dampened for the same reasons they are magnified when $\epsilon \neq 1$ - asset accumulation. This feature is also noted in Kollmann (2001) and it is possible to generate a similar result in the economy described above, when $\epsilon = 1$, because in that case the solution for the exchange rate is given by the following expression.

$$
\hat{s}_0 = \frac{(1 + (1 - n) [(1 - \beta) \Psi + \beta \Upsilon]) / \Theta}{1 - (1 - \beta) n \Psi - \beta [n + 1 / (\alpha \kappa - 1)] \Upsilon}
$$

(55)

where $\Psi$, $\Upsilon$ and $\Theta$ are composite values of the underlying structural parameters, defined in the Appendix. Note when $\lambda = 1$, $\Psi = \Upsilon = 0$ and $\Theta = 1$, such that $\hat{s}_0 - 1 = 0$, i.e. CD preferences with $\epsilon = 1$, allow a proportional change in the exchange rate for a given change in the money supply. With $\lambda \neq 1$, it is then possible to show $d\hat{s}_0 / d\hat{M} < 1$ when $\lambda > 1$ for $\lambda$ in the neighborhood of 1, which implies that the change in the exchange rate is less than proportional to the change in the money supply. As a result of changes in the net foreign asset position of the economy (that derive from the interaction of the money shock and sticky wages) the exchange rate is less sensitive to changes in the money supply than in the flex-wage environment. This is contrary to the effect asset accumulation has when the money demand is interest inelastic despite the fact that the net foreign asset position reacts in essentially the same way.39

**III. A Parameterized Example**

There is no simple analytical expression for overshooting in the most general case I consider and neither is there a simple diagrammatic representation of the economy that corresponds to

39It is also far easier to verify, in this particular case, that $\hat{q}_0 > 0$ and $\hat{q} < 0$. 

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(52) and (53). To gain some idea about the behavior of the exchange rate in the short-run it is easier to parameterize the solution in (54) (of which $\lambda = 1$ corresponds to (52) and (53)). I choose parameters set at $\alpha = 2$, $\kappa = 2$, $n = 0.6$. The steady-state interest rate is assumed to be 4.2% (i.e. $\beta = 0.96$) with values for the interest elasticity set at $\epsilon = \{1.1, 2, 4\}$ consistent with Chari et al. (2002) who choose a value of $\epsilon = 2.56$. The parameterization is independent of the weight of monetary frictions in utility. I also consider the range $\lambda \in (1, 3)$ for the elasticity of substitution between goods in line with values used in Chari et al. (2002) and Smets and Wouters (2002).

![Figure 3: Exchange Rate Dynamics when $\lambda \in (1, 3)$](image)

Any point above the horizontal axis shows that the exchange rate overshoots in response to the change in the money supply as the horizontal axis is equivalent to $\epsilon = 1$.\footnote{This result is robust to change in other parameter values.} The origin then corresponds to the $\epsilon = \lambda = 1$ case ($\hat{s}_0 - \hat{s} = 0$). From inspection it is clear how $\lambda$ magnifies
the initial dynamics of the nominal exchange rate. The factor by which overshooting changes is also not trivial. For example, moving from $\lambda = 1 \rightarrow 1.5$ when $\epsilon = 4$ changes the value of overshooting 1.05 to 1.2, an increase of around 14%. For the space considered in figure three it is also possible to verify the claims made regarding the behavior of the real exchange rate which drive the result, that is, it is possible to compute numerical values for the real exchange rate such that, $\hat{q}_0 < 0$ and $\hat{q} > 0$.

The idea that asset accumulation magnifies the reaction of the nominal exchange rate is something that has not been highlighted before. And it is also clear that this mechanism is powerful enough, in some cases, to produce a 'perverse' result. That is, even when money demand is relatively interest inelastic, when traded goods are compliments, the economy runs a long-run trade surplus and the long-run reaction of output (which would be positive) is larger than that of current output. This throws the liquidity effect into reverse and can produce undershooting. Naturally, the same is true for the case, $\lambda > 1$ and $\epsilon < 1$, although neither of these cases seem particularly interesting on empirical grounds.

