Present-Biased Individuals, Optimal Paternalism, and Transfers in Kind

Jes Winther Hansen

2005-11
Present-Biased Individuals, Optimal Paternalism, and Transfers in Kind

Jes Winther Hansen

Department of Economics, University of Copenhagen & EPRU

September 22, 2005

Abstract

Present-biased preferences cause distortions in consumption that can motivate the use of paternalistic in-kind transfers. Empirically, goods are consumed to different degrees when consumption outlay changes. Economists distinguish between necessary goods and luxury goods. A present-biased individual has an intertemporal distortion of consumption toward the present, which in turn distorts present consumption toward luxury goods. In-kind transfers of necessary goods, such as food stamps, can alleviate the intertemporal distortion and make present-biased transfer recipients better off. Further, transfers in kind are asymmetrical in the sense that they can target present-biased recipients without affecting fully rational recipients.

Keywords: Paternalism; In-Kind Transfers; Time Preference

JEL classification: D91; H21; H42; I38
1 Introduction

Most means-tested government transfers in the United States are provided in kind, for example as food stamps, Medicaid, and housing aid. In 2002, cash aid was only 20 percent of total expenditure on state and federal means-tested welfare programs (Burke, 2003). Economists are typically skeptical of transfers in kind since they may violate the principle of consumer sovereignty. No rational transfer recipient would prefer a transfer in kind to a cash transfer of equivalent value. If income maintenance programs are to help recipients in the best possible way, it is tempting to conclude that the transfers should be provided in cash and not tied to consumption of certain goods.

A commonly held notion among practitioners of public policy is that the use of in-kind transfers reflects paternalism. If the government has preferences directly on the recipients’ consumption patterns, then transfers in kind can be used to ensure consumption of goods that the government for some reason finds desirable. Following Musgrave (1959), the literature on public economics calls such goods merit goods. This view leaves little guidance for policy design. First, we need a theory that motivates why the government has preferences directly on consumption. Second, we need a framework allowing us to distinguish between good paternalistic policies and bad paternalistic policies, where recipients would be better off deciding for themselves. Third, we need a framework helping us to design paternalistic policies. The existing literature gives little help with these issues.

The growing literature on behavioral economics systematically examines deviations from rational economic behavior. By using psychological insights, as well as controlled experiments, this literature has pointed out several common and persistent behavior rules that conflict with economists’ understanding of rationality.¹ For instance, there is ample evidence that people are impulsive and tend to desire immediate rewards, even if this is contrary to their long run interests.² In an interesting new paper, Shapiro (2005) provides

¹ See Camerer and Loewenstein (2003) for a survey of the literature.
² Thaler and Loewenstein (1992) provide an overview of some of the empirical evidence.
evidence that such behavior is prevalent on a daily basis among U.S. benefit recipients.

Economists have modeled impulsive behavior by *present-biased preferences*, where the relative discount rate between two time periods increases the closer they are to the present.\(^3\) An individual with present-biased preferences is said to have a *self-control problem* since there is a conflict between the individual’s present preferences and the preferences that the individual will have in the future. The present-bias leads to distortions in the intertemporal allocation of consumption; particularly, a present-biased individual would be better off if she could commit herself to save more for later consumption. These distortions imply that there can be scope for government intervention.

The objective of this paper is to demonstrate how present-biased preferences can motivate the use of transfers in kind, such as food stamps, for paternalistic reasons and to provide a framework for the design of such policies. The key argument is that, empirically, goods are consumed to different degrees when income or consumption outlay changes. Economists distinguish between *necessary goods* and *luxury goods*. For example, necessary goods, such as food, are consumed relatively less when outlay is high. One implication of this is that intertemporal distortions in consumption also result in a change in the relative consumption shares of goods. Present-biased preferences cause an intertemporal distortion toward the present which in turn distorts present consumption toward luxury goods. This motivates why in-kind transfers of necessary goods can help present-biased transfer recipients. When the recipients are forced to consume more necessary goods, they are in effect also forced to postpone consumption.

This kind of regulation is paternalistic in the sense that it overrides the recipients’ own decisions. Still, a benevolent government would want to engage in such policies if they could benefit the recipients in the longer run by helping to correct the self-control problem. If present-biased recipients are *sophisticated*, meaning that they are aware of their future self-control problems, they will in fact prefer to receive future benefits partly

\(^3\)Such preferences are also called time-inconsistent preferences. Classic references are Strotz (1956) and Phelps and Pollak (1968).
in kind since this can help to restrain their future behavior. The advantage of in-kind transfers over traditional corrective instruments, such as a luxury tax, is that they are likely to target exactly those recipients who suffer from the self-control problem without interfering with the choices of perfectly rational recipients. This constitutes an example of asymmetric paternalism, as defined by Camerer et al. (2003).

Recent studies on present-biased preferences, with specific relation to this paper, include Shapiro (2005) who provides empirical evidence in favor of impulsiveness among benefit recipients. Using data on the daily caloric intake of U.S. food stamp recipients, he finds that food consumption declines over the food stamp month. This decline is too sharp to be explained by reasonable exponential time discount rates but supports that recipients have present-biased preferences. Other papers have used the theory of present-biased preferences for recommendations on tax policy. Gruber and Köszegi (2001, 2004) use present-biased preferences to model cigarette consumption in order to analyze the implications for optimal excise taxes on cigarettes. In a somewhat similar approach, O’Donoghue and Rabin (2003) study optimal taxation in a model where present-biased individuals consume an inefficiently large amount of a good involving negative health consequences in the long run. One common feature of these papers is that the government should use paternalistic Pigouvian taxes on unhealthy goods to correct for negative “internalities” caused by self-control problems. Recently, Bertrand et al. (2004) have argued that behavioral insights may be of particular importance for poverty-alleviating policies. The present paper should be viewed as a first step toward modeling the optimal design of income maintenance programs when behavioral issues are important.

Another strand of literature related to this paper has sought to justify the use of in-kind transfers within the rational choice framework. One result is that cash transfers can encourage recipients to behave inefficiently in order to manipulate the size of future transfers, see Bruce and Waldman (1991). For example, a transfer recipient may deliberately choose to save too little or to underinvest in education in order to induce
a larger future transfer from a benevolent government (the Samaritan’s Dilemma). The
government can possibly avoid this inefficiency by providing transfers in kind, e.g., by
tying part of the transfer to education or other illiquid assets. In fact, the case for in-kind
transfers of necessary goods that is presented in this paper also extends to their model.
If the recipient chooses to save too little for strategic reasons, then the government can
force the recipient to save more by tying part of the initial transfer to necessary goods.
A second insight in this literature is that transfers in kind can be used to screen re-
cipients in second-best environments of incomplete information, see for instance Nichols
and Zeckhauser (1982) and Blackorby and Donaldson (1988). Small distortions of the
benefit recipients’ consumption can increase transfer efficiency because the self-selection
constraints of the transfer program are eased. The ideas presented in the present paper
add a new dimension to this line of reasoning. As will be explained below, present-biased
preferences may influence self-selection into transfer programs but the effect depends
crucially on whether the recipients are sophisticated or not.

The paper is organized as follows. Section 2 presents a simple two-period model that
explains the main argument while Section 3 analyzes an $N$-period model that includes
the strategic effects of self-control. Section 4 considers how paternalistic transfers in kind
affect the possibilities for income redistribution. Section 5 concludes.

## 2 In-Kind Transfers of Necessary Goods

The main focus of the paper is on how transfer recipients allocate consumption in the
short run. The analysis rests on two basic premises. The first premise is that recipients
are unable to borrow against future income and hence are constrained by their current
disposable income. If government benefits are their only source of income and they
discount future utility, their planning horizon will be the time between transfers and,
consistent with the empirical findings, their consumption will be declining over the trans-
fer period. The second premise is that some, or all, of the intended transfer recipients
have present-biased preferences. If we think of recipients who receive a cash allowance at the beginning of each month, the present-bias implies that they will go on a consumption binge and spend too much money during the first couple of weeks such that there is little money left at the end of the month.

The government can help the present-biased recipients by forcing them to smooth consumption. If cash benefits were smaller but paid out more frequently, for example biweekly instead of monthly, the recipients would have less opportunity to overconsume (a possibility mentioned by, e.g., Shapiro, 2005). However, there is a limit to the frequency of transfer payments. At some point transaction costs will outweigh the advantages from curbing the self-control problem. Eventually, recipients are left to decide on their own how to allocate consumption until the next transfer payment.

Because different goods are related in different ways to consumption outlay, the government can help the present-biased recipients in an alternative way. In particular, the intertemporal distortions of outlay distort consumption toward luxury goods at the beginning of the planning horizon and toward necessary goods at the end. For the sake of argument, assume that a recipient can spend money on two goods: a luxury good and a necessary good, for instance food. The government can help recipients to restrain consumption at the beginning of the month by providing part of the transfer as vouchers for food (food stamps). For this to work, the value of the food stamps would have to exceed the value of the recipient’s food consumption before the intervention, such that the vouchers force the individual to consume more food. Since food consumption is distorted toward the end of the planning horizon because of the present-bias, the marginal propensity to consume the additional units of food is higher at the end. By substituting cash for food stamps and keeping the value of the allowance constant, the government in effect forces the recipient to postpone consumption.
2.1 A Two-Period Model

In order to demonstrate how in-kind transfers of a necessary good can affect the intertemporal allocation of consumption, we first consider a single transfer recipient in a model where transfers are awarded every other period. This simple model disregards the strategic effects of self-control. A model with $N$ periods that captures these strategic aspects is presented in Section 3. For simplicity we assume that there are only two goods: food, $F$, and taxis, $T$. The recipient has the intertemporal utility function

$$U = u(F_1, T_1) + \beta \delta u(F_2, T_2),$$

where $0 < \beta \leq 1$ and $0 < \delta < 1$. At this point, we can think of $\beta \delta$ as a normal time discount factor, but later on (from Section 2.2 onwards) we will interpret $\beta$ as representing the present-bias, whereas $\delta$ will represent the standard time-consistent discount factor. The instantaneous utility function $u(F, T)$ is strictly increasing in each argument, twice differentiable, and strictly concave. Both $F$ and $T$ are normal goods and the marginal utility of either good is infinitely large when consumption is zero.

