The Political Economy of Post-Compulsory Education Policy with Endogenous Credit Constraints*

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Abstract

Altruistic parents, who differ in income, make financial transfers to their children, who differ in ability. The children make investments in post-compulsory education; their investment choices may be endogenously credit-constrained, as lenders will consider a child’s incentives to repay the loan and will only lend amounts that the child can credibly promise to repay. The parents, prior to making transfers, vote over subsidies to those who participate in education, financed by a proportional tax on income. A voting equilibrium, if it exists, tends be such that voters in the two tails of the income distribution would support a reduction in the education subsidy: the “poor” support a reduction since they have a low participation rate and the “rich” support a reduction in the subsidy since they are paying a particularly high tax price. The support for an expansion of the education subsidy comes primarily from the “middle-class”. We first provide a necessary condition that has to be satisfied by an interior equilibrium. We then verify its existence in a calibrated version of the model. Finally we perform a number of comparative statics exercises tracing the effect on the equilibrium policy as well as other outcomes such as participation and intergenerational social mobility.

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I Introduction

In recent years, the role played by the State in the financing, regulation and provision of education has become increasingly dominant. It is also widely recognised that understanding the educational process is key for addressing strongly felt issues such as intergenerational mobility, growth and development, and income equality.

The choices made by the families for the education of their children must be based, among other things, on the existing education policies, and in turn these policies react to the political choices of the families. In the present paper, we address this circular relationship with a political economy model that investigates the political support for post-compulsory education. We seek to determine the shape taken by the public intervention, and its effects on participation and inter-and intragenerational inequality. There is a general perception that post-compulsory education policies are at least partially regressive, redistributing income from the lower income groups to middle- and high income groups. We confirm this reading in our model.

It has to be noted that post-compulsory education has a number of features that distinguishes it from basic education.

- In most Western countries, by the time an individual reaches post-compulsory education she is old enough to make economic decisions; hence parents and children should reasonably be treated as separate individuals each with their own individual preferences.

- A key decision for an young individual is whether or not to participate in post-compulsory education; the individual has the option of leaving education at the minimum school leaving age. Moreover, in line with the stylized facts, the participation decision should be related to parental income.

- Young individuals may be financially constrained with respect to post-compulsory educational choices, leading either to non-participation or to a downward bias in the chosen quality/expenditure. Parents can, however, financially assist their children and hence mitigate any potential credit constraints.

This paper presents a simple model that contains the above features. In the model the parents, who are altruistic, make financial transfers to the children who, in turn, decide whether or not to participate in post-compulsory education and, if so, how much to spend. A standard way of incorporating credit constraints is simply to assume that children cannot borrow against
future income. We do not follow this approach; instead we adopt an approach where children can borrow any amount that they can credibly promise to repay. This has the effect that e.g. parental transfers and/or government subsidies can boost a child’s credit limit.

Prior to the children entering post-compulsory education, the parents vote on educational policy. We assume that the government can observe only the participation decision, not how much a child spends on education. An educational policy hence consists of a fixed subsidy that is received by any participating child, financed via a proportional tax on income. Despite the relative complexity of the model, the parents’ preferences over policy has a surprisingly simple structure. Indeed, the modeling of parents and children as separate individuals linked via the parents’ financial transfers is key to this result: the assumptions imply that the parents are, in the language of Becker, “effective altruists”, implying that they have simple preferences defined over total family resources.

In line with Fernandez and Rogerson (1995) we find that existence of a majority voting equilibrium is not guaranteed. However, following the approach of Epple and Romano (1996), we can give necessary conditions that must be satisfied by any majority voting equilibrium. Indeed, in line with Epple and Romano we find that a political equilibrium, when one exists, will typically be of an “ends-against-the-middle” type, where parents in both tails of the income distribution want to reduce the tax/subsidy policy. However, the intuition for this is quite different from the Epple and Romano result. In the current context the poor do not support the subsidy policy since they will not be participating in post-compulsory education. The very rich would like to see the policy scaled back since they pay a large tax price. The support for an expansion of the education subsidy comes primarily from the “middle-class”. So, in this sense, we have the regressive effect we mentioned above: redistribution flows not only from the high-income but also from the low-income to the middle-income individuals.

Finally, we investigate the properties and the comparative statics of the political equilibrium using a numerical example. We check the existence of the equilibrium and its "ends-against-the-middle" nature in a baseline model that can be thought of as representing a typical western country. Then we note the effect of changing various parameters; we find e.g. that increases in the return to education boost participation as well as the support for policy, increases in inequality among parents severely reduce the support for policy, and relaxing credit restrictions also reduces the support for policy. This latter result points out a role for policies limiting the imperfections of the credit market (US-style policies) as a valid alternative against educational
policies based on fiscal incentives (European-style policies).

II The Model

Consider an economy populated by a large number of households, each made of a parent and a child. The parents are distinguished by their earnings $y$; children have different abilities $\theta$. Each household is identified by a $(y, \theta)$ pair; the parent observes her own earnings as well as her child’s ability. Both variables are continuously distributed on the supports $[y, \bar{y}]$ (with $y > 0$) and $[\underline{\theta}, \bar{\theta}]$ (with $\underline{\theta} > 0$) and the number of households is normalised to unity, $\int \int f(y, \theta)dyd\theta = 1$.

In line with the empirical literature [refs] we allow $y$ and $\theta$ to be positively, but not perfectly, correlated. Parents are altruistic towards their offspring: each adult makes a transfer $b$ to her child. The child decides whether or not to participate in post-compulsory education. If she does participate she makes a decision about how much to spend, $z$, on that education. Hence a child that attends education and spends $z$ obtains the final income $w(z, \theta)$. The earnings function satisfies

$$w_z > 0; w_{\theta} > 0; w_{zz} < 0; w_{yy} < 0; w_{z\theta} > 0.$$ (1)

We assumed that the two inputs in the earnings function are complements, $w_{z\theta} > 0$; this reflects an idea that people with higher innate ability can profit more from their own human capital investments. For simplicity, we take it that a child that does not participate in education obtains fixed earnings $w_0 > 0$.\footnote{In fact, this simplification is extreme. The model would work as long as ability has a larger impact on the earnings of those who acquire an education than on the earnings of those who don’t. In the numerical model below, we will relax the assumption that uneducated workers earn a fixed income.}

The timing of the model is as follows. First the parents vote over policy, consisting of a fixed subsidy $\sigma$, conditioned on participation only, and a proportional income tax $\tau$ levied on the parents’ income (we disregard any taxes on income that the children may face). Then each parent makes a transfer $b$ to her children. Finally, the child makes her decision about education. The model is solved by backward induction.

The Investment in Education

Consider a child who has received a transfer $b$ from her parent. She faces the decision of whether or not to participate in post-compulsory education, and, if she decides to participate, how much
to invest. We start by exploring the latter decision and its consequences in terms of the child’s final resources.