V. Conclusion

The Mundell-Fleming approach assumes that wage rigidities allow monetary policy to affect output in line with the standard ISLM model. But in an open economy the CPI is a function of both domestic and foreign prices. The CPI changes even when domestic prices are rigid so that monetary policy has a direct link to prices via changes in the exchange rate (pass-through) and the terms of trade influence output (expenditure switching). In the closed economy ISLM model an increase in the money supply reduces the nominal interest rate through a liquidity effect also raising output. Here, I assume nominal wage rigidity and a goods market structure consistent with the Mundell-Fleming model. Home-bias is then also shown to lead to a liquidity effect. The paper therefore makes two main contributions.

First, and consistent with older analysis, I show that using the small open economy assumption with an import-export structure there are additional dynamics in the exchange rate in response to monetary shocks when the interest elasticity of money demand is sufficiently low and the substitution elasticity between goods is unity (this is solved exactly). This suggests the introduction of a traded-nontraded goods structure is not required to generate overshooting because the assumption of specialized outputs delivers the same type of dynamics when using a fully specified dynamic general equilibrium model.

Second, in a general equilibrium setting, overshooting depends on the degree of substitution between goods - this is not evident in a directly postulated model. When the substitution elasticity between goods has a non-unit value there may be under or overshooting when money demand is not too responsive to changes in the nominal interest. Overshooting in the exchange rate, for empirically reasonable values of $\lambda$, is magnified because asset accumulation allows monetary policy to have an affect beyond the length of the rigidity and this increases the strength of the liquidity effect working on the nominal interest rate.
Appendix

Here I provide a formal derivation of the overshooting result, showing the additional steps required to implicitly differentiate the final ADAS system (34) and (37). I also fill in some of the steps needed to derive the CES version of the model.

A. Deriving the Overshooting Multiplier for the CD Case

The Cobb-Douglas final ADAS system can be written as,

\[ 0 = F_{AS}(w_0; y_0, s_0) \]  
\[ 0 = F_{AD}(M_1; y_0, s_0) \]

where \( F_{AS} \) describes the equilibrium supply-side relations, and \( F_{AD} \) the equilibrium demand-side relations. To determine the effect of a permanent unanticipated monetary expansion on the economy I implicitly differentiate (56) and (57). The reaction of the economy to a change in the money supply is captured by,

\[
\left( \frac{\partial F_{AS}}{\partial y_0} \frac{\partial F_{AS}}{\partial s_0} \right) \left( \frac{\partial y_0}{\partial M_1} \frac{\partial s_0}{\partial M_1} \right) = - \left( \frac{\partial F_{AS}}{\partial M_1} \frac{\partial F_{AD}}{\partial y_0} \frac{\partial F_{AD}}{\partial s_0} \right) (58)
\]

From here it is straightforward to recover \( \frac{\partial y_0}{\partial M_1} \) and \( \frac{\partial s_0}{\partial M_1} \). Applying Cramer’s rule to (58) the reaction of the exchange rate and output to a change in the money supply is given by,

\[
\frac{\partial s_0}{\partial M_1} = \left( \frac{\partial F_{AD}}{\partial y_0} \frac{\partial F_{AS}}{\partial M_1} - (\partial F_{AS}/\partial y_0)(\partial F_{AD}/\partial M_1) \right) / (\partial F_{AD}/\partial y_0)(\partial F_{AS}/\partial s_0) - (\partial F_{AD}/\partial y_0)(\partial F_{AS}/\partial s_0)
\]

\[
\frac{\partial y_0}{\partial M_1} = \left( \frac{\partial F_{AS}}{\partial y_0} \frac{\partial F_{AD}}{\partial s_0} - (\partial F_{AS}/\partial y_0)(\partial F_{AD}/\partial M_1) \right) / (\partial F_{AS}/\partial y_0)(\partial F_{AD}/\partial s_0) - (\partial F_{AD}/\partial y_0)(\partial F_{AS}/\partial s_0)
\]