For now, we assume that the recipient receives a cash transfer payment $B$ in period 1 (we consider a simple model of income redistribution in Section 4). It is possible to transfer income from period 1 to period 2 by saving, but it is assumed that savings do not accrue interest. This is an unrealistic assumption but it can be motivated by our focus on a short planning horizon, where the interest rate presumably has a negligible influence on intertemporal decisions. Importantly, there is corroborating empirical evidence that many benefit recipients in the U.S. do not hold interest bearing assets. In the 2002 sample from the *Survey of Program Dynamics* by the U.S. Census Bureau, only 14 percent of households receiving food stamps report that they own an interest bearing account. The assumption may not be innocuous, however. This will be discussed in Section 5.

The recipient maximizes intertemporal utility subject to the budget constraint $B \geq \ldots$

---

4 Additionally, in the March 2004 supplement to the *Current Population Survey* by the U.S. Census Bureau and the Bureau of Labor Statistics, 14 percent of households receiving food stamps have positive gross interest income. The median level of annual interest income among those households is $28.
\[ p_F (F_1 + F_2) + p_T (T_1 + T_2), \] where \( p_F \) and \( p_T \) are the prices of \( F \) and \( T \), respectively. We call the solution to this program \([(F_1^*, T_1^*), (F_2^*, T_2^*)]\). From the first order conditions we can obtain the following necessary conditions

\[
\frac{u_F (F_t^*, T_t^*)}{u_T (F_t^*, T_t^*)} = \frac{p_F}{p_T}, \quad t = 1, 2, \quad (1)
\]

\[
\frac{u_F (F_2^*, T_2^*)}{u_F (F_1^*, T_1^*)} = \frac{u_T (F_2^*, T_2^*)}{u_T (F_1^*, T_1^*)} = \frac{1}{\beta \delta}, \quad (2)
\]

where a subscript on \( u(F, T) \) denotes the partial derivative. In the optimum the recipient sets the marginal rate of substitution in each period equal to the price ratio. Consumption of both \( F \) and \( T \) is higher in period 1 than in period 2. The relative size of consumption outlay in period 1 increases when \( \beta \delta \) decreases, since period 2 is discounted more.

Now assume that part of the cash transfer \( B \) is replaced with vouchers for \( F \), which we can think of as food stamps. The vouchers have value \( V \) and can only be used to purchase \( F \). We assume that the vouchers cannot be exchanged for money. However, the recipient can freely decide how to divide the vouchers between \( F_1 \) and \( F_2 \) and she can purchase additional units of \( F \) in the market if she wishes to do so. The residual part of the transfer is available as money income \( M \), such that \( M + V = B \).

The recipient will spend all vouchers since the marginal utility of \( F \) is strictly positive. Hence, the recipient faces a voucher constraint, \( V \leq p_F (F_1 + F_2) \) in addition to the cash budget constraint, which we can rewrite as \( M \geq p_F (F_1 + F_2) + p_T (T_1 + T_2) - V \). When intertemporal utility is maximized subject to the two constraints, condition (2) still applies but the first order conditions yield a new expression for the intratemporal allocation

\[
\frac{u_F (F_t, T_t)}{u_T (F_t, T_t)} = \left( \frac{\lambda - \gamma}{\lambda} \right) \frac{p_F}{p_T}, \quad t = 1, 2. \quad (3)
\]

The parameters \( \lambda > 0 \) and \( \gamma \geq 0 \) are the Lagrange multipliers of, respectively, the money budget constraint and the voucher constraint. We have \( \gamma = 0 \) if and only if the voucher constraint is not binding. In this case, the recipient chooses \([(F_1^*, T_1^*), (F_2^*, T_2^*)]\) and is said to be inframarginal. The voucher constraint will be binding, such that the recipient is rationed, if the value of the vouchers exceeds the inframarginal outlay on \( F \); that is, if
V > p_F (F_1^* + F_2^*). Rationing distorts the intratemporal tradeoff between goods since a rationed recipient is, in effect, forced to substitute toward F.

The intratemporal distortion created by rationing will affect the intertemporal allocation if the propensities to consume each good differ across periods. To see this, suppose that the recipient is rationed and conduct the following policy experiment: raise the value of the vouchers marginally and reduce the cash transfer such that we still have $M + V = B$. This forces the recipient to consume strictly more F and strictly less T. If the marginal propensity to consume F is relatively high in period 2, this change will shift consumption outlay toward period 2.

**Lemma 1.** For a rationed recipient, a marginal increase in the value of the vouchers, $dV > 0$, and a decrease in the cash transfer, $dM$, such that $dM = -dV$, will reduce consumption outlay in period 1, $B_1 = p_F F_1 + p_T T_1$, if and only if

$$\frac{p_T u_{FF} (F_1, T_1) - p_F u_{TF} (F_1, T_1)}{p_F u_{TT} (F_1, T_1) - p_T u_{TF} (F_1, T_1)} > \frac{p_T u_{FF} (F_2, T_2) - p_F u_{TF} (F_2, T_2)}{p_F u_{TT} (F_2, T_2) - p_T u_{TF} (F_2, T_2)}.$$  \hspace{1cm} (4)

**Proof.** See appendix B.

The condition (4) has an intuitive interpretation when the recipient is only marginally rationed. First, differentiate the intratemporal first order condition (1) for an inframarginal recipient. This gives an expression for the slope of the income expansion path in $(F, T)$ space

$$\frac{dT_i}{dF_i} = \frac{p_T u_{FF} (F_i, T_i) - p_F u_{TF} (F_i, T_i)}{p_F u_{TT} (F_i, T_i) - p_T u_{TF} (F_i, T_i)}.$$  \hspace{1cm}

The slope of the income expansion path expresses the relative changes in $F$ and $T$ that are necessary in order to fulfill the intratemporal first order condition when outlay changes. The numerator and the denominator are both negative when $F$ and $T$ are normal, such that the income expansion path has a positive slope. Now suppose that the transfer bundle consists of a cash part, $M^* = p_T (T_1^* + T_2^*)$, and vouchers for $F$ with value $V^* =
Figure 1: Income Expansion Path

$p_F (F_1^* + F_2^*)$. The voucher constraint is fulfilled with equality but the recipient chooses the inframarginal allocation $[(F_1^*, T_1^*), (F_2^*, T_2^*)]$. If the value of the vouchers increases by $dV > 0$ and the cash transfer changes by $dM = -dV$, the recipient becomes marginally rationed and Lemma 1 tells us that period 1 outlay is reduced if and only if

\[
\frac{dT_1^*}{dF_1^*} > \frac{dT_2^*}{dF_2^*}
\]

This is equivalent to saying that the slope of the income expansion path is greater in period 1 than in period 2, when evaluated in the inframarginal optimum. The condition is fulfilled if the income expansion path bends toward $T$ as outlay increases since consumption outlay is greater in period 1 (see Figure 1). With an income expansion path that slopes toward $T$, the relative consumption of $T$ is high when outlay is high. In this sense, we say that $T$ is a luxury good and $F$ is a necessary good.

The usual definition of luxury goods and necessary goods does not use the income expansion path. Rather, the common definition is that the budget share of a luxury good is increasing in outlay (an income elasticity larger than one) whereas the budget share of a necessary good is decreasing in outlay (an income elasticity less than one). It is fairly

---

5Strictly speaking, the method used for deriving Lemma 1 cannot be guaranteed to be valid in $[(F_1^*, T_1^*), (F_2^*, T_2^*)]$, because this allocation is exactly on the boundary between two sets of constraints. This is a well-established concern in the literature on rationing and the reader is referred to Tobin and Houthakker (1951) and Pollak (1969) for discussions of the issue.
easy to relate this to the slope of the income expansion path. In any period outlay equals the value of consumption. If we differentiate this totally we find that the budget share of $T$ is increasing in outlay if and only if $dT_t/dF_t > T_t/F_t$. Hence, $T$ is a luxury good according to the usual definition if and only if the slope of the income expansion path exceeds the slope of the secant line from the origin to $(F, T)$. If the income expansion path $T = f (F)$ is strictly convex and passes through the origin (as in Figure 1), then $T$ is a luxury good in the usual sense for all outlay levels. The two definitions are not identical, however. It is possible to find examples where the slope of the income expansion path is increasing in outlay even though the budget share of $T$ is decreasing and vice versa.