If a child participates in education and invests the amount \( z \), her final resources will be \( x = w(z, \theta) - z + \sigma + b \). In order to finance the investment \( z \) she will need to borrow \( z - \sigma - b \); assuming a zero interest rate, she will have to repay exactly this amount. A particular loan will be available if and only if it is in the child’s interest to subsequently repay that loan. This in turn hinges on the consequences of defaulting. Following Lochner and Monge-Naranjo (2002), we assume that if a child defaults all her assets will be seized by the lender, and she will also have to pay a penalty equal to a fraction \( \gamma \in (0, 1) \) of her future earnings: final resources for a defaulter are thus \( (1 - \gamma)w(z, y) \). Hence a child will be able to borrow the amount \( z - \sigma - b \) if and only if she is better off by not defaulting, i.e. \( w(z, \theta) - z + \sigma + b \geq (1 - \gamma)w(z, \theta) \) or simply

\[
\gamma w(z, \theta) - z + \sigma + b \geq 0. \tag{2}
\]

The objective of the child is to maximize her final resources \( x \). The child’s maximal final resources, conditional on participation, can then be written as

\[
x^p(\sigma + b, \theta) = \max_z (w(z, \theta) - z + \sigma + b) \quad \text{s.t.} \quad (2) \tag{3}
\]

Note that \( \sigma \) and \( b \) enter as argument in \( x^p(\cdot) \) in the form of the sum \( \sigma + b \). Hence we can define \( a \equiv \sigma + b \) as the child assets and write \( x^p = x^p(a, \theta) \) and \( z(a, \theta) \) as the solution to the above problem. Letting \( \mu \) denote the Lagrange multiplier (to be interpreted as the marginal value of credit), the first order condition can then be written as\(^2\)

\[
(w_z - 1) + \mu (\gamma w_z - 1) = 0 \tag{4}
\]

If the credit constraint does not bind, \( \mu = 0 \) and (4) reduces to \( w_z = 1 \). From (1), we have:

\[
z_\theta = -\frac{w_z \theta}{w_{zz}} > 0; \quad z_a = 0. \tag{5}
\]

Thus, the unconstrained child’s educational investment is increasing in her own ability; also, neither the government grant nor the domestic transfer affect the child’s optimal level of investment (it may, of course, affect the child’s decision to participate). Parental income (ability) acts indirectly through the positive correlation with the child’s ability, tendentially boosting educational spending.

\(^2\)Subscripts denote partial derivatives.
If the constraint binds, the optimal investment satisfies the credit constraint (2) with equality, and can be obtained as an implicit solution to $\gamma w (z, \theta) - z + a = 0$. Note that we can determine bounds on the marginal return to $z$ as (i) it must be that $w_z > 1$ since the unconstrained investment is not available, and, (ii) it must be that $\gamma w_z < 1$ since a marginal increase in the investment necessarily leads the credit constraint to be violated.\(^3\) Combining the two inequalities yields that
\[ 1 < w_z < \frac{1}{\gamma} \]  
(6)
Treating $\gamma w (z, \theta) - z + a = 0$ as an identity and using (6) and (1) we can sign the following comparative statics in the constrained case
\[ z_a = (1 - \gamma w_z)^{-1} > 0 \]  
(7)
\[ z_\theta = \gamma w_\theta (1 - \gamma w_z)^{-1} > 0 \]  
(8)
that is, if the child is credit constrained, then her investment is increasing in her initial assets and in her ability (and thus tendentially in her parent’s income). Note that the constrained investment is increasing in the child’s ability for any given transfer $b$. This implies that a high ability child (tendentially, then, a child to a rich parent) can borrow more than a low ability child given the same transfer; the reason is that the child’s innate ability will boost the child’s earnings, which allows the credit to the child to be extended without violating the constraint.

For future use, we need a few results concerning the marginal value to the child of her initial assets and of her innate ability:

**Lemma 1** For unconstrained children, the marginal value of the assets is constant ($x_a^P = 1$ and $x_{aa}^P = x_{a\theta}^P = 0$), and that of ability is positive. For constrained children, the marginal value of assets exceeds unity, since it also has the effect of relaxing the credit constraint, and is decreasing, since the marginal value of credit $\mu$ decreases as initial assets increase; that of ability is positive. Formally:

\[ x_a^P = 1 + \mu \geq 1 > 0; \quad x_\theta^P = w_\theta (1 + \gamma \mu) \geq w_\theta > 0. \]  
(9)

and, for constrained children,

\[ x_{aa}^P = \mu_a < 0; \quad x_{a\theta}^P = \mu_\theta. \]  
(10)

\(^3\)The constraint’s derivative w.r.t. $z$ is $\gamma w_z - 1$; the constraint has a positive intercept ($a$) and to be binding it must cross the abscissa from above, hence $\gamma w_z < 1$. 

6
The Parent’s Transfer Decision

We assume that all parents make “interior” (strictly positive) transfers. This will imply that the parents are, in Beckerian parlance, "effective altruists", a fact that has strong implications for the structure of their indirect utilities. The parent cares about her own consumption and the child’s final resources, and makes a transfer to the child, taking policy as given. For simplicity, we let the parents’ utility be additively separable.

For a participating family, we can write
\[ U(b, y; \tau, \sigma) = u((1 - \tau)y - b) + v(x^p(\sigma + b, \theta)), \]
with \( u(\cdot) \) and \( v(\cdot) \) strictly concave. The optimal transfer \( b^p(y; \theta; \tau, \sigma) \) will satisfy
\[ -u' + v'_x x^p_a = 0; \quad (11) \]
(it is easy to check that the second order condition is satisfied). We are interested first in determining how the transfer varies with the parent’s earnings. We expect that richer parents make larger transfers (everything else, and in particular child’s ability, being equal), and in fact:
\[ b^p_y = \frac{-u''(1 - \tau)}{u'' + v''(x^p_a)^2 + v'x^p_{a\theta}} > 0. \quad (12) \]
Second, we want to investigate the role of the child’s ability; a standard result is that altruistic parents compensate for the children’s failures, hence transfers should be decreasing in the child’s ability. In our case, the outcome is complicated by the presence of the credit constraint:
\[ b^p_\theta = -\frac{v''x^p_{a\theta} + v'x^p_{a\theta}}{u'' + v''(x^p_a)^2 + v'x^p_{a\theta}}. \quad (13) \]
The second term at the numerator, \( v'x^p_{a\theta} \), equals zero for unconstrained children, see the discussion of (10) above; then, \( b^p_\theta < 0 \) by concavity of \( v(\cdot) \) and (9). Thus, the standard result is confirmed for unconstrained children. For constrained ones, the effect remains ambiguous as the term \( v'x^p_{a\theta} \) cannot be signed; on the one hand, the parent do try and compensate for reduced ability, but they have also to account for the fact that changes in \( \theta \) affect the child’s capability to obtain a loan. Hence, we have:
Lemma 2 The parental transfer is increasing in the parent’s earnings for all families; it is decreasing in the child’s ability for unconstrained families, but is ambiguously related to the child’s ability for constrained families.

As for the effects of policy on the transfer, we have

\[ b_p = -\frac{u''}{u'' + v''(x_a)^2 + v'x_{aa}}y < 0; \quad |b_p| \leq y; \]  

\[ b_a = -\frac{v''(x_a)^2 + v'x_{aa}}{u'' + v''(x_a)^2 + v'x_{aa}} < 0; \quad |b_a| < 1 \]  

Intuitively, an increase in the tax rate affects negatively the transfer due to an income effect, although an increase in the tax by 1% leads to a reduction in the transfer of less than 1%, as \( \frac{\partial (b_p)}{\partial \tau} = \frac{b_p}{y} \in (-0, 1) \). Also, increasing the education grant by one euro crowds out the domestic transfer, although by less than one euro.

Finally, note that parent’s problem can be rewritten using \( c \equiv (1 - \tau) y - b \) and \( a = b + \sigma \) to eliminate \( b \). We thus have:

\[ V^p ((1 - \tau) y + \sigma; y, \theta) = \max_{c,a} \{ u(c) + v(x^p(a, \theta)) \mid c + a = (1 - \tau) y + \sigma \} \]

This is how the assumption of effective altruism comes into play: conditional on the child participating in education, the parent evaluates alternative policies only by how they affect net family resources, \( m^p = (1 - \tau) y + \sigma \).