The derivatives of interest are,

\[
\frac{\partial F_{AS}}{\partial y_0} = \alpha y_0^{n-1} \]  
(61)

\[
\frac{\partial F_{AS}}{\partial s_0} = g^* / (n - 1) \alpha w_0 \]  
(62)

\[
\frac{\partial F_{AS}}{\partial M_1} = 0 \]  
(63)

\[
\frac{\partial F_{AD}}{\partial y_0} = \gamma n (1 - \epsilon) y_0^{n(1-\epsilon)-1} \]  
(64)

\[
\frac{\partial F_{AD}}{\partial s_0} = (\eta M_1 / s_0 \beta)^{\epsilon} \left( (1 - \epsilon) (\eta M_1 / s_0 \beta)^{-1} + \epsilon \right) / s_0 \]  
(65)

\[
\frac{\partial F_{AD}}{\partial M_1} = (\eta M_1 / s_0 \beta)^{\epsilon} \left( (\epsilon - 1) (\eta M_1 / s_0 \beta)^{-1} - \epsilon \right) / M_1 \]  
(66)

It is possible sign all of these derivatives; (61) is positive and (62) is negative. Equation (64) is positive if \( \epsilon < 1 \), and negative if \( \epsilon > 1 \). As the gross nominal interest rate \( (1 + i_0) = (\eta M_1 / s_0 \beta) > (\epsilon - 1) / \epsilon \forall \epsilon > 0 \), (65) is positive and (66) negative.
Before deriving the multiplier it is useful to check that the denominator in (59) and (60) is positive, as it must be. Re-substituting \( 0 = F_s(y_{h,0}, 0) \) and \( 0 = F_d(y_{h,0}, s_0; M_1) \) back into this condition I can rewrite it in terms of exogenous parameters and the nominal interest rate. A positive denominator requires,

\[
(1 + i_0) > (1 - n/\alpha) / (\epsilon/(\epsilon - 1) - n/\alpha)
\]

This is trivially satisfied as \( n/\alpha < 1 \) and \( \epsilon/(\epsilon - 1) > 1 \) for \( \epsilon > 1 \). The right hand side of (67) is always less that one, whereas the left hand side, which is the gross nominal interest rate, is always greater than one. In economic terms, output always rises and the exchange rate always depreciates when there is a change in the money supply.

To derive the overshooting multiplier, I essentially perform the same substitutions to the numerator of (59). Replacing \( \eta M_1/s_0 \beta \) with the nominal interest rate and after some simplifications,

\[
\left( \partial s_0 / \partial M_1 \right) M_1 / s_0 = \frac{\alpha (\epsilon + (1 - \epsilon) / (1 + i_0))}{(1 - \epsilon) (n + (\alpha - n) / (1 + i_0)) + \alpha \epsilon}
\]

It is clear now that when \( \epsilon = 1 \) the multiplier is unity. Whether or not the multiplier is greater or less than one for \( \epsilon \neq 1 \) depends on whether \( n (1 - \epsilon) i_0 / (1 + i_0) \leq 0 \). When \( \epsilon > 1 \), \( n (1 - \epsilon) i_0 / (1 + i_0) < 0 \) and the multiplier is greater than one.

B. Additional Derivations for the CES case

The key relationships that change as a result of assuming a CES sub-utility function are the goods market clearing condition and the trade balance. Respectively these are,

\[
y_t = n C_t (P_t/P_{h,t})^\lambda + g^* (s_t/P_{h,t})^\lambda
\]

\[
\Gamma_t = s_t^{\lambda} g^* P_{h,t}^{1-\lambda} - (1 - n) s_t^{1-\lambda} P_t^\lambda C_t
\]

It is also worth noting that I assume a zero inflation, balanced trade steady-state.

