Marginal wage subsidies: a redistributive instrument for employment creation

Andreas Knabe*

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Abstract

Critics of marginal wage subsidies claim that their inter-firm displacement effects eliminate their employment and fiscal advantages over general wage subsidies. We develop a model in which we show that the contrary is correct. Inter-firm displacement with marginal subsidies forces down prices and raises output and employment more than under general subsidies. Moreover, we show that marginal subsidization is less expensive for the public budget than general subsidies and that it serves as a redistribution device by raising the labor share in the functional income distribution. A numerical illustration is used to support the theoretical results and to estimate their effects on unemployment in Germany.

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*Otto-von-Guericke-University Magdeburg, P.O. Box 4120, D-39016 Magdeburg (Germany), Telephone (+49) 391 67 18518, Fax (+49) 391 67 11218, Email: Andreas.Knabe@ww.uni-magdeburg.de

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1 Introduction

Low-skilled labor in industrialized countries has come under pressure, because skill-biased technological progress and international economic integration have decreased the productive capacity of low-skilled individuals below what is deemed an acceptable standard of living. Without government intervention, this segment of the labor market would become working poor. If, however, government intervention takes the form of introducing minimum wages or allowing other rigidities on the labor market, such as wage-replacing unemployment benefits, the result will be unemployment among the low-skilled.

As a remedy to this problem, wage subsidies have been proposed to restore employment opportunities and incomes in the low-skill segment. In flexible labor markets with working poor, such subsidies lift up net wages to a socially acceptable level. It does not matter whether the subsidy is given to employers or employees – market forces will always shift the subsidy such that an identical equilibrium outcome is obtained. With rigid gross wages, however, only wage subsidies to employers will promote employment while wage subsidies for employees will only raise the net income of incumbent workers. Therefore, only employer-oriented wage subsidies can reduce unemployment both in rigid and flexible labor markets. Kaldor (1936) was the first to propose such an employer-oriented wage subsidy. More recently, various types of wage subsidies were suggested by Haveman (1996), Orszag and Snower (2000, 2003), Phelps (1997), and Snower (1994).

One of the main objections to general wage subsidization is its fiscal costs. To overcome this problem, policy proposals known as marginal wage subsidies (MS) recommend to restrict the subsidy to a firm’s additional employment only. This would reduce fiscal costs while maintaining the same marginal stimulus to employment creation as a general subsidy. Marginal wage subsidization has been proposed by Chiarella and Steinherr (1982), Haveman (1996), Layard and Nickell (1980), and Rehn (1982). The MS-concept was recently reintroduced to the political discussion in Germany by Schöb and Weimann (2003, 2005) under the name Magdeburg Alternative, and is now intensively debated among economists as well as politicians (Bothfeld et al. 2006, German Council of Economic Experts 2005, Sinn et al. 2006).

There is an obvious way how firms could circumvent the marginal character of MS and thus counteract the efficiency of marginal subsidy schemes: firms could outsource their entire incumbent workforce to a newly established firm, in which all workers would count as newly hired and receive the subsidy. This circumvention can be avoided with double marginal subsidies (DMS), a concept first developed by Schöb and Weimann.
(2003, 2005), where for each new employee also one incumbent employee is subsidized. This doubles the marginal stimulus to employment creation in incumbent firms and prevents them from outsourcing.

Even though DMS are effective in preventing direct outsourcing, they cannot prevent competitive between-firm displacement. It has been pointed out by Layard and Nickell (1980) that if all firms in a competitive industry are offered a (single) marginal wage subsidy, they will all want to expand their production. This will cause the price in this industry to fall. Since average costs will have fallen only little, the firms will be making losses, so that some firms will leave the industry. In the resulting equilibrium, most of the employment expansion in remaining firms will have taken place at the expense of employment in exiting firms. Since most workers have to be subsidized in equilibrium, the favorability of DMS compared to GS vanishes. Knabe (2006) explicitly models this type of intra-industry competition and shows, using a partial equilibrium model of long-run industry equilibrium, that DMS generally create more employment at less fiscal costs than general subsidies (GS) despite inter-firm displacement.

In this paper, we extend the model of Knabe (2006) in three respects. First, we embed the industry-level analysis into an economy-wide model of a small open economy. Second, besides comparing the employment and fiscal effects of DMS compared to GS, we examine the distributive effects of the two subsidy schemes. In particular, we analyze the differential impact of DMS and GS on incumbent firms’ rents, incumbent workers wages, the public budget, and the functional distribution of income. Third, we use the model developed in this paper to conduct a numerical illustration of the employment and fiscal effects of DMS, exemplified by the proposal Magdeburg Alternative for Germany. The numerical illustration can be used to check the robustness of previously derived results that did not explicitly account for intra-industry displacement (Knabe, Schöb and Weimann 2006).

We will proceed as follows. In the next section, we briefly develop the theoretical model. In Section 3, we analyze and compare the differential impact of DMS and GS on employment, fiscal, and distributive variables. Section 4 contains the numerical illustration. Section 5 concludes.

2 The model

Suppose the economy consists of a large number of competitive firms that produce a homogeneous, tradable good. We assume for simplicity that all of the good is exported and that the proceeds from exporting are used to import a bundle of consumption
goods, the world market price of which is fixed and normalized to one. World demand for the output good is given by the isoelastic demand function
\[ D(p) = Ap^{-\varepsilon}, \]
where \( p \) is the output price in terms of the imported consumption good, \( \varepsilon > 0 \) is the price elasticity of demand, and \( A \) is a scaling parameter.

We assume that labor is the only (variable) factor of production. We thus neglect firm-level substitution effects between capital and labor. Even though these effects are surely very important, the debate on the efficacy of DMS and GS has been concerned only with the intra-industry effects of such subsidies in a competitive environment (see Bothfeld et al. 2006, Layard and Nickell 1980, Sinn et al. 2006). By restricting our attention to a one-factor production technology, we can focus specifically on the differential effects of DMS and GS on between-firm competition and industry structure.

A firm’s production function is given by
\[ y = f(l) = l^\alpha, \]
where \( y \) is a firm’s output level, \( l \) denotes its labor input, and \( \alpha \in [0, 1] \). Before starting production, firms face start-up costs \( F \). These costs are sunk after entry.\(^2\)

Households, the number of which is normalized to one, supply one unit of labor and receive wage income \( w \) if employed or unemployment benefits \( b \) if unemployed. Moreover, they receive firms’ profits \( \pi \) and finance government expenditures – for unemployment benefits and employment subsidies – via a labor tax \( \tau \). The government is required to balance its budget.

Unemployment arises because the wage rate \( w \) (in terms of the consumption good) is fixed above its full-employment level.

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\(^1\)These assumptions allow us to ignore income effects on demand and to focus on the production of the economy’s output good. A similar setup can be found in Ethier (1985).