In a non-participating family, the final resources for the child will be \( x^{np}(b) = w_0 + b \) since she is not eligible for the subsidy \( \sigma \). We thus have \( U(b; y, \tau) = u((1 - \tau) y - b) + v(w_0 + b) \), with first order condition (both necessary and sufficient),

\[ -u' + v' = 0 \]

We can now compute:

\[ b_{yp} = \frac{u''(1 - \tau)}{u'' + v''} > 0; \quad b_{yp} = -\frac{u''y}{u'' + v''} < 0, \]

i.e. the transfer is increasing in parental income and decreasing in the income tax rate for non-participating families. Note that the child’s earnings do not depend on her ability; hence, the parental transfer is also independent from the child’s ability. Finally, writing

\[ V^{np} ((1 - \tau) y) = \max_{c,b} \{ u(c) + v(w_0 + b) \mid c + b = (1 - \tau) y \} \]

shows that the indirect utility can be written simply as a function of the parent’s net-of-tax income.
The Participation Choice

The participation choice formally rests with the child and the child can take this decision after the parent has made the financial transfer. Hence, we assume that the parent cannot condition the transfer on the child’s education choice. The reason for this is that, if the child needs the transfer to finance her investment, it must occur upfront, i.e. before the child implements her education choice. It is however easy to see that there is no conflict between the parent and the child, in general and specifically where the participation choice is concerned. Whatever maximises the child’s resources also maximises the parent’s utility, as the latter cares for the child’s objective. This has the useful implication that we can study the participation decision from the point of view of the parent, because in equilibrium, the child will always make the educational choice most preferred by the parent. This approach implies that we can analyse a family’s participation choice using the parent’s indirect utility rather than focusing on the child’s final resources.

We establish first a preliminary result.

Lemma 3 Consider a family indifferent between participating and not participating in education: if the child is not credit constrained, both $x$ and $c$ are the same no matter whether the child participates or not; if she is constrained, $c$ is larger and $x$ is smaller when non-participating than when not participating.

Proof. See the Appendix. ■

We can now show that the participation choice is strictly monotonic in the child’s ability level within each parental income class:

Proposition 4 Within any group of parents with the same income, there exists a cut-off child’s ability level $\hat{\theta}$ such that all parents whose children are of ability $\theta > \hat{\theta}$ will prefer them to enter education, while all parents whose children are of ability $\theta < \hat{\theta}$ will prefer them not to enter education.

Proof. See the Appendix. ■

Strict monotonicity in the participation choice follows the fact that high-ability children obtain a larger income from education than low-ability children at the same level of investment.

The cut-off income level $\hat{\theta}$ is in general also a function of policy, $\hat{\theta}(y; \tau, \sigma)$. How does it vary with policy? We can implicitly differentiate $V^p \left( (1 - \tau) \tilde{y} + \sigma; y, \hat{\theta} \right) - V^{np} ((1 - \tau) y) = 0$; then
since for the critical family, $V_p = -u'(\bar{c}_p) y$ and $V_{np} = -u'(\bar{c}_{np}) y$, we have that

$$\hat{\theta}_\tau = -\frac{\tilde{y} [u' (\bar{c}_{np}) - u' (\bar{c}_p)]}{V_p - V_{np}} \geq 0;$$

and

$$\hat{\theta}_{\sigma} = -\frac{\tilde{y} x_p}{V_p - V_{np}} < 0.$$  

where the signs follow from Lemma 3 and from (9). An increase in $\tau$, everything else being equal, will reduce participation within the $y$-class if the family at $\tilde{y}$ is credit constrained, but will leave it unaffected if the family is unconstrained; indeed, in the latter case the utility of participating and that of non-participating vary in the same way, so the indifference remains, while in the former case it becomes less attractive to participate. An increase in $\sigma$, everything else being equal, will trivially boost participation within the $y$-class as only participating families enjoy the education grant.

Next, we show that the participation choice is weakly monotonic in parental income keeping $\theta$ constant.

**Proposition 5** Within any group of parents of children of the same ability level, there exists a cut-off income level $\tilde{y}(\theta)$ at which parents are indifferent between participation or not, and such that i) if the parents with income $\tilde{y}$ are constrained, then a parent with income $y > \tilde{y}$ wants her child to enter education, while a parent with income $y < \tilde{y}$ does not want her child to enter education; ii) if the parents with income $\tilde{y}$ are unconstrained, then a parent with income $y > \tilde{y}$ either is indifferent also or wants her child to enter education, while a parent with income $y < \tilde{y}$ either is indifferent also or does not want her child to enter education.

**Proof.** See the Appendix.

Weak monotonicity in the participation choice follows because when there are credit constraints, high income families are less financially constrained than poor families, making education more attractive the larger are the parental earnings.

**III On the Political Equilibrium**

A parent’s indirect utility, taking into account the endogenous participation choice, is

$$V(\sigma, \tau; y, \theta) = \max \{V_p ((1 - \tau) y + \sigma, y, \theta), V_{np} ((1 - \tau) y)\}$$  

For participating families, the marginal rate of substitution between policy tools is

$$\frac{u_\tau}{u_\sigma} = y;$$

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for a non-participating family, there is no way in which an increase in \( \sigma \) can compensate an increase in \( \tau \), since they are not entitled to the grant. Note that for each family we can identify, in the \((\tau, \sigma)\)-space, a locus of indifference between participating or not; it is the locus of all \((\tau, \sigma)\) pairs such that \( V^p ((1 - \tau) y + \sigma, y, \theta) = V^{np} ((1 - \tau) y, \theta) \), as we know that the child’s choice agrees with that of the parent. Note that an increase in \( \sigma \), everything else being the same, will trivially break the indifference in favour of participation; hence, all policies “above” the indifference locus will determine participation, and all policies “below” the locus will determine non-participation.

### Preferences over policy

The above formulation makes it easy to see that a typical indifference curve is vertical below the families participation locus and has slope \( y \) above the locus (importantly, children’s ability does not affect policy preferences; the marginal rate of substitution only depends on \( y \)). The revenue constraint can be written, in per-capita terms, as

\[
\tau y^a - \sigma Q(\tau, \sigma) = 0,
\]

where \( y^a \) is average income and \( Q(\cdot) \) is the total share of the population participating in post-compulsory education. This can be seen as implicitly defining e.g. \( \sigma \) as a function of \( \tau \). For the ideal policy problem (conditional on participation) to be well-behaved, we need the revenue curve in the \((\tau, \sigma)\)-space to be strictly concave. We can prove that, as expected:

**Proposition 6** The revenue curve in the \((\tau, \sigma)\)-space is increasing,

\[
\sigma'(\tau) > 0.
\]

**Proof.** See the Appendix.

As for strict concavity, we can provide the intuitive argument that as the tax rate grows, so does the budget-balancing grant, but at a decreasing pace, since more and more people are entitled to it – aggregate participation increases with \( \sigma \), as can be inferred by (21). We will need to determine whether this is actually the case numerically.

\[\text{It is also possible to show that the locus is flat as long as the family is unconstrained, and positively sloped when it becomes constrained.}\]
Using the revenue constraint to eliminate one policy tool, we may rewrite the indirect utility functions of participating and non-participating families as follows:

\[ V_p(\tau; y) = u((1 - \tau) y - b) + v(x^p(\sigma(\tau) + b, y)) ; \]
\[ V_{np}(y; \tau) = u((1 - \tau) y - b) + v(x_0 + b) , \]

where in both cases it is understood that \( b \) is chosen optimally. The derivatives w.r.t. \( \tau \) are:

\[ V_p^\tau = -yu' + v'x^p_\sigma \sigma' ; \quad V_{np}^\tau = -yu' . \]

For families outside education, it is immediate to see that the preferred policy is no policy at all: they only lose from an income tax liability that gives them noting in return. For these families, the highest attainable indifference curve is the one through the origin – the revenue curve is all below such indifference curve. For participating families, the ideal tax rate satisfies, using that \( u' = v'x^p_0 \) by (11):

\[ y = \sigma' . \]

The indifference curve is tangent to the revenue curve somewhere along the former’s positively sloped tract. Note that it does not make a difference for the policy preferences whether a family is credit-constrained or not as long as it is participating in education. The ideal tax rate has then the standard interpretation of being the tax rate that equates the MRS with the slope of the revenue curve.

**The Ideal Policy as a Function of Income**

If concavity of \( \sigma(\cdot) \) holds the following “algorithm” can be used to obtain the ideal policy for each family \( y \) given that the budget set is concave (over the relevant region): for any \( y \) (i) find the policy that maximizes the participation utility \( V_p((1 - \tau) y + \sigma, y, \theta) \) over the budget set \( \sigma(\tau) \) using the first order condition \( \sigma'(\tau) \leq y \) with complementary slackness (i.e. \( [\sigma'(\tau) - y] y = 0 \)); then (ii) compare the “participation utility” at this policy to the “non-participation” utility at \( \text{laiss\'ez-faire} \) \( V_{np}(y) \).

