Monetary Policy with Sectoral Trade-offs
Technical Appendix

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APPENDIX A: First Order Conditions from Households' Utility Maximization

Maximizing (1) subject to (3), (4), (5), and (6) leads to a set of first-order conditions that can be re-arranged to obtain:

$$\begin{aligned}
\mu_n H_t^{1-\sigma}\left(C_t^n\right)^{-1} &= \beta R_t E_t\left[\frac{\mu_n H_{t+1}^{1-\sigma}\left(C_{t+1}^n\right)^{-1}}{\Pi_{t+1}^n}\right], && \text{(1a)}\\
\frac{\mu_n H_t^{1-\sigma} P_t^d}{C_t^n P_t^n} &= E_t\left\{\beta\left(1-\delta\right)\mu_n\frac{H_{t+1}^{1-\sigma}P_{t+1}^d}{C_{t+1}^n P_{t+1}^n}+\right. && \text{(1b)}\\
&\quad \left.+\frac{\mu_d H_t^{1-\sigma}}{\mathfrak{D}_t\left[1-\frac{\Xi}{D}\left(D_t-D_{t-1}\right)\right]^{-1}}+\beta\frac{\Xi}{D}\frac{\mu_d H_{t+1}^{1-\sigma}}{\mathfrak{D}_{t+1}\left(D_{t+1}-D_t\right)^{-1}}\right\}, &&\\
W_t^n\frac{\mu_n H_t^{1-\sigma}\left(C_t^n\right)^{-1}}{P_t^n} &= \varrho\phi^{-\frac{1}{\lambda}}L_t^{v-\frac{1}{\lambda}}\left(L_t^n\right)^{\frac{1}{\lambda}}, && \text{(1c)}\\
W_t^d\frac{\mu_n H_t^{1-\sigma}\left(C_t^n\right)^{-1}}{P_t^n} &= \varrho\left(1-\phi\right)^{-\frac{1}{\lambda}}L_t^{v-\frac{1}{\lambda}}\left(L_t^d\right)^{\frac{1}{\lambda}}. && \text{(1d)}
\end{aligned}$$

APPENDIX B: Some Useful Steady State Relationships

As in the competitive equilibrium, real wage in each sector equals the marginal product of labor. Thus, we can derive the following relationship between the production of non-durables and that of durables in the steady state:

$$\frac{Y^n}{Y^d}=\frac{\alpha_{Ld}\phi}{\alpha_{Ln}\left(1-\phi\right)}Q^{-1}.$$

Furthermore, the following relationship between durable and non-durable consumption can be derived from the Euler conditions:

$$\frac{C^n}{C^d}=\left(1-\beta\left(1-\delta\right)\right)\frac{\mu_n}{\mu_d}\frac{1}{\delta}Q^{-1}.$$

Moreover, the following shares of consumption and intermediate goods over total production are determined for the non-durable goods sector:

$$\begin{aligned}
\frac{M^{nn}}{Y^n} &= \alpha_{Mn}\gamma_{nn},\\
\frac{M^{nd}}{Y^n} &= \frac{\alpha_{Ln}}{\alpha_{Ld}}\frac{1-\phi}{\phi}\alpha_{Md}\gamma_{nd},\\
\frac{S^n O^n}{Y^n} &= \alpha_{On},\\
\frac{C^n}{Y^n} &= 1-\frac{M^{nn}}{Y^n}-\frac{M^{nd}}{Y^n}-\frac{S^n O^n}{Y^n}.
\end{aligned}$$

Analogously, for the durable goods sector:

$$
\begin{aligned}
\frac{M^{dn}}{Y^d} &= \frac{\alpha_{Ld}}{\alpha_{Ln}}\frac{\phi}{1-\phi}\alpha_{Mn}\gamma_{dn}, \\
\frac{M^{dd}}{Y^d} &= \alpha_{Md}\gamma_{dd}, \\
\frac{S^d O^d}{Y^d} &= \alpha_{Od}, \\
\frac{C^d}{Y^d} &= 1-\frac{M^{dn}}{Y^d}-\frac{M^{dd}}{Y^d}-\frac{S^d O^d}{Y^d}.
\end{aligned}
$$

These conditions prove to be crucial in the second-order approximation of consumers' utility to eliminate the linear terms. Moreover, they allow us to derive the steady state ratio of labor supply in the non-durable goods sector over the total labor supply (ϕ). To this end, we take the ratio between the following equations:

$$
\begin{aligned}
C^n &= \left(1-\beta\left(1-\delta\right)\right)\frac{\mu_n}{\mu_d}\frac{1}{\delta}Q^{-1}C^d, \\
Y^n &= \frac{\alpha_{Ld}\phi}{\alpha_{Ln}\left(1-\phi\right)}Q^{-1}Y^d.
\end{aligned}
$$

Thus:

$$
\frac{C^n}{Y^n} = \left(1-\beta\left(1-\delta\right)\right)\frac{\alpha_{Ln}\left(1-\phi\right)}{\alpha_{Ld}\phi}\frac{\mu_n}{\mu_d}\frac{1}{\delta}\frac{C^d}{Y^d}
$$

which returns a coherent value of ϕ.