\(^2\)Examples for sunk start-up costs are investments in highly specific physical assets, the gathering of information before a firm enters a specific industry, costs of organizing the new operation, product-differentiating sales efforts etc. (Martin 1993, p. 306). In the context of our model, one could also think of \( F \) being a shadow, rather than direct, cost arising from legal and administrative barriers to entry (Blanchard and Giavazzi 2003).

Empirical evidence suggests that sunk costs are an important determinant of firm and industry behavior. For example, Roberts and Tybout (1997) observed that firms are more likely to to remain in the export market (having paid the associated sunk costs) than to exit the market if they face unfavorable, but temporary shocks. Lambson and Jensen (1995, 1998) found that firm value is more variable in industries exhibiting higher sunk costs. Bresnahan and Reiss (1994) found that the minimum price that triggers entry by rural dentists is strictly higher than the maximum price that induces exit.
2.1 Initial equilibrium

The initial equilibrium is characterized by the zero-profit condition for newly entering firms. The output price in the initial equilibrium has to be equal to the firms’ minimum average cost. From the production function (2), a firm’s variable cost function is given by \( C(y) = wy^{1/\alpha} \), so that the price in the initial equilibrium is

\[
p_0 = \min_y \left[ \frac{F + C(y)}{y} \right] = \frac{F}{(1 - \alpha) y_0},
\]

where each firm produces

\[
y_0 = \left( \frac{\alpha F}{1 - \alpha w} \right)^{\alpha}.
\]

The demand function (1) then determines the total output of the economy. The number of firms \( n \) is such that this total output will be supplied by individual firms, each of which produces \( y_0 \). Hence,

\[
n_0 = \frac{D(p_0)}{y_0}.
\]

Total employment is then given by \( L_0 = n_0 y_0^{1/\alpha} \).

We have to determine the initial income distribution to examine the redistributive consequences of employment subsidization policies. Government expenditures in the initial equilibrium are \( \tau_0 = (1 - L_0) b \), operating firms obtain rents which exactly cover their start-up costs \( F \), and aggregate net labor income is given by \( wL_0 + b (1 - L_0) - \tau_0 \). The labor share in the functional distribution of income is given by the ratio of aggregate net labor income to total output, \( wL_0 / [p_0 D(p_0)] = \alpha \).

2.2 Equilibrium with general subsidies

If GS are introduced, all firms – incumbents as well as new entrants – receive the subsidy \( s \) for all their employees. Hence, their variable cost function becomes \( C(y) = (1 - s) wy^{1/\alpha} \). Because the start-up costs of incumbent firms are sunk, only newly entering firms take them into account in their entry decision. Only if the output price is high enough to ensure that rents cover the start-up costs, new firms will enter. The minimum price at which new firms are induced to enter is given by

\[
\bar{p} = \min_y \left[ \frac{F + (1 - s) C(y)}{y} \right] = \frac{F}{(1 - \alpha) \bar{y}},
\]

where each firm produces

\[
\bar{y} = \left( \frac{\alpha F}{1 - \alpha (1 - s) w} \right)^{\alpha}.
\]

\(^3\)This is the standard procedure to determine the long-run competitive equilibrium with free entry (cf. Mas-Colell et al. 1995, Section 10F).
The aggregate supply curve thus consists of two parts. For all prices less than $\bar{p}$, new firms will stay out of the market and incumbent firms supply a quantity at which price equals marginal variable costs. For all prices at and above $\bar{p}$, any aggregate output quantity can be supplied by incumbent and newly entering firms. Hence, aggregate supply with GS is given by

$$Y_{GS}(p) = \begin{cases} n_0 \left( \frac{\alpha p}{(1-s)w} \right)^{\alpha/(1-\alpha)} & \text{if } p < \bar{p} \\ n_0 \left( \frac{\alpha p}{(1-s)w} \right)^{\alpha/(1-\alpha)}, \infty & \text{if } p \geq \bar{p} \end{cases} \quad (8)$$

The upper part of Figure 1 illustrates the effects of GS at the level of an individual firm. $MC$ and $AC$ are the unsubsidized marginal and average cost curves, so that $(y_0, p_0)$ is a firm’s initial equilibrium. With GS, marginal costs are reduced to $MC_S$, while a new firm’s average cost curve is given by $AC_{new}$. For all prices below $\bar{p}$, incumbent firms choose to supply along the $MC_S$ curve. At $\bar{p}$ – the minimum average cost of a new firm – new firms enter the market and aggregate supply becomes perfectly price-elastic.

The aggregate effect of GS is shown in the bottom part of Figure 1. The dashed line indicates the aggregate supply curve $Y_{GS}(p)$. The new equilibrium price with GS is then determined by $Y_{GS}(p_{GS}) = D(p_{GS})$.

### 2.3 Equilibrium with single marginal subsidies

To aid the understanding of marginal subsidization, we will shortly discuss the functioning of single marginal subsidies ($MS$).

Upon introduction of $MS$, incumbent firms can either expand their workforce and receive the subsidy for their additional employees, or they can maintain, or even reduce, their workforce. In the latter case, none of their employees will be subsidized. The resulting marginal cost schedule can be read off in the upper part of Figure 1. For output level below $y_0$, incumbent firms are unsubsidized, so that their marginal costs are given by the $MC$-curve. If firms expand their output above $y_0$, the additional employment receives the subsidy, so that the marginal production costs drop to the $MC_S$-curve. If the output price stayed at $p_0$, all firms would want to expand their output to $MC_S^{-1}(p_0)$. For $p < p_0$, however, the price equals marginal cost at two output levels. Expanding firms obtain marginal profits for those output units in the interval $[y_0, MC_S^{-1}(p)]$, but face intra-marginal losses from those output units in the interval $[MC^{-1}(p), y_0]$. There will be a price level $\hat{p}$ (not plotted in Figure 1) at which incumbent firms are just indifferent between expanding and contracting.
Figure 1: Firm-level and aggregate effects of GS and DMS
The aggregate supply curve with MS is made up of four parts.⁴ If the output price is below \( \hat{p} \), all incumbent firms will want to reduce their production compared to \( y_0 \). At \( \hat{p} \), some firms will expand while others will contract. The supply curve is horizontal and the shares of expanding and contracting firms are determined by total demand. If the output price is between \( \hat{p} \) and \( \bar{p} \) (as defined in equation 6) all incumbent firms will expand but no new firms will enter. \( MC^{-1}_S(p) \) will then determine aggregate supply. At \( \bar{p} \), new firms will enter the market and supply any output demanded at that price.