### 2.2 Welfare

The principle of consumer sovereignty tells us that a rational transfer recipient never prefers an in-kind transfer to a cash transfer of equivalent value. Hence, transfers should not be tied to goods if the government wishes to help a rational recipient in the best possible way. In the presence of self-control problems, however, it may be desirable to provide the transfers partly in kind in order to restrict the recipient’s choice set. A recipient with present-biased preferences has an impulsive urge to spend much of the transfer upon receipt. The recipient disapproves of this urge in the longer run and would prefer to smooth consumption over the planning horizon. The government can help to achieve this by tying part of the transfer to the necessary good.

We take $\beta$ to represent the self-control problem. A fully rational, time-consistent recipient has $\beta = 1$ and only discounts period 2 consumption with $\delta$. In contrast, a present-biased recipient has $\beta < 1$ in period 1 and therefore has an additional fondness for present consumption. A present-biased recipient chooses to allocate more income to consumption in period 1 than a rational recipient with the same income would choose, since period 2 is discounted more. This framework is a crude example of the quasi-hyperbolic model, which was originally proposed by Phelps and Pollak (1968) and has been reintroduced to the literature by Laibson (1997). The two-period model in this
section can be obtained as a special case of the quasi-hyperbolic model, where transfer recipients are credit constrained and transfers are awarded every other period.

There is no unambiguous way of evaluating welfare when the recipient has a self-control problem, since preferences differ over time. Throughout this paper, we assume that the recipient’s welfare can be expressed by setting $\beta = 1$ in the intertemporal utility function. These are the preferences that the recipient would use if she were to evaluate consumption in period 1 and 2 at any point in time before entering period 1. They capture the fact that there is no special preference for period 1 consumption in itself in the longer run and that the recipient disapproves of the impulsive period 1 behavior at all other times. This welfare measure is called long-run utility and has been used by O’Donoghue and Rabin (1999, 2003) and Gruber and Köszegi (2001, 2004) in order to make normative statements about paternalistic policies.

A marginal amount of rationing with respect to the necessary good $F$ can increase the long-run utility of a present-biased recipient. Let $[\left(F_1^P, T_1^P\right), \left(F_2^P, T_2^P\right)]$ denote the allocation chosen by an inframarginal present-biased recipient who receives a transfer of value $B$.

**Proposition 1.** Starting from $V = p_F \left(F_1^P + F_2^P\right)$, a marginal increase in the value of the vouchers, $dV > 0$, and a decrease in the cash transfer, $dM$, such that $dM = -dV$, will increase the long-run utility of a present-biased recipient if and only if $dT_1^P/dF_1^P > dT_2^P/dF_2^P$.

**Proof.** See appendix B.

The intuition behind the result is that the long-run utility welfare measure treats the present-bias as a decision error. A small amount of rationing causes a small intratemporal distortion which is dominated by the first order welfare gain from a reduction of the intertemporal distortion. It is worth mentioning that this result does not hinge on the specific assumption of long-run utility as the true welfare metric. The crucial assumption is that the recipient will be better off by smoothing consumption.
2.3 Asymmetric Paternalism

Having considered the case of a single transfer recipient, we now turn to the issue of a heterogeneous population of recipients. While transfers in kind can benefit present-biased recipients by helping them to curb their self-control problems, the government should be wary of interfering with the choices of fully rational recipients who are better off deciding for themselves.

Vouchers for $F$ only affect the choices of recipients who are rationed. This makes it possible to target present-biased recipients if they on average demand less $F$ than rational recipients, even if the government is unable to distinguish one type from the other. There is a restriction on the income expansion path, closely connected to increasing slope, which ensures this. Let $(F_1^R, F_2^R)$ and $(F_1^P, F_2^P)$ be the demands for $F$ by, respectively, a rational recipient with $\beta = 1$ and a present-biased recipient with $\beta < 1$, when both recipients are inframarginal and receive identical transfer bundles.

Lemma 2. If the income expansion path $T = f(F)$ is convex on the interval $[F_2^P, F_1^P]$, then $F_1^R + F_2^R \geq F_1^P + F_2^P$. The inequality is strict if the income expansion path is strictly convex.

Proof. See appendix B.

Proposition 2. If the income expansion path $T = f(F)$ is strictly convex on the interval $[F_2^P, F_1^P]$, there exists a range of values of $V$ that will increase the long-run utility of a present-biased recipient while leaving a rational recipient inframarginal.

Proof. See appendix B.

In other words, if the income expansion path is “well-behaved,” a present-biased recipient demands less of the necessary good than a rational recipient does, such that it is possible to help present-biased recipients without affecting the choices of rational ones.

The possibility of targeting exactly those recipients who suffer from the self-control problem is a very attractive feature. Economists are normally cautious of policies that interfere with people’s choices. On the other hand, individuals who make poor decisions
may benefit immensely from a guiding hand. Considerations such as these have led some economists to argue that a yardstick by which to measure benign paternalistic policies is that they help people who make decision errors, while imposing very small costs on people who can decide for themselves. Camerer et al. (2003) use the term “asymmetric paternalism” to describe such policies. Within this mindset, a policy of in-kind transfers of the necessary good is an example of desirable paternalism, in the sense that it can leave rational recipients unaffected. A tax on the luxury good would distort intratemporal preferences for all individuals in the economy, not only for the intended target group, and hence may not be desirable.\footnote{However, as demonstrated by O’Donoghue and Rabin (2003), a marginal excise tax on a “sinful” good will only have adverse utility effects of second-order for fully rational individuals while possibly having positive first-order effects for individuals with self-control problems.}

### 3 An $N$-Period Model

The consumption choices made at any point in time will influence the choice set in future periods through the level and composition of savings. A recipient takes this into account when maximizing intertemporal utility, since the recipient has preferences on future consumption. The simple two-period model from the previous section paid no heed to the fact that recipients may be well aware of their future self control problems. If a recipient knows that she will be present-biased in the future, such that there is a conflict between present and future preferences, she may use savings strategically to manipulate her choices in the future. This section develops an $N$-period model that allows for strategic behavior and considers how in-kind transfers affect the recipients’ choices.

Following the literature, the consumption decision is modeled as a non-cooperative game between the different incarnations of the recipient at different points in time. We assume that a present-biased recipient has quasi-hyperbolic preferences and we let “self $t$” denote the decision maker in period $t$.\footnote{However, as demonstrated by O’Donoghue and Rabin (2003), a marginal excise tax on a “sinful” good will only have adverse utility effects of second-order for fully rational individuals while possibly having positive first-order effects for individuals with self-control problems.}
The equilibrium strategies of a recipient's selves are required to be subgame perfect. These strategies have the property that the choice made by any self is required to be an optimal response to the choices made by all subsequent selves. It is often a formidable task to characterize these strategies since the maximization problem is quite complex.\(^7\) In order to simplify, we restrict recipients to have quasi-linear preferences on consumption, such that instantaneous utility is 
\[ T + u(F), \]
where \( u(F) \) is an increasing, twice differentiable, and strictly concave function. It is assumed that 
\[ \lim_{F \to 0} u_F(F) = \infty \]
and 
\[ \lim_{F \to \infty} u_F(F) = 0. \]
The income expansion path has an inverted L-shape when preferences are quasi-linear (see Figure 2). There is a threshold level of outlay, corresponding to the kink in the expansion path, such that recipient solely consumes \( F \) if outlay is at or below this level. If outlay is higher than the threshold level, consumption of \( F \) is fixed and all additional outlay is spent on \( T \). Hence, \( F \) is a necessary good and \( T \) is a luxury good in a very strong sense.

The economic environment is exactly the same as in Section 2, except that we now consider a recipient with a planning horizon of \( N \) periods, \( t \in \{1, 2, \ldots, N\} \). The recipi-

---

\(^7\)In general, the first order conditions are not sufficient since the choice sets of early selves need not be convex. Further restrictions on the problem are needed in order to guarantee sufficiency. See Laibson (1997) and Morris (2002), among others, for discussions of this issue.
ent’s self $t$ has the intertemporal utility function

$$U_t = T_t + u(F_t) + \beta \sum_{i=1}^{N-t} \delta^i [T_{t+i} + u(F_{t+i})] ,$$

where $0 < \beta \leq 1$ and $0 < \delta < 1$ are constants. As before, the parameter $\delta$ is the time-consistent time discount factor, whereas $\beta$ is an additional discount factor that expresses self $t$’s desire for immediate consumption. The recipient receives a transfer bundle of value $B$ in period 1. The bundle consists of two assets: cash and vouchers for $F$. The value of the cash transfer is $M$ and the vouchers have a value of $V$, such that the recipient faces two constraints in period one: a money budget constraint, $M \geq \sum_{i=1}^{N} (p_F F_i + p_T T_i) - V$, and a voucher constraint $V \leq p_F \sum_{i=1}^{N} F_i$.

Following O’Donoghue and Rabin (1999), we distinguish between three types of recipients. *Rational* recipients, denoted by superscript $R$, have $\beta = 1$, are time-consistent, and do not suffer from the self-control problem. *Naïve* present-biased recipients, denoted by superscript $P$, have $\beta < 1$ but are not aware that they will have the self-control problem in the future. Specifically, a naïve self $t$ believes that all selves $t+1, \ldots, N$ are rational. *Sophisticated* present-biased recipients, whom we denote by superscript $S$, have $\beta < 1$ and are aware that subsequent selves are present-biased as well.