Applying part (i) of this algorithm will generally identify a group of families for whom \( y \geq \sigma'(0) \). These are families that are rich enough that they always participate no matter what is their child’s ability, but will never support any active policy. This is because the subsidy they receive is simply not enough to compensate the taxes they pay (the subsidy is the same for all but the tax liability is increasing in income). Hence, any family \( y \geq y^\circ \), where \( y^\circ \equiv \sigma'(0) \), will
have the ideal policy \( \tau = \sigma = 0 \). Note that all these families have incomes above the average. evaluating (25) at the origin yields
\[
y^o = \frac{y^a}{Q(0,0)},
\]
where \( Q(0,0) \) is the participation in laissez-faire. Since \( Q(0,0) \) is in the interval \( (0,1) \), then \( y^o > y^a \).

For families with income below \( y^o \), refer to the above algorithm, and think about part (i) as generating a “participating” ideal tax \( \tau^p(y) \) (which is a well-behaved decreasing function in \( y \)) and part (ii) as generating a “non-participating” ideal tax \( \tau^{np}(y) \) (which is trivially zero). Then the family’s ideal tax is
\[
\tau(y) = \begin{cases} 
\tau^p(y) > 0 & \text{if } V^p(\tau^p(y), y, \theta) \geq V^{np}(\tau^{np}(y), y, \theta) \\
\tau^{np}(y) = 0 & \text{if } V^p(\tau^p(y), y, \theta) < V^{np}(\tau^{np}(y), y, \theta)
\end{cases}
\]

In a well-behaved scenario, we would expect that families up to a threshold income level have an ideal zero tax rate, while the families above the threshold have a positive and decreasing ideal tax rate. We cannot prove that this is the case in general, since given a family whose ideal policy is active, it will be possible to find a family with higher parental income whose ideal policy is laissez-faire.\(^5\) The most general statement we can make is therefore – with the proof coming trivially from concavity of \( \sigma(\tau) \):

**Proposition 7** If two families with income \( y^2 \) and \( y^1 \), \( y^1 < y^2 \), have active ideal policies, then the family with parental income \( y^1 \) prefers a higher tax rate than the family with parental income \( y^2 \).

**Necessary Conditions for Local Equilibria**

We now discuss political equilibria under majority voting. For the purpose of this section, we postulate that all parents attend elections; in the numerical example below, we relax this assumption.

Voting is over \( \tau \), with \( \sigma \) implicitly defined through the budget constraint; we refer to \( \tau^* \) as the equilibrium tax rate, and to \( \sigma^* = \sigma(\tau^*) \) as the equilibrium, budget-balancing, subsidy. Given \(^5\)This depends on how the shape of the indifference curves changes with income: as income increases the positively sloped part becomes steeper, but the vertical part becomes shorter. Hence, given an interior equilibrium at some income \( y \), it may well be that for income \( y' > y \) the budget set lies entirely beneath the indifference curve passing through the origin. This means that the family with income \( y \) has an ideal tax \( \tau > 0 \), while the family with income \( y' > y \) has an ideal tax \( \tau = 0 \).
the nature of the policy preferences, and the way they vary with income, it is difficult to make
general statements. We cannot invoke neither single-peakedness nor single-crossing, and hence
the median voter theorem does not apply. Another difficulty is that "large" policy changes (i.e.
non-marginal changes) induce a correspondingly "large" change in participation. This implies
possibly complex voting behaviour: for example, a typical reaction for a parent would be that
if a given tax-subsidy pair that induces non-participation is taken as a starting point, she will
certainly oppose a marginal expansion but may well favour a non-marginal one that makes her
jump out of the non-participation area.

With this in mind, we proceed in our discussion focusing on "small" policy changes and
local equilibria. We distinguish between a trivial equilibrium, in which the implemented policy
is in fact the laissez-faire, \( \tau^* = \sigma^* = 0 \), and a non-trivial one in which \( \tau^* > 0 \) and \( \sigma^* > 0 \).
There are two possibilities:

1. If more than half the population have a "non-participating" ideal tax \( \tau^{np} = 0 \), then \( \tau = 0 \)
can be an equilibrium. The no-policy coalition is made of non-participating families with
low incomes \( y < \tilde{g}(0, \theta) \) for all \( \theta \), families with high incomes \( y > y^0 \), plus families with
incomes between \( \tilde{g}(0, \theta) \) and \( y^0 \) (for all \( \theta \)) which prefer laissez-faire over any other policy.
This coalition would block a marginal tax increase. The condition is only necessary as it
implies that \( \tau = 0 \) beats a "small" tax increases, but not necessarily a "large" tax increase
– in this sense \( \tau = 0 \) is a local equilibrium.

2. If more than half the population have a "participating" ideal tax \( \tau^p > 0 \), this is sufficient
for \( \tau = 0 \) not to be a majority voting equilibrium. The pro-policy coalition is made of the
families that, for all \( \theta \), have incomes between \( \tilde{g}(0, \theta) \) and \( y^p \), who would normally favor a
marginal tax increase unless they have a zero ideal tax rate – see the discussion of (31)
above. Then, if there is an equilibrium it must be at some \( \tau^* > 0 \). For \( \tau^* \) to be a local
equilibrium, it must be that at least half the population is made of families which, for all
\( \theta \), have incomes between \( \tilde{g}(0, \theta) \) and \( y^p \) where \( y^p = \sigma^p(\tau^*) \) – strictly more than half if not
all these families have a positive ideal tax rate.

In order to interpret the above conditions for local equilibria, let us imagine that voting is
over marginal reforms, and that a reform only wins if it collects more than half the votes. There
is a status quo, an arbitrary active policy \( \tau^k > 0 \) (and \( \sigma^k = \sigma^k(\tau^k) > 0 \)); let \( y^k = \sigma^k(\tau^k) \) –
clearly, \( \tilde{g}(\tau^k, \theta) < y^k < y^0 \) for all \( \theta \). Consider the group made of the following families: for all
\( \theta \), those with income below \( \tilde{y}(\tau^k, \theta) \), plus those with income above \( y^k \), plus those with income in the \( (\tilde{y}(\cdot), y^k) \) interval whose ideal tax rate is zero. Suppose this group makes exactly half the population: then, if a marginal tax decrease is proposed, it will get exactly 50% of the votes, and thus will not go through. The other half of the population includes, for all \( \theta \), the families in the \( (\tilde{y}(\cdot), y^k) \) interval whose ideal tax rate is positive; thus, if a marginal tax increase is proposed, it will get also exactly 50% of the votes, and will not go through either. If \( y^k \) is such that the population is partitioned in the way described above, then \( (\tau^k, \sigma^k) \) is a local equilibrium in the sense that it will beat all marginal reforms.

IV The Computational Experiment

In this section we present a numerical version of the model. The purpose of the numerical analysis is (i) to verify that a political equilibrium exists in a reasonable specification, and (ii) to explore how the outcome varies with the key parameters in the model. Given this general aim, we have chosen to calibrate the model to resemble an “average Western economy” rather than any particular country, and then rely on the comparative statics exercises to cover a variety of parameter values and outcomes. Since the model is strongly focused on distributional issues such as inequality among parents, the link between parental income and participation in education, and intergenerational transmission of economic success, we will try to construct an example that looks reasonable along these key dimensions.