The above reasoning about the functioning of MS ignores that incumbent firms can circumvent the restrictions imposed by the initial employment level as the subsidy threshold. Any incumbent firm could establish a second, new firm to which it outsources all its labor inputs. Since the new firm’s employment level at the reference date would be zero, all workers hired in the new firm had to be subsidized, but net employment would not have changed. By receiving the subsidy for its complete workforce, an incumbent firm could substantially increase its profits by outsourcing. A counteracting force is that outsourcing is not costless: a new firm has to be established, administrative costs increase, production plans have to be reorganized, new employees need to receive some training etc.

If outsourcing costs were equal to start-up costs \( F \), outsourcing would never be a profitable strategy for incumbent firms. Outsourcing firms would be faced with the same payoffs as new entrants, so that they could at best achieve zero profits. It is, however, plausible to assume that outsourcing part of a firm’s workforce costs much less than establishing a completely new firm. Under the assumption that outsourcing costs are too low to prevent incumbent firms from outsourcing, one has to set economic incentives that make it more profitable to expand employment in the already existing firm instead of outsourcing to a new firm. DMS – which combine a subsidy for additional employment with a stock subsidy for incumbent workers – provide such incentives because subsidizing part of the existing workforce does not affect the profit from outsourcing, but it increases the profit from expanding employment in the existing firm. As is shown in Knabe (2006), this substantially reduces the minimum level of outsourcing costs necessary to prevent the evasion of marginal subsidization via outsourcing. In the remainder of this paper, we will assume that outsourcing costs are sufficiently large to prevent such behavior under DMS. With this assumption, we can restrict our attention to the DMS-case.

⁴We abstract from the case that \( \bar{p} \geq \hat{p} \).
2.4 Equilibrium with double marginal subsidies

If DMS are introduced, firms have the choice either to expand their workforce and obtain the double subsidy for their new employees, or to maintain or reduce their employment level without receiving a subsidy for any of their employees. The upper part of Figure 1 shows the resulting marginal cost curve for an incumbent firm: below $y_0$, its marginal costs are given by the $MC$–curve. If output exceeds $y_0$, the additional employment receives the double subsidy, so that marginal costs drop to $MC_{DMS}$. Only if output is expanded beyond $y_2 = f(2f^{-1}(y_0))$, i.e. beyond the level at which a firm’s employment doubles compared to its initial level, additional employment receives only the single subsidy and marginal costs rise to $MC_S$. Since start-up costs are sunk, an incumbent firm’s average variable costs are given by $AC_{inc}$.

There will be a price level $\tilde{p}$ at which an incumbent firm makes the same profit by either contracting or expanding. In Figure 1, this price level can be read off where the distances between output price $\tilde{p}$ and average variable costs $AC_{inc}$, multiplied by the output level at which price equals marginal costs, are equal for expanding ($y_0^+$) and contracting ($y_0^-$) firms (the two hatched areas are of equal size). Formally, $\tilde{p}$ is implicitly defined by

$$\tilde{p} \left( \frac{\alpha \tilde{p}}{w} \right)^{\alpha/(1-\alpha)} - w \left( \frac{\alpha \tilde{p}}{w} \right)^{1/(1-\alpha)} = \begin{cases} \tilde{p} \left( \frac{\alpha \tilde{p}}{(1-2s)w} \right)^{\alpha/(1-\alpha)} - (1-2s)w \left( \frac{\alpha \tilde{p}}{(1-2s)w} \right)^{1/(1-\alpha)} & \text{if } \tilde{p} < (1-2s)C'(y_2) \\ -2sw_0^{1/\alpha} & \text{if } \tilde{p} \geq (1-2s)C'(y_2) \end{cases}$$

where the LHS and the RHS of (9) are the profit of a contracting and an expanding firm, respectively.

The aggregate supply function with DMS, depicted by the solid line in the bottom part of Figure 1, thus consists of five sections. If the market price is very low

\footnote{Note that we restrict $s$ to the interval $[0, 1/2]$. We thus rule out the case where the single subsidy exceeds 50 percent because the marginal labor cost under DMS would then become negative.}

\footnote{The price level $\tilde{p}$ has to defined piece-wise, since it could either be where expanding firms increase their employment to less than twice the initial level ($\tilde{p} < (1-2s)C'(y_2)$), or where they double their initial employment ($\tilde{p} \geq (1-2s)C'(y_2)$).}

\footnote{Depending on the chosen functional specification, some of these sections might not exist, see Appendix A. Without loss of generality for our formal analysis, we will restrict our attention to the most general case in which all five sections feasible with our functional specification exist. In the numerical simulations in Section 4, we will include all different variants.}
$(p < \bar{p})$, all incumbent firms contract because it is not profitable to expand production despite the subsidy (Section 1). If $p = \bar{p}$ (Section 2), some incumbent firms expand and others contract, but all firms make the same profit. If $\bar{p} < p < (1 - 2s) C''(y_2)$, all incumbent firms increase their employment (Section 3). As long as the price is less than $(1 - 2s) C''(y_2)$ – the marginal production cost at twice the initial employment level –, it is not profit-maximizing for the firm to double employment. If $(1 - 2s) C''(y_2) \leq p < \bar{p}$, all incumbent firms exactly double their employment (Section 4). Further employment expansion would only be profitable if the output price exceeded $(1 - s) C''(y_2)$, because only the single subsidy rate would be paid at the margin. Finally, if $p = \bar{p}$, all incumbent firms double their employment and new, completely subsidized firms enter the market (Section 5). With these considerations, one obtains a piecewise-defined aggregate supply function $Y_{DMS}(p)$ (see Appendix A). With $DMS$, the new equilibrium price is then determined by $Y_{DMS}(p_{DMS}) = D(p_{DMS})$.

3 Comparing double marginal and general subsidies

Equilibria with $GS$ and $DMS$ are quite different. In this section, we will compare the output, employment, and aggregate income effects of $DMS$ compared to $GS$ and the initial equilibrium, and examine the impact on incumbents’ rents, government deficit, and the functional distribution of income.

Let us start with the output effect.

**Proposition 1 (Output)** Aggregate output with $DMS$ is at least as large as that with $GS$, if both are provided at an equal rate. For both types of subsidy, the aggregate output level exceeds that in the initial equilibrium.

**Proof.** See Appendix B. ■

The main force driving the differential effects of $DMS$ and $GS$ is the displacement competition between incumbent firms triggered by $DMS$. The large reduction in marginal costs under $DMS$ makes all firms want to expand production compared to the initial, unsubsidized equilibrium. This drives down the market price. Since incumbent workers are not subsidized, average costs fall less than marginal cost. Firms will make losses, so some firms will significantly decrease their production level to reduce average (variable) costs. The sunkness of start-up costs, however, prevents incumbent firms from exiting the market completely despite the losses they make compared to the initial equilibrium. The displacement competition drives down output prices significantly, and thus raises demand and aggregate output.
Under GS, no such displacement competition occurs. Since all firms are subsidized independently of whether they expand production or reduce it, firms cannot profitably underbid their competitors by expanding production. Since the subsidy is only paid at the single rate, firms’ desire to expand production is smaller than under DMS, prices do not fall as much, and hence demand and aggregate output do not increase as much as under DMS.

