Chained Financial Frictions and Credit Cycles: Technical Appendix

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Appendix A. Proofs

Proof of Proposition 1

As borrowers' marginal product of capital equals one in the steady state, we restrict our analysis to the impact of ξ on mpk^I:

$$\frac{\partial mpk^I}{\partial \xi} = \frac{\partial mpk^I}{\partial R^B}\frac{\partial R^B}{\partial \xi}. \tag{1}$$

As for the partial derivative of bankers' marginal product of capital with respect to the loan rate:

$$\frac{\partial mpk^I}{\partial R^B} = -\frac{\varkappa \omega \beta^B}{\kappa^2 R^S \beta^I}, \tag{2}$$

where $\kappa \equiv R^F\left(1-\beta^F\right) - \omega\left(1-\beta^F R^F\right) > 0$ and $\varkappa \equiv R^S\left(1-\beta^I\right) - \chi\left(1-\beta^I R^S\right) > 0$, so that $\partial mpk^I/\partial R^B < 0$.

As for $\partial R^B/\partial \xi < 0$, this is negative, in light of assuming $\beta^I R^S < 1$:

$$\frac{\partial R^B}{\partial \xi} = -\frac{\chi\left(1-\beta^I R^S\right)}{\beta^I R^S}. \tag{3}$$

Thus, both factors on the right-hand side of (1) are negative and, since $\partial\Delta/\partial\xi = -\partial mpk^I/\partial\xi$, increasing ξ inevitably reduces the productivity gap.■

Proof of Proposition 2

We first prove that increasing ξ attenuates the impact of the technology shock on borrowers' capital-holdings. According to Equation (35) in the main text, v quantifies the pass-through of $\hat{\alpha}_t$ on $\hat{k}_t^B$. In turn, the marginal impact of ξ on v can be computed as:

$$\frac{\partial v}{\partial \xi} = \frac{(\lambda-\phi)(1-\rho)\rho}{(1-\lambda)(1-\phi\rho)}\frac{\partial \eta}{\partial \xi} + \frac{(\lambda\rho-1)(1-\rho)\rho}{(1-\phi\rho)^2}\frac{\partial \phi}{\partial \xi}, \tag{4}$$

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where:

$$\frac{\partial \eta}{\partial \xi} = \frac{\partial \eta}{\partial k^B}\frac{\partial k^B}{\partial R^B}\frac{\partial R^B}{\partial \xi} \quad \text{and} \quad \frac{\partial \phi}{\partial \xi} = \frac{\partial \phi}{\partial R^B}\frac{\partial R^B}{\partial \xi}. \tag{5}$$

Focusing on the second term on the right-hand side of (4), we can show this is negative, as: (i) $\frac{(\lambda\rho-1)(1-\rho)\rho}{(1-\phi\rho)^2} < 0$, given that $\lambda\rho < 1$; (ii) $\partial\phi/\partial R^B = -\omega/\left(R^B\right)^2 < 0$; (iii) $\partial R^B/\partial\xi < 0$, as implied by (3).

As for the first term on the right-hand side of (4): $\frac{(\lambda-\phi)(1-\rho)\rho}{(1-\lambda)(1-\phi\rho)} > 0$. Furthermore:

$$\frac{\partial \eta}{\partial k^B} = -\frac{1}{(1-\mu)\left(k^B\right)^2} < 0$$

and

$$\frac{\partial k^B}{\partial R^B} = \frac{\omega}{\kappa R^B(\mu-1)}\left(\frac{1}{\mu}\frac{R^B\beta^B\varkappa}{R^S\beta^I\kappa}\right)^{\frac{1}{\mu-1}} < 0, \tag{6}$$

where $\kappa \equiv R^B\left(1-\beta^B\right) - \omega\left(1-\beta^B R^B\right)$ and $\varkappa \equiv R^S\left(1-\beta^I\right) - \chi\left(1-\beta^I R^S\right)$. As $\partial R^B/\partial\xi < 0$, also the first term on the right-hand side of (4) is negative. Therefore, ν is a negative function of ξ.

As for the impact of technology shocks on the capital price:

$$\frac{\partial \gamma}{\partial \xi} = \frac{\partial \gamma}{\partial \phi}\frac{\partial \phi}{\partial \xi}. \tag{7}$$

As for $\partial\gamma/\partial\phi$:

$$\frac{\partial \gamma}{\partial \phi} = -\frac{1-\rho}{(1-\phi\rho)^2}\rho < 0, \tag{8}$$

while we already know that $\partial\phi/\partial\xi > 0$. Therefore, the overall effect of ξ on γ is negative.■

Proof of Proposition 3

We know that $G'(k^I)$ is a decreasing function of θ. Thus, we aim to prove that the gap between bankers' and borrowers' marginal product of capital is greater than zero at $\theta = 0$. To this end, we combine the capital Euler equations of bankers and borrowers, obtaining:

$$G'(k^I)\Big|_{\theta=0} = \frac{R^B\beta^B\left(R^S-1\right)}{\left(1-\beta^B\right)R^B - \omega\left(1-\beta^B R^B\right)}.$$

We then impose $G'(k^I)\Big|_{\theta=0} < 1$ to obtain

$$R^B > \frac{\omega}{1-\beta^B\left(R^S-\omega\right)}.$$

As $R^B\big|_{\theta=0} = \frac{R^S\left(1+\beta^I\right)-1}{\beta^I}$, all we need to prove is that

$$\frac{R^S\left(1+\beta^I\right)-1}{\beta^I} > \frac{\omega}{1-\beta^B\left(R^S-\omega\right)},$$

which can be manipulated to obtain

$$\left(1-\beta^B R^S\right)\left[\beta^I\left(R^S-\omega\right)+R^S-1\right]+\left(R^S-1\right)\beta^B\omega>0.$$

As $\beta^B R^S < 1$, it is immediate to verify that both terms on the left-hand side of the last inequality are positive.