The model is solved in Appendix A.\textsuperscript{8} The solution has the same structure for all three types: there exists a type-dependent threshold transfer value $\bar{B}$ such that the recipient solely consumes $F$ in all periods if $B \leq \bar{B}$. In this case, the distinction between cash and vouchers does not matter since the entire transfer is spent on $F$. We say that the recipient is *income constrained*. If the transfer value exceeds the threshold, an inframarginal recipient spends $B - \bar{B}$ on $T$ in period 1. It is never optimal to consume $T$ in later periods because utility is linear in $T$ and future utility is discounted.

Consumption of $F$ is spread out over all $N$ periods. The allocation of $F$ is also

\textsuperscript{8}To ensure sufficiency of the first order conditions for the sophisticated recipient, we assume that the marginal propensity to consume out of savings is weakly increasing in wealth for all selves. Morris (2002) describes a fourth-order property of the instantaneous utility function that ensures this in a three-period model. If $u(F)$ is of the HARA-class, which nests, e.g., logarithmic and power functions, the marginal propensity to consume is independent of wealth such that the first order conditions are sufficient.
type-dependent and can be characterized by the Euler equations

\[ u_F(F_t^R) = \delta u_F(F_{t+1}^R), \]
\[ u_F(F_t^P) = \beta \delta u_F(F_{t+1}^P), \]
\[ u_F(F_t^S) = \left(1 - (1 - \beta) p_F \frac{\partial F_{t+1}^S}{\partial S_{t+1}}\right) \delta u_F(F_{t+1}^S). \]

A rational recipient discounts instantaneous utility in the next period by \( \delta \), such that consumption of \( F \) is decreasing over time. The naïve present-biased recipient discounts instantaneous utility in the next period by \( \beta \delta \), and will therefore choose to save less for consumption in the subsequent periods than a rational recipient with the same available income. The consumption of \( F \) is also decreasing in time for a naïve recipient. In addition to the immediate present-bias, a sophisticated recipient also takes account of how savings affect the self-control problem in the next period. From the viewpoint of a sophisticated self \( t \), self \( t + 1 \) does not optimize since the future is discounted too heavily. This implies that a marginal change in the savings available in period \( t + 1 \) has a first order effect on self \( t \)'s utility. This effect enters the Euler equation through the marginal propensity to consume in period \( t + 1 \), \( p_F \left( \frac{\partial F_{t+1}^S}{\partial S_{t+1}} \right) \), which measures the proportion of a marginal increase in savings, \( S_{t+1} \), that is consumed. In a well-behaved equilibrium, where the marginal propensity to consume is between zero and one, the sophisticated recipient has decreasing consumption of \( F \) over time but puts more relative weight on the next period than a naïve recipient does.

Rationing affects behavior in an intuitive way when preferences are quasi-linear. A recipient who is not income constrained will be inframarginal as long as the value of the vouchers do not exceed the threshold transfer value, that is, when \( V \leq \bar{B} \). The recipient is rationed when \( V > \bar{B} \). If \( V \) is raised above \( \bar{B} \) and the cash transfer \( M \) is reduced, such that the value of the transfer bundle is kept constant, the amount of cash available for consumption of \( T \) in period 1 is reduced. On the other hand, the additional voucher income raises consumption of \( F \) in all periods. Since the value of the entire transfer
bundle is held constant, this change will reduce outlay in period 1 and raise outlay in all subsequent periods.

It is possible to make present-biased recipients better off, measured by the long-run utility metric from Section 2.2, through in-kind transfers of the necessary good. In this case, long-run utility represents the preferences of a recipient with quasi-hyperbolic preferences who evaluates consumption choices in the \( N \) periods before entering period 1. This welfare measure, obtained by setting \( \beta = 1 \) in the intertemporal utility function of self 1, captures the fact that the recipient disapproves of any future present-bias and in case of bias would like to restrain or commit her future behavior. The attractive feature of asymmetric paternalism also extends to the \( N \)-period model. The threshold transfer value \( \bar{B} \) is larger for a rational recipient than for any type of present-biased recipient. This makes it possible to choose \( V \) such that only present-biased recipients are rationed and rational recipients are unaffected.

**Proposition 3.** Suppose that no recipients are income constrained. Let \( \bar{B}^R \), \( \bar{B}^P \), and \( \bar{B}^S \) be the threshold transfer values of a rational, a naïve present-biased, and a sophisticated present-biased recipient, respectively. Then,

(i) \( \bar{B}^R > \bar{B}^S \) and \( \bar{B}^R > \bar{B}^P \).

(ii) Raising \( V \) marginally above \( \bar{B}^S \), keeping the transfer value constant, increases the long-run utility of a sophisticated present-biased recipient.

(iii) Raising \( V \) marginally above \( \bar{B}^P \), keeping the transfer value constant, increases the long-run utility of a naïve present-biased recipient.

(iv) There exists a range of values of \( V \) that will increase the long-run utility of either a sophisticated or a naïve present-biased recipient while leaving a rational recipient inframarginal.

**Proof.** See appendix B.
The fact that in-kind transfers can improve long-run utility implies that sophisticated recipients will prefer to receive future transfers partly in kind. These recipients will readily agree to be rationed by future transfers in order to limit their self-control problems. Hence, sophisticated recipients can be helped separately if the government offers two transfer bundles, differing only in the amount of vouchers, and requires that recipients choose their preferred bundle before receiving the transfer. The level of $V$ in the bundle intended for the sophisticated recipients should maximize their long-run utility given the value of the transfer. Rational and naïve recipients have the same preferences over future transfers since both types believe they will act rationally. The bundle designated for these recipients can then be designed to help the naïve present-biased recipients in the best possible way.9

More generally, the above argument calls for attention to the possibility of providing benefit recipients with the option of tying (some of) their benefits to consumption of specific goods. In view of the multitude of in-kind transfer programs and the existing administrative apparatus, this may be a simple and comparably inexpensive way of providing these individuals with commitment devices.

4 In-Kind Transfers and Redistribution

So far in the analysis, the only source of heterogeneity has been the degree of the self-control problem. We focused solely on the efficient design of transfers, cash versus in-kind, in a world where individuals were simply assumed to be benefit recipients and where the total value of the transfer $B$ was fixed. The reason for giving out transfers in the first place, usually justified by differences in earnings abilities, was not considered. Moreover, the earnings decision, featuring prominently in the optimal tax-transfer literature, was...
not modeled. These choices were made to focus on the novel aspect of this paper, relating to the implications of self-control problems for the use of in-kind transfers. However, it is relevant to consider if the use of in-kind transfers can have adverse (or reinforcing) effects on self-selection into transfer programs and the possibility for income redistribution.

We extend the $N$-period model from the previous section with a static labor earnings choice. For simplicity, we consider an income taxation problem with only two types of earnings abilities like in, e.g., Stiglitz (1982). The two types of individuals are denoted $H$ and $L$ and have earnings abilities $w_H$ and $w_L$, respectively. We assume that type $H$ is most able, $w_H > w_L$, and that there are the same number of individuals of each type. Individuals decide on labor earnings, $I$, which are paid out in period 1 and make up total earnings over periods $1, \ldots, N$. Earnings give disutility $h(I/w)$, where $h(\cdot)$ is a strictly increasing and strictly convex function, such that the able type $H$ has less disutility from a given level of earnings than type $L$ has. The cash budget constraint in period 1 is $\sum_{i=1}^{N} (p_F F_i + p_T T_i) = I + B(I)$, where $B(I)$ is net cash transfers from the government, integrating both taxes and transfers, as a function of labor earnings.

The timing of the earnings decision is important because our model allows for time-inconsistent preferences. Since we are ultimately interested in investigating how the transfers affect the decision on whether to become a benefit recipient or not, it is assumed that the earnings decision is made before the transfers are given out (i.e., before period 1). Hence, we should think of an individual who decides on a level of labor earnings (e.g., an occupation) before the beginning of the transfer month. The individual receives labor earnings and net transfers on the first day of the month and then allocates consumption over the month through a series of consecutive optimization problems. This framework allows us to focus on the effect of in-kind transfers while disregarding the possibility of present-bias in the allocation of work effort.\footnote{Obviously, present-biased preferences may be very important for decisions on when to provide effort, see for instance O’Donoghue and Rabin (1999). The assumption should rather be viewed as resulting from features of the transfer program: the certification decision on eligibility for transfer benefits is made before the recipient receives the transfer bundle.}
Earnings are chosen to maximize long-run utility of consumption less the disutility of earnings

\[ \psi_j (I + B(I), V) - h \left( \frac{I}{w} \right), \]

where \( \psi_j (I + B(I), V) \) is the indirect long-run utility function, conditional on cash income, \( I + B(I) \), and voucher income, \( V \). We need to distinguish between two different indirect utility functions (hence the index), since the evaluation of long-run utility depends on whether the individual accounts for the present-bias or not. Rational and naïve present-biased individuals have the same indirect utility function, since both types believe that they will act rationally in the future. In contrast, sophisticated individuals know that they will be present-biased and evaluate utility accordingly. This distinction becomes crucial when introducing in-kind transfers.