If demand is sufficiently elastic, so that the output price reaches \( \bar{p} \), new firms enter the market. Only in this case, both DMS and GS yield the same total output.

Let us turn to the employment effects.

**Proposition 2 (Employment)** Aggregate employment with DMS is always larger than that with GS, if both are provided at an equal rate. For both types of subsidy, the aggregate employment level exceeds that in the initial equilibrium.

**Proof.** See Appendix B. □

DMS induce a technical inefficiency in the industry structure. Since they reward only the expansion of firms, even if it takes place at the expense of other firms, they result in larger firms than GS. Since marginal productivity is smaller in larger firms, this means that average labor productivity is smaller under DMS, so that more labor is needed to produce the same amount of output. This technical inefficiency, combined with the generally favorable output effects of DMS, implies that DMS also cause larger employment effects than GS.

Our measure of aggregate domestic income \( I \) comprises aggregate net labor incomes and rents minus start-up costs. Since taxes and the payment of unemployment benefits and subsidies net out in aggregate incomes, the sum of aggregate firm rents and aggregate labor incomes is the same as aggregate firm revenues. The aggregate domestic income thus reduces to aggregate firm revenues minus start-up costs. In our model, the relationship between aggregate domestic income in the two subsidized equilibria, \( I_{DMS} \) and \( I_{GS} \) respectively, and the initial income level \( I_0 \) depends on the elasticity of world output demand.

**Proposition 3 (Aggregate domestic income)** If world output demand is

\[
\begin{align*}
\begin{cases}
\text{elastic} & (\varepsilon > 1) \\
\text{unit elastic} & (\varepsilon = 1) \\
\text{inelastic} & (\varepsilon < 1)
\end{cases}, \text{ the income relation is}
\end{align*}
\begin{align*}
\begin{cases}
I_{DMS} > I_{GS} > I_0 \\
I_{DMS} = I_{GS} = I_0 \\
I_{DMS} < I_{GS} < I_0
\end{cases}.
\end{align*}
\]

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Proof. See Appendix B. ■

By paying a subsidy, the government acts like a monopolistic intermediary because it directly influences the equilibrium output price. If demand is unit elastic ($\varepsilon = 1$), any change in the output price causes a proportional negative change in output demand, so that aggregate firm revenues stay constant. As is shown in the proof, the increased output of incumbent firms exactly suffices to satisfy the increased demand, so that no new firms enter and no additional start-up costs have to be expended. Hence, aggregate domestic income is not affected by the subsidy. If demand is elastic ($\varepsilon > 1$), an output price change leads to an overproportional negative change in demand, so that a price reduction will increase aggregate firm revenues. Since DMS cause competition between incumbent firms to drive down prices more and/or create larger firms (and thus leaves less room for new entrants which would cause additional start-up costs), aggregate domestic income will be larger with DMS than with GS. Vice versa, an inelastic demand ($\varepsilon < 1$) would call for higher prices and less output to maximize aggregate domestic income. In such a situation, a social planner is faced with a trade-off between employment and aggregate income effects.\footnote{In other words, even though $p_{DMS} = p_{GS} = \bar{p}$ implies that the output effect is the same for GS and DMS if demand is elastic or inelastic, aggregate domestic income differs between the two cases. If demand is elastic and $p_{DMS} = p_{GS} = \bar{p}$, GS will cause more new firms to enter than DMS. Hence, more entry costs have to be paid that reduce incomes. With inelastic demand, it always holds that $p_{GS} < \bar{p}$ (see the proof of Proposition 3), so the case $p_{DMS} = p_{GS} = \bar{p}$ cannot occur. Graphically, the demand curve at $\varepsilon = 1$ always intersects the supply curve with GS exactly in the kink where the supply curves becomes horizontal.}

We now turn to the distributive effects of DMS and GS. Our first result shows that incumbent firms lose under DMS.

Proposition 4 (Incumbent firms’ rents) The rent obtained by incumbent firms under DMS is always less than under GS or in the initial equilibrium. Incumbents’ rents under GS are at most as large as those in the initial equilibrium.

Proof. See Appendix B. ■

This again is a consequence of the displacement competition between incumbent firms when DMS are introduced. Firms have an incentive to expand production by using the subsidy to undercut their unsubsidized competitors. Since all firms attempt to do that, they compete prices down more than they would under GS, and thereby destroy their rents. Under GS, incumbent firms’ rents are unaffected and suffice to
cover start-up costs if \( p_{GS} = \bar{p} \), while these firms also suffer from rent reduction if \( p_{GS} < \bar{p} \).

Since political acceptability of a subsidy program depends on its fiscal effects, it is worthwhile to examine the effect of DMS and GS on government expenditures.

**Proposition 5 (Government expenditures)** DMS will always result in smaller government expenditures than GS if \( sw < b \).

**Proof.** Proposition 2 shows that employment under DMS always exceeds that under GS. If \( sw < b \), any additional employee reduces government expenditures. Moreover, while GS always subsidize the entire workforce, the share of subsidized employees under DMS can never exceed unity. ■

If \( sw \) exceeded \( b \), or in other words, if the single subsidy rate exceeded unemployment benefits’ replacement ratio, each additionally employed worker would cost the government more than it saves. Since DMS always cause a larger employment effect than GS, this would raise government expenditures more under DMS than under GS if a sufficiently large number of incumbent employees has to be (double-)subsidized. It is, however, very plausible to assume that \( sw < b \). As mentioned before, DMS restrict \( s \) to be less than 50 percent, because otherwise the double subsidy would result in negative marginal labor costs. Moreover, benefit replacement rates in OECD countries are typically far above 50 percent, especially in Europe (Carone et al. 2004, Table 8).

The net income of incumbent workers and those persons who remain unemployed will be larger under DMS than under GS. This follows directly from Proposition 5. Since government expenditures are lower under DMS than under GS if \( sw < b \), those persons, whose employment status has not changed, also have to contribute less under DMS than under GS.\(^9\)

The final distributive measure in our analysis is the labor share in the functional distribution of income.

**Proposition 6 (Labor share)** DMS cause the labor share to rise compared to the initial equilibrium, while GS have no effect on the functional distribution of income.