■

Appendix B. The Model with Capital Requirements

Preliminary, note that combining the capital requirement with the definition of equity returns the following constraint: $b_t^S \leq q_t k_t^I + (1-\theta)\, b_t^B$. Thus, the Lagrangian for bankers' optimization may be written as:

$$\begin{aligned}\mathcal{L}_t^I &= E_0 \sum_{t=0}^{\infty} \left(\beta^I\right)^t \left\{c_t^I - \vartheta_t^I [c_t^I + R^S b_{t-1}^S + b_t^B + q_t(k_t^I - k_{t-1}^I) \right. \\ & \quad \left. - b_t^S - R^B b_{t-1}^B - \alpha_t G(k_{t-1}^I)] - \delta_t \left[b_t^S - q_t k_t^I - (1-\theta)\, b_t^B\right]\right\}, \end{aligned} \tag{9}$$

where $\vartheta_t^I = 1$ and δ_t are the multipliers associated with bankers' budget constraint and the capital requirement constraint, respectively. The first-order conditions are:

$$\frac{\partial \mathcal{L}_t^I}{\partial b_t^S} = 0 \Rightarrow -R^S \beta^I E_t \vartheta_{t+1}^I + \vartheta_t^I - \delta_t = 0; \tag{10}$$

$$\frac{\partial \mathcal{L}_t^I}{\partial b_t^B} = 0 \Rightarrow R^B \beta^I E_t \vartheta_{t+1}^I - \vartheta_t^I + (1-\theta)\,\delta_t = 0; \tag{11}$$

$$\frac{\partial \mathcal{L}_t^I}{\partial k_t^I} = 0 \Rightarrow -\vartheta_t^I q_t + \beta^I E_t\left[\vartheta_{t+1}^I q_{t+1}\right] + \beta^I E_t\left[\vartheta_{t+1}^I \alpha_{t+1} G^{'}(k_t^I)\right] + \delta_t q_t = 0. \tag{12}$$

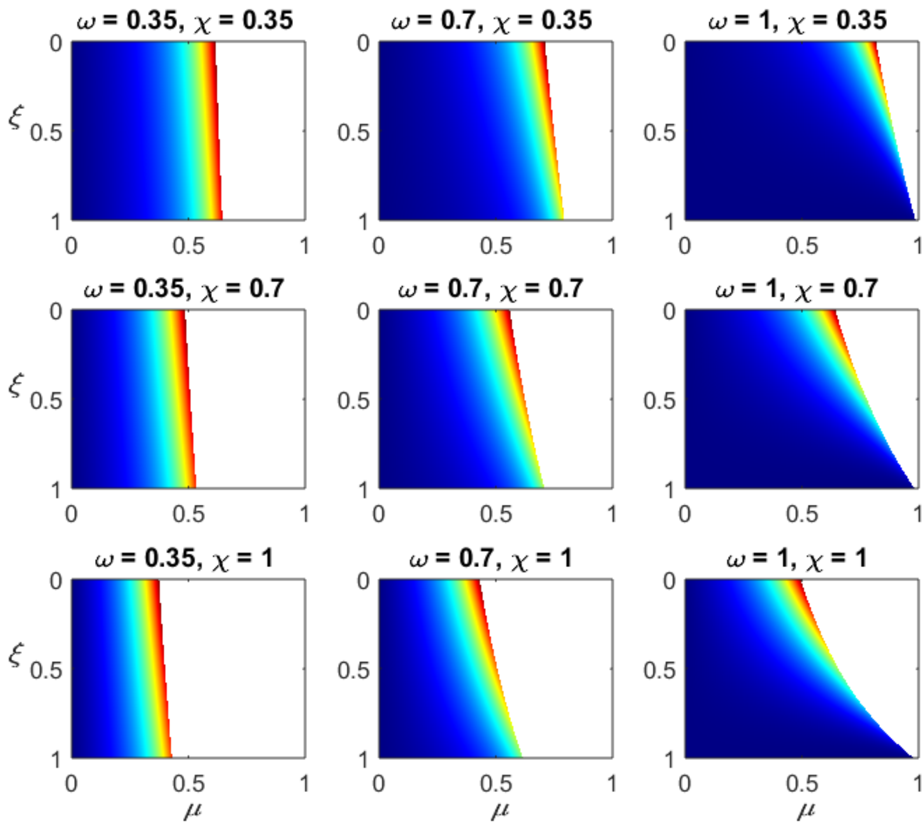
In light of these conditions we can derive expressions for both R_t^B and q_t in the presence of a binding capital requirement constraint:

$$R^B = \frac{R^S - (1-\theta)\left(1-\beta^I R^S\right)}{\beta^I}, \tag{13}$$

$$q_t = \frac{1}{R^S} E_t q_{t+1} + \frac{1}{R^S} E_t\left[\alpha_{t+1} I^{'}(k_t^I)\right]. \tag{14}$$

Appendix C. Robustness Exercises

Figure C.1 Business cycle amplification.



Notes. Figure C.1 graphs ϖ as a function of ξ (y-axis) and μ (x-axis), and for different values of χ and ω, under the following parameterization: $\beta^S = 0.99$, $\beta^I = 0.98$, $\beta^B = 0.97$, $\rho = 0.95$. The white area denotes inadmissible equilibria where bankers' capital-holdings are virtually negative.