Suppose the government wishes to redistribute income from the able type \( H \) to the less able \( L \). The government cannot observe abilities and instead implements a pure income tax. For simplicity, we assume that all type \( H \) individuals are identical with respect to the self-control problem. When there are only two ability types, it can be shown that the optimal income tax consists of a cut-off income level \( \bar{I} \) and a transfer value \( \tau \). Individuals with income \( \bar{I} \) or less receive a cash transfer, such that \( B(I) = \tau \) for \( I \leq \bar{I} \), while individuals with higher income pays \( \tau \) in taxes, \( B(I) = -\tau \) for \( I > \bar{I} \). The income taxation problem is constrained by a self-selection constraint: it must be optimal for the able type to pay the tax. Hence, the utility of \( H \) from having high earnings and paying the tax must exceed the utility from masquerading as type \( L \) by earning \( \bar{I} \) and receiving the transfer. A masquerading type \( H \) will choose exactly the same consumption path as the true type \( L \), since the disutility of earnings is additively separable, such that the self-selection constraint for type \( H \) under the pure income tax is

\[ \psi_j (I_H - \tau, 0) - h \left( \frac{I_H}{w_H} \right) \geq \psi_j (\bar{I} + \tau, 0) - h \left( \frac{\bar{I}}{w_H} \right). \quad (5) \]

When the government is constrained to cash transfers, \( (5) \) must hold with equality in the optimum. It is assumed that the optimal tax system fulfills \( \bar{I} + \tau \geq \bar{B}^R \), such that no
recipients are income constrained.

Now suppose that the government replaces part of the cash transfer with vouchers for $F$. There are two cases: (a) If type $H$ individuals are rational or naïve, present-biased transfer recipients can be helped without affecting the self-selection constraint. The argument follows from noticing that (5) is unchanged for rational and naïve type $H$ individuals when $V \leq \bar{B}^R$. Since it in fact is possible to ration both sophisticated and naïve recipients for some $V < \bar{B}^R$, vouchers can target recipients who have the self-control problem without affecting the choices of anyone else. (b) If individuals of type $H$ are sophisticated present-biased it is possible that the self-selection constraint tightens. Since vouchers offer a possibility of self-control, sophisticated individuals may find the transfer bundle more attractive if $V$ exceeds their threshold value. In this case, the value of the transfer will have to decrease in order to fulfill the constraint, which surely makes rational recipients worse off. Hence, the government may face a trade-off. Since we cannot a priori say whether naïve or sophisticated present-biased recipients have the largest threshold value, it may be possible to help naïve recipients without affecting the transfer size.

The model is very simple but it highlights a potential concern: government measures to correct self-control problems through the design of taxes and transfers may affect self-selection into benefit programs.\footnote{The idea that transfers in kind can be used to target benefits has a long history in the literature. Nichols and Zeckhauser (1982) demonstrate how in-kind transfers of certain \textit{indicator goods}, where consumption is negatively correlated with ability (such as medicine), may facilitate redistribution. By including indicator goods in the transfer bundle, able types will find the transfer bundle less attractive such that the self-selection constraint is eased (the model in this paper precludes this possibility because disutility of earnings is separable in the utility function). The argument in this paper is in many ways the exact opposite. If recipients are present-biased, it is desirable in itself to restrict the recipients’ choices. Further, transfers should be tied to a good that the intended target group is less likely to consume for a given level of income. Finally, as this section shows, the transfers may affect self-selection adversely if individuals are aware that they will have a self-control problem in the future.}\ The implications for the deadweight loss of taxation and the costs of income redistribution should be taken into account when comparing the pros and cons of such paternalistic policies.
5 Conclusion

This paper has considered the efficient design of transfers in the presence of self-control problems. Intertemporal distortions in consumption arising from present-biased preferences also distort the consumption bundle toward luxury goods. Transfers that are partly tied to a necessary good can help recipients with present-biased preferences to curb their self-control problems while at the same time leave fully rational recipients unaffected. In consequence, the analysis has provided a behavioral foundation for the concept of merit goods. To the extent that in-kind transfers help to reduce self-control problems, such transfers may be more efficient than analysis within the rational choice framework would suggest.

A number of details that have been left out above are worth discussing. First, the analysis only considered the case with two goods. It is possible to extend the argument to cases with more goods, where there may be several necessary goods. In these cases it is not sufficient to provide vouchers for any necessary good; it has to be the necessary good that is least responsive with respect to outlay. Except in knife-edge cases, including the case with a linear income expansion path, in-kind transfers can force recipients to smooth consumption.

Second, the analysis presumed that there was no interest rate. The benefits from tying transfers to vouchers come at a cost if cash savings accrue interest but vouchers do not. In this case, voucher income may in itself distort consumption toward the present such that the beneficial effects are lessened. This is arguably a minor cost, however, bearing in mind the empirical indication of a high percentage of recipients without savings accounts or interest income.

Third, we assumed that vouchers could not be exchanged for money. Obviously, the restrictions that tied transfers place on recipients may result in the creation of a black market for vouchers. If vouchers can be costlessly converted into cash, rationing has no effect on consumption. However, traders in the black market face search costs and risks
of detection which tend to lower the resale price and reduce the attractiveness of voucher trafficking. Whitmore (2002) uses survey evidence to investigate the black market for U.S. food stamps and finds that food stamps are, on average, traded at 64 percent of their nominal value. In addition, the government may take appropriate action in order to reduce the possibilities for voucher trade. As an example, Whitmore (op.cit.) argues that the implementation of the Electronic Benefit Transfer system, where food stamp benefits are distributed with a debit card instead of actual stamps, has reduced large-scale black market trading by food merchants, since transactions can be monitored easily.

One final issue that we touched upon in Section 4, but which deserves further scrutiny, is the take up of benefits. The incentive to work is only one aspect of the take up decision. In reality, not all individuals or households eligible for benefits choose to enroll in the benefit programs. Possible explanations include social “stigma,” hassles and transaction costs, as well as informational constraints, all of which may be affected by the design of transfers. Behavioral factors constitute an additional explanation for low take up rates in welfare programs (see Currie, 2004) and offer an interesting avenue for future research.
Appendix

A The $N$-Period Model

A.1 Self $N$

In order to characterize the recipient’s behavior, we proceed sequentially and start with the utility maximization problem of the last self, self $N$. Let $S^M_N$ and $S^V_N$ denote the savings in cash and vouchers, respectively, of self $N-1$. The recipient maximizes $U_N = T_N + u(F_N)$ with respect to $F_N$ and $T_N$ subject to three constraints: the cash budget constraint $S^M_N \geq p_NT_N + p_F F_N - S^V_N$, the voucher constraint $S^V_N \leq p_F F_N$ and a non-negativity constraint on $T$, $T_N \geq 0$. The first order conditions for self $N$’s optimum are

$$u_F(F_N) = (\lambda - \gamma)p_F,$$
$$1 + \mu_{T_N} = \lambda p_T,$$

where $\lambda$, $\gamma$, and $\mu_{T_N}$ are the Lagrange multipliers of the cash budget constraint, the voucher constraint, and the non-negativity constraint, respectively. Define $\hat{F}$ by $u_F(\hat{F}) = p_F/p_T$. There are three possible types of solutions: i) if $S^M_N + S^V_N < p_F \hat{F}$, the recipient is income constrained and spends all income on $F_N$. ii) If $S^M_N + S^V_N \geq p_F \hat{F}$ and $S^V_N \leq p_F \hat{F}$, the recipient chooses $F_N = \hat{F}$ and spends residual cash savings on $T$, $p_NT_N = S^M_N - p_F \hat{F}$. iii) If $S^V_N > p_F \hat{F}$, the recipient is rationed and chooses $S^V_N = p_F F_N$. Any cash savings are spent on $T_N$, such that $p_NT_N = S^M_N$.

The intuition behind this is quite simple. If the level of savings is very low, the recipient solely consumes the necessary good $F$ (case i). If savings are higher the recipient would like to choose $F_N = \hat{F}$ and spend residual savings on the luxury good $T$. This is only possible if voucher savings are sufficiently low (case ii). Otherwise, the recipient is constrained to spend voucher savings on $F$ (case iii), such that consumption of $F$ exceeds $\hat{F}$. The important lesson learned from this exercise is that self $N$ only consumes $T$ if savings are sufficiently high and cash savings are available.
A.2 Sophisticated Selves

Self $t$ receives cash savings $S^M_t$ and voucher savings $S^V_t$. The savings available for self $t + 1$ can be found by subtracting period $t$ consumption: $S^M_{t+1} = S^M_t - p_T T_t$ and $S^V_{t+1} = S^V_t - p_F F_t$. A sophisticated self $t$, $t \in \{1, \ldots, N - 1\}$ is aware of the time-inconsistency problem and knows that subsequent selves are present-biased. Consequently, self $t$ knows how savings $S^M_{t+1}$ and $S^V_{t+1}$ influence the decisions of future selves. We assume that a Markov perfect equilibrium with differentiable strategies for $F$ exists. Hence, we can think of the choices of selves $t + 1, \ldots, N$ as functions of $S^M_{t+1}$ and $S^V_{t+1}$. We further assume that the marginal propensity to consume $F$ (abbr. MPC) is weakly increasing in wealth for each self. As mentioned in Section 3, Morris (2002) derives a property that ensures this in a three period model. If $u(F)$ is HARA, e.g., a logarithmic or power function, the MPC is independent of wealth.

We characterize the equilibrium strategy by going through the following four steps:

1. We first assume that selves $k, \ldots, N$ solely consume $F$. We find the first order conditions for these selves under this restriction.

2. We then solve the maximization problem for self $t$ under the restriction that selves $t + 1, \ldots, N$ solely consume $F$. This allows us to find conditions for $T_t > 0$.

3. Then we turn to self $t - 1$. Self $t - 1$ prefers that $T_t = 0$ such that savings fulfill the conditions we found under pt. 2. Self $t - 1$’s problem is then similar to the problem we have just solved for self $t$. We check whether the solution is consistent with $T_t = 0$.