Functional Forms

Three functional forms need to be specified. First, the parents’ preferences: we adopt a standard iso-elastic specification,

\[
U = \frac{c^{1-\rho} + \mu x^{1-\rho}}{1 - \rho}.
\]

(32)

The risk-aversion parameter \( \rho \) is set at a level that is common in the literature \( \rho = 1.5 \) and is then varied in the comparative statics exercise. \( \mu \) is the “altruism” parameter. There are obviously few direct estimates of \( \mu \) in the literature; here we choose to set \( \mu \) in order to generate reasonable income transfers from parents to children. Anderberg (2005) reports that 55 percent of children that do not stay-on past the compulsory age of 16 in the UK (when asked at the age of 26) state having received financial support from their parents. In order to match this value we find that \( \mu = 0.9 \) works well.
Second, we need to specify how ability is transmitted across generations. We use a standard log-linear mean-reverting stochastic process,

\[ \ln(\theta) = 1 + k \ln(y) - E[\ln(y)] + \varepsilon, \]  

(33)

where \( \varepsilon \) is i.i.d. \( N(0, \sigma^2) \) and independent of \( y \). In the above formulation we have normalized the scale of ability so that log ability has unit mean in the population. The two parameters \( k \) and \( \sigma^2 \) need to be assigned values. Together they imply a correlation between parental income and children’s abilities. The literature suggests that the correlation between parents’ and children’s abilities may be as high as 0.5. [refs]. Here we use data... [Use PISA data, compare with BCS data] We set the parameters so as to generate a correlation of 0.25. This leaves one parameter to be determined; we return to this below.

Finally, we need to specify the education technology. We adopt a standard Cobb-Douglas specification: the future earnings of a child with ability \( \theta \) who invests \( z \) is

\[ w^p(z, \theta) = \eta^p z^\alpha \theta^{\beta^p}. \]  

(34)

Estimates of the elasticity of earnings with respect to educational spending are available in the literature. Our reading of the literature is that estimates tend to vary between 0.1 to 0.18, with several estimates at the lower end of this range [refs] In our baseline specification we set \( \alpha = 0.12 \) and then vary it in the comparative statics.

While we assumed in the theoretical analysis that all children who choose not to participate would earn the same income, here we let ability enter in the unskilled children’s future earnings (see fn. 1). This is important in order to generate a reasonable picture of the lower tail in income and hence the degree of inequality among the children. Moreover, it would also seem justified on empirical grounds. We adopt a similar iso-elastic specification for the earnings of the unskilled children,

\[ w^{np}(\theta) = \eta^{np} \theta^{\beta^{np}}. \]  

(35)

It is easy to see that allowing ability to affect the children’s unskilled earnings does not affect the necessary conditions for a political equilibrium. However, in order to obtain the result that the most able children choose to participate in education it is clearly crucial that ability has a larger proportional impact on skilled earnings than on unskilled earnings, \( \beta^p > \beta^{np} \).

To summarize, we need to assign values to the parameters of the earnings functions \( \eta^p, \eta^{np}, \beta^p, \beta^{np} \), the parameter \( k \) in the ability regression, and the fraction \( \gamma \) of future earnings that can
be seized by lenders in case a child defaults on a loan. The parameters of the earnings functions will be chosen to match the aggregate participation rate in education and the aggregate spending on post-compulsory education, and to ensure that distribution of earnings among the children have the same mean and median as the distribution of earnings among the parents (see below). The parameter $k$ is key to generating a positive link between participation and parental income. Hence this parameter will used to generate a reasonable pattern of participation across income quartiles. Finally, the parameter $\gamma$ controls the tightness of credit constraints. Below we will summarize findings in the literature that suggests that no more than 7-8 percent of children are short-run credit constrained. Hence $\gamma$ will be used to match the estimates of the fraction of children that are credit constrained.

In addition, we also need to specify the distribution of parental income and the participation in voting. Details on these, and a more detailed description of the information used to assign parameter values can be found in the Appendix.

**The Baseline Model**

The parameters used in the baseline model are summarized in table 1. First, we want to verify that the model is not only capable of replicating the above stylized facts, but also that the outcome corresponds to a political equilibrium. The approach is straightforward and involves the following steps: i) we compute the government budget constraint $\sigma = \sigma(\tau)$ and its derivative $\sigma'(\tau)$; ii) we look for a policy $\tau^*$ and $\sigma^* = \sigma(\tau^*)$ that satisfies the necessary condition for an (interior) equilibrium policy: the fraction of families that (i) participate and (ii) have income below $\sigma(\tau^*)$ should be exactly 0.5; iii) we put the candidate equilibrium policy to the global test by computing its political support against all other budget balance policies, $\tau$ and $\sigma = \sigma(\tau)$.

Figure 1 illustrate the government budget constraint in the baseline model. The concave shape of the budget constraint stems from the fact that a more generous policy encourages participation by more families. In the baseline specification there is only one policy that satisfies the necessary condition; thus we have a unique equilibrium candidate policy $\tau^* = 0.0155$. Even though there is only one candidate equilibrium, there is no guarantee that this policy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha$</th>
<th>$\beta^p$</th>
<th>$\beta^{pp}$</th>
<th>$\eta^p$</th>
<th>$\eta^{pp}$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
<th>$\mu$</th>
<th>$\theta_0$</th>
<th>$\theta_1$</th>
<th>$k$</th>
<th>$\sigma^2_x$</th>
<th>$\sigma^2_y$</th>
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</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.12</td>
<td>0.5</td>
<td>0.2</td>
<td>42</td>
<td>52</td>
<td>0.075</td>
<td>1.5</td>
<td>0.9</td>
<td>0.35</td>
<td>0.9</td>
<td>0.5</td>
<td>1.0</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 1: Baseline calibration
will be a global political equilibrium. We still need to verify that it will be supported by a majority of voters against every other alternative. Hence we perform a number of pairwise comparisons where we compute the political support for the candidate equilibrium against a grid of alternative tax rates (and associated subsidies). When we plot the support for the candidate equilibrium policy, if it is indeed a global equilibrium, we should find that the support is always at least 0.5. Indeed, this turns out to be the case here (see Fig 2). In order to interpret the

Figure 2: The political support for the equilibrium candidate tax $\tau_e$. 
figure, note that the identity of the group of voters who support $\tau^*$ against the challenger $\tau$ depends on whether the challenging tax is above or below $\tau^*$. When the challenging tax $\tau$ is below $\tau^*$, $\tau^*$ represents a more generous policy option; hence $\tau^*$ will be supported by families who are participating but do not have “too high” income. Conversely when the challenging tax $\tau$ is above $\tau^*$, $\tau^*$ represents the less generous policy option; hence $\tau^*$ will be supported by families who are either not participating or have particularly high income.

Support for a reduction in the generosity of the policy tends to come from the two ends of the income distribution – from the relatively poor since they are less likely to be participating and from the rich since they pay a particularly high tax price. Conversely, support for an increase in the generosity of the policy tends to come from the “middle-class”.

Having established that there is indeed a political equilibrium in the benchmark model, we will next describe the properties of that equilibrium. These are summarized in Table 2. Aggregate participation in the benchmark economy is, in equilibrium, 71 percent. The equilibrium tax rate in the benchmark model is 1.55 percent. Since the average parental income is 100 and the aggregate participation rate is 0.71, the equilibrium subsidy is $\sigma = (1.55/0.71) = 2.15$. Average total spending on education by participants is 10.25. Since the public spending per participant is 1.55, average private spending is 8.7 (the largest part of which corresponds to foregone earnings).

However, there is a strong link between participation and parental income: the participation rate in the top quartile is nearly twice that of the bottom quartile. As noted above, the benchmark model was calibrated to generate the same mean and median earnings among the children as among the parents, in this case 100 and 83 respectively.

Despite the fact that the parents in the model care nearly as much for their children as for themselves, only little over half of the parents make positive transfers to the children. This is true also for the participating families since the children have the option of borrowing to finance their investments. The distribution of transfers is also highly skewed.

A value that was not matched is the degree of intergenerational social mobility in earnings. In the literature the persistence of economic success across generations is commonly measured using regressions of log earnings of the child on the log earnings of the parent. In the benchmark

\footnote{The number of studies of intergenerational social mobility has grown quite rapidly recently and there is by now sufficient evidence to make some cautious international comparisons. Indeed, there seems to be an emerging consensus that (i) the degree of intergenerational persistence is somewhere around 0.4 for the US and possibly the UK, (ii) it may be somewhat lower continental Europe but the evidence is not clearcut, and (iii) the persistence}
model we find that the regression coefficient of children’s log earnings on parents’ log earnings comes to 0.28. This is of course very similar to the correlation between children’s abilities and parent’s earnings.

