Status Effects, Public Goods Provision, and the Excess Burden

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Abstract

Most studies of the optimal provision of public goods or the excess burden from taxation assume that individual utility is independent of other individuals’ consumption. This paper investigates public good provision and excess burden in a model that allows for interdependence in consumption in the form of status (relative consumption) effects. In the presence of such effects, consumption and labor taxes no longer are pure distortionary taxes but have a corrective tax element that addresses an externality from consumption. As a result, the marginal excess burden of consumption taxes is lower than in the absence of status effects, and will be negative if the consumption tax rate is below the "Pigouvian" rate. Correspondingly, when consumption or labor tax rates are below the Pigouvian rate, the second-best level of public goods provision is above the first-best level, contrary to findings from models without status effects. For plausible functional forms and parameters relating to status effects, the marginal excess burden from existing U.S. labor taxes is substantially lower than in most prior studies, and is negative in some cases.

Keywords and Phrases: status effects, (marginal) excess burden, deadweight loss, public goods provision, corrective taxation, consumption externality, first best, second best.

JEL Classification Numbers: D62, H23, H41
1 Introduction

Most analyses of optimal provision of public goods or of the excess burden of taxation regard individual utility as depending directly on one’s own consumption and leisure. However, utility can depend directly on the consumption or income of others. Several studies have explored the significance of this interdependence in consumption. The earliest work tended to be theoretical. For example, almost 30 years ago Boskin and Sheshinski (1978) explored theoretically how the optimal redistributive taxation is affected when individual utility depends on one’s relative income or consumption. Recently, however, a number of studies have aimed to assess empirically the extent to which individual utility depends on others’ consumption or income. In particular, several studies have sought to determine the strength of a particular form of interdependence here termed the status effect – the utility-impact of one’s consumption relative to others’ consumption.1 Such studies can be divided into two categories: studies based on survey-experimental methods, and studies based on econometric analyses of panel data on individuals’ incomes and self reported happiness. Studies falling into the former category include Alpizar et al. (2005), Carlsson et al. (2003), Johansson-Stenman et al. (2002, 2006), Solnick and Hemenway (1998, 2005). Studies falling into the latter category include Ferrer-i-Carbonell (2005), Luttmer (2005), McBride (2001), and Neumark et al. (1998).

Prior theoretical and empirical studies have shed important light on the implications of status effects for happiness (Easterlin 1995, Frank 1985, Frank 1999, Scitovsky 1976), economic growth (Abel 2005, Brekke and Howarth 2002, Carroll et al. 1997, Liu and Turnovsky 2005), asset pricing (Abel 1990, 1999, Campbell and Cochrane 1999, Dupor and Liu 2003), optimal tax policy over the business cycle (Ljungqvist and Uhlig 2000), and for optimal redistributive taxation (Boskin and Sheshinski 1978). Yet status effects also have important implications for the excess burden of taxation and the optimal provision of public goods. Although other authors have raised this point2, we know of no prior study that rigorously analyzes how status effects

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1 The concern for one’s economic status relative to that of others is sometimes termed a “positional concern.” (See, for example, Alpizar et al. (2005), Brekke and Howarth (2002), Hirsch (1976), Frank (1985, 1999), Solnick and Hemenway (2005).) The status effects discussed in this paper derive from a particular positional concern: namely, the concern for one’s consumption relative to others’.

2 Important references include Howarth (1996), Ng (1987), Ng and Wang (1993).
influence the excess burden and the optimal first-best and second-best levels of public goods provision. This is the focus of the present paper. We develop a theoretical model to examine optimal public goods provision and excess burden in the presence of status effects. In addition, we incorporate recent estimates of status effects in the model to explore their quantitative implications for excess burden.

The paper offers four results. First, status effects lower the excess burden of a given consumption or labor tax. Such effects imply that an individual’s increase in consumption imposes a negative externality on other individuals by reducing others’ relative economic position. Under these conditions, a consumption or labor tax functions both as a device for raising revenue and as an instrument for correcting the negative consumption externality.

Second, the sign of the marginal excess burden from a consumption (labor) tax depends on the magnitude of the tax relative to the marginal consumption externality. If the consumption tax rate equals the “Pigouvian” rate (marginal external cost), the marginal excess burden from the tax is zero. The marginal excess burden is negative (positive) if the tax rate is below (above) the Pigouvian rate.

Third, if the second-best optimum involves consumption tax rates above (below) the corrective rate, the marginal excess burden of the tax is positive (negative) and the second-best optimal level of public goods provision is below (above) the first-best level.

Finally, empirical evidence suggests that status effects are large enough to imply a marginal excess burden of consumption (labor) taxes significantly lower than the value obtained in studies that assume no such effects. Indeed, the marginal excess burden is negative in some plausible cases.

Our paper is related as follows to the prior literature on excess burden and public goods provision. Applying a general framework similar to that in Gronberg and Liu (2001), it extends the discussion of the the first-best and second-best levels of optimal public goods provision (Atkinson and Stern (1974), Bartolomé (2001), Batina and Ihori (2005), Gaube (2000), Gronberg and Liu (2001), Stiglitz and Dasgupta (1971), Wildasin (1984), Wilson (1991a, 1991b)). These papers demonstrate that the second-best level of public good provision generally is below the first-best level.³

³ An exception is a result by Gronberg and Liu (2001), who show that under some circumstances the second-best level can exceed the first-best level. They provide an instructive example where indifference curves exhibit a kink at the equilibrium, and consumers must be taxed to be induced to consume at this kink. Gronberg and Liu, however, suggest
Our paper also contributes to the literature on optimal taxation in the presence of externalities. Status effects imply that an individual’s consumption generates negative externalities by lowering others’ relative consumption. We derive optimal consumption taxes in the presence of this externality, in first-best and second-best settings. In some respects our approach resembles the seminal work of Sandmo (1975), who derived the optimal first- and second-best taxes when production or consumption of one of the goods involves an externality. However, in contrast with Sandmo’s analysis our paper considers not only optimal tax rates but also the marginal excess burden of a consumption (or labor) tax, and treats the level of public goods provision as endogenous rather than fixed. We show that status effects unambiguously reduce the excess burden relative to what would be the case if such effects were absent. This confirms an idea suggested informally by Ng (2000, p.263).

Section 2 of the paper presents the model. Section 3 considers optimal tax rates in the presence of status effects, first-best, and second-best allocations. Section 4 discusses the impact of status effects on the excess burden. Section 5 establishes the relationship between the level of the consumption tax rate, the sign of the marginal excess burden, and the relation of the first-best to the second-best level of public goods provision. Section 6 incorporates empirical information from other studies on the strength of status effects to suggest the quantitative implications of such effects for public good provision and excess burden. Section 7 offers conclusions. The appendix provides proofs for all propositions.

2 The Economy

We consider an economy with \( N > 0 \) consumers (households), two private commodities, and a pure public good. The private commodities, a consumption good and leisure, are respectively denoted by \( c \) and \( l \). The public good is denoted by \( G \). A representative household\(^5\) has preferences over consumption (including relative consumption), leisure, and a pure public good. The

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\(^4\)Also, we focus on a broad-based consumption (or labor) tax, whereas Sandmo focused on the optimal system of differentiated commodity taxes.

\(^5\)The assumption of a representative household implies uniformity of after-tax incomes. Consumption externalities (status effects) arise nevertheless, as discussed below.
public good is strictly separable from private goods in the household’s utility function \( U \):

\[
U = u(c, l, \bar{c}; \gamma) + g(G; \Psi).
\]  

(1)

Subutility \( u(c, l, \bar{c}; \gamma) \) is a function of one’s own absolute consumption, \( c \), of leisure, \( l \), and of average consumption of the society, \( \bar{c} \). The latter element accounts for status effects, which here are represented as concerns about relative consumption \( c/\bar{c} \). The parameter \( \gamma \in [0,1] \) measures the strength of the impact of relative consumption on individual utility. In particular, \( \gamma \) is chosen such as to represent the marginal degree of positionality, i.e. that fraction of marginal utility of consumption stemming from increased relative consumption.\(^6\) If \( \gamma > 0 \) utility is a positive function of relative consumption, or a negative function of \( \bar{c} \). E.g., if \( \gamma = 0.2 \), 20% of marginal utility of consumption comes from increased relative consumption, whereas 80% stems from increased absolute consumption (holding fixed the level of relative consumption).

Various studies indicate that status effects are stronger for consumption goods than for leisure. In particular, Carlsson et al. (2003), Solnick and Hemenway (1998), and Solnick and Hemenway (2005), find that leisure time is the least “positional” (that is, has the lowest status effect) of all the goods investigated. As pointed out by Carlsson et al. (2003, p.15), “the marginal degree of positionality for leisure ... is not statistically larger than zero at the 10% levels.” In accordance with the empirical evidence, we adopt the simplified assumption that households care about status with regard to the consumption good, but not so with regard to leisure.

The following assumptions are imposed on the subutility function \( u \):

(A.1) \( u \) is twice continuously differentiable on \( \mathbb{R}^3_+ \);

(A.2) \( u_i(c, l, \bar{c}; \gamma) > 0, \ (i = 1, 2), \ u_3(c, l, \bar{c}; \gamma) < 0; \)\(^7\)

(A.3) \( u(c, l, \bar{c}; \gamma) \) and \( u(c, l, c; \gamma) \) are strictly concave in \( (c, l) \);

(A.4) \( u(c, l, c; \gamma) \equiv u_1(c, l, \bar{c}; \gamma) + u_3(c, l, c; \gamma) > 0; \)

(A.5) \( u(c, l, c; \gamma) = u(c, l, 0) \) is homothetic.

Utility increases in consumption and leisure. If \( \gamma > 0 \), a rise in average consumption (for a given level of individual consumption) lowers utility of

\(^6\)Define \( r \equiv c/\bar{c}, \) and \( w(c, l, r; \gamma) \equiv u(c, l, \bar{c}; \gamma). \) The marginal degree of positionality is given by: \( \gamma = (\partial w/\partial r)(\partial r/\partial c)/[(\partial w/\partial c) + (\partial w/\partial r)(\partial r/\partial c)]. \)

\(^7\)Partial derivatives are denoted as follows: \( u_1 \equiv \partial u(c, l, \bar{c}; \gamma)/\partial c, \ u_2 \equiv \partial u(c, l, \bar{c}; \gamma)/\partial l, u_3 \equiv \partial u(c, l, \bar{c}; \gamma)/\partial \bar{c}. \)
the household (A.2). Assumption (A.4) is a regularity assumption. If in a symmetric allocation ($\bar{c} = c$) both own consumption and average consumption increase by the same amount, utility rises. That is, the marginal utility gained from raising one’s own consumption exceeds the status effect (from increased average consumption). Assumption (A.5) restricts the specification of status effects. If $\bar{c} = c$, then $\gamma$ does not affect utility: if one’s own consumption equals average consumption ($\bar{c} = c$), then $u$ is independent of the strength of status effects. Assumption (A.5) requires the subutility function to be homothetic if $\bar{c} = c$.