I. Real Side of the Economy

With CES preferences, aggregate supply-side of the economy is unchanged because the labor demand and labor supply conditions, (4) and (9), are unaffected. But the relationship between the CPI and the GDP deflator, which determines the real exchange rate, does change. This means that although the aggregate supply curve is an equation in \( (y_{h,t}, C_t, q_t) \) the relation between all three variables changes when \( \lambda \neq 1 \). Again, the important point is that output still only depends on consumption and relative prices. As in the main text, the goods and labor market clearing relationships for \( t \geq 1 \) form an implicit system in output, consumption and the real exchange rate. An approximation of the CPI implies, \( \hat{P}_t = n \hat{P}_{h,t} + (1 - n) \hat{s}_t \) for
all \( t \). The period \( t \geq 1 \) goods and labor market clearing equations can then be rewritten up to a first-order approximation as,

\[
\hat{y}_t = n\hat{C}_t + \left(\frac{\lambda}{n}\right) (1 - n^2) \hat{q}_t \tag{71}
\]
\[
\hat{y}_t = \left(\frac{1}{1 - \kappa\alpha}\right) \left(\hat{C}_t + \hat{q}_t ((1 - n)/n)\right) \tag{72}
\]

These can be written as \( \hat{q}_t = q\left(\hat{C}_t\right) \) and \( \hat{y}_t = y\left(\hat{C}_t\right) \).

II. Current and Future Exchange Rate

As is clear from equation (70) with the more general CES preferences the behavior of the trade balance is no longer independent of relative prices (specifically a \( P_h \) term enters). However, the forward solution for the national budget constraint (NIBC) is as in (27) above. That this solution is linearized around a zero trade balance steady-state plays an important role and in particular it eliminates any interest rate differentials from directly entering the NIBC, cf. (44).

Once all equilibrium conditions have been linearized, solving for the exchange rate from (44), there are two stages. First, I need to eliminate the GDP deflator (for both the long and short-run) using an ‘intermediate’ AD-AS system, given by (71) with either (72) or labor demand. Second, I need to eliminate the current period domestic nominal interest rate using the current period money demand function.

The intermediate AD-AS system can be represented schematically in the following way.

\[
\hat{P}_{h,t} = P_h \left(\hat{\Omega}_t, \hat{s}_t\right) \quad \text{for all } t \tag{73}
\]

The function \( \hat{P}_{h,t} = P_h (\cdot) \) captures the idea of an intermediate AD-AS system - the period \( t = 0 \) version combines labor demand and goods market clearing, whilst the period \( t \geq 1 \) simply uses (71) and (72) along with the CPI. Using (73) with (44) there is an equivalent simplified national intertemporal constraint.

\[
\hat{s}_0 (1 - \beta) \Theta \left[1 + (1 - n) \Psi\right] + \hat{s} \beta \Theta \left[1 + (1 - n) \Upsilon\right]
\]
\[
\quad = \hat{\Omega}_0 (1 - \beta) \left[1 + n\Psi\right] + \hat{\Omega} \beta \left[1 - \Upsilon \left(n + 1/(\alpha\kappa - 1)\right)\right] \tag{74}
\]

where \( \Theta \equiv (\lambda + n (\lambda - 1)) \) and \( \Phi \equiv (n^2 (1 - \lambda) + \lambda) \) are functions of openness and the consumption elasticity of intratemporal substitution and \( \Psi \equiv (1 + n) (1 - \lambda)/(1 + (\alpha - 1) \Phi) \) and \( \Upsilon \equiv (1 + n) (1 - \lambda) (\alpha\kappa - 1)/(1 + (\alpha\kappa - 1) \Phi) \) are composites, also including measures the elasticity of substitution of labor and the returns to scale in production. Note that this expression is significantly more complicated than when \( \lambda = 1 \). Intuitively, it helps to go back and think about the Cobb-Douglas case. If, \( \lambda = 1 \) then \( \Theta = \Phi = 1 \) and \( \Psi = \Upsilon = 0 \) and so the NIBC reduces to \((\beta - 1) \left(\hat{s}_0 - \hat{\Omega}_0\right) = -\beta \hat{\Omega} + \beta \hat{s}\). Previously I used the UIP and consumption Euler equations (in linear form these are given by \( \hat{s} = \hat{s}_0 + (1 - \beta) \hat{i}_0 \) and \( \hat{\Omega} = \hat{\Omega}_0 + (1 - \beta) \hat{i}_0 \)
respectively) to reduce this condition to $\hat{s}_t = \hat{\Omega}_t$ for all $t$. This simplification is possible because there is a one-to-one relationship between the current period nominal exchange rate and nominal consumption; a unit coefficient property in which there is no additional interest rate effect. When $\lambda \neq 1$ this simplification is lost and any expression for the nominal exchange rate will involve an expression in current nominal consumption and the current period nominal interest rate.