The Relative Price in the Steady State

We start from the steady-state condition for the marginal cost in the non-durable goods sector:

$$
MC^n = \bar{\phi}_n\left[\left(P^n\right)^{\gamma_{nn}}\left(P^d\right)^{\gamma_{dn}}\right]^{\alpha_{Mn}}\left(W^n\right)^{\alpha_{Ln}}S^{\alpha_{On}}.
$$

where $\bar{\phi}_n = \frac{1}{\alpha_{Mn}^{\alpha_{Mn}}\alpha_{Ln}^{\alpha_{Ln}}\alpha_{On}^{\alpha_{On}}}$. As in the steady state the production subsidies neutralize distortions due to imperfect competition:

$$
P^n = MC^n.
$$

After some trivial manipulations it can be shown that:

$$
\bar{\phi}_n Q^{-\alpha_{Mn}\gamma_{dn}}\left(RW^n\right)^{\alpha_{Ln}}\left(S^n\right)^{\alpha_{On}} = 1. \tag{2}
$$

Analogously, for the durable goods sector:

$$
\bar{\phi}_d Q^{\alpha_{Md}\gamma_{nd}}\left(RW^d\right)^{\alpha_{Ld}}\left(S^d\right)^{\alpha_{Od}} = 1. \tag{3}
$$

Using the fact that $S^n/S^d = 1/Q$:

$$
\frac{\left(\bar{\phi}_n\right)^{\frac{1}{\alpha_{On}}}Q^{-\frac{\alpha_{Mn}\gamma_{dn}}{\alpha_{On}}}\left(RW^n\right)^{\frac{\alpha_{Ln}}{\alpha_{On}}}}{\left(\bar{\phi}_d\right)^{\frac{1}{\alpha_{Od}}}Q^{\frac{\alpha_{Md}\gamma_{nd}}{\alpha_{Od}}+1}\left(RW^d\right)^{\frac{\alpha_{Ld}}{\alpha_{Od}}}} = 1
$$

Moreover, as in the steady state $W^n = W^d = W$, $RW^d = RW^n Q$. This allows us to find a closed form expression for Q.

APPENDIX C: Relative Price in the Efficient Equilibrium with Perfect labor Mobility

We now define the efficient equilibrium in the model with no frictions in both the goods and the labor market. On the labor market this condition, obtained for $\lambda \to \infty$, ensures that nominal salaries are equalized across sectors of the economy:

$$W_t^{n^*} = W_t^{d^*} = W_t^*. \tag{4}$$

Moreover, given the production subsidies that eliminate sectoral distortions due to monopolistic competition:

$$P_t^{n^*} = MC_t^{n^*} \qquad P_t^{d^*} = MC_t^{d^*}. \tag{5}$$

Conditions (4) and (5) imply that:

$$\bar{\phi}_n \frac{(Q_t^*)^{-\alpha_{Mn}\gamma_{dn}} (W_t^*)^{\alpha_{Ln}} \left(\frac{1}{P_t^{*n}}\right)^{\alpha_{Ln}} (S_t^{*n})^{\alpha_{On}}}{Z_t^n} = 1, \tag{6}$$

$$\bar{\phi}_d \frac{(Q_t^*)^{\alpha_{Md}\gamma_{nd}} (W_t^*)^{\alpha_{Ld}} \left(\frac{1}{P_t^{*d}}\right)^{\alpha_{Ld}} \left(S_t^{*d}\right)^{\alpha_{Od}}}{Z_t^d} = 1 \tag{7}$$

We then use both conditions to eliminate W_t^*:

$$(Q_t^*)^{\frac{\alpha_{Mn}\gamma_{dn}}{\alpha_{Ln}} + \frac{\alpha_{Md}\gamma_{nd}}{\alpha_{Ld}} + 1} = \frac{\left(\bar{\phi}_n \frac{(S_t^{*n})^{\alpha_{On}}}{Z_t^n}\right)^{\frac{1}{\alpha_{Ln}}}}{\left(\bar{\phi}_d \frac{\left(S_t^{*d}\right)^{\alpha_{Od}}}{Z_t^d}\right)^{\frac{1}{\alpha_{Ld}}}}$$