4. Finally, we find the equilibrium strategy using backward induction.

1. Suppose that selves $k, \ldots, N$ do not consume $T$. The behavior of these selves only depends on total savings $S_k = S^M_k + S^F_k$, since $F$ can be purchased using either cash or vouchers. Self $k$ maximizes $U_k = u(F_k) + \beta \sum_{i=1}^{N-k} \delta^i u(F_{k+i})$ with respect to $F_k$ and
subject to the cash budget constraint. In the optimum, the cash budget constraint will be fulfilled with equality since \( u(F) \) is increasing. This allows us to substitute for \( F_N \) by using the budget constraint. The first order condition for \( F_k \) is

\[
\beta \delta^{N-k} u(F_N) = u_F(F_k) - \beta \left[ \delta u_F(F_{k+1}) - \delta^{N-k} u(F_N) \right] p_F \frac{\partial F_{k+1}}{\partial S_{k+1}}
\]

(6)

We can obtain the first order condition for \( F_{k+1} \) in a similar manner. Multiplying this by \( \delta (\partial S_{k+2}/\partial S_{k+1}) = \delta [1 - p_F (\partial F_{k+1}/\partial S_{k+1})] \) and subtracting it from (6) gives

\[
u_F(F_k) = \left( 1 - (1 - \beta) p_F \frac{\partial F_{k+1}}{\partial S_{k+1}} \right) \delta u_F(F_{k+1}),
\]

(7)

which is the Euler equation. The marginal utility of \( F_{k+1} \) is discounted by the exponential time discount factor \( \delta \) and a term correcting for the effect on \( F_{k+1} \) from a marginal increase in savings. If self \( k+1 \)’s MPC, \( p_F (\partial F_{k+1}/\partial S_{k+1}) \), is positive, then a lower value of \( \beta \) implies that self \( k \) discounts future consumption more heavily.

We can make four observations: (a) if the MPC of self \( k+1 \) is positive and weakly increasing in wealth, \( p_F [\partial^2 F_{k+1}/(\partial S_{k+1})^2] \geq 0 \), then the MPC of self \( k \) is strictly between zero and one and the first order condition for \( F_k \) is sufficient. (b) The MPC of all selves \( k, \ldots, N-1 \) are strictly between zero and one if they are weakly increasing in wealth. (c) Aggregate consumption of \( F \), \( \sum_{i=0}^{N-k} F_{k+i} \), is increasing in \( S_k \) if the MPC of selves \( k, \ldots, N-1 \) are weakly increasing in wealth. (d) If the MPC of all selves are strictly between zero and one, \( F_{k+i} \) is decreasing in \( i \) (decreasing consumption path).

2. Under the restriction that selves \( t+1, \ldots, N \) solely consume \( F \), self \( t \) maximizes

\[
T_t + u(F_t) + \beta \sum_{i=1}^{N-t} \delta u(F_{t+i})
\]

with respect to \( F_t \) and \( T_t \) subject to the constraints

\[
\begin{align*}
S_t^M & \geq p_T T_t + \sum_{i=0}^{N-t} p_F F_{t+i} - S_t^V, \\
S_t^V & \leq p_F \sum_{i=0}^{N-t} F_{t+i}, \\
T_t & \geq 0.
\end{align*}
\]

26
Since the cash budget constraint will be fulfilled with equality in the optimum, we can rewrite the voucher constraint as \( S_t^M \geq T_t \). The behavior of subsequent selves only depends on total savings \( S_{t+1} \), such that there is just one state variable in self \( t \)'s maximization problem. We can use \((6)\) with \( k = t \) to characterize the first order condition for \( F_t \), and \((7)\) to characterize the Euler equation. Let \( \gamma \) and \( \mu_{T_t} \) be the Lagrange multipliers of the voucher constraint and the non-negativity constraint, respectively. The first order condition for \( T_t \) is

\[
u_F\left(F_t\right) = (1 + \mu_{T_t}) \frac{p_F}{p_T} - \gamma p_F,\]  

(8)

where we have used the first order condition for \( F_t \).

The solution to self \( t \)'s problem is characterized by \((7)\) for \( k = t, \ldots, N - 1 \), \((8)\), and the constraints. Our assumption that the MPC is weakly increasing in wealth ensures that the first order conditions are sufficient. First, assume that \( T_t > 0 \) and that the voucher constraint is not binding. From \((8)\) we see that \( u_F(F_t) = p_F/p_T \), such that \( F_t = \hat{F} \). In this case, the Euler equations for selves \( t + 1, \ldots, N - 1 \) and the budget constraints describe a unique solution for \( F_{t+1}, \ldots, F_N \), which we denote \((F^*_t, \ldots, F^*_N)\).

Define \( \bar{S}_t = p_F \left(\hat{F} + \sum_{i=1}^{N-t} F^*_{t+i}\right) \). This is the threshold level of savings for self \( t \). As in Section A.1, there are three possible types of solutions:

\[ i) \text{ If } S_t^M + S_t^V < \bar{S}_t, \text{ self } t \text{ is income constrained and chooses } T_t = 0. \text{ To see this, suppose that } T_t > 0. \text{ From } (8) \text{ we get that } F_t \geq \hat{F}. \text{ It then follows from the Euler equation and properties (a)--(c) from above that } p_F \left(F_t + \sum_{i=1}^{N-t} F_{t+i}\right) \geq \bar{S}_t, \text{ which violates the budget constraint.} \]

\[ ii) \text{ If } S_t^M + S_t^V \geq \bar{S}_t \text{ and } S_t^V \leq \bar{S}_t, \text{ self } t \text{ chooses } F_t = \hat{F} \text{ and } (F^*_t, \ldots, F^*_N). \text{ All residual cash savings are spent on } T_t. \text{ To see this, suppose that } F_t < \hat{F}. \text{ This requires that } \mu_{T_t} > 0 \text{ such that } T_t = 0. \text{ Using the Euler equation and the properties mentioned above, we find that } p_F \left(F_t + \sum_{i=1}^{N-t} F_{t+i}\right) < \bar{S}_t, \text{ which would imply that not all savings are spent. This is not possible. On the contrary, suppose that } F_t > \hat{F}. \text{ This would require that } \gamma > 0 \text{ such that the voucher constraint is binding, } S_t^V = p_F \sum_{i=0}^{N-t} F_{t+i}. \text{ Since } F_t > \hat{F} \text{ we must have } S_t^V > \bar{S}_t, \text{ which was ruled out by assumption.} \]

27
iii) If \( S_t^V > \bar{S}_t \) self \( t \) is rationed and spends all voucher savings on \( F \). Any cash savings are spent on \( T_t \). To see this, note that we get \( p_F \sum_{i=0}^{N-t} F_{t+i} > \bar{S}_t \) from the voucher constraint. This implies that \( F_t > \hat{F} \), which in turn requires \( \gamma > 0 \), such that the voucher constraint is binding. From the cash budget constraint we then find \( S_t^M = p_T T_t \).

3. The intertemporal utility of self \( t-1 \) is \( T_{t-1} + u(F_{t-1}) + \beta \delta \left( T_t + \sum_{i=0}^{N-t} \delta^i u(F_{t+i}) \right) \).

Self \( t-1 \) never wants \( T_t > 0 \), as long as \( \beta \delta < 1 \), since any income spent on \( T_t \) would be better spent on \( T_{t-1} \). Hence, the savings of self \( t-1 \) must satisfy either \( S_t^M + S_t^V \leq \bar{S}_t \) or \( S_t^V > \bar{S}_t \) and \( S_t^M = 0 \). That is, self \( t \) must either be income constrained or savings must be entirely in vouchers. When \( T_t = 0 \) the maximization problem of self \( t-1 \) is similar to the problem we solved above. All we need to do now is to check that this solution is indeed consistent with \( T_t = 0 \).

If \( S_{t-1}^M + S_{t-1}^V < \bar{S}_{t-1} \) (case \( i \)) or \( S_{t-1}^M + S_{t-1}^V \geq \bar{S}_{t-1} \) and \( S_{t-1}^V \leq \bar{S}_{t-1} - \bar{F} \) (case \( ii \)), we know that \( F_{t-1} \leq \hat{F} \). Hence, from the Euler equation, savings fulfill \( S_t^M + S_t^V \leq \bar{S}_t - \bar{F} \).

When the MPC of self \( t \) is between zero and one we have \( \bar{S}_{t-1} - \hat{F} < \bar{S}_t \). Hence, self \( t \) is income constrained and chooses \( T_t = 0 \). If \( S_{t-1}^V > \bar{S}_{t-1} \) (case \( iii \)) we know that any cash savings are spent on \( T_{t-1} \). Hence, self \( t \) must choose \( T_t = 0 \).

4. Self \( N-1 \) wants \( T_N = 0 \) and chooses \( S_N^M + S_N^V < p_F \hat{F} \) or \( S_N^M = 0 \). We can describe the solution to self \( N-1 \)'s problem by (7), (8), and the constraints. The equilibrium strategy follows from backward induction.