Examples of utility functions satisfying assumptions (A.1) to (A.5) are the following:

$$u = \left[ \alpha \hat{c}^{\sigma} \sigma - 1 + (1 - \alpha) l^{\sigma} \right]^{\sigma \gamma} \sigma$$

where $\hat{c} \equiv c (\bar{c})^{\gamma / (1 - \gamma)}$, or where $\hat{c} \equiv (1 - \gamma) c + \gamma (c - \bar{c}) = c - \gamma \bar{c}$. A more general example satisfying (A.1) – (A.5) is provided by Dupor and Liu (2003), who use $\hat{c} \equiv \left[ \left( c^{\theta} - \gamma \bar{c}^{\theta} \right) / (1 - \gamma) \right]^{1 / \theta}$, which contains the two former specifications as special cases (as $\rho$ is respectively approaching zero and unity).  

The subutility function $g(G; \Psi)$ is twice continuously differentiable, increasing, and concave, with $g(G; 0) = 0$. The parameter $\Psi$ determines the strength of the household’s preference for the public good $G$. We also assume:

(A.6) $g_{\Psi}(G; \Psi) > 0$, $g_{G,\Psi}(G; \Psi) > 0$.

We can think of $\Psi$ as the $G$-elasticity of utility $g(.)$, which is how $g(.)$ is parameterized for Figure 1 in Section 5, and for the numerical simulations in Section 6. Any other specifications according with (A.6) are equally acceptable.

The consumption good as well as the public good are produced by private firms that use labor as the only input. The aggregate production constraint is characterized by a fixed-coefficients transformation function. Without loss of generality, the units of all goods can be normalized such that the marginal rates of transformation equal unity:

$$N (\omega - l) - C - G = 0,$$  \hspace{1cm} (2)

where $\omega$ is the total amount of time (labor and leisure) available to each
household, and $C$ is the total quantity of the consumption good produced.

## 3 Optimal Tax Rates in the Presence of Status Effects

Here we consider optimal tax rates and first-best and second-best allocations.

### 3.1 The Planner’s Solution

We first use the model to study conditions for social welfare maximization, assuming that social welfare can be evaluated by means of a Benthamite social welfare function:

\[
W(u^1 \cdots, u^N) = u^1(c, l, c; \gamma) + g^1(G; \Psi) + \cdots + u^N(c, l, c; \gamma) + g^N(G; \Psi)
\]

\[
= N u(c, l, c; \gamma) + N g(G; \Psi),
\]

where superindex $i \in \{1, \ldots, N\}$ denotes individual households and where the last part of the expression makes use of the assumption of identical utility functions. A social planner, taking fully into account the externality on all households generated by individual consumption, would choose \{c, l, G\} such as to maximize $W(u^1 \cdots, u^N)$. Since each household has the same preferences, and the welfare function is utilitarian, the optimum will be described by equal treatment: $C = \sum_{i=1}^{N} c_i = N c$. It is important to recognize that the externality remains despite equal treatment: each household experiences a status effect despite its equal status with other households ex post. At the margin, the consumption of other households has a negative impact on a given household's utility.

Assumption (A.5) implies: $W(u^1 \cdots, u^N) = N u(c, l, c; 0) + N g(G; \Psi)$. Consumption, leisure, and public goods provision are derived from:

\[
\{c, l, G\} = \arg \max_{c, l, G} \{ W | N (\omega - l) - N c - G = 0 \}.
\]

The planner’s outcome can be characterized by the following conditions:

\[
\frac{u_1}{u_2} \left[ 1 + \frac{u_3}{u_1} \right] = 1, \quad (4)
\]

\[
N \frac{g(G; \Psi)}{u_2} = 1, \quad (5)
\]

\[
c(\omega - G/N) + l(\omega - G/N) = \omega - G/N, \quad (6)
\]

6
where \( u_i \equiv u_i(c(\omega - G/N), l(\omega - G/N), c(\omega - G/N)), i = 1, 2, 3 \). Equation (4) states that the marginal rate of substitution of consumption for leisure equals the marginal rate of transformation (unity), corrected for an externality factor \( 1/(1 + u_3/u_1) \), which accounts for the marginal rate of substitution between \( c \) as a private good and \( c \) as a public good. Equation (5) is the Samuelson rule for optimal public goods supply, requiring the equality between the sum (over all households) of the marginal rate of substitution of the public good for leisure and the marginal rate of transformation (unity). Equation (6) restates the resource constraint.

Notice that assumption (A.5) implies that \( c \) and \( l \) are independent of \( \gamma \) in the planner’s outcome. The planner implements a symmetric allocation, because preferences are assumed to be homogeneous across households. Therefore, by (A.5), the optimal allocation \( \{c, l, G\} \) is not affected by \( \gamma \). This facilitates the following analysis. The market outcome, however, is affected by \( \gamma \), as individual households assume \( \bar{c} \) to be fixed and not to be equal to their individual consumptions, regardless of their respective consumption choice. The distortion from the consumption externality increases in \( \gamma \) and so does the optimal tax, as will be shown below.

### 3.2 Optimal Taxes in a Market Economy

Next, we characterize market equilibria with taxes and transfers.\(^9\) The wage rate (numeraire) is set equal to one.\(^10\) The consumer price of the private good (in terms of hours of work) is \( q = 1/(1 - \tau) \), where \( \tau \) is the consumption tax rate. A lump-sum tax (transfer) is denoted by \( t \).

As the public good enters the individual utility functions in a weakly separable way, the optimization problem can be solved on two levels by embedding a household problem within the government’s problem (see, for example, Barten and Boehm 1982, p.400).

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\(^9\) Denote exogenously given producer prices of \( c \) and \( G \) by \( (p, p_G) \). By the normalization, \( (p, p_G) = (1, 1) \).

\(^10\) The problem can be equivalently restated with a wage tax instead of a consumption tax. Common practice in the literature, however, is to adopt the commodity taxation model in which labor (leisure) is taken to be the numeraire. See, e.g., Atkinson and Stern (1974), Gronberg and Liu (2001), Stiglitz and Dasgupta (1971).
3.2.1 First-Best Solution

The government can achieve the first-best if it has a lump-sum tax as well as the consumption tax as instruments. A household’s budget constraint is \( \omega - t - q c - l = 0 \). Because the public good enters the utility function \( U \) in a separable way, the Marshallian demands of \( c \) and \( l \) are independent of \( G \).

The household’s problem consists of choosing respectively \( c \) and \( l \):

\[
\{c, l\} \equiv \arg \max_{c, l} \{u(c, l, \bar{c}; \gamma) | \omega - t - q c - l = 0 \}.
\]

The first-order condition for the market outcome is:

\[
\frac{u_1}{u_2} = q. \tag{7}
\]

Let \( v \) denote the indirect utility function. The resulting Marshallian demand and indirect utility functions are:

\[
c = c(q, \omega - t, \bar{c}; \gamma), \quad l = l(q, \omega - t, \bar{c}; \gamma), \quad v = v(q, \omega - t, \bar{c}; \gamma),
\]

where \( (\omega - t) \) is the net full income (after-tax value of the labor endowment) of a household.

**Ex post solutions.** Since preferences and endowments of all households are equal, the equal treatment property holds ex post, and \( c = \bar{c} \). Note that even though the market outcome will involve equality between \( \bar{c} \) and \( c \), the household regards \( c \) as its choice variable (and thus endogenous), while considering \( \bar{c} \) as exogenous. Let a tilde denote ex post solutions: that is, \( \tilde{c}(q, \omega - t; \gamma) \) is the solution to \( c - c(q, \omega - t, c; \gamma) = 0 \), and \( \tilde{l}(q, \omega - t; \gamma) = l(q, \omega - t, \tilde{c}(q, \omega - t; \gamma); \gamma) \). Ex post solutions can then be written as:

\[
\tilde{c} = \tilde{c}(q, \omega - t; \gamma), \quad \tilde{l} = \tilde{l}(q, \omega - t; \gamma), \quad \tilde{v} = \tilde{v}(q, \omega - t; \gamma).
\]

Let \( P \) and \( M \) respectively signify the planner’s and the market outcome. The corrective (Pigouvian) consumption tax rate, \( \hat{\tau} \), is:

\[
\hat{\tau} \equiv -\frac{u_3}{u_1}|_P, \tag{8}
\]

thus, the corrective consumer price becomes: \( \hat{q} \equiv 1/(1-\hat{\tau}) = (u_1)/(u_1+u_3)|_P \).

For the consumer price to be positive, we must have \( \hat{\tau} < 1 \Leftrightarrow u_1 + u_3 > 0 \), which is ensured by assumption (A.4).
Lemma 1 The market economy can be induced to attain the first-best marginal rate of substitution of consumption for leisure, (4), by implementing the corrective tax \( \hat{\tau} \).

The Lemma follows directly from (4), and (7). Observe that for \( \gamma = 0 \) we have \( u_3 = 0 \) (i.e., average consumption does not affect individual utility), in which case \( \hat{\tau} = 0 \), and \( \hat{q} = p = 1 \). As Lemma 1 implies \( (u_1 + u_3)/(u_2)\big|_P = 1 \), we know that \( (u_1 + u_3)/(u_2)\big|_M \leq 1 \iff q \leq \hat{q} \).

Equation (4) can also be written in the following way:

\[
\frac{u_1^i}{u_2^i} + \sum_{i=1}^{N} \frac{u_3^i}{u_1^i} \frac{\partial \bar{c}}{\partial c^i} = \frac{u_1^i}{u_2^i} \left( 1 + \sum_{i=1}^{N} \frac{u_3^i}{u_1^i} \frac{\partial \bar{c}}{\partial c^i} \right) = p = 1.
\]

Together with equation (7), the corrective tax rate turns out to be:

\[
\hat{\tau} = -\sum_{i=1}^{N} \frac{u_3^i}{u_1^i} \frac{\partial \bar{c}}{\partial c^i} = -\frac{u_3}{u_1} \sum_{i=1}^{N} \frac{\partial \bar{c}}{\partial c^i} = -\frac{u_3}{u_1}.
\]

The corrective tax rate amounts to the marginal social damage of an extra unit of consumption (by every household). A general increase in consumption implies a one-unit increase in \( \bar{c} \). The social damage from this increase is equal to the sum over all households of the marginal willingnesses to pay (in terms of the unit of account) for a lower level of average consumption.