Finally, consider the tightness of credit constraints. In order to assess the fraction of children whose participation choices are credit constrained we perform the following exercise: we hold the policy \( \tau^*, \sigma^* \) at its equilibrium level and the recompute the behaviour under the assumption that there is full commitment to borrowing. The increase in participation rate measures the fraction of families that in the original equilibrium who are credit constrained on the participation margin.\(^7\) As noted above, the number of credit constrained families should not exceed a couple of percentage points.

**Comparative Statics**

We now turn to the second main aim of the numerical analysis, namely the comparative statics exercises. For each exercise we report the same summary statistics as for the benchmark case. See Table 3.

- **Increasing Returns to Education.** One of the most important economic trends in the recent decades has been the increasing returns to education caused by skill-biased technological change. The increasing returns to education, as reflected e.g. in increased college wage premia, has also increased participation. However, it has been noted that the increased participation has been socially biased in the sense that it has occurred disproportionately in relatively well-off families. Here we explore the impact of an increased return to education using two comparative statics exercises: in the first exercise (labelled “CS1” in Table 3) we increase the elasticity \( \alpha \) of skilled earnings with respect to spending from 0.12 to 0.13. The response is easily predicted: the increase in the return to education boosts participation and hence also the support for policy (which, in turn, further increases participation). The overall result is an increase in the equilibrium tax rate to 1.9 percent and a three percentage point increase in aggregate participation. The model however predicts that this increase comes predominantly from the bottom two income quartiles. Children’s earnings naturally increase. This also causes the parents to trans-

\(^7\)In addition, there will of course be participating families who are credit constrained on the “intensive” margin in the sense that their levels of investment are affected.
<table>
<thead>
<tr>
<th>Benchmark Equilibrium</th>
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<tbody>
<tr>
<td><strong>Investment levels</strong></td>
</tr>
<tr>
<td>Average total</td>
</tr>
<tr>
<td>Average private</td>
</tr>
<tr>
<td><strong>Policy</strong></td>
</tr>
<tr>
<td>Tax rate ((\tau\ %))</td>
</tr>
<tr>
<td>Public subsidy</td>
</tr>
<tr>
<td><strong>Participation rates</strong></td>
</tr>
<tr>
<td>1st income quartile</td>
</tr>
<tr>
<td>2nd income quartile</td>
</tr>
<tr>
<td>3rd income quartile</td>
</tr>
<tr>
<td>4th income quartile</td>
</tr>
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<td><strong>Children’s earnings</strong></td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Median</td>
</tr>
<tr>
<td><strong>Transfers</strong></td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Median</td>
</tr>
<tr>
<td>Positive transfers: participants</td>
</tr>
<tr>
<td>Positive transfers: non-participants</td>
</tr>
<tr>
<td><strong>Intergenerational links</strong></td>
</tr>
<tr>
<td>Correlation: parental earnings/ability</td>
</tr>
<tr>
<td>ISM coefficient</td>
</tr>
<tr>
<td><strong>“Credit tightness”</strong></td>
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<tr>
<td>Fraction families constrained (%)</td>
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Table 2: Participation rates and investments in the benchmark equilibrium
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<thead>
<tr>
<th></th>
<th>CS1</th>
<th>CS2</th>
<th>CS3</th>
<th>CS4</th>
<th>CS5</th>
<th>CS6</th>
<th>CS7</th>
<th>CS8</th>
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<td></td>
<td></td>
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<tr>
<td>Av. private</td>
<td>8.10</td>
<td>8.40</td>
<td>6.90</td>
<td>9.93</td>
<td>7.72</td>
<td>6.70</td>
<td>9.60</td>
<td>7.42</td>
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<tr>
<td><strong>Policy</strong></td>
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<td></td>
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<td></td>
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<tr>
<td>Tax rate (%)</td>
<td>1.55</td>
<td>1.90</td>
<td>2.51</td>
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<td>2.06</td>
<td>0.59</td>
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<td>Aggregate</td>
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<td>0.74</td>
<td>0.76</td>
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<td>0.38</td>
<td>0.47</td>
<td>0.51</td>
<td>0.32</td>
<td>0.53</td>
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<td>2nd inc. quart.</td>
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<td>0.73</td>
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<td>0.71</td>
<td>0.78</td>
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<tr>
<td>3rd inc. quart.</td>
<td>0.77</td>
<td>0.80</td>
<td>0.81</td>
<td>0.75</td>
<td>0.78</td>
<td>0.85</td>
<td>0.75</td>
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<tr>
<td>4th inc. quart.</td>
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<td>0.90</td>
<td>0.91</td>
<td>0.88</td>
<td>0.90</td>
<td>0.94</td>
<td>0.88</td>
<td>0.90</td>
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<tr>
<td><strong>Children’s earnings</strong></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Mean</td>
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<td>102</td>
<td>99</td>
<td>100</td>
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<td>98</td>
<td>100</td>
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<tr>
<td>Median</td>
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<td>86</td>
<td>83</td>
<td>84</td>
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<tr>
<td><strong>Transfers</strong></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Mean</td>
<td>9.68</td>
<td>9.23</td>
<td>8.57</td>
<td>11.19</td>
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<td>9.00</td>
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<td>Median</td>
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<td>0.06</td>
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<td>0.08</td>
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<td>Positive transf.</td>
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<td>0.58</td>
<td>0.58</td>
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<td>0.55</td>
<td>0.65</td>
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<tr>
<td>Positive transf.</td>
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<td>0.47</td>
<td>0.43</td>
<td>0.56</td>
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<td><strong>Intergen. linkages</strong></td>
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<td>Correlation</td>
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<td>0.25</td>
<td>0.25</td>
<td>0.26</td>
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<td>0.34</td>
<td>0.25</td>
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<tr>
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<td>0.28</td>
<td>0.27</td>
<td>0.29</td>
<td>0.32</td>
<td>0.28</td>
<td>0.31</td>
<td>0.26</td>
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<tr>
<td><strong>Credit Const.</strong></td>
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<tr>
<td>Fract. const.</td>
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<td>??</td>
<td>??</td>
<td>??</td>
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<td>??</td>
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</tr>
</tbody>
</table>

Table 3: Comparative statics results (To be completed)
fers less income to their children. Despite the relatively higher participation among lower income families, the intergenerational persistence of income, if anything, increases. In the second exercise we (labelled “CS2”) we increase the total productivity of skilled workers by increasing $\eta^p$ from 42 to 43. The reponse is similar in nature: participation is encouraged and the support for policy grows. Participation again increases particularly in the lower income quartiles. Public spending increases more than total spending; hence public spending relative to private spending increases. Parental transfers are reduced as kids earnings are increased. The degree of intergenerational persistence in earnings is essentially unaffected in this case.

- **Increasing Inequality among Parents.** The next factor we consider is the degree of inequality among the parents. We would naturally predict that higher inequality will reduce the support for policy since it will tend to increase the number of families with incomes high enough to oppose public spending. This will definitely be true if the participation rate was unaffected by inequality. However, we must also take into account that children’s abilities are linked to the parents’ earnings: an increase in parental income inequality also implies an increased inequality in children’s abilities, which may affect participation. Hence consider the impact on the equilibrium of increasing the variance of log-parental income from $\sigma_y^2 = 0.6$ to $\sigma_y^2 = 0.65$ (labelled as “CS3”). As predicted, this severely reduces the support for policy. Indeed, no positive policy will garner support when $\sigma_y^2$ exceeds [to be completed]. The aggregate participation rate drops significantly and the differences in participation over the income quartiles becomes much more pronounced. Due to this composition effect, average total investment per participating student increases somewhat and, as predicted, public spending is now a much smaller share of total spending. The average earnings among the children decreases somewhat. Moreover, part of the increased inequality among the parents is carried forward also to the children; e.g. that the ratio of mean to median earnings among the children is higher in CS3 than in the benchmark economy. Parental transfers increase somewhat but more importantly, there is an increase in the persistence of earnings across generations.