i. Solving for $\hat{s}_0$

Using the simplified NIBC and linear approximations for the Euler and UIP conditions, $\hat{s}_0 = s(\hat{\Omega}_0, \hat{i}_0)$. The change in the interest rate is then captured by appealing to the current period money demand function (also using the appropriate supply-side equation to describe the reaction of the domestic price level), which can be written as,

$$\hat{i}_0 = F_1 \hat{\Omega}_0 + F_2 \hat{s}_0 - \epsilon \hat{M}$$

$$F_1 \equiv (1 - \epsilon) (1 - n) [1 + (\alpha - 1) (\Phi + n\Theta)] / (1 + (\alpha - 1) \Phi)$$

$$F_2 \equiv (1 + (\alpha - 1) (\Phi + (\epsilon - 1) n^2)) / (1 + (\alpha - 1) \Phi)$$

and so $\hat{i}_0 = i(\hat{\Omega}_0, \hat{s}_0, \hat{M})$. Note that when so that when the interest elasticity of money demand is unity, i.e. $\epsilon = 1$, $F_1 = 0$ and $F_2 = 1$, so $\hat{i}_0 = 0$. The reason $\hat{i}_0 = 0$ when $\epsilon = 1$ is that logarithmic preferences over real balances implies the interest rate jumps immediately to it’s steady-state despite the change in the money supply (and $\hat{\Omega}_0 = \hat{\Omega} = \hat{M}$). Using the money demand function, the Euler and UIP conditions and the simplified NIBC I can then write (45) in the main text.

ii. Solving for $\hat{s}$

The NIBC, UIP and consumption Euler equations also imply, $\hat{s} = s(\hat{\Omega}, \hat{i}_0)$. Again, I use the current period money demand function, (75), but now I also substitute in the Euler and UIP conditions to derive an equation for money demand such that $\hat{i}_0 = i(\hat{\Omega}, \hat{s}, \hat{M})$. Finally, it is possible to eliminate $\hat{\Omega}$ because the future period money demand function can be expressed as,

$$\epsilon \hat{M} = F_3 \hat{\Omega} + F_4 \hat{s}$$

$$F_3 \equiv (1 + (\epsilon - 1) n + (\alpha - 1) (\Phi + n^2)) / (1 + (\alpha - 1) \Phi)$$

$$F_4 \equiv (\epsilon - 1) (1 - n) (1 + (\alpha - 1) (\Phi + \Theta n)) / (1 + (\alpha - 1) \Phi)$$

where $F_3 = 1$ and $F_4 = 0$ when $\epsilon = 1$. Thus I now have $\hat{i}_0 = i(\hat{s}; \hat{M})$. Combining equations, I can then solve for the future exchange rate as (46) in the text.

III. Exchange Rate Dynamics when $\epsilon = 1$
In the text I claim that \( \frac{\partial}{\partial \lambda} \left( \frac{d\tilde{s}_0/d\tilde{M}}{d\tilde{M}} \right) \bigg|_{\lambda=1} < 0 \). Using (74) when \( \hat{\Omega}_0 = \hat{\Omega} \) and \( \hat{s}_0 = \hat{s} \), it is possible to show that,

\[
\left. \frac{\partial}{\partial \lambda} \left( \frac{d\tilde{s}_0/d\tilde{M}}{d\tilde{M}} \right) \right|_{\lambda=1} = \frac{n (1 - \beta) (\alpha - 1)}{\alpha/\left[ n^2 - (2 + n) \right]} - \left[ \left( \frac{\alpha \kappa - 1}{\alpha \kappa} \right) + \beta (\alpha \kappa + 1) \right] (1 + n) \quad (77)
\]

which is negative as \( n \in (0, 1) \). Given that \( d\tilde{s}_0/d\tilde{M} = 1 \) (as it is when \( \lambda = 1 \)), raising \( \lambda \) slightly above unity lowers \( d\tilde{s}_0/d\tilde{M} \) slightly below one.
References