In the first-best case (with a lump-sum tax available), the government sets \( \tau = \hat{\tau} \): that is, it sets the consumption tax equal to the corrective consumption tax rate. The government’s problem is to choose optimal values for \( t \) and \( G \):

\[
\{t, G\} \equiv \arg \max_{t, G} \{\tilde{v}(\hat{q}, \omega-t; \gamma) + g(G; \Psi) \mid N t + N (\hat{q}-1) \tilde{c}(\hat{q}, \omega-t; \gamma) = G\}.
\]

The resulting Samuelson condition is

\[
N \frac{g\gamma(G; \Psi)}{u_2} = 1,
\]

which, together with the government budget constraint, yields

\[
N t + N (\hat{q}-1) \tilde{c}(\hat{q}, \omega-t; \gamma) = G,
\]
the first best level of public goods provision, $G^*$, and the optimal lump-sum tax (transfer), $t^*$. It holds that $N t^* + N (\hat{q} - 1) \tilde{c}(\hat{q}, \omega - t^*; \gamma) = G^*$. In the first best case, the government sets the consumption tax rate equal to $\hat{\tau}$, and the level of public goods provision equal to $G^*$.

Consider the special case where the revenues from corrective consumption taxation exactly equal the necessary revenue to make up for the first-best level of public goods provision. In this case $t^* = 0$ and $G^* = N \omega (1 - \zeta)$, where $\zeta \equiv [1 - (\hat{q} - 1) \tilde{c}(\hat{q}, 1; \gamma)]$ is the share of income net of corrective taxation to full income ($\omega$). More generally, if revenues from corrective consumption taxation fall short of (exceed) the revenue needed for the first-best level of public goods provision, then $t^* > 0$ ($t^* < 0$).

The conditions characterizing the market outcome in the first-best case are the two first order conditions (7), (10), the government budget constraint (11), and the household budget constraint:

$$\hat{q} \tilde{c}(\hat{q}, \omega - t; \gamma) + \tilde{l}(\hat{q}, \omega - t; \gamma) = \omega - t. \quad (12)$$

**Lemma 2** By implementing the corrective tax $\hat{\tau}$, and considering the government budget constraint, $N t^* + N (\hat{q} - 1) \tilde{c}(\hat{q}, \omega - t^*; \gamma) = G^*$, the market economy can be induced to attain the first-best optimal allocation $\{c, l, G\}$, as characterized by conditions (4) to (6).

Lemma 2 shows that $\{\hat{\tau}, t^*\}$, as defined for the government’s problem above, represents the first-best policy. The proof of Lemma 2 is provided in the Appendix.

### 3.2.2 Second-Best Solution

In the second-best case, no lump-sum taxes (or transfers) are available. The only revenue instrument available to the government is a consumption tax. With a consumption tax in place, the household’s problem becomes:

$$\{c, l\} \equiv \arg \max_{c, l} \{u(c, l, \bar{c}; \gamma) \mid \omega - q c - l = 0\}. \quad (10')$$

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11 First order condition (10), which can equivalently be written as

$$\frac{\tilde{v}(\hat{q}, 1; \gamma)}{N [1 - (\hat{q} - 1) \tilde{c}(\hat{q}, 1; \gamma)]} = g_{\mathcal{G}}(G; \Psi), \quad (10')$$

is derived in the Appendix.
For a given tax rate $\tau$, the conditions describing a second-best equilibrium are:

\begin{align}
\frac{u_1}{u_2} &= q, \\
\omega - qc - l &= 0.
\end{align}

The resulting Marshallian ex post demand and indirect utility functions are:

\[ \tilde{c} = \tilde{c}(q, \omega; \gamma), \quad \tilde{l} = \tilde{l}(q, \omega; \gamma), \quad \tilde{v} = \tilde{v}(q, \omega; \gamma). \]

As the utility function is separable, the demand functions are independent of $G$.

The government’s problem in the second-best case consists of choosing $\{\tau, G\}$ or, equivalently, $\{q, G\}$:

\[ \{q, G\} \equiv \arg \max_{q, G} \{\tilde{v}(q, \omega; \gamma) + g(G; \Psi) | N (q - 1) \tilde{c}(q, \omega; \gamma) = G\}. \]

The resulting second-best level of public goods provision is denoted $G^{**}$. The level of $G^{**}$ is determined by the following first-order condition:

\[ N g_G(G^{**}, \Psi) R_q(q, \omega; \gamma) dq = -\tilde{v}_q(q, \omega; \gamma) dq, \]

where $R_q(q, \omega; \gamma) = \tilde{c}(q, \omega; \gamma) + (q - 1)\tilde{c}_q(q, \omega; \gamma)$ is the change in revenue due to a marginal increase in the consumption tax rate. To understand condition (15), notice the following. A marginal increase of the tax rate raises revenue (thus the level of public goods) by $R_q$. Thus the marginal utility of the increase in $G$ financed by a marginal increase of the tax rate is given by $N g_G(.) R_q(.)$. The right hand side of (15) represents the loss in utility when the consumption tax rate is marginally raised. The increase of $q$ by $dq$ is equivalent (in terms of utility) to a decline in income by both the tax revenue and the associated excess burden: $dR + dEB$. The excess burden, in turn, will be lower than in models without status effects, for reasons given in the following section. As the public good benefits all households, first order condition (15) requires the marginal benefit to society of an increase of the public good due to a marginal rise of the tax rate to be equal to every household’s loss in marginal utility due to the marginal rise of the tax rate.\footnote{Similar to Sandmo (1975), the second best tax rate on $c$ is equal to $\tau^{**} = (1 - \mu)(-\varepsilon\omega^{-1}) + \mu(-u_3/u_1)$, where $\mu = \lambda/\beta$, $\lambda$ is the marginal utility of income, and $\beta$ is the marginal benefit to society of an increase of the public good. The first best and second best levels of $\tau$ coincide if and only if $\lambda = \beta = N g_G(G; \Psi) \Leftrightarrow u_2 = N g_G(G; \Psi)$, which corresponds to (10) above.}
4 Excess Burden and Status Effects

In this section we show that status effects reduce the excess burden of a consumption tax. Moreover, the marginal excess burden is negative (positive) when \( q < \hat{q} \) (when \( q > \hat{q} \)). Here we consider the excess burden associated with a given magnitude of the status effect (as measured by \( \gamma \)) and a given consumption tax rate, \( \tau \) (or corresponding \( q \)). Here we apply arbitrary values for \( \tau \): Section 5 will examine optimal second-best levels for \( \tau \) and the public good.

**Definition 1** The excess burden, \( EB \), of a tax is the difference between the negative of the equivalent variation and the tax revenue collected.

Thus, the excess burden is the loss to the private sector over and above the revenue collected by the tax.

The marginal excess burden (MEB) measures the change in the excess burden per marginal unit of tax revenue.

**Definition 2** The marginal excess burden is defined as

\[
\text{MEB}(q, \omega; \gamma) = \frac{d \text{EB}(q, \omega; \gamma)}{d \text{R}(q, \omega; \gamma)}.
\]

Throughout the rest of the paper, we employ the following additional assumption:

(A.7) \( d \text{R}(q, \omega; \gamma)/d q > 0 \).

That is, the tax revenue is increasing in \( q \), so we consider the increasing part of the Laffer curve. By (A.7), the sign of the MEB equals the sign of \( d \text{EB}/d q \).

The excess burden is zero at price \( \hat{q} \) and income \((\omega - t^*)\): for this price-income pair, the resulting allocation is socially optimal, by Lemma 2. Let \( \bar{R} = (q - 1)\hat{c}(q, y; \gamma) \) denote the commodity tax revenue, which can be decomposed into two components: the corrective tax revenue, \( \bar{R} \), and the net tax revenue, \( R^\alpha \) (the revenue in excess of \( \bar{R} \)). Clearly, \( R = \bar{R} + R^\alpha \). The excess burden is implicitly defined by:\(^{14}\)

\[
\tilde{v}(\hat{q}, \omega - t^* - EB - R^\alpha; \gamma) = \tilde{v}(q, \omega - t^*; \gamma), \quad (16)
\]

\(^{13}\)As \( R_q(q) > 0 \), we can express \( q \) as a function of \( R \): \( q = q(R) \), with \( q'(R) = 1/R'(q) > 0 \) by the Inverse Function Rule. Thus, \( MEB = EB_R = EB_q q'(R) = EB_q / R'(q) \).

\(^{14}\)Observe that the excess burden is zero with \( q = \hat{q} \) and \( t = t^* \). However, with \( q = \hat{q} \), the government receives a revenue \( \bar{R} \geq 0 \).
and, as shown in the Appendix, explicitly given by:

\[ EB(q, \omega; \gamma) = \omega - \epsilon(\hat{q}, \bar{v}(q, \omega; \gamma); \gamma) - R^e(q, \omega; \gamma), \tag{17} \]

where \( \epsilon(.) \) denotes the expenditure function. Clearly, \( EB(q, \omega; \gamma) \geq 0 \), and \( EB(\hat{q}, \omega; \gamma) = 0 \). The equality stems from the fact that \( R^e|_{\eta=\hat{q}} = 0 \).\(^{15}\) The inequality becomes obvious when we reformulate the excess burden as:\(^{16}\)

\[ EB(q, \omega; \gamma) = \hat{v}(q, \omega; \gamma) \times \left[ \hat{c}^h(q, 1; \gamma) + \hat{t}^h(q, 1; \gamma) - \hat{c}^h(\hat{q}, 1; \gamma) - \hat{t}^h(\hat{q}, 1; \gamma) \right], \tag{18} \]

where superscript \( h \) denotes Hicksian (compensated) demands. Consider \( \hat{v}(q, \omega; \gamma) > 0 \). Also,

\[
\frac{d}{dq} \left[ \hat{c}^h(q, 1; \gamma) + \hat{t}^h(q, 1; \gamma) \right] = \hat{c}^h_q + \hat{t}^h_q = \hat{c}^h_q - (u_1 + u_3)/u_2 \hat{c}^h_q
\]

\[
= \hat{c}^h_q \left( 1 - (u_1 + u_3)/u_2 \right) \geq 0 \Leftrightarrow q \geq \hat{q}. \]

The inequalities follow from Lemma 1, which implies \( (1 - (u_1 + u_3)/u_2) \leq 0 \Leftrightarrow q \geq \hat{q} \). Therefore \( EB(q, \omega; \gamma) \) is nonnegative throughout.\(^{17}\)

Intuitively, if \( \tau < \hat{\tau} \), the tax system is characterized by a Pigouvian tax plus a distortionary subsidy. If, however, \( \tau > \hat{\tau} \), the tax system is characterized by a Pigouvian tax plus a distortionary tax. In both cases, the distortion leads to a positive excess burden.