\(^9\)The exact distributitional consequences depend on the specific type of taxation. If, for example, taxation is lump-sum, employment rises, and total government expenditures do not change compared to their initial level, the lump-sum tax per person does not change, so that the income of incumbent workers and the remaining unemployed is not affected. If, however, taxation is proportional, employment rises, and government expenditures do not change, the newly employed pay higher taxes, so that the tax rates for incumbent workers and the remaining unemployed can be reduced, thereby making them better off.
Proof. See Appendix B. ■

One of the well-known properties of the group of Cobb-Douglas-type production functions, to which (2) belongs, is the constancy of the functional distribution of income if all factors of production receive a competitive compensation (see Mankiw 2003, 71). In our case, however, firm incomes are not competitive compensations, but rents arising from the sunkness of start-up costs. The firms’ share in total income can thus only stay constant as long as their ability to secure these rents is unaffected. Since GS treat all firms identically, the subsidies do not affect the rent-capturing ability of incumbent firms and the labor share stays constant. With DMS, on the other hand, competition between incumbent firms reduces their rent-capturing abilities and thus leads to an increased labor share. Hence, in addition to their employment-promoting effect, DMS also serve as a redistribution device.

To sum up, we have shown that DMS cause larger output and employment effects and are less expensive for the government than GS. DMS increase (decrease) aggregate domestic income more than GS if output demand is elastic (inelastic). Finally, DMS change the functional distribution of income in favor of the labor share by reducing the ability of incumbent firms to capture rents. Under GS, no such redistributive effect occurs.

4 Numerical illustration

In the first part of this section, we will present a numerical illustration of our model, the purpose of which is to illustrate the general magnitude of the differential effects of DMS and GS on employment and distributional variables. In the second part, we will use this model to estimate the employment and fiscal effects of the DMS-proposal for Germany. The results serve to check the robustness of previous estimations.

We set the parameters of the model such that the resulting initial equilibrium exhibits an unemployment rate of 11.7 percent, a labor share of 67 percent, and a benefit replacement rate of 60 percent. These values reflect the situation in Germany in 2005. This initial equilibrium is obtained by setting $\alpha = 0.67$, $w = 1$, $b = 0.6$, $F = 1$, $\varepsilon = 1$, and $A = 1.381$. The unit-elastic output demand is chosen to abstract from aggregate income considerations (see Proposition 3).

Figure 2 shows how DMS and GS affect the unemployment rate, the public budget, incumbents’ rents, and the labor share for the chosen numerical specification. Graph (a) shows the effect on the unemployment rate. As stated in Proposition 2, DMS create more employment than an equal-rate GS. In our example, a 7 percent DMS would be sufficient to virtually restore full employment. A GS at the same rate would reduce
Figure 2: Numerical illustration
the unemployment rate only to 5.1 percent.

Proposition 5 states that \textit{DMS} will always be cheaper to the government than \textit{GS}. Graph (b) in Figure 2 clearly supports this point. For the chosen specifications, \textit{DMS} even result in savings for the public budget, while \textit{GS} put an additional burden on it. With the 7 percent subsidy, \textit{DMS} would reduce government expenditures by 64 percent, while \textit{GS} would increase them by 38 percent. This also means that incumbent workers would see their net incomes rise under \textit{DMS}, but fall if \textit{GS} were introduced.

The distributional consequences are as predicted by Propositions 4 and 6. As is shown in graphs (c) and (d), \textit{DMS} reduce the rent obtained by incumbent firms by 21 percent and increase the labor share from 67 to 74 percent if the 7 percent subsidy is introduced. \textit{GS} neither affect incumbents’ rents nor the functional income distribution.

A numerical illustration could also be used to check the robustness of previously derived predictions about the employment and fiscal effects of double marginal subsidies. The method used by Schöb and Weimann (2003, 2005) and Knabe, Schöb and Weimann (2006), henceforth abbreviated \textit{KSW}, to determine the effects of a double marginal subsidy relies on the principle that firms produce where prices equal marginal cost. A straightforward extension of this principle is that a double marginal subsidy has the same employment effect as a general subsidy at the double rate, because both types of subsidies reduce marginal costs by the same amount. This approach would be valid if individual firms operated on separate markets or if they were collective price-takers, for example because the world-market price could not be affected by domestic firms. In both cases, there could be no inter-firm displacement effects, and only the marginal cost reduction would be relevant for a firm’s production decision. Since \textit{DMS} offer the double reduction in marginal costs, the \textit{KSW}-approach predicts that \textit{DMS} provide large employment gains and fiscal relief.

Critics of \textit{DMS} claim that the alleged favorable effects of marginal subsidization are only due to the failure of the \textit{KSW}-approach to take inter-firm displacement effects into account (Bothfeld et al. 2006, Sinn et al. 2006). They argue that inter-firm displacement will force all firms with some unsubsidized employment out of the market, so that only completely subsidized firms remain. Then, \textit{DMS} would effectively be equivalent to \textit{GS}. In this paper, we have developed a model that shows that the solution methods of both the \textit{KSW}-approach and its critics are too simplistic. \textit{DMS} will indeed trigger strong inter-firm displacement, but this does not make them equivalent to \textit{GS}. Quite to the contrary, it is exactly the displacement competition, and the resulting rent reduction, that causes \textit{DMS} to yield favorable effects.

We will now quantitatively compare the differential effects of \textit{GS} and \textit{DMS}. For the double marginal subsidy, we apply both the \textit{KSW}-approach and the model developed in
this paper, henceforth called DMS-approach. We revert to the same base parameters as the most recent calculations on the German DMS-proposal (Knabe, Schöb and Weimann 2006). The number of target group, i.e. low-skilled, unemployed persons in Germany is estimated to be 2.8 million, while 2.1 million low-skilled persons are employed. For the numerical simulation, we thus set the initial unemployment rate to 57.1 percent. Bringing one additional low-skilled unemployed back into work saves the public budget 15,309 Euro p.a. (gross savings without subsidy costs), while annual gross labor costs (including employer’s social security contributions) are 17,359 Euro.\footnote{By bringing one low-skilled unemployed back into work at a monthly wage of 1,200 Euro, the government saves on average 626 Euro in welfare benefits. Moreover, the average amount of taxes and social security contributions paid is 536 Euro, while no new claims to the social insurances arise (welfare recipients were covered by health and basic pension insurances already; cf. Knabe, Schöb and Weimann 2006). In addition, past experiences show that some welfare recipients prefer to decline a job offer and forfeit future benefits (Feist and Schöb 1998). This saves, on average, another 113.75 Euro per new job. Thus, the total amount saved is 1,275.75 Euro per month, or 15,309 Euro per annum. Annual labor costs of 17,359.20 Euro are obtained by adding employer’s social security contributions (20.55 percent) to the monthly wage of 1,200 Euro.}

To reproduce these parameters in our model, we set $b = 0.76$ and $w = 1$. Knabe, Schöb and Weimann (2006) assume a wage elasticity of labor demand of 0.5, which corresponds to a price elasticity of output demand of $\varepsilon = 0.4$, as measured by the employment reaction under GS. The other parameters are set at $\alpha = 0.67$, $F = 1$, and $A = 0.438$. With these parameter values, our numerical illustration will result in the same outcomes for general subsidization both with the model developed in this paper and with the KSW–method, so that we can analyze whether the outcomes for double marginal subsidization differ between the two methods.