The threshold transfer value is \( \bar{B}^S = p_F \left( \hat{F} + \sum_{i=1}^{N-1} F_{1+i}^* \right) \), where \( F_2, \ldots, F_N^* \) solve

\[
\begin{align*}
    u_F \left( F_{1+i}^* \right) &= \frac{p_F \delta u_F \left( F_{1+i}^* \right)}{\prod_{j=1}^i \left( 1 - (1 - \beta) p_F \frac{\partial F_{1+i}^*}{\partial S_{1+j}} \right)}, \quad i = 1, \ldots, N - 2, \\
    u_F \left( F_{N-1}^* \right) &= \beta \delta u_F \left( F_N^* \right), \\
    S_{2+i} &= S_{1+i} - p_F F_{1+i}^*, \quad i = 1, \ldots, N - 2, \quad (9) \\
    S_2 &= p_F \sum_{i=1}^{N-1} F_{1+i}^*.
\end{align*}
\]

This value determines whether a sophisticated recipient is income constrained or rationed.
The recipient is income constrained if $B \leq \bar{B}$. An inframarginal recipient ($B > \bar{B}$ and $V \leq \bar{B}$) chooses $T_1 = B - \bar{B}$ and $(\hat{F}, F_2, \ldots, N)$. A rationed recipient ($B > \bar{B}$ and $V > \bar{B}$) chooses $T_1 = M$ and an allocation of $(F_1, F_2, \ldots, N)$ that fulfills the Euler equations, given by (7), and the voucher constraint.

### A.3 Rational Selves

A rational recipient does not have present-biased preferences, which is captured by $\beta = 1$. A rational self $t$ maximizes $U_t = T_t + u(F_t) + \sum_{i=1}^{N-t} \delta^i [T_{t+i} + u(F_{t+i})]$ with respect to $\{F_{t+j}\}_{j=0}^{N-t}$ and $\{T_{t+j}\}_{j=0}^{N-t}$, subject to the constraints

\[
S_t^M \geq \sum_{i=0}^{N-t} (p_T T_{t+i} + p_F F_{t+i}) - S_t^V,
\]
\[
S_t^V \leq p_F \sum_{i=0}^{N-t} F_{t+i},
\]
\[
T_{t+j} \geq 0, \quad j = 0, \ldots, N - t.
\]

The first order conditions for $(F_{t+j}, T_{t+j})$ are

\[
\delta^j u_F(F_j) = (\lambda - \gamma) p_F,
\]
\[
\delta^j = \lambda p_T - \mu_{T_{t+j}},
\]

where $\lambda$ and $\gamma$ are the Lagrange multipliers of the cash budget constraint and the voucher constraint, respectively, and $\mu_{T_{t+j}}$ is the multiplier of the non-negativity constraint on $T_{t+j}$. From the first order conditions we can obtain the Euler equation for $F_t$

\[
u_F(F_t) = \delta u_F(F_{t+1}),
\]

and an expression for the intratemporal allocation in period $t$

\[
u_F(F_t) = (1 + \mu_{T_t}) \frac{p_F}{p_T} - \gamma p_F.
\]

Because future utility is discounted, $\delta < 1$, we have $T_{t+1}, \ldots, T_N = 0$ and a decreasing consumption path for $F$. 
The threshold transfer value of a rational recipient is \( \bar{B}^R = p_F \left( \hat{F} + \sum_{i=1}^{N-1} F_{t+i}^* \right) \), where \( F_2^*, \ldots, F_N^* \) solve

\[
u_F \left( F_{t+i}^* \right) = \frac{p_F}{p_T} \hat{F}_{t+i}, \quad i = 1, \ldots, N - 1.
\]

\[ (10) \]

A.4 Naïve Selves

The naïve present-biased self \( t \) has \( \beta < 1 \) but believes that subsequent selves behave rationally. The maximization problem is similar to the problem of a rational self, except that intertemporal utility is \( T_t + u (F_t) + \beta \sum_{i=1}^{N-t} \delta^i \left[ T_{t+i} + u (F_{t+i}) \right] \). Similar to the analysis in the previous section, we can find the Euler condition

\[
u_F (F_t) = \beta \delta u_F (F_{t+1}),
\]

and an expression for \( T_t \)

\[
u_F (F_t) = \left(1 + \mu_{T_t}\right) \frac{p_F}{p_T} - \gamma p_F,
\]

from the first order conditions. It follows from the Euler condition and the properties of \( u (F) \) that the MPC of each self is strictly between zero and one. Self \( t \) is rationed if

\[
\bar{S}_t > \bar{S}_t = \hat{F} + \sum_{i=1}^{N-t} \hat{F}_{t+i}, \quad \text{where} \quad \hat{F}_{t+1}, \ldots, \hat{F}_N \text{ solve}
\]

\[
\frac{p_F}{p_T} = \beta \delta u_F (F_{t+i}), \quad i = 1, \ldots, N - t.
\]

A naïve self \( t \) does not wish subsequent selves to consume \( T \) because \( \beta \delta < 1 \). This implies that self \( t \) will tie all savings to vouchers or make sure that future selves are income constrained. However, since the naïve self \( t \) misperceives her future behavior, we need to check whether self \( t + 1 \) indeed will be constrained by the savings of self \( t \).

All savings will be in vouchers if self \( t \) is rationed which implies that subsequent selves cannot consume \( T \). If self \( t \) is not rationed, savings \( S_{t+1} \) will never exceed \( \bar{S}_t - \hat{F} \). Self \( t + 1 \) is income constrained if \( S_{t+1} \leq \bar{S}_{t+1} = \hat{F} + \sum_{i=1}^{N-t-1} \hat{F}_{t+i} \). If we subtract \( \bar{S}_{t+1} \) from \( \bar{S}_t - \hat{F} \) we get

\[
\bar{S}_t - \hat{F} - \bar{S}_{t+1} = \hat{F}_N - \hat{F} < 0,
\]

30
such that we always have \( S_{t+1} < \bar{S}_{t+1} \). Hence, self \( t + 1 \) will be income constrained.

The threshold transfer value for a naïve recipient is 
\[
B^p = p_F \left( \hat{F} + \sum_{i=1}^{N-1} F^*_i \right),
\]
where \( F_2^*, \ldots, F_N^* \) solve
\[
u_F (F^*_i) = \frac{p_F/p_T}{\beta^2}, \quad i = 1, \ldots, N-1.
\]

### B Proofs

**Proof of Lemma 1.** The optimum is characterized by four equations in four unknowns when the voucher constraint is binding. If we linearize these equations we get the following system of equations
\[
\begin{pmatrix}
u_{TT}(T_1, F_1) & u_{TF}(T_1, F_1) & -\beta \delta u_{TT}(T_2, F_2) & -\beta \delta u_{TF}(T_2, F_2) \\
u_{TF}(T_1, F_1) & u_{FF}(T_1, F_1) & -\beta \delta u_{TF}(T_2, F_2) & -\beta \delta u_{FF}(T_2, F_2) \\
p_T & 0 & p_T & 0 \\
0 & p_F & 0 & p_F
\end{pmatrix}\begin{pmatrix}dT_1 \\ dF_1 \\ dT_2 \\ dF_2\end{pmatrix} = \begin{pmatrix}0 \\ 0 \\ dM \\ dV\end{pmatrix}.
\]

Our main concern is the change in outlay in period one if \( V \) increases and \( M \) decreases such that the sum \( B = M + V \) is constant. Under the restriction that \( dM = -dV \), period one outlay changes by 
\[
 dB_1 = [p_F (\partial F_1/\partial V - \partial F_1/\partial M) + p_T (\partial T_1/\partial V - \partial T_1/\partial M)] dV.
\]
This is negative if and only if \( p_F (\partial F_1/\partial V - \partial F_1/\partial M) < p_T (\partial T_1/\partial V - \partial T_1/\partial M) \). The voucher constraint is still binding after the change. The concavity of \( u(F, T) \) ensures that the Jacobian has a non-negative determinant. We assume that the recipient is in a regular optimum where the determinant of the Jacobian is different from zero, such that we can use the implicit function theorem to solve for \( \partial T_1/\partial V \) and \( \partial F_1/\partial V \). After some algebra we find that \( dB_1 < 0 \) if and only if
\[
[p_T u_{FF}(F_1, T_1) - p_F u_{TF}(F_1, T_1)] [p_F u_{TT}(F_2, T_2) - p_T u_{TF}(F_2, T_2)] > [p_T u_{FF}(F_2, T_2) - p_F u_{TF}(F_2, T_2)] [p_F u_{TT}(F_1, T_1) - p_T u_{TF}(F_1, T_1)].
\]

All parentheses are negative if both \( F \) and \( T \) are normal goods. This allows us to rewrite the condition to (4). \( \square \)
Proof of Proposition 1. Differentiation of the long-run utility function yields
\[ dU = \left( \frac{1}{\beta} - 1 \right) \left[ p_F \left( \frac{\partial F_1}{\partial M} - \frac{\partial F_1}{\partial V} \right) + p_T \left( \frac{\partial T_1}{\partial M} - \frac{\partial T_1}{\partial V} \right) \right] \frac{u_F(F_1^p, T_1^p)}{p_F} dV, \]
where we have used the first order conditions. The first term is only positive if \( \beta < 1 \). The expression in the square brackets is the change in period 1 outlay, \( dB_1 \), from marginal rationing in the inframarginal optimum. From Lemma 1 and the derivation of the slope of the income expansion path we know that \( dB_1 < 0 \) if and only if \( dT_1^P/dF_1^P > dT_2^P/dF_2^P \). □

Proof of Lemma 2. The recipients face identical budget constraints because they receive identical transfers. Their consumption bundles fulfill
\[ p_T \left[ T_1^R + T_2^R - (T_1^P + T_2^P) \right] + p_F \left[ F_1^R + F_2^R - (F_1^P + F_2^P) \right] = 0, \]
where \((T_1^R, T_2^R)\) and \((T_1^P, T_2^P)\) are the inframarginal demands for \( T \) of the rational and the present-biased recipient, respectively. We know that a present-biased recipient chooses a larger outlay in period one than a rational recipient since \( u(T, F) \) is strictly concave. The assumption that both \( T \) and \( F \) are normal goods then implies that \( F_2^P < F_2^R < F_1^R < F_1^P \) and \( T_2^P < T_2^R < T_1^R < T_1^P \). There exists constants \( \alpha, \beta \in (0,1) \) such that we can write \( F_1^R \) and \( F_2^R \) as convex combinations of \( F_1^P \) and \( F_2^P \); \( F_1^R = \alpha F_1^P + (1 - \alpha) F_2^P \) and \( F_2^R = \beta F_1^P + (1 - \beta) F_2^P \).