**Proposition 1** For \( q > \hat{q} \), the excess burden is lower, the stronger are status effects: \( \frac{d}{dq} EB(q, \omega; \gamma)|_{q>\hat{q}} < 0 \).

Status effects lower the excess burden associated with a given consumption (labor) tax rate. In particular, the excess burden is lower in an economy with status effects (\( \gamma > 0 \)) than in an economy without status effects (\( \gamma = 0 \), \( \hat{q} = \hat{p} \)). The reason is that the tax corrects for the negative externality

\(^{15}\)One might expect that \( EB(\hat{q}, \omega - \hat{t}^*; \gamma) = 0 \). As \( u(c, l, e; \gamma) \) is homothetic, \( EB(\hat{q}, \omega - \hat{t}^*; \gamma) = EB(\hat{q}, 1; \gamma) (\omega - \hat{t}^*) = EB(\hat{q}, \omega; \gamma) (\omega - \hat{t}^*)/\omega = 0 \). It follows that \( EB(\hat{q}, \omega; \gamma) = 0 \).

\(^{16}\)For obtaining (18), we make use of the fact that \( \bar{v}(q, \omega; \gamma) = \bar{v}(\hat{q}, \omega - EB - R^e; \gamma) \). We observe that \( EB = \epsilon(q, \bar{v}(q, \omega; \gamma); \gamma) - \epsilon(\hat{q}, \bar{v}(\hat{q}, \omega; \gamma); \gamma) - R^e = (\epsilon(q, \bar{v}(q, \omega; \gamma); \gamma) - R) - (\epsilon(\hat{q}, \bar{v}(\hat{q}, \omega - EB - R^e; \gamma); \gamma) - \hat{R}). \)

\(^{17}\)The minimum of the excess burden can be found where \( d EB/d q = 0 \Leftrightarrow (u_1 + u_3)/u_2 = 1 \Leftrightarrow q = \hat{q} \). From the inequalities shown above, it follows that the EB cannot be negative.
associated with consumption, which worsens the relative position of other individuals. The externality-correcting feature of the tax yields an improvement in allocative efficiency. Thus, the tax implies a lower excess burden than would have resulted without status effects.

Notice that the result of Proposition 1 holds if $q > \hat{q}$. It need not hold for the case that $q < \hat{q}$. To gain some insight, suppose the excess burden, as a function of $q$, is U-shaped, with its minimum at $q = \hat{q}$. Then, a rise in $\gamma$ shifts the excess burden curve to the right. For tax rates below the corrective level, a rise in the strength of status effects puts the actual tax rate further away from the corrective one, and the excess burden rises.

**Proposition 2** The marginal excess burden is negative for tax rates below the corrective tax rate ($q < \hat{q}$) and zero at $q = \hat{q}$. For tax rates above the corrective tax rate, the marginal excess burden is positive.

(The proof is in the Appendix.) The consumption externality introduces the possibility of a negative excess burden. If $q < \hat{q}$, the tax system is characterized by a Pigouvian tax plus a distortionary subsidy. A rise in the consumption tax rate lowers the distortionary subsidy (thus, implying a negative marginal excess burden). If $q > \hat{q}$, the marginal excess burden is strictly positive, when preferences are strictly concave and $u(.)$ is twice continuously differentiable. In particular, this holds for the class of CES utility functions, as was previously shown by Wilson (1991a).  

5 Preferences, Excess Burden and Optimal Public Goods Provision

We now examine how preferences for the public good, along with the strength of status effects, influence the second-best consumption tax rate, its excess burden, and the relation of first-best to the second-best level of public goods provision. We proceed in two steps. First, we show that in the second-best setting, the sign of the marginal excess burden of the consumption tax is positive (negative) when the second-best level of public goods provision is below (above) the first-best level. Second, we indicate the effects of the

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18A counterexample is given by Gronberg and Liu (2001). In their example, however, indifference curves exhibit a kink, that is, $u(.)$ is not twice continuously differentiable everywhere.
two exogenous preference parameters $\gamma$ and $\Psi$ on the relation between the first-best and the second-best consumption tax rates.

**Step 1.** We first consider, in the second-best setting, the relation of first-best to the second-best level of public goods provision and the sign of the marginal excess burden (irrespective of the values of the exogenous parameters $\gamma$ and $\Psi$).

**Proposition 3** In the model with status effects, the second-best level of provision of a public good falls short of the first-best level if and only if the sign of the marginal excess burden is positive.

The proof of Proposition 3 (see the Appendix) shows the following first order conditions of the government’s problems in the first-best and second-best cases:

$$g_G(G^*; \Psi) = \frac{\hat{v}(\hat{q}, 1; \gamma)}{N[1 - (\hat{q} - 1)\hat{c}(\hat{q}, 1; \gamma)]},$$

$$g_G(G^{**}; \Psi) = \frac{\hat{v}(\hat{q}, 1; \gamma)}{N[1 - (\hat{q} - 1)\hat{c}(\hat{q}, 1; \gamma)] [1 + MEB(q, \omega; \gamma)].$$

These expressions indicate that the second-best level of provision of public goods exceeds the first-best level if the sign of the marginal excess burden is negative.\(^{19}\) For an economy with (status) externalities, Proposition 2 indicates that the second-best level of provision of public goods exceeds the first-best level ($G^{**} > G^*$) if $q < \hat{q}$. Likewise, the second-best level of provision of public goods is lower than the first-best level if $q > \hat{q}$ (in case $MEB > 0$).

**Step 2.** Here, we consider the impact of the two exogenous parameters $\gamma$ and $\Psi$ on the relation between the first-best and the second-best consumption tax rates. The relation between the first-best and the second-best consumption tax rates determines the sign of the marginal excess burden (Proposition 2), which determines the relationship between the first-best and second-best levels of public goods provision (Proposition 3).

The parameter $\gamma$ determines the first-best consumption tax rate, $\hat{\tau}$. According to (8), $\hat{\tau}(0) = 0$, and $\hat{\tau}(\gamma)$ is rising in $\gamma$ and independent of $\Psi$. That is, every exogenously given value of $\gamma$ gives rise to a unique $\hat{\tau}(\gamma)$.

\(^{19}\)For an economy without externalities, this result was previously shown by Gronberg and Liu (2001).
The parameter $\Psi$ (strength of preference for $G$), determines the demand for the public good. The second-best tax rate (tax revenue) needed to finance the public good increases in $\Psi$. Let $\tau(\Psi)$ stand for the second-best consumption tax rate. Then, $\tau(0) = 0$, and $\tau(\Psi)$ is rising in $\Psi$.

The key issue is whether Pigouvian taxation generates enough revenue to meet the demand for the public good or not. If, for a given value of $\Psi$, Pigouvian taxation generates revenue that exactly meets the demand for the public good, the marginal excess burden is zero, and the second-best public good level is equal to the first-best level. However, if, for a given value of $\Psi$, Pigouvian taxation generates revenue less than (in excess of) the demand for the public good, the marginal excess burden is positive (negative), and the second-best public good level falls short of (exceeds) the first-best level.

In other words, any exogenously given value of $\gamma$ implies a specific value for $\hat{\tau}$$\gamma$). Given this value for $\gamma$, there exists a unique value of $\Psi$, such that $\tau(\Psi) = \hat{\tau}$$\gamma$). For any lower $\Psi$, $\tau(\Psi) < \hat{\tau}$$\gamma$); likewise, for any higher $\Psi$, $\tau(\Psi) > \hat{\tau}$$\gamma$).

The stronger the preference for the public good (the higher the $\Psi$), the higher is the second-best tax rate (revenue) needed to finance the public good. The stronger the preference for status (the higher the $\gamma$), the higher is the Pigouvian tax rate. Clearly, there exist parameter-pairs of $\gamma$ and $\Psi$, such that $\tau(\Psi) = \hat{\tau}$$\gamma$), or $q = \hat{q}$. For all such pairs, the marginal excess burden equals zero. In Figure 1 below, all such parameter-pairs are represented by the $\Psi(\gamma)|_{\text{MEB}=0}$-curve in $(\gamma, \Psi)$ space.

**Proposition 4** In $(\gamma, \Psi)$ space, the $\Psi(\gamma)|_{\text{MEB}=0}$-curve has a positive slope. Moreover, $\Psi(0)|_{\text{MEB}=0} = 0$. Along this curve, $G^* = G^{**}$. For all $(\gamma, \Psi)$ pairs above this curve, $\text{MEB} > 0$, and $G^* > G^{**}$. For all $(\gamma, \Psi)$ pairs below this curve, $\text{MEB} < 0$, and $G^* < G^{**}$.

Figure 1 illustrates the results of this section. It shows that along the $\Psi(\gamma)|_{\text{MEB}=0}$-curve, $\tau = \hat{\tau}$, or $q = \hat{q}$ (by Proposition 2), and $G^* = G^{**}$ (by Proposition 3), as the Pigouvian taxation generates revenue that exactly

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20Figure 1 is based on a CES utility function: $u = [\alpha \hat{c}^{(\sigma-1)/\sigma} + \beta \hat{l}^{(\sigma-1)/\sigma}]^{\sigma/\sigma-1} + G^\Psi$, where $\hat{c} = c(e/c)^{1/1-\gamma}$. The parameters are assigned the following values: $\omega = 1$, $\alpha = 1$, $\beta = 1.5$, $\sigma = 2$, which are in line with the numerical estimates presented in Section 6. Numerical experimentation shows that the $\Psi(\gamma)|_{\text{MEB}=0}$-curve is not sensitive with respect to changes in these parameter values.
meets the first-best (second-best) level of the public good. The $\Psi(\gamma)|_{MEB=0}$ curve has a positive slope, as a higher $\gamma$ implies a higher Pigouvian tax rate, and therefore allows for a higher level of revenues (a stronger preference for public goods, $\Psi$). Thus, the larger the strength of status effects, the larger is the range of tax rates for which the marginal excess burden is negative. In the region above the $\Psi(\gamma)|_{MEB=0}$-curve, $q > \hat{q}$, and the sign of the marginal excess burden is positive. However, by Proposition 1, the excess burden is smaller compared to an economy without status effects (vertical axis). In the region below the $\Psi(\gamma)|_{MEB=0}$-curve, $q < \hat{q}$, and the sign of the marginal excess burden is negative.