- **Ability Transmission.** How does the equilibrium depend on the strength of the link between the abilities of the children and the economic success of their parents? Since much of the differences in participation across income groups stem from differences in abilities we would expect those differences to be accentuated. We would also expect the
degree of intergenerational income mobility to decrease. Consider first what happens as the rate of regression to the mean of log ability decreases; to explore this we increase the parameter $k$ from 0.5 to 0.6. The above predicted results come through (see the column labelled “CS4” in Table 3). However, it is less clearcut what will happen to the political support for policy. Perhaps somewhat surprisingly we find that it increases the equilibrium policy. One intuition behind this finding is that the stronger intergenerational link in ability reduces the responsiveness of the aggregate participation rate to policy. Hence, the politically influential participating “middle-class” families can increase the generosity of the policy without this attracting large scores of new participants from the lower income groups. Indeed, note that even with the more generous policy, the aggregate participation rate has only increased marginally and has not increased at all in the lowest income quartile, and total investment per participant is barely affected. Hence the main effect on investments has been to crowd out private spending. The link between parents’ earnings and children’s abilities will also be stronger when the noise term in $\varepsilon$ in (33) becomes smaller. Thus, consider the effect of reducing the standard deviation $\sigma_\varepsilon$. Note that since we have normalized the average log ability to unity, a reduction in $\sigma_\varepsilon$ also implies an increase in average ability. Hence, as we reduce $\sigma_\varepsilon$ from 1 to 0.75 we are not only reducing the heterogeneity among children, we are also increasing their average ability. The impact on participation is illuminating (see column “CS5” in Table 3): the aggregate participation rate increases significantly. Moreover, the largest increases occurs in the middle-income groups, not in the lowest income quartile. The support for policy increases substantially and the equilibrium tax rate increases markedly. The inequality among the children is also reduced substantially; the upper tail of the income distribution in particular is markedly cut back.

- **Preferences.** We consider changes in risk-aversion (CS6) [to be completed] and altruism (CS7). The latter change takes the form increasing $\mu$ from 0.9 to 1.0. We have that this increases participation, especially in the lowest income quartile, as well as the support for policy, raises total investment per participating child marginally (but note that there is a compositional effect here since the participation increases particularly among those making small investments); naturally it also increases the transfers made to children.

- **Credit Restrictions.** Increasing the maximum debt-to-earnings ratio from 7.5 percent to 12.5 percent reduces the support for policy, thus indicating that some part of the
support for policy was due to the fact that it helps to relax credit constraints. Negligible increase in participation (due to the reduction in the policy). Marked increase in average investment; this indicates that many participating families were credit constrained on the margin. No other significant impacts.

V Conclusions

[To be written]

Appendix

Proofs

Proof of Lemma 1. We first consider the behaviour of the return to educational spending as the child’s own ability changes. In equilibrium, we write \( w_z(z(a, \theta), \theta) \) to emphasize that \( \theta \) affects \( w_z \) through the optimal educational investment as well as directly. The total derivative of \( w_z \) w.r.t. \( \theta \) is

\[
\frac{dw_z}{d\theta} = w_{zz\theta} + w_{z\theta}.
\]  

(A1)

Using (5), we find that (A1) equals zero for unconstrained families. (This is natural since the unconstrained solution satisfies \( w_z = 1 \) as identity). For constrained families instead, it is positive only if there is enough complementarity between \( z \) and \( \theta \) to overcome the effect of the decreasing returns of \( z \) (\(|w_{zz\theta}| < w_{z\theta}\)), and negative (or zero) otherwise. We proceed now by solving the first order condition (4) for the multiplier \( \mu \), that can be viewed as the marginal value of credit. This yields

\[
\mu(a, \theta) = \frac{1 - w_z(z(a, \theta), \theta)}{\gamma w_z(z(a, \theta), \theta) - 1} \geq 0,
\]  

(A2)

where it is emphasized that the first order condition is evaluated at the optimal (unconstrained or constrained) value of \( z \). Trivially \( \mu = 0 \) if the choice is unconstrained, i.e. \( w_z(\cdot) = 1 \). On the other hand, \( \mu > 0 \) by (6) if the choice is constrained. Focusing on this case and differentiating \( \mu \) with respect to \( a \) and to \( y \) yields

\[
\mu_a = \frac{(1 + \gamma \mu) w_{zz\theta}}{(1 - \gamma w_z)} < 0;
\]  

(A3)

\[
\mu_\theta = \frac{(1 + \gamma \mu) (w_{zz\theta} + w_{z\theta})}{(1 - \gamma w_\theta)}.
\]  

(A4)
For constrained children, we have that \( \mu_a < 0 \) by (6), (A2), (7) and (1); intuitively, the value of credit will decrease as the child’s assets go up. As for \( \mu_q \), the sign depends on the sign of (A1) for constrained families – see above. For example, the value of credit for constrained children will increase in parental income (\( \mu_q > 0 \)) if complementarity in the earnings function is strong enough, as it induces the child to want to match her own higher ability by spending more on her own education.

We can now recover an expression for the marginal value of the child’s assets. Applying the envelope theorem yields that:

\[
x^p_a = 1 + \mu \geq 1 > 0; \quad x^p_\theta = w_\theta (1 + \gamma \mu) \geq w_\theta > 0.
\]

Furthermore, (9), (A3) and (A4) allow us to write

\[
x^p_a = \mu_a < 0; \quad x^p_\theta = \mu_\theta.
\]

Proof of Lemma 3. Let \( (\hat{c}^p, \hat{x}^p) \) and \( (\hat{c}^{np}, \hat{x}^{np}) \) denote an indifferent family’s allocation under participation and non-participation respectively. We then write \( \hat{V}^p = u(\hat{c}^p) + v(\hat{x}^p) \) and \( \hat{V}^{np} = u(\hat{c}^{np}) + v(\hat{x}^{np}) \) for the utility functions when participating and non-participating respectively; we know that \( \hat{V}^p = \hat{V}^{np} \). Consider the locus of \((c, x)\) generating the common value \( \hat{V}^p \); formally define

\[
\pi \equiv \{(c, x) | u(c) + v(x) = \hat{V}^p\}.
\]

By construction \((\hat{c}^p, \hat{x}^p)\) and \((\hat{c}^{np}, \hat{x}^{np})\) both belong to the locus \(\pi\) which is downward sloping. We can now show that \((\hat{c}^p, \hat{x}^p)\) lies to the “north-east” of \((\hat{c}^{np}, \hat{x}^{np})\) if the credit constraint binds on the indifferent family and they coincide if the credit constraint is slack. To see this note from (11) and (17) that

\[
\frac{u'(\hat{c}^p)}{v'(\hat{x}^p)} = x^p_a \geq 1 \quad \text{and} \quad \frac{u'(\hat{c}^{np})}{v'(\hat{x}^{np})} = 1.
\]

If the credit constraint binds, \( x^p_a > 1 \) by (9) and then it follows from (A8) and from \( \hat{V}^p = \hat{V}^{np} \) that \( \hat{c}^p < \hat{c}^{np} \) while \( \hat{x}^p > \hat{x}^{np} \). If the credit constraint is slack, \( x_a = 1 \) and \((\hat{c}^p, \hat{x}^p)\) and \((\hat{c}^{np}, \hat{x}^{np})\) coincide.

Proof of Proposition 4. A parent will prefer the child to attend education if

\[
V^p ((1 - \tau) y + \sigma; y, \theta) \geq V^{np} ((1 - \tau) y)
\]

Some tedious computations will show that Young’s theorem holds, i.e. \( x^p_{\sigma y} = x^{np}_{\sigma y} = \mu_y \).
If $V^p$ increases faster in $\theta$ than $V^{np}$ at $\hat{\theta}$ (i.e. when $V^p = V^{np}$) it follows that $V^p$ will only ever cut $V^{np}$ from below in the $(\theta, V)$ space and the result follows.