Suppose that \( F_1^P + F_2^P > F_1^R + F_2^R \). Then \( F_1^P + F_2^P > (\alpha + \beta) F_1^P + [2 - (\alpha + \beta)] F_2^P \), such that \( \alpha + \beta < 1 \) because \( F_1^P > F_2^P \). The income expansion path can be expressed as a strictly increasing function \( T = f(F) \), such that there is a unique corresponding value of \( T \) to any \( F \). It follows from Jensen’s inequality that if \( f(F) \) is convex on \([F_2^P, F_1^P]\) then
\[ f \left( F_1^R \right) = f \left( \alpha F_1^P + (1 - \alpha) F_2^P \right) \leq \alpha f \left( F_1^P \right) + (1 - \alpha) f \left( F_2^P \right) , \]
\[ f \left( F_2^R \right) = f \left( \beta F_1^P + (1 - \beta) F_2^P \right) \leq \beta f \left( F_1^P \right) + (1 - \beta) f \left( F_2^P \right) . \]
This implies that
\[ f \left( F_1^R \right) + f \left( F_2^R \right) \leq (\alpha + \beta) f \left( F_1^P \right) + [2 - (\alpha + \beta)] f \left( F_2^P \right) < f \left( F_1^P \right) + f \left( F_2^P \right) , \]
because $\alpha + \beta < 1$ and $f(F)$ is a strictly increasing function. This violates the budget constraint. Thus, we must have $F_1^R + F_2^R \geq F_1^P + F_2^P$. The proof is analogous if the income expansion path is strictly convex. □

**Proof of Proposition 2.** The result follows from Proposition 1 and Lemma 2. □

**Proof of Proposition 3.**

(i) **Threshold transfer values** The threshold transfer value for fully rational recipients, $\bar{B}^R$, is given by (10). It is fairly easy to see that $\bar{B}^S$, which is defined by (9), is smaller than $\bar{B}^R$ when the MPC of the sophisticated recipient is between zero and one. The assumption of weakly increasing MPC ensures this. Similarly, $\bar{B}^P$, defined by (11), is smaller than $\bar{B}^R$.

(ii) **Sophisticated recipients** The long-run utility welfare measure is

$$U = T_1 + u(F_1) + \sum_{i=1}^{N-1} \delta^i u(F_{1+i}). \quad (12)$$

When $B > \bar{B}^R$ we have $B > \bar{B}^S$, such that the recipient is not income constrained. The sophisticated recipient chooses the inframarginal optimum if $V = \bar{B}^S$. The effect on welfare from raising $V$ by $dV > 0$, and lowering the cash transfer by a similar amount, is

$$dU = \left( u_F(F_1) \frac{\partial F_1}{\partial V} + \sum_{i=1}^{N-1} \delta^i u_F(F_{1+i}) \frac{\partial F_{1+i}}{\partial S_{1+i}} \frac{\partial S_{1+i}}{\partial S_2} \left( 1 - p_F \frac{\partial F_1}{\partial V} \right) - \frac{1}{p_T} \right) dV,$$

where we have used that the recipient becomes rationed, such that all cash income is spent on $T_1$. Differentiate the voucher constraint in order to substitute for $\partial F_N/\partial S_2$ and rearrange to get

$$\frac{dU}{dV} = \left( + \sum_{i=2}^{N-2} \left[ \delta^i u_F(F_{1+i}) - \delta^{N-1} u_F(F_N) \right] \frac{p_F \delta S_2}{\delta F_{1+i} \delta S_{1+i}} \frac{\partial F_{1+i}}{\delta S_{1+i}} \frac{\partial S_{1+i}}{\partial S_2} \right) \frac{(1 - p_F \frac{\partial F_1}{\partial V})}{p_F},$$

$$+ u_F(F_1) \frac{\partial F_1}{\partial V} - \frac{1}{p_T}.$$
The expression in the large parenthesis equals \((1/\beta) u_F(F_1)\), which can be seen by using (6). Inserting this yields

\[
\frac{dU}{dV} = \frac{u_F(F_1)}{p_F} \frac{1}{\beta} \left[ 1 - (1 - \beta) \frac{p_F}{p_T} \frac{\partial F_1}{\partial V} \right] - \frac{1}{p_T}.
\]

Since the recipient is only marginally rationed we know that \(u_F(F_1) = \frac{p_F}{p_T}\), which we use to obtain the final expression for \(dU\)

\[
dU = \left( \frac{1}{\beta} - 1 \right) \left( 1 - p_F \frac{\partial F_1}{\partial V} \right) \frac{dV}{p_T}.
\]

This is positive when \(\beta < 1\) and the MPC in period 1 is between zero and one. The latter condition is fulfilled when the MPC is weakly increasing in wealth.

(iii) **Naïve recipients** Recall that the long-run utility measure is given by (12). When \(B > \bar{B}^R\) we have \(B > \bar{B}^P\), such that the recipient is not income constrained. The effect on long run utility from raising \(V\) marginally above \(\bar{B}^P\), keeping the transfer value constant, is

\[
dU = \left( \frac{p_F}{p_T} \frac{\partial F_1}{\partial V} + \sum_{i=1}^{N-1} \delta^i u_F(F_{1+i}) \frac{\partial F_{1+i}}{\partial S_{1+i}} \frac{\partial S_{1+i}}{\partial S_2} \left( 1 - p_F \frac{\partial F_1}{\partial V} \right) - \frac{1}{p_T} \right) dV,
\]

where we have used that \(u_F(F_1) = \frac{p_F}{p_T}\). Because all naïve selves 1, ..., \(N - 2\) are “cheated” by subsequent selves, there is no straightforward way to compare the marginal utilities across selves by using the first order conditions. However, it is possible to find a sufficient condition for a welfare improvement by using that self \(N - 1\) predicts the marginal utility of self \(N\) correctly: we know that \(u_F(F_{N-1}) = \beta \delta u_F(F_N)\). A sufficient condition for \(dU/dV > 0\) is

\[
\frac{p_F}{p_T} \frac{\partial F_1}{\partial V} + \sum_{i=1}^{N-2} \delta^i u_F(F_{1+i}) \frac{\partial F_{1+i}}{\partial S_{1+i}} \frac{\partial S_{1+i}}{\partial S_2} \left( 1 - p_F \frac{\partial F_1}{\partial V} \right) + \delta^{N-2} u_F(F_{N-1}) \frac{1}{\beta} \frac{\partial F_N}{\partial S_N} \frac{\partial S_N}{\partial S_2} \left( 1 - p_F \frac{\partial F_1}{\partial V} \right) > \frac{1}{p_T}.
\]

Next, observe that each naïve self \(1 + i\), where \(i = 1, \ldots, N - 2\), plans to have \(F_N < F_N^*\). This follows from the fact that self 2 saves less than \(\sum_{i=2}^{N-1} F_1^{*}\) and that the
MPC is between zero and one. The first order conditions for self $1 + i$ then imply that $u_F(F_{1+i}) > \beta \delta^{N-1-i} u_F(F_N^*)$. Multiply this by $\delta^i$ to find

$$\delta^i u_F(F_{1+i}) > \beta \delta^{N-1} u_F(F_N^*) = \frac{p_F}{p_T}.$$ 

This means that we can substitute $p_F/p_T$ for $\delta^i u_F(F_{1+i})$ in (13), since the MPC is between zero and one, such that the condition reduces to

$$\left(\frac{1}{\beta} - 1\right) p_F \frac{\partial F_N}{\partial S_N} \frac{\partial S_N}{\partial S_2} \left(1 - p_F \frac{\partial F_1}{\partial V}\right) > \left(1 - \sum_{i=1}^{N-1} p_F \frac{\partial F_{1+i}}{\partial S_{1+i}} \frac{\partial S_{1+i}}{\partial S_2} \left(1 - p_F \frac{\partial F_1}{\partial V}\right) \right).$$

The right hand side is equal to zero, which can be seen by differentiating the voucher constraint. By using that $\partial F_N/\partial S_N = 1/p_F$ and $\partial S_N/\partial S_2 = \prod_{j=1}^{N-2} \left[1 - p_F (\partial F_{1+j}/\partial S_{1+j})\right] \in (0, 1)$, the condition reduces to $(1/\beta - 1) [1 - p_F (\partial F_1/\partial V)] > 0$, which is fulfilled since $\beta < 1$ and the MPC of a naïve self 1 is between zero and one.

(iv) **Asymmetric Paternalism** Follows from (i)–(iii). □

References