If $q < \hat{q}$, according to Proposition 2, $G^{**} > G^*$, which is contrary to Pigou’s conjecture that the second-best level of public good would be below the first-best level. If $q > \hat{q}$, Pigou’s conjecture holds. Finally, if $q = \hat{q}$, the marginal excess burden equals zero, and $G^{**} = G^*$:

\begin{align*}
q < \hat{q} & \Rightarrow MEB(q, \omega; \gamma) < 0 \& g_G(G^*; \Psi) > g_G(G^{**}; \Psi) \Rightarrow G^* < G^{**}, \\
q > \hat{q} & \Rightarrow MEB(q, \omega; \gamma) > 0 \& g_G(G^*; \Psi) < g_G(G^{**}; \Psi) \Rightarrow G^* > G^{**}, \\
q = \hat{q} & \Rightarrow MEB(q, \omega; \gamma) = 0 \& g_G(G^*; \Psi) = g_G(G^{**}; \Psi) \Rightarrow G^* = G^{**}.
\end{align*}
6 Empirical Evidence

In this Section, we provide some evidence about what location in the \((\gamma, \Psi)\) plane might represent that of a typical industrialized country. Based on the empirical evidence regarding the magnitude of status effects, we suggest that the marginal excess burden lies well below estimates based on the assumption that households do not derive utility from relative consumption. The strength of status effects from various studies suggests that the corrective Pigouvian tax rate on consumption can be as high as 40\% or 50\%. Since many actual tax rates\(^{21}\) are below 40 percent, this suggests that in many instances the marginal excess burden of labor taxes may well be negative.

6.1 Relative Income and Consumption

Many studies provide empirical evidence supporting the importance of relative consumption (or relative income) for deriving utility.\(^{22}\) A typical finding is that an increase in neighbors’ earnings and a similarly sized decrease in own income each lead to a reduction in happiness of about the same order (Luttmer, 2005). Another aspect pointed out by various studies is that positional concerns (status effects) might be smaller at low income levels than at high income levels. McBride (2001), for example, uses an econometric approach to find significant evidence in support of the importance of relative income, however, the impact of relative income on assessments of subjective well-being is smaller for households with low income levels. While this evidence is supported by Solnick and Hemenway (2005), Johansson-Stenman et al. (2002) do not find evidence for stronger status effects at higher income levels (see Table 1).

6.2 Empirical values for the status parameter

The studies employing survey-experimental methods generally confront an individual with two states of the world, state \(A\), and state \(R\). These states

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\(^{21}\)That is, the consumption tax rates equivalent to existing labor taxes and consumption taxes combined.

differ with respect to two dimensions: absolute income (consumption) of the individual, and income (consumption) of the individual relative to average income (consumption). In state A, an individual is better off in absolute terms, compared to the other state. However, relative income (consumption) is smaller compared to state R. In state R, an individual is better off in relative terms.

The subjects are asked to indicate which of the two states they prefer. Solnick and Hemenway (2005) use the following hypothetical situation. In state R, an individual would earn an annual income of $50,000 while a typical member of society would earn an income of $25,000. In state A, the individual would earn an annual income of $100,000 while a typical member of society would earn an income of $200,000. Everything else would be equal in both states. A similar question was posed for higher income values (state R: $200,000 versus $100,000, and state A: $400,000 versus $800,000). Regarding the low-income (high-income) question, 33% (48%) of the respondents preferred state R over state A.

Given two states of the world that differ in absolute and relative income (consumption), the implicit degree of positionality, $\gamma$, is defined to be that value of $\gamma$ for which an individual is indifferent between state A and state R:

$$u_A(\cdot; \gamma) = u_R(\cdot; \gamma).$$

If a respondent prefers state R over state A, it must be the case that $u_A(\cdot; \gamma) < u_R(\cdot; \gamma)$, or equivalently, $\gamma > \gamma$. Otherwise, if a respondent prefers state A over state R, $\gamma < \gamma$.

Given two states of the world that differ in absolute and relative income (consumption), the value of $\gamma$ depends on the specification of the utility function. For the estimations of $\gamma$ shown below, we use the status formulation offered by Dupor and Liu (2003) and consider the following CES utility function:

$$u = [\alpha \hat{c}^{(\sigma-1)/\sigma} + \beta l^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)} + g(G; \Psi), \quad \hat{c} \equiv c^{1/(1-\gamma)} \bar{c}^{-\gamma/(1-\gamma)},$$

where $\sigma$ denotes the constant elasticity of substitution between $\hat{c}$ and $l$. As the whole income is consumed, a respondent is equivalent between states A and R if and only if:

$$[\alpha (c_A^{1/(1-\gamma)} \bar{c}_A^{\gamma/(1-\gamma)})^{\sigma/(\sigma-1)} + \beta l^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)} + g(G; \Psi)$$

$$= [\alpha (c_R^{1/(1-\gamma)} \bar{c}_R^{\gamma/(1-\gamma)})^{\sigma/(\sigma-1)} + \beta l^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)} + g(G; \Psi),$$

where $\hat{c} \equiv (c^\sigma [1 + \gamma (c^\rho - \bar{c}^\rho)/(1-\gamma)])^{1/\rho}$, and $\lim_{\rho \to \infty} \hat{c} = c^{1/(1-\gamma)} \bar{c}^{-\gamma/(1-\gamma)}$. 

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where subindexes respectively indicate states $R$ and $A$, and $\bar{c}$ is average consumption. Notice that $G$ and $l$ is the same (and fixed) in both states, by design of the questions in the survey-experimental studies. Thus, (20) is equivalent to:

$$c_A \left(\frac{c_A}{\bar{c}_A}\right)^{\gamma/(1-\gamma)} = c_R \left(\frac{c_R}{\bar{c}_R}\right)^{\gamma/(1-\gamma)}.$$  

(21)

In the particular example just provided from Solnick and Hemenway (2005), our assumed utility function implies an implicit degree of positionality (in both the low- and the high-income question) of $\bar{\gamma} = 1/3$. Thus, regarding the low-income (high-income) question, for 33% (48%) of the respondents, $\gamma > \bar{\gamma} = 1/3$.

Most empirical studies employ several $R$-states that differ in the respective level of $\bar{\gamma}$. We use this information to infer values for $\gamma$. We apply two methods: a parametric method (binary probit analysis), and a non-parametric one (Spearman-Karber method). In what follows we briefly describe both methods and present the estimates in Table 1.

**Probit analysis.** We formulate a random parameter model (see Carlsson et al., 2003) and introduce a stochastic term, $\varepsilon$, reflecting preference uncertainty and choice errors. Specifically,

$$\gamma = \theta + \varepsilon,$$  

(22)

where $\varepsilon \sim N(0, s^2)$ has a Normal distribution, and $E[\gamma] = \theta$. Given two specific states, $A$ and $R$, the probability, $P$, of choosing (preferring) state $A$ equals $P[\gamma < \bar{\gamma}] = P[\theta + \varepsilon < \bar{\gamma}] = P[\varepsilon < \bar{\gamma} - \theta]$. Let $F(.)$ be the cumulative density function. Then, $P[\varepsilon < \bar{\gamma} - \theta] = F(\beta_0 + \beta_1 \bar{\gamma})$, where $\beta_0 \equiv -\theta/s$ and $\beta_1 \equiv 1/s$. Parameters $\beta_0$ and $\beta_1$ are estimated as maximum likelihood estimators of the associated probit model (method A). The mean value of

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24 This approach is similar to the random utility approach.

25 As seen in Table 1, for some estimates a similar method (method B) is employed. A few empirical studies consider only states associated with a uniform implicit degree of positionality. For these studies, only one parameter can be identified by the log likelihood function. In these cases, we set $\beta_1$ (the absolute of the inverse of the standard error of $\varepsilon$) equal to the mean of all other studies’ estimates of $\beta_1$, and determine $\beta_0$ as maximum likelihood estimate (method B). Specifically, we set $\beta_1 = -1.68$. To consider the sensitivity of this assumption, we provide additional estimates when applying method B. We estimate $\beta_0$ again under the assumption that $\beta_1$ is equal to the mean of estimated values for $\beta_1$ (of -1.68) plus one standard deviation, which amounts to a value of -1.226. The estimated values for $\gamma$ are lower under the assumption that $\beta_1 = -1.226$, as compared to $\beta_1 = -1.68$. 

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the strength of the status effect is then given by: \( E[\gamma] = -\beta_0/\beta_1 \).

**Spearman-Karber method.** The results obtained from the probit analysis can be questioned for their dependence on a particular distributional assumption; namely, the assumption that \( \gamma \) follows a normal distribution. Our second approach employs a non-parametric method in which the distribution of tastes is determined by the data rather than assumed. In particular, we use the non-parametric Spearman-Karber method, which in many cases has been shown to be more powerful than probit analysis for estimating parameters in psychometric functions (Miller and Ulrich, 2001). In Table 1, we also provide Spearman-Karber estimates for \( E[\gamma] \), and associated 95% confidence intervals.\(^{26}\)

Table 1 reports parametric and non-parametric estimates for \( \gamma \) (following the methods described above), based on data of several studies employing survey-experimental methods. The estimates of the mean of \( \gamma \) vary between 0.20 and 0.65. Standard deviations are calculated by the multiparameter delta method. The 95 percent confidence intervals are reported in Table 1 in brackets below the estimates of \( \gamma \).

\(^{26}\)A detailed description of the approach is offered, e.g., in USEPA (1993). Briefly, mean and variance are calculated as follows. Consider \( k \) states \( A \) and \( R \) such that the associated implicit degrees of positionality are \( \bar{\gamma}_0, \bar{\gamma}_1, \ldots, \bar{\gamma}_i, \ldots, \bar{\gamma}_k \). Let \( p_i \) be the proportion of group \( i \) individuals that consider \( \gamma < \bar{\gamma}_i \). Make sure that \( \bar{\gamma}_0 \) and \( \bar{\gamma}_k \) are such that \( p_0 = 0 \) and \( p_k = 1 \). Then, \( E[\gamma] = \sum_{i=1}^{k-1} (p_{i+1} - p_i)(\bar{\gamma}_i + \bar{\gamma}_{i+1})/2 \). Let \( n_i \) be the number of group \( i \) individuals. Then, the variance is calculated as \( V[E[\gamma]] = \sum_{i=2}^{k-1} p_i(1 - p_i)(\bar{\gamma}_{i+1} - \bar{\gamma}_{i-1})^2/(4(n_i - 1)). \)
TABLE 1