Applying the envelope theorem on (16) and on (19) yields (note that this refers to the total derivative of the indirect utility w.r.t. child’s ability)

$$V^p_\theta = v'(x^p) x^p_\theta > 0; V^{np}_\theta = 0,$$

where the sign of the first expression follows from (9). It is then easy to see that

$$V^p_\theta - V^{np}_\theta|_{\theta=\hat{\theta}} > 0$$

for a parent who is indifferent within each income class.

**Proof of Proposition 5.** A parent will prefer the child to attend education if

$$V^p ((1 - \tau) y + \sigma; y, \theta) \geq V^{np} ((1 - \tau) y)$$

For an indifferent family, this holds with equality. Given $\theta$, if $V^p$ increases in $y$ at least as fast as $V^{np}$ whenever $V^p = V^{np}$ it follows that $V^p$ will either cut $V^{np}$ from below or overlap for a tract in the $(y, V)$ space; if overlapping only occurs when the family is unconstrained, the result follows.

Applying the envelope theorem on (16) and on (19) yields (note that this refers to the total derivative of the indirect utility w.r.t. parental ability)

$$V^p_y = u'(c^p) (1 - \tau); V^{np}_y = u'(c^{np}) (1 - \tau).$$

To evaluate $V^p_y - V^{np}_y$ for a parent who is indifferent, we compute

$$V^p_y - V^{np}_y|_{y=y^*} = [u'(\hat{c}^p) - u'(\hat{c}^{np})] (1 - \tau) \geq 0$$

where the inequality follows from Lemma 3; note that (A14) holds with equality only when the indifferent family is constrained, and holds with strict inequality otherwise.

**Proof of Proposition 6.** Let

$$q(y; \tau, \sigma) = \int_{\hat{\theta}(\tau, \sigma)}^{\hat{\theta}} f(y, \theta) \, d\theta,$$

be the fraction of participating families within a given $y$-class. Note that, by the Leibniz rule,

$$q_\tau = -\hat{\theta}_\tau f(y, \hat{\theta}) \leq 0; q_\sigma = -\hat{\theta}_\sigma f(y, \hat{\theta}) > 0,$$
where the signs follow from (20) and (21) and the interpretation is obvious. The total share of participating families is simply

\[ Q(\tau, \sigma) = \int_{\mathcal{Y}} q(\tau; \sigma) \, dy, \]  

(A17)

with derivatives

\[ Q_\tau = \int_{\mathcal{Y}} q_\tau \, dy \leq 0; \quad Q_\sigma = \int_{\mathcal{Y}} q_\sigma \, dy > 0, \]  

(A18)

where the signs follow from (A16). Using the government budget constraint (24) and applying the implicit theorem function we get

\[ \sigma'(\tau) = \frac{y^\alpha - \sigma Q_\tau}{Q + \sigma Q_\sigma} > 0, \]  

(A19)

where we used (A18) to arrive at the sign.

**Stylized Facts used to Parameterize the Computational Model**

**Income Distribution.** The model does not endogenize the income distribution as we want to perform comparative statics with respect to the degree of inequality. Nevertheless, we calibrate the benchmark model such that the income distribution among the children and the parents share the same mean and median. We assume that parental income is log-normally distributed, \( \ln y \sim N(\mu_y, \sigma^2_y) \). It is convenient to normalize the scale of income so that average parental earnings is 100; several magnitudes in the model can then be directly interpreted as fractions. More crucial is the degree of inequality, measured in terms of the variance of log-parental income. As we will see, this parameter has significant impact on the political equilibrium. Moreover, its value is difficult to choose because parental income should be interpreted as lifetime income and therefore the degree of inequality at any given moment may significantly overestimate the degree of inequality in lifetime income if there is income mobility over the lifecycle. The available evidence suggests that mobility may be significant and, moreover, may differ across countries. [refs] Here we take a pragmatic approach: since one purpose of the current exercise is to explore the comparative statics of the equilibrium, we vary \( \sigma^2_y \) over a reasonable range, covering values between 0.4 to 0.8 (see below). [refs].

**Staying-on Rates.** The aggregate staying-on rates are in the range of 65 - 90 percent. There rates divided according to parental background are to be computed. For now use the following data from the UK: participation by income quartile equal to roughly .47, .66, .77 and .89 (to be completed)
**Children’ Abilities.** In (33) we have normalized the scale of ability so that log ability has unit mean in the population. Since we also take the distribution of parental income to be log-normally distributed, it follows that log-ability and log-parental income follow a bivariate normal distribution. The two parameters $k$ and $\sigma_\varepsilon^2$ need to be assigned values. Together they imply a correlation between parental income and children’s abilities. The literature suggests that the correlation between parents’ and children’s abilities may be as high as 0.5. [refs]. [Here we use PISA data, compare with BCS data]. We set the parameters so as to generate a correlation of 0.25-0.3. This leaves one parameter to be determined (see below).

**Credit Constraints.** The model allows for endogenous credit constraints to affect the children’s educational choices. There is a growing literature that tries to quantify the fraction of children whose participation choices are affected by the presence of short-run liquidity constraints. Here we base our calibration on findings from Heckman and Carneiro (2003), Heckman and Masterov (2005), Dearden et al. (2004) and Anderberg (2005). These studies estimate the fraction of students whose participation decision is affected by credit constraints by controlling for measured ability and long-run family environmental factors.\(^9\) The studies present evidence on three sets of decisions: (i) the decision to enroll in college in the US, (ii) the decision to stay-on past the compulsory age of 16 in the UK, and (iii) the decision to participate in tertiary education in the UK. The findings quite consistently suggests that at most 7-8 percent of children are short-run credit constrained in the sense that their participation decisions are negatively affected. Indeed, several estimates fall in the range of only two to three percentage points. We use these findings in the following way: the model is calibrated such that if, at the equilibrium, full commitment to borrowing suddenly became possible, the increase in participation should not significantly exceed three percent.

**Spending on Education** The largest component of the cost of post-compulsory education is foregone earnings. However, there are also direct costs. Both types of costs need to be added up in order to measure the average total spending on post-compulsory education by participants. Consider first foregone earnings. What would the participating children have earned during the time they spent in post-compulsory education had they not participated? A reasonable estimate for average foregone earnings among participants can be arrived at using the following back-of-the-envelope calculation. Given that the average earnings in the population is set to 100, the average earnings for unskilled workers should be little over half of this value, say 60.

\(^9\)The technique is described in detail in Heckman and Carneiro (2003).
Workers spend about 40 years in the labour force, implying annual earnings of 1.5 (we are ignoring discounting throughout). The children that do choose to participate in education have higher average ability and hence would have earned a higher average income as unskilled, say an 25% ability premium. Moreover, the average number of years spent in post-compulsory education among participants is somewhere in the range of four to five years. [data from e.g. ISSP] Hence the cost of the average foregone earnings among the participants would be from 7.5 to 9 (percent of average lifetime income). We target a value in this range. To the foregone earnings we need to add direct costs. These direct costs are in most countries for the main part covered by public spending [refs]. However, there is important cross-country variation, with the continental European and Nordic countries leaning more towards public spending and the Anglo-Saxon countries having a larger fraction of private spending to cover direct costs. We estimate direct costs to be in the range of 2-3 percent. [refs], hence we target a total average spending per participating student of 10 with public spending being 2.

**Participation in Voting.** The final piece of information that is included in the model is the pattern of participation in voting. We include this in the model since we want to capture the political bias that obtains since participation in voting is positively related to income. In order to make the analysis comparable across economies, we assume that the probability of voting is positively related to an individual’s income *rank* rather than to the individual’s income level. Hence let the income of individual *i* be *y* and let the CDF of parental income by *F*. Then we assume that the probability that individual *i* actively votes is \( \Phi (\theta_0 + \theta_1 F (y_i)) \), where \( \Phi \) is the standard normal CDF. The two parameters \( \theta_0 \) and \( \theta_1 \) are hence standard probit parameters that can be estimated from the data .. [To be completed using data from TPCE ISSP. For now, we use estimates from TPCE UK, \( \theta_0 = 0.35 \) and \( \theta_1 = 0.9 \), see Anderberg, 2005].

**References**

[To be done]