The numerical results of the comparison between the model developed in this paper (DMS) and the KSW–method are presented in Figure 3. The KSW–method predicts that double marginal subsidies have large employment gains and significantly reduce government expenditures. For a 35 percent subsidy rate examined by Knabe, Schöb and Weimann (2006), employment would rise by 1.7 million persons. Government expenditures would fall by 5.5 billion Euro.\footnote{It is noticable that the KSW–method predicts a constant unemployment rate and sharply rising government expenditures for $s \geq 0.375$. For these subsidy rates, all firms would double their employment and experience a sharp increase in their marginal cost (because further employment expansions receive only the single subsidy). Hence, increases in the subsidy rate do not create more employment, but raise the necessary expenditures.}

While the employment effect obtained by the KSW–method depends only on the reduction in marginal costs, the DMS–model developed in this paper demonstrates that the employment reaction to a double marginal subsidy depends on the interplay of
Figure 3: Comparison between the DMS- and KSW-methods and general subsidies.

Marginal costs, average costs, and between-firm competition. The DMS-line in Figure 3 shows that double marginal subsidies nevertheless have a strong employment effect, but the numerical illustration suggests that it is not as strong as that predicted by the KSW-method.\(^{12}\) In particular, employment expands by only 1.03 million jobs.\(^{13}\) Also with respect to the government budget, the numerical results are less favorable to double marginal subsidization than those predicted by the KSW-method. At a subsidy rate of 35 percent, the DMS-method predicts that government expenditures rise by 2.5 billion Euro per annum.

Figure 3 shows that policymakers are offered a “free-lunch” at small subsidy rates, where employment gains can be achieved with fiscal savings, while they face a trade-off between employment expansion and fiscal consolidation for larger subsidy rates. For example, a subsidy rate of 13 percent would maximize fiscal savings. Government expenditures could be reduced by 1.37 billion Euro, and 340,000 new jobs could be created. At a subsidy rate of 25 percent, DMS would be revenue-neutral, but bring about 780,000 new jobs. With subsidy rates above 25 percent, the employment effects are even larger, but come at the cost of a rising public budget.\(^{14}\)

\(^{12}\) As can be seen in Figure 3, the employment-enhancing effect of larger subsidy rates under DMS becomes weaker for \(s \geq 0.2\). The reason is that, for subsidy rates below 20 percent, an increase in \(s\) causes expanding firms to become larger, while for \(s\) exceeding 20 percent, they will already have doubled their employment and will not expand further. Hence, the price and employment effects of higher subsidy rates are smaller if \(s \geq 0.2\).

\(^{13}\) This number shows the net employment effect. It is comprised of 1.5 million additional jobs in expanding firms and an employment reduction of 470,000 jobs in shrinking firms.

\(^{14}\) For comparison, the KSW-method at a subsidy rate of 13 percent predicts 340,000 new jobs and
Compared to GS, double marginal wage subsidies score better under both methods. For any rate of subsidy, general subsidies lead to smaller employment effects at larger fiscal costs than double marginal subsidies. According to our numerical results, GS at a subsidy rate of 35 percent cause employment to rise by half a million new jobs and the government budget to rise by 8 billion Euro per annum, independently of the method used.\textsuperscript{15}

To sum up, double marginal wage subsidies create more employment at smaller fiscal costs than general subsidies. Our results suggest, however, that the quantitative effects are smaller than those previously suggested. The choice of the subsidy rate confronts policymakers with a trade-off. At small subsidy rates, they can enjoy both employment gains and fiscal savings. The employment effect, however, is fairly small. If policymakers want to achieve larger employment gains, these can only be achieved at higher fiscal costs.

5 Conclusion

Proponents of marginal wage subsidies claim that, if wages are downward rigid, such subsidies could create more employment at lower fiscal costs than general wage subsidies because it is the marginal cost reduction that matters for firms. Critics, however, point to the inter-firm displacement effects. They object that some firms might expand due to the marginal subsidy, but their expansion will mainly force other firms out of the market. The resulting net employment gain is small and since most workers have to be subsidized, the fiscal costs are high. According to the critics, the long-run equilibrium effects of DMS and GS are very similar because its the average cost reduction that determines the new equilibrium.

Our model's results lie in between these two extremes. We model a small open economy in which competitive firms produce an internationally differentiated good and fiscal savings of 3.71 billion Euro, while at a rate of 25 percent, the respective predictions are 870,000 new jobs and savings of 5.77 billion Euro.

For a more complete evaluation of the fiscal effects of the different subsidy schemes, one also has to take into account savings in other active labor market programs, the draining of the shadow economy, the reversal of international outsourcing movements, and institutional peculiarities, such as the conversion of unsubsidized part-time jobs for secondary earners to subsidized full-time jobs for welfare recipients in Germany. These effects will, however, only affect the magnitude of the fiscal effects, without changing the ordering of the differential effects between GS and the KSW-/DMS-approaches.

\textsuperscript{15}At the abovementioned subsidy rate of 13 percent, GS would create only 150,000 new jobs at 2.77 billion Euro additional costs. At a subsidy rate of 25 percent, 330,000 new jobs at 5.57 billion Euro costs could be created by GS.
in which market entry is restricted by sunk start-up costs. In this model, inter-firm displacement of considerable magnitude takes place in equilibrium under DMS. After the introduction of double marginal subsidies, all firms have an incentive to expand production. This drives down the market price until firms are indifferent between expanding or contracting. Some firms will have increased their employment level, but mostly to cover the former supply of now contracted firms. Hence, even though the subsidy is only for additional employment at the firm-level, in the aggregate equilibrium most employees have to be subsidized.

Nevertheless, DMS and GS differ substantially in their effects. The fiercer competition between incumbent firms triggered by DMS reduces the output price further than under GS. This causes a larger output demand and employment gain. As long as subsidy expenditures do not exceed spending on unemployment benefits and foregone taxes, a net increase in subsidized employment reduces public expenditures. Hence, DMS are always cheaper for the government than GS because they create more employment without necessarily subsidizing all employees. This also means that incumbent workers, who have to finance public expenditures, are better off under DMS than under GS.

Inter-firm displacement triggered by DMS reduces the ability of incumbent firms to capture the rents associated with their cost of market entry. Consequently, DMS cause a strong redistribution from firm rents to labor income, which causes the labor share in the functional distribution of income to rise. Under GS, no such redistributive effect occurs because the rent-capturing ability of incumbent firms is not affected.