**Empirical Estimates of Status Effects**

<table>
<thead>
<tr>
<th>Study</th>
<th>Remarks</th>
<th>#</th>
<th>Method</th>
<th>Probit Analysis</th>
<th>Spearman-Karber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpizar et al. (2005)</td>
<td>7 R-states with differing implicit degrees of positionality ($\gamma_i$)</td>
<td>283</td>
<td>A</td>
<td>0.43</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.37 – 0.48)</td>
<td>(0.47 – 0.52)</td>
</tr>
<tr>
<td>Carlsson et al. (2003)</td>
<td>3 data sets with different $\gamma_i$</td>
<td>329</td>
<td>A</td>
<td>0.65</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.54 – 0.76)</td>
<td>(0.50 – 0.57)</td>
</tr>
<tr>
<td>Johansson-Stenman et al. (2002)</td>
<td>7 different $\gamma_i$; low income and high income versions of the questionnaire in addition</td>
<td>356</td>
<td>A</td>
<td>0.36</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.33 – 0.39)</td>
<td>(0.37 – 0.41)</td>
</tr>
<tr>
<td></td>
<td>“Low income” sample, same $\gamma_i$ as above but lower individual income levels</td>
<td>90</td>
<td>A</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.33 – 0.42)</td>
<td>(0.34 – 0.42)</td>
</tr>
<tr>
<td></td>
<td>“High income” sample, same $\gamma_i$ as above but higher individual income levels</td>
<td>90</td>
<td>A</td>
<td>0.22</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.15 – 0.28)</td>
<td>(0.27 – 0.35)</td>
</tr>
<tr>
<td>Solnick and Hemenway (1998)</td>
<td>1 R state; probit sensitivity (see Footnote 25): $E[\gamma] = 0.26 (0.12 – 0.39)$</td>
<td>238</td>
<td>B</td>
<td>0.28</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.18 – 0.37)</td>
<td>(0.21 – 0.25)</td>
</tr>
<tr>
<td>Solnick and Hemenway (2005)</td>
<td>1 implicit degree of positionality; probit sensitivity (see Footnote 25): $E[\gamma] = 0.15 (0.02 – 0.29)$</td>
<td>226</td>
<td>B</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.10 – 0.30)</td>
<td>(0.18 – 0.23)</td>
</tr>
</tbody>
</table>

**Notes.** – # refers to the number of respondents. Numbers in brackets below the estimates of $E[\gamma]$ indicate the 95% confidence intervals. $\hat{\gamma}$ is the implicit degree of positionality ($\gamma$ for which, according to (20), the data yield the same utility for both states A and R).

According to the estimates presented in Table 1, we consider $\gamma \in [0.2, 0.4]$ a realistic range for the status parameter.
6.3 Status Effects, Corrective Tax Rate, and the Excess Burden

In the following, we compare the marginal excess burden under the assumption that $\gamma = 0$, with that occurring in a situation where $\gamma > 0$, for four parameter sets, which are calibrated as follows.

The calibration is based on utility function (19). First, we fix the share of labor supply to full-time labor endowment to 40 percent: $\left(\omega - l\right)/\omega = 0.4$. We base our calculations on a full-time labor endowment of 5000 hours a year. A household works for about 2000 hours a year, or about 40 hours per week. Second, we develop four different calibrated data sets according to the compensated elasticity of labor supply ($\varepsilon_{cL}$). Sets IA and IB employ a more conservative estimate of the compensated elasticity of labor supply of $\varepsilon_{cL} = 0.3$. Sets IIA and IIB employ an estimate of $\varepsilon_{cL} = 0.7$. These estimates correspond well with the empirical values reported in Blundell (1992). Third, we distinguish data sets according to the base value of $\gamma$. Sets IA and IIA employ $\gamma = 0$, and sets IB and IIB employ $\gamma = 0.3$. Normalizing $\alpha$ to unity, we calibrate values for $\beta$ and $\sigma$ for all four parameter sets at $\tau = 0.2$. Those values are shown in Table A.1 in the Appendix (A.8).

For all four parameter sets — given the respective values for $\alpha$, $\beta$, $\sigma$ — we consider five values for the status parameter ($\gamma = 0, 0.1, 0.2, 0.3, 0.4$), and four values for the tax rate ($\tau = 0.2, 0.3, 0.4, 0.5$) each, and calculate the marginal excess burden.\footnote{Among others, this estimate is used by Auerbach and Kotlikoff (1987) for their analyses of dynamic fiscal policy.}

For $\gamma = 0$, parameter sets IA and IB (low compensated elasticity of labor supply) imply a marginal excess burden of 7 to 27 cents (depending on the marginal tax rate), which corresponds to estimates offered by Håansson and Stuart (1985). These estimates can be viewed as a lower bound for empirical estimates of the marginal excess burden. For $\gamma = 0$, parameter sets IIA and IIB (high compensated elasticity of labor supply) imply a marginal excess burden of 17 cents to US$ 1.57, which is more in line with estimates given

\footnote{For parameter set IA, $\varepsilon_{cL} = 0.3$ and the labor share equals 0.4 (as calibrated) only when $\gamma = 0$ and $\tau = 0.2$. The compensated labor elasticity and the labor share attain slightly different values when $\gamma > 0$ or $\tau > 0.2$. This follows from the fact that the same calibrated values of $\alpha$, $\beta$, $\sigma$ are applied for all calculations of the marginal excess burdens (for all considered values of $\tau$ and $\gamma$), based on parameter set IA. Analogue reasoning holds for the other parameter sets.}
by Browning (1976) that we view as an upper bound on empirical estimates of the marginal excess burden.\textsuperscript{29}

Table 2 presents the marginal excess burdens (in US$) per additional dollar of revenue raised, for four tax rates and several levels of $\gamma$.

### Table 2

**Marginal Excess Burden and Status Effects**

<table>
<thead>
<tr>
<th>Parameter Set IA</th>
<th>Parameter Set IB</th>
<th>Parameter Set IIA</th>
<th>Parameter Set IIB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$\gamma$</td>
<td>$\gamma$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>0.0 0.1 0.2 0.3 0.4</td>
<td>0.0 0.1 0.2 0.3 0.4</td>
<td>0.0 0.1 0.2 0.3 0.4</td>
<td>0.0 0.1 0.2 0.3 0.4</td>
</tr>
<tr>
<td>0.2 0.07 0.03 0.00 -0.04 -0.08</td>
<td>0.09 0.05 0.00 -0.05 -0.11</td>
<td>0.09 0.05 0.00 -0.05 -0.11</td>
<td>0.09 0.05 0.00 -0.05 -0.11</td>
</tr>
<tr>
<td>$\tau$ 0.07 0.03 0.00 -0.04 -0.08</td>
<td>0.09 0.05 0.00 -0.05 -0.11</td>
<td>0.09 0.05 0.00 -0.05 -0.11</td>
<td>0.09 0.05 0.00 -0.05 -0.11</td>
</tr>
<tr>
<td>0.3 0.10 0.07 0.04 0.00 -0.04</td>
<td>0.15 0.10 0.05 0.00 -0.06</td>
<td>0.15 0.10 0.05 0.00 -0.06</td>
<td>0.15 0.10 0.05 0.00 -0.06</td>
</tr>
<tr>
<td>0.4 0.14 0.11 0.08 0.04 0.00</td>
<td>0.21 0.16 0.11 0.06 0.00</td>
<td>0.21 0.16 0.11 0.06 0.00</td>
<td>0.21 0.16 0.11 0.06 0.00</td>
</tr>
<tr>
<td>0.5 0.18 0.15 0.12 0.09 0.05</td>
<td>0.27 0.23 0.18 0.13 0.07</td>
<td>0.27 0.23 0.18 0.13 0.07</td>
<td>0.27 0.23 0.18 0.13 0.07</td>
</tr>
</tbody>
</table>

**Note.** The marginal excess burden equals zero when the tax rate corresponds to the corrective tax rate. This is the case where $\tau = \gamma$ for the employed utility function. The parameters underlying the simulations are shown in Table A.1 in the Appendix.

Table 2 shows that for all parameter sets, the marginal excess burden decreases substantially in the status parameter $\gamma$. Moreover, it becomes negative, when the actual tax rate falls short of the corrective tax rate.\textsuperscript{30}

In view of the uncertainties regarding the values for $\gamma$ and $\varepsilon_{cL}$, it is not possible to pinpoint the excess burden associated with any given tax rate $\tau$. However, in light of our assessment that $\gamma$ is likely to fall within the range of .2 to .4, Table 2 suggests that status effects have a significant impact on excess burden. In IA, when the consumption tax rate $\tau$ is 0.4, the excess burden is 0.14 when there are no status effects, but falls to 0.08 or 0 when $\gamma$ is 0.2 or 0.4, respectively. Moreover, the possibility of a negative excess burden is not very sensitive to the compensated elasticity of labor supply. All percentage changes derived from Table 2 for $\varepsilon_{cL} = 0.3$ are quite similar to those for $\varepsilon_{cL} = 0.7$.

\textsuperscript{29}In between are most other estimates of the marginal excess burden, including Ballard et al. (1985), or Campbell (1975).

\textsuperscript{30}The impact of status effects on the percentage change of the marginal excess burden is not very sensitive to the compensated elasticity of labor supply. All percentage changes derived from Table 2 for $\varepsilon_{cL} = 0.3$ are quite similar to those for $\varepsilon_{cL} = 0.7$. 24
burden cannot be ruled out. In this case, “level reversal” occurs: that is, according to Proposition 3, the second-best level of public goods provision exceeds the first best level.

7 Conclusions

This paper addresses analytically and empirically the implications of status effects for the excess burden of a consumption tax and for the optimal levels of public goods provision.

The analytical framework indicates that the excess burden of a consumption tax is reduced by the status externality, because the tax not only serves a revenue-raising purpose but also an externality-correcting purpose. Moreover, for tax rates below the Pigouvian corrective tax rate, the marginal excess burden becomes negative. A negative marginal excess burden implies that the second-best level of optimal public goods provision exceeds the first-best level.

In our empirical investigation we find that a plausible range for the status parameter $\gamma$ is between 0.2 to 0.4. When $\gamma$ is in this range and when utility functions have the CES form, even moderate levels of the status parameter substantially reduce the marginal excess burden. In fact, one cannot rule out the possibility that the marginal excess burden is negative.

The model can be extended to account for other potential interdependencies and related externalities, such as network externalities. Moreover, with minor changes in notation, the model can be applied to a wage tax instead of a consumption tax.

These results raise some general philosophical issues. Even if status effects imply much lower (or negative) excess burdens than would be implied by analyses that assume independent preferences, some might question the normative standing of these results. That is, it can be argued that envy and other concerns about relative position should not form a basis for deciding on tax rates and public good levels. This ethical issue is beyond the scope of this paper. However, we would point out that, to the extent that one puts weight on the criterion of economic efficiency, one seems obliged to take these excess burden results seriously.

We note the following limitations in this analysis. First, households are homogeneous. It would be useful to extend the model to consider cases involving heterogeneous households. In addition, while we have taken some
steps toward estimating the magnitude of status effects, considerable scope remains for developing better estimates. Our analysis relied on studies in which individuals merely indicated which of two states is preferred. A great deal more information could be obtained – and estimates of the status parameter could be much improved – if households were asked for preferences across a range of states in which relative and absolute consumption (or income) were varied systematically.

A related issue regards differing marginal degrees of positionality across consumption goods, in which case a broad-based consumption tax would not generate a first-best outcome. One question for future research then is, under what conditions is the marginal excess burden negative.