Proof of Proposition 1

Suppose there were a monetary policy under which the equilibrium allocation under sticky prices would be Pareto optimal. Then, in such an equilibrium, the gaps would be completely closed for every period. That is, $\widetilde{rmc}_t^n = \widetilde{rmc}_t^d = 0$, $\forall t$. It follows from the pricing conditions that $\pi_t^n = \pi_t^d = 0$, $\forall t$. The relative price evolves as:

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^*.$$

Since we also have that $\Delta \widetilde{q}_t = 0$, the equation above implies that $\pi_t^n - \pi_t^d = \Delta q_t^*$. From the analysis above:

$$q_t^* = \frac{1}{1+\varkappa} \left[\frac{1}{\alpha_{Ld}} z_t^d - \frac{1}{\alpha_{Ln}} z_t^n + \frac{\alpha_{On}}{\alpha_{Ln}} (s_t - p_t^{*n}) - \frac{\alpha_{Od}}{\alpha_{Ld}} \left(s_t - p_t^{*d}\right) \right],$$

Therefore, it cannot be that $\pi_t^n = \pi_t^d = 0$, unless $\Delta q_t^* = 0$, which translates into:

$$\frac{z_t^d - \alpha_{Od} s_t^{*d}}{z_t^n - \alpha_{On} s_t^{*n}} = \frac{\alpha_{Ld}}{\alpha_{Ln}}.$$

APPENDIX D: Equilibrium Dynamics in the Efficient Equilibrium

This appendix details the linearized system in the efficient equilibrium:

$$
\begin{aligned}
c_t^{d*} &= \frac{1}{\delta} d_t^* - \frac{1-\delta}{\delta} d_{t-1}^*, && (8)\\
c_t^{n*} &= \frac{1}{\gamma} r_t^* + E_t c_{t+1}^{n*} + \frac{(1-\sigma)\,\mu_d}{\gamma} E_t \Delta d_{t+1}^* && (9)\\
c_t^{n*} &= \frac{1}{\mu_n (1-\sigma)} \Big\{ \left[1-\mu_d (1-\sigma)\right] d_t^* + \frac{1}{1-\beta(1-\delta)} \left[\left(\mu_n (1-\sigma) - 1\right) c_t^{n*} + \mu_d (1-\sigma)\, d_t^* - q_t^*\right] + \\
& \quad - \frac{(1-\delta)\,\beta}{[1-\beta(1-\delta)]} \left[\left(\mu_n (1-\sigma) - 1\right) E_t c_{t+1}^{n*} + \mu_d (1-\sigma)\, E_t d_{t+1}^* - E_t q_{t+1}^*\right] + \\
& \quad + \Xi \left(d_t^* - d_{t-1}^*\right) - \beta \Xi \left(E_t d_{t+1}^* - d_t^*\right) \Big\} && (10)\\
rw_t^{n*} &= -\gamma c_t^{n*} - (1-\sigma)\,\mu_d d_t^* + \left(\vartheta \phi + \frac{1}{\lambda}\right) l_t^{n*} + \vartheta (1-\phi)\, l_t^{d*}, && (11)\\
l_t^{n*} &= \lambda \left(rw_t^{n*} - rw_t^{d*} + q_t^*\right) + l_t^{d*}, && (12)\\
y_t^{n*} &= z_t^n + \alpha_{Mn} \gamma_{nn} m_t^{nn*} + \alpha_{Mn} \gamma_{dn} m_t^{dn*} + \alpha_{Ln} l_t^{n*} + \alpha_{On} o_t^{n*}, && (13)\\
y_t^{d*} &= z_t^d + \alpha_{Md} \gamma_{nd} m_t^{nd*} + \alpha_{Md} \gamma_{dd} m_t^{dd*} + \alpha_{Ld} l_t^{d*} + \alpha_{Od} o_t^{d*}, && (14)\\
y_t^{n*} &= \frac{C^n}{Y^n} c_t^{n*} + \frac{M^{nn}}{Y^n} m_t^{nn*} + \frac{M^{nd}}{Y^n} m_t^{nd*} + \frac{S^n O^n}{Y^n} \left(s_t^{n*} + o_t^{n*}\right), && (15)\\
y_t^{d*} &= \frac{C^d}{Y^d} c_t^{d*} + \frac{M^{dn}}{Y^d} m_t^{dn*} + \frac{M^{dd}}{Y^d} m_t^{dd*} + \frac{S^d O^d}{Y^d} \left(s_t^{d*} + o_t^{d*}\right), && (16)\\
0 &= rw_t^{n*} + l_t^{n*} - y_t^{n*}, && (17)\\
0 &= rw_t^{d*} + l_t^{d*} - y_t^{d*}, && (18)\\
0 &= m_t^{nn*} - y_t^{n*}, && (19)\\
0 &= m_t^{nd*} + q_t^* - y_t^{d*}, && (20