The numerical illustration of our model supports the theoretical findings. DMS can substantially reduce unemployment, are less expensive for the public budget than GS, and raise the labor share. Our model, however, yields more cautious results about the quantitative effects of recent DMS-proposals than those predicted by previously used methods (Knabe, Schöb and Weimann 2006). The employment effects are still substantial, albeit smaller than previously predicted. Moreover, policymakers are offered a “free-lunch” at small subsidy rates because small employment gains and fiscal savings can be realized at the same time. At larger subsidy rates, however, policymakers face a trade-off between employment expansion and fiscal consolidation. If they aim at larger employment gains, they have to accept higher fiscal costs. Nevertheless, double marginal wage subsidies can always create more jobs at smaller fiscal costs than equal-rate general wage subsidies.

To conclude, our model supports the general belief that marginal wage subsidies, under the condition that they are designed as double marginal subsidies, create more employment at less fiscal costs than general wage subsidy programs. Additionally, we
have shown that marginal wage subsidies redistribute income from firm rents to labor income, while general subsidies leave the functional income distribution unaffected. In this respect, marginal wage subsidies also serve as a redistribution device. Given that politicians are restricted by fiscal and distributional considerations, this makes double marginal wage subsidies a superior policy instrument for employment creation compared to general wage subsidies.

Appendix A

In Sections 2 and 3, we restricted our attention to the case where the aggregate supply function with \( DMS \) consists of five parts. Formally, the five-part aggregate supply function with \( DMS \) can be written as

\[
Y_{DMS}(p) = \begin{cases} 
    n_0 \left( \frac{op}{w} \right)^{\alpha/(1-\alpha)} & \text{if } p < \tilde{p} \\
    \left[ n_0 \left( \frac{op}{w} \right)^{\alpha/(1-\alpha)}, n_0 \left( \frac{op}{(1-2s)w} \right)^{\alpha/(1-\alpha)} \right] & \text{if } p = \tilde{p} \\
    n_0 \left( \frac{op}{(1-2s)w} \right)^{\alpha/(1-\alpha)} & \text{if } \tilde{p} < p < (1 - 2s) C'(y_2) \\
    n_0 y_2 & \text{if } (1 - 2s) C'(y_2) \leq p < \bar{p} \\
    [n_0 y_2, \infty[ & \text{if } p \geq \bar{p}
\end{cases}
\]

Here, we will show under which conditions this is correct and which other cases might occur.

Generally, the aggregate supply function could consist of up to six different sections. The first section is where the market price is so low that all incumbent firms want to reduce production. In the second section, the output price is where firms are indifferent between expanding or contracting, so that a horizontal supply curve results. The third and fourth sections describe the case where all incumbent firm expand production, but do not (Section 3) or do (Section 4) double their employment. A fifth section could arise if incumbent firms find it profitable to expand their employment to more than twice its initial level, while new entrants do not find it profitable to enter the market. The sixth section arises where the output price is at the minimum average costs of new entrants. New firms enter the market, and the supply curve becomes perfectly price-elastic.

Depending on the form of the production function, not all of these section necessarily occur. With the functional specification laid out in Section 2, we can rule out the existence of some of these sections and show that other sections exist independently of
the chosen parameter values.

Section 1 always exists because $MC(0) = 0$. Hence, at no positive output price will incumbent firms find it optimal to leave the market completely.

Section 2 will exist for any admissible parameter values, because, as can easily be seen by contradiction, $\bar{p} < \check{p}$ always holds. If $\check{p} \geq \bar{p}$ and $\check{p} < (1 - 2s) C'(y_2)$, it must hold that

$$
\begin{align*}
\left[ \check{p} \left( \frac{\alpha \check{p}}{w} \right)^{\alpha/(1-\alpha)} \right] & + (1 - 2s) \left[ \frac{\alpha}{1-2s} \right] \left( \frac{1}{1-2s} \right) \alpha/(1-\alpha) \right] + 2\alpha s \geq 0.
\end{align*}
\right.

(12)

(12)

which, after inserting (6), becomes

$$
(1 - \alpha)(1 - s)^{\alpha/(1-\alpha)} \left[ 1 - \left( \frac{1}{1-2s} \right)^{\alpha/(1-\alpha)} \right] + 2\alpha s \geq 0.
$$

(12)

This condition is never fulfilled for $\alpha \in [0, 1]$ and $s \in [0, 1/2]$. If $\check{p} \geq \bar{p}$ and $\check{p} \geq (1 - 2s) C'(y_2)$, the same logic leads to the condition

$$
(1 - \alpha)(1 - s)^{\alpha/(1-\alpha)} - 2^\alpha (1 - s) + 2\alpha (1 - s) \geq 0.
$$

(13)

Moreover, $\check{p} \geq (1 - 2s) C'(y_2)$ requires that

$$
2^{1-\alpha} - \frac{(1 - s)\alpha}{1-2s} \leq 0.
$$

(14)

Conditions (13) and (14) cannot be fulfilled at the same time for $\alpha \in [0, 1]$ and $s \in [0, 1/2]$. Therefore, we always have $\check{p} < \bar{p}$, and Section 2 always exists.

The third section exists if $\check{p} < (1 - 2s) C'(y_2)$, while the fourth section exists if $\check{p} > (1 - 2s) C'(y_2)$. Whether these conditions are fulfilled depends on the parameter values chosen. It is possible that one of the two sections does not exist. Both sections, however, cannot be non-existing at the same time because $\check{p} < \bar{p}$.

The fifth section (where incumbent firms more than double their employment, but new firms do not enter) cannot exist with our functional specification. This section would only exist if $\check{p} > (1 - s) C'(y_2)$, which holds if $[2(1-s)]^{1-\alpha} - 1 < 0$. This condition is never fulfilled for $\alpha \in [0, 1]$ and $s \in [0, 1/2]$.

Since the sixth section always exists, our functional specification restricts the multitude of aggregate supply curves to three different cases. If $\check{p} < (1 - 2s) C'(y_2) < \bar{p}$, all sections, except the fifth, exist. If $\check{p} < \bar{p} \leq (1 - 2s) C'(y_2)$, the fourth and fifth sections do not exist, and if $(1 - 2s) C'(y_2) \leq \bar{p} < \check{p}$, the third and fifth sections do not exist.
Appendix B

Proof of Proposition 1. Since the production function (2) is homogenous of degree $\alpha$, we have

$$(1 - s) C'(y_0) \geq \tilde{p}. \quad (15)$$

This can easily be seen by inserting this inequality into (9), which, if the condition holds, yields

$$\frac{1 - (1 - 2s)^{\alpha/(\alpha-1)}}{(1 - s)^{1/(\alpha-1)}} \left( \frac{1}{\alpha} - 1 \right) + 2s \leq 0. \quad (16)$$

Since this holds for all values of $\alpha \in [0,1]$ and $s \in [0,1/2]$, condition (15) must hold.