Notwithstanding these limitations, we hope this study clarifies the impact of status effects on excess burden and public good provision, both theoretically and empirically, and can contribute to future discussions of tax reform and public goods evaluation.

Appendix

A.1 Derivation of FOC (10). By homotheticity of the ex post Marshallian demand functions and ex post indirect utility function, we know that \( \tilde{c}(\hat{q}, \omega - t; \gamma) = (\omega - t) \tilde{c}(\hat{q}, 1; \gamma) \), \( \tilde{l}(\hat{q}, \omega - t; \gamma) = (\omega - t) \tilde{l}(\hat{q}, 1; \gamma) \), \( \tilde{v}(\hat{q}, \omega - t; \gamma) = (\omega - t) \tilde{v}(\hat{q}, 1; \gamma) \). Define \( \zeta \equiv [1 - (\hat{q} - 1) \tilde{c}(\hat{q}, 1; \gamma)] \). Then, the government budget constraint can be written as:

\[
t(G) = \frac{G}{N \zeta} - \frac{(1 - \zeta)}{\zeta} \omega.
\]

Notice that \( t_G \equiv \left( \frac{\partial t}{\partial G} \right) = 1/(N \zeta) \).

The government chooses \( G \) such as to maximize (indirect) utility:

\[
V = \tilde{v}(\hat{q}, \omega - t(G); \gamma) + g(G; \Psi).
\]

Notice that \( \partial \tilde{v}(\hat{q}, \omega - t(G); \gamma)/\partial G = -t_G \tilde{v}(\hat{q}, 1; \gamma) \). Obviously, \( \partial V/\partial G = 0 \) directly implies (10’).

Equivalently, \( \partial V/\partial G = 0 \) implies

\[
u_2 \left[ \frac{u_1 + u_3}{u_2} \right]_{\{M, q = \hat{q}\}} \tilde{c}(\hat{q}, 1; \gamma) + \tilde{l}(\hat{q}, 1; \gamma) \right] \frac{1}{N \zeta} = g_G(G; \Psi),
\]
which, by Lemma 1, amounts to
\[ u_2 \left[ \hat{c}(\hat{q}, 1; \gamma) + \bar{l}(\hat{q}, 1; \gamma) \right] \frac{1}{N \zeta} = g_G(G; \Psi). \]

The household budget constraint together with homotheticity of ex post demand functions implies: \( \hat{q} \hat{c}(\hat{q}, 1; \gamma) + \bar{l}(\hat{q}, 1; \gamma) = 1. \) Add \( [1 - \hat{q} \hat{c}(\hat{q}, 1; \gamma) - \bar{l}(\hat{q}, 1; \gamma)] \), which equals zero, to the expression in square brackets above:
\[ u_2 [1 - (\hat{q} - 1) \hat{c}(\hat{q}, 1; \gamma)] \frac{1}{N \zeta} = g_G(G; \Psi). \]

Consider the definition of \( \zeta \). Then,
\[ N \frac{g_G(G; \Psi)}{u_2} = 1, \]
which is the first order condition (10).

A.2 Proof of Lemma 2. Conditions (4) to (6) characterize the planner’s outcome. We have to show that (7), (10), (11), and (12), imply (4), (5), and (6). Consider \( q = \hat{q} \), Lemma 1, and \( t = t^* \). Then, We have to prove that the government budget constraint together with the household budget constraint (both at \( \tau = \hat{\tau} \) and \( t = t^* \)) imply the resource constraint. I.e.:
\[
\begin{align*}
[\omega - t^* = \hat{q} \hat{c}(\hat{q}, \omega - t^*; \gamma) + \bar{l}(\hat{q}, \omega - t^*; \gamma)] & \land \\
[t^* + (\hat{q} - 1) \hat{c}(\hat{q}, \omega - t^*; \gamma) = G^*/N] & \Rightarrow \\
[\omega - G^*/N = c(1, \omega - G^*/N) + l(1, \omega - G^*/N)] ,
\end{align*}
\]

In the household budget constraint substitute for the first \( t^* \): \( t^* = G^*/N - (\hat{q} - 1)(\omega - t^*) \hat{c}(\hat{q}, 1; \gamma) \). The household budget constraint becomes:
\[ \omega - G^*/N = \hat{c}(\hat{q}, \omega - t^*; \gamma) + \bar{l}(\hat{q}, \omega - t^*; \gamma). \]

It remains to show that
\[ \hat{c}(\hat{q}, \omega - t^*; \gamma) + \bar{l}(\hat{q}, \omega - t^*; \gamma) = c(1, \omega - G^*/N) + l(1, \omega - G^*/N). \]

The government budget constraint implies: \( \omega - t^* = (\omega - G^*/N)/\zeta \). Therefore, the household budget constraint becomes:
\[ \hat{c}(\hat{q}, \omega - t^*; \gamma) + \bar{l}(\hat{q}, \omega - t^*; \gamma) = \omega - G^*/N = \zeta (\omega - t^*) \text{ thus,} \]
\[ \hat{c}(\hat{q}, 1; \gamma) + \bar{l}(\hat{q}, 1; \gamma) = \zeta. \] (23)
Monotonicity assumptions (A.2) and (A.4) imply that for all prices and incomes, the budget is completely spent:

\[ c(1, \zeta; \gamma) + l(1, \zeta; \gamma) = \zeta. \]  

Equations (23) and (24) together:

\[ \check{c}(\hat{q}, 1; \gamma) + \check{l}(\hat{q}, 1; \gamma) = c(1, \zeta; \gamma) + l(1, \zeta; \gamma), \text{ or,} \]

\[ \check{c}(\hat{q}, \omega - t^*; \gamma) + \check{l}(\hat{q}, \omega - t^*; \gamma) = c(1, \zeta (\omega - t^*); \gamma) + l(1, \zeta (\omega - t^*); \gamma). \]

As \( \zeta (\omega - t^*) = (\omega - G^*/N) \):

\[ \check{c}(\hat{q}, \omega - t^*; \gamma) + \check{l}(\hat{q}, \omega - t^*; \gamma) = c(1, \omega - G^*/N) + l(1, \omega - G^*/N). \]

Q.E.D.

A.3 Derivation of the Excess Burden. We can (implicitly) define the excess burden as follows:

\[ \check{v}(\hat{q}, \omega - t^* - EB - R^n; \gamma) = \check{v}(q, \omega - t^*; \gamma), \]  

where \( R \) is consumption tax revenue, \( R^n \) (net revenue) denotes the consumption tax revenue collected in excess of “corrective tax revenue”, \( \hat{R} \). Formally, \( R^n = R - \hat{R} = (q - 1) \check{c}(q, \omega - t^*; \gamma) - \hat{R} \), where we define \( \hat{R} \) as the solution to \( \hat{R} - (\hat{q} - 1) \check{c}(\hat{q}, \omega - t^* - R^n - EB; \gamma) = 0. \)

Denote the expenditure function by \( e(.). \) Then:

\[ e(\hat{q}, \check{v}(\hat{q}, \omega - t^* - EB - R^n; \gamma); \gamma) = \omega - t^* - EB - R^n. \]

By (25),

\[ EB(q, \omega - t^*; \gamma) = (\omega - t^*) - e(\hat{q}, \check{v}(\hat{q}, \omega - t^*; \gamma); \gamma) - R^n. \]

Notice that, by homotheticity, \( e(\hat{q}, \check{v}(q, \omega - t^*; \gamma); \gamma) = (\omega - t^*) e(\hat{q}, \check{v}(q, 1; \gamma); \gamma) \), and \( R^n(q, \omega - t^*; \gamma) = (\omega - t^*) R^n(q, 1; \gamma) \). Thus, \( EB(q, \omega - t^*; \gamma) = (\omega - t^*) EB(q, 1; \gamma) \), and so, \( EB(q, 1; \gamma) = 1 - e(\hat{q}, \check{v}(q, 1; \gamma); \gamma) - R^n(q, 1; \gamma) \). Therefore, we can define the excess burden for any given income \( \omega \) as:

\[ EB(q, \omega; \gamma) = \omega - e(\hat{q}, \check{v}(q, \omega; \gamma); \gamma) - R^n(q, \omega; \gamma). \]
A.4 Proof of Proposition 1. Define $\Delta \equiv [\tilde{c}_h(q, 1; \gamma) + \tilde{l}_h(q, 1; \gamma) - \tilde{c}_h(\hat{q}, 1; \gamma) - \tilde{l}_h(\hat{q}, 1; \gamma)]$. Then, by (18), the excess burden is:

$$EB(q, \omega; \gamma) = \tilde{v}(q, \omega; \gamma) \Delta,$$

and

$$\frac{d EB(q, \omega; \gamma)}{d \gamma} = \tilde{v}_\gamma(q, \omega; \gamma) \frac{EB(q, \omega; \gamma)}{\tilde{v}(q, \omega; \gamma)} + \tilde{v}(q, \omega; \gamma) \Delta_\gamma.$$

As status effects represent a negative externality, $\tilde{v}(q, \omega; \gamma) < 0$. Moreover, both the excess burden and indirect utility are nonnegative. The first term on the right hand side, thus, is negative. It remains to show that $\Delta_{\gamma|q}>\hat{q}$ is negative as well. Observe that $\tilde{c}_h > 0$, as every consumer needs to rise consumption to keep utility constant at unity. Moreover, $\frac{1}{2} + \frac{1}{3} = 0$. Lemma 1 implies: $(u_1 + u_3)/u_2 \geq 1 \iff q \geq \hat{q}$. 

$$d \Delta d \gamma = \tilde{c}_h(q, 1; \gamma) + \tilde{l}_h(q, 1; \gamma) = \tilde{c}_h(q, 1; \gamma) - \frac{u_1 + u_3}{u_2} \tilde{c}_h(q, 1; \gamma)$$

$$= \tilde{c}_h(q, 1; \gamma) \left( 1 - \frac{u_1 + u_3}{u_2} \right) \geq 0 \iff q \geq \hat{q}.$$

Q.E.D.

A.5 Proof of Proposition 2. The sign of the marginal excess burden is equal to the sign of $d EB/d q$ (see Footnote 13). Without loss of generality, we assume: $v(.) > 0$. We distinguish two main cases: $q \leq \hat{q}$ (Case 1), and $q > \hat{q}$ (Case 2). We proceed as follows.

Step 1. Show $MEB \leq 0$ if $q \leq \hat{q}$ (Case 1).

Step 2. Develop a general condition for $MEB > 0$ when $q > \hat{q}$ (Case 2).

Step 3. Show $MEB > 0$ when $q > \hat{q}$.

**Step 1.** Show $MEB \leq 0$ when $q \leq \hat{q}$.

$$\frac{d EB(q, \omega; \gamma)}{d q} = \tilde{v}_\gamma(q, \omega; \gamma) \frac{EB(q, \omega; \gamma)}{\tilde{v}(q, \omega; \gamma)} + \tilde{v}(q, \omega; \gamma) \Delta_q,$$

where $\Delta$ is defined as in the proof of Proposition 1. Indirect utility is non-increasing in $q$: $\tilde{v}_\gamma \leq 0$. That is, the first expression on the right hand side above is nonpositive. As $v(.) > 0$, we need to show that $\Delta_q \leq 0$ when $q \leq \hat{q}$.

First, we note that $u(.)$ is twice continuously differentiable and strictly concave. As shown by Dierker (1982, p. 573), it follows that compensated
consumption is strictly decreasing in $q$: $\tilde{c}_q^b < 0$. Next, Lemma 1 implies: 

\[(u_1 + u_3)/u_2 \geq 1 \Leftrightarrow \hat{q} \geq 0.\]

Therefore, $\Delta$ is decreasing in $q$ when $q < \hat{q}$, and it does not change in $q$ if $q = \hat{q}$:

\[
\Delta_q = \tilde{c}_q^b(q, 1; \gamma) + \tilde{h}_q^b(q, 1; \gamma) = \tilde{c}_q^b(q, 1; \gamma) - (u_1 + u_3)/u_2 \tilde{c}_q^b(q, 1; \gamma)
\]

\[
= \tilde{c}_q^b(q, 1; \gamma) \left(1 - \frac{u_1 + u_3}{u_2}\right) \geq 0 \Leftrightarrow q \geq \hat{q}.
\]

Thus, if $q < \hat{q}$, $\text{MEB} < 0$. If $q = \hat{q}$, $\text{MEB} = 0$, as $\text{EB}|_{q=\hat{q}} = 0$.

**Step 2. Develop a general condition for $\text{MEB} > 0$, when $q > \hat{q}$.**

For $q > \hat{q}$, we want to demonstrate that:

\[
\text{EB}_q = v_q(.) \Delta(q) + v(.) \Delta_q(.) = v(.) \left[\frac{v_q(.)}{v(.)} \Delta(q) + \Delta_q(q)\right] > 0.
\]

We introduce the unit expenditure function: $b(q) \equiv e(q, 1; \gamma) = q \tilde{c}_q^b(q, 1; \gamma) + \tilde{h}_q^b(q, 1; \gamma)$. As $e(q, v(q, \omega; \gamma); \gamma) \equiv \omega$, we know that $v(q, \omega; \gamma) e(q, 1; \gamma) \equiv \omega$ (by homotheticity), thus, $v(q, \omega; \gamma) b(q) \equiv \omega$. Therefore, we know that $v_q(q, \omega; \gamma) b(q) + v(q, \omega; \gamma) b'(q) = 0$, and $v_\omega(q, \omega; \gamma) b(q) = 1$. From these considerations, it follows that we need to show:

\[
-\frac{b'(q)}{b(q)} \Delta(q) + \Delta_q(q) > 0 \Leftrightarrow \Delta_q(q) q b(q)/q > b'(q) \Delta(q).
\]

(26)

**Step 3. Show $\text{MEB} > 0$ when $q > \hat{q}$.**

(i) As $b(q)$ is strictly concave, it follows that $b(q)/q > b'(q)$. As $q > \hat{q}$, it follows: $b(q)/(q - \hat{q}) > b'(q)$. Thus, a sufficient condition for (26) to hold is:

\[
\Delta_q(q) q \geq \Delta(q) , q > \hat{q}.
\]

(27)

If $\Delta(q)$ is (weakly) convex for $q \geq \hat{q}$, then (27) holds. To see this, notice that $\Delta(\hat{q}) = 0$. Convexity is given by:

\[
\frac{\Delta(q) - \Delta(\hat{q})}{q - \hat{q}} \leq \Delta_q(q) , q > \hat{q}.
\]

(28)

The left hand side of (28) amounts to:

\[
\frac{\Delta(q) - \Delta(\hat{q})}{q - \hat{q}} = \frac{\Delta(q)}{q - \hat{q}} > \frac{\Delta(q)}{q} , q > \hat{q}.
\]
Thus, (28) implies $\Delta_q(q) > \Delta(q)/q$ (by convexity), which implies sufficient condition (27). In the following steps, we show that $\Delta(q)$ is convex and, therefore, sufficient condition (27) holds. 

(ii) Let $dq \equiv q - \hat{q} > 0$. Then, $\Delta(q)$ is convex if for all $q > \hat{q}$:

$$
\Delta(q) - \Delta(q) \geq \Delta_q(q) dq .
$$

Also, define $dc \equiv \hat{c}^h(q) - \hat{c}(\hat{q})$, and $dl \equiv \hat{l}^h(q) - \hat{l}(\hat{q})$. By definition of $\Delta(q)$, 

$$
\Delta(q) - \Delta(q) = dc + dl .
$$

Therefore, convexity of $\Delta(q)$ amounts to:

$$
dc + dl \geq \Delta_q(q) dq , q > \hat{q} .
$$

(iii) From $u(.) \equiv 1$, define $i(\hat{c}) \equiv u^{-1}(\hat{c}, 1, \hat{c}; \gamma)$, which is a strictly convex indifference curve by strict convexity of preferences. Along the indifference curve: $\hat{l} = i(\hat{c})$, and $dl > i_c(.) dc$ (by strict convexity). Thus, at $\hat{q}$:

$$
dc + dl > dc[1 + i_c(\hat{c}(\hat{q}))] = \frac{dc}{dq}[1 + i_c(\hat{c}(\hat{q}))] dq .
$$

By definition of $\Delta(q)$:

$$
\Delta_q(q) dq = \frac{d(c(\hat{q}) + l(\hat{q}))}{dq} dq = \frac{dc}{dq}[1 + i_c(\hat{c}(\hat{q}))] dq .
$$

From (29) and (30) it follows: $dc + dl > \Delta_q(q) dq$, for $q > \hat{q}$. Therefore, we conclude that $\Delta(q)$ is convex, the sufficient condition (27) holds, thus, $MEB > 0$ when $q > \hat{q}$.

A.6 Proof of Proposition 3. According to (10'), the first best level of public goods provision is given by

$$
g_G^G(N, \gamma) = \frac{\tilde{v}(\hat{q}, 1; \gamma)}{N \zeta} ,
$$

where $\zeta$ is defined as in the derivation of FOC (10) above.

From (16) we know that $\tilde{v}(q, \omega; \gamma) = \tilde{v}(\hat{q}, \omega - EB - R + \hat{R}; \gamma)$. Moreover, 

$$
\hat{R} = (\hat{q} - 1) \hat{c}(\hat{q}, \omega - EB - R + \hat{R}; \gamma) = (\hat{q} - 1)(\omega - EB - R) \hat{c}(\hat{q}, 1; \gamma) + (\hat{q} - 1) \hat{R} \hat{c}(\hat{q}, 1; \gamma) \>(by\ homotheticity\ of\ the\ ex-post\ demand\ function).\ Using\ \zeta,\ 
$$

$$
\hat{R} = (1 - \zeta) / \zeta (\omega - EB - R),\ thus,\ \omega - EB - R + \hat{R} = (\omega - EB - R) / \zeta .
$$

It follows $\tilde{v}(q, \omega; \gamma) = \tilde{v}(\hat{q}, \omega - EB - R; \gamma) / \zeta$ (using homotheticity again). The government’s problem, in the second-best case, therefore becomes:

$$
\{q, G\} \equiv \arg\max_{q, G} \{\tilde{v}(\hat{q}, \omega - EB - R; \gamma) / \zeta + g(G; \Psi) | N R = G\} .
$$
Notice that \( R_q > 0 \) by assumption (A.7), and \( MEB = EB_q/R_q \) (see Footnote 13). Thus, the government’s problem gives rise to the following first order condition:

\[
g_G(G^{**}; \Psi) = \frac{\hat{v}(\hat{q}, 1; \gamma)}{N\zeta} [1 + MEB(q, \omega; \gamma)]. \tag{32}
\]

From (31) and (32) follows that \( G^{**} \gtrless G^* \Leftrightarrow MEB(q, \omega; \gamma) \lesssim 0 \). Q.E.D.

**A.7 Proof of Proposition 4.** The first order condition \((10')\) determines the optimal level of public goods provision. Notice that the government budget constraint implies: \((\omega - t^* \zeta) = \omega - G^*/N\). Thus, \((10')\) can be written as:

\[
N g_G(G; \Psi) = \frac{\hat{v}(\hat{q}, 1; \gamma)}{\zeta} = \frac{(\omega - t^*) \hat{v}(\hat{q}, 1; \gamma)}{(\omega - t^*) \zeta} = \frac{\omega - G^*/N}{\omega - G^*/N} \frac{\hat{v}(1, \omega - G^*/N; 0)}{\omega - G^*/N} = \hat{v}(1, 1; 0).
\]

Observe that the right hand side of the first order condition is independent of both \( \gamma \) and \( \Psi \). Implicit differentiation of the first order condition with respect to \( \Psi \) and \( \gamma \) yields:

\[
\frac{d\Psi}{d\gamma} \bigg|_{MEB=0} = -\frac{g_{G,G}(G; \Psi) N [R_q|_{\eta=\hat{q}} \hat{q}_\gamma + (\hat{q} - 1) \tilde{c}_\gamma(\hat{q}, \omega; \gamma)]}{g_{G,\Psi}(G; \Psi)} > 0. \tag{33}
\]

Observe that \( g_{G,G}(G; \Psi) < 0, g_{G,\Psi}(G; \Psi) > 0 \) (from (A.6)), \( R_q > 0 \) (by (A.7)), \( \tilde{c}_\gamma > 0 \), and the corrective tax rate increases in \( \gamma, (\hat{q}_\gamma > 0) \). Thus, \( (d\Psi)/(d\gamma)\bigg|_{MEB=0} > 0 \). I.e., (33) implicitly defines a relationship: \( \Psi = \Psi(\gamma) \).

Finally, \( \Psi(0) = 0 \). As along \( \Psi = \Psi(\gamma), MEB = 0 \), we know that \( G^* = G^{**} \). At \( \gamma = 0, G^* = G^{**} = (\hat{q} - 1) \tilde{c}(\hat{q}, \omega; 0) = (1 - 1)\tilde{c}(1, \omega; 0) = 0 \), which is obviously fulfilled for \( \Psi = 0 \) only. Q.E.D.

**A.8 Benchmark Data.**

<table>
<thead>
<tr>
<th>Table A.1</th>
<th>Calibrated Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_{cL} )</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>0.7</td>
<td>0.0</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Note. \( \alpha = 1, \tau = 0.2, (\omega - l)/\omega = 0.4. \)

References