From (15), it follows that $Y^{-1}_{DMS} (n_0y_0) \leq Y^{-1}_{GS} (n_0y_0)$, and $Y^{-1}_{DMS} (Y) \leq Y^{-1}_{GS} (Y) < p_0 \forall Y \geq n_0y_0$. Since $D' (p) < 0$, this implies that $p_0 > p_{GS} \geq p_{DMS}$ and $D (p_0) < D (p_{GS}) \leq D (p_{DMS}).$ ■

Proof of Proposition 2. It always requires more workers to produce a given output with $DMS$ than with $GS$ because the firms’ production function (2) exhibits decreasing marginal productivity of labor, and the number of firms is smaller with $DMS$ than with $GS$, and/or firms are of unequal sizes with $DMS$ while they are of equal size with $GS$. (A general proof of this result is provided by Knabe 2006). Since Proposition 1 states that output with $DMS$ is at least as large as with $GS$, the employment effect of $DMS$ always has to be larger than that of $GS.$ ■

Proof of Proposition 3. First note that the critical demand elasticity which separates the two parts of the $GS$–supply curve is exactly $\varepsilon = 1$. This critical elasticity is determined by the condition $A \tilde{p}^{-\varepsilon} = n_0 \tilde{y}$. From (6), (7), and (4), we have $\tilde{p} = (1 - s)^{\alpha} p_0$ and $\tilde{y} = (1 - s)^{-\alpha} y_0$, so that the condition becomes $(1 - s)^{-\alpha \varepsilon} = (1 - s)^{-\alpha}$, which is only fulfilled for $\varepsilon = 1$.

From Proposition 1, we know that $p_{DMS} \leq p_{GS} < p_0 \forall \varepsilon > 0$. In case of elastic demand ($\varepsilon > 1$), $p_{DMS} < p_{GS}$ implies that revenues with $DMS$ are at least as large as with $GS$. Moreover, we know that for $\varepsilon > 1$, more new firms will enter under $GS$ than under $DMS$. Hence, $n_{DMS} < n_{GS}$, which causes larger start-up costs with $GS$. Therefore, $I_{DMS} > I_{GS}$. Comparing $GS$ to the initial equilibrium, we have $I_{GS} = \tilde{p} D (\tilde{p}) - \tilde{n} F$ and $I_0 = p_0 D (p_0) - n_0 F$, which results in $\text{sgn} (I_{GS} - I_0) = \text{sgn} \left( (1 - s)^{\alpha (1-\varepsilon)} - 1 \right) > 0 \forall \varepsilon > 1$. Hence, $I_0 < I_{GS} < I_{DMS}$.

For $\varepsilon = 1$, firm revenues and the number of firms are the same under $DMS$, $GS$, and initially. Hence, $I_0 = I_{GS} = I_{DMS}$.

In case of inelastic demand ($\varepsilon < 1$), we have a strict inequality $p_{DMS} < p_{GS} < p_0$, which implies that firm revenues are smallest with $DMS$ and second smallest with $GS$. 22
Since the number of firms is always \( n_0 \), we have \( I_0 > I_{GS} > I_{DMS} \). □

**Proof of Proposition 4.** Let \( R_{DMS} (p) \), \( R_{GS} (p) \), and \( R_0 \) be the rents of an incumbent firm under DMS, GS, and in the initial equilibrium, respectively. Since \( R_{GS} (\bar{p}) = F \) and \( R_0 = F \) by the zero-profit condition for new entrants, and \( \partial R_{GS} (p) / \partial p > 0 \), we have \( R_{GS} (p) \leq R_0 \forall p \leq \bar{p} \).

Since \( p_{DMS} \leq p_{GS} \) and \( \partial R_{GS} (p) / \partial p > 0 \), it is sufficient to show that \( R_{GS} (p) > R_{DMS} (p) \) for any given \( p \in [\bar{p}, \bar{p}] \) to prove the proposition. We check whether this condition holds for the three relevant price intervals separately.

For \( p = \bar{p} \), \( R_{DMS} (p) = R_{GS} (p) |_{s=0} \). Since \( \partial R_{GS} / \partial s > 0 \) and \( s > 0 \), we have \( R_{GS} (p) > R_{DMS} (p) \).

For \( p \in [\bar{p}, (1 - 2s) C' (y_2)] \), an incumbent’s rent under DMS can be written as

\[
R_{DMS} (p) = p \left[ y_{DMS} - (1 - s) wy_{DMS}^{1/\alpha} \right] - sw \left[ 2y_0^{1/\alpha} - y_{DMS}^{1/\alpha} \right]
\]

where \( y_{DMS} = [\alpha / ((1 - 2s) w)]^{1/(1-\alpha)} \). It follows that \( R_{DMS} (p) < R_{GS} (p) |_{y=y_{DMS}} \).

Moreover, \( R_{GS} (p) \geq R_{GS} (p) |_{y=y_{DMS}} \), since the rent would not be maximized otherwise. Hence, \( R_{GS} (p) > R_{DMS} (p) \).

In the interval \( p \in [(1 - 2s) C' (y_2), \bar{p}] \), \( R_{DMS} (p) = R_{GS} (p) |_{y=y_2} \). Since \( y_2 > [\alpha / ((1 - s) w)]^{1/(1-\alpha)} \) in the relevant price interval, we have \( R_{GS} (p) > R_{GS} (p) |_{y=y_2} \).

Therefore, \( R_{GS} (p) > R_{DMS} (p) \forall p \in [\bar{p}, \bar{p}] \). □

**Proof of Proposition 6.** As has been shown in equation (8), the labor share in the initial equilibrium is \( \alpha \). Under GS, each firm produces \( y_{GS} = [\alpha p_{GS} / ((1 - s) w)]^{1/(1-\alpha)} \). The number of firms is then given by \( n_{GS} = D (p_{GS}) / y_{GS} \). Making use of these expressions, the labor share under GS becomes

\[
\frac{(1 - s) w L_{GS}}{p_{GS} D (p_{GS})} = \frac{(1 - s) w n_{GS} y_{GS}^{1/\alpha}}{Ap_{GS}^{1-\frac{1}{\alpha}}} = \alpha.
\]

Let \( \phi \) be the share of subsidized workers under DMS, and \( L_{DMS} (p) \) and \( L_{GS} (p) \) be the total employment under DMS and GS if \( p \) is the equilibrium price. From Proposition 2, (18), and \( \phi \leq 1 \), it follows that

\[
\frac{(1 - \phi s) w L_{DMS} (p_{DMS})}{p_{DMS} D (p_{DMS})} > \frac{(1 - s) w L_{GS} (p_{DMS})}{p_{DMS} D (p_{DMS})} = \alpha.
\]

This shows that DMS always results in a larger labor share than GS. □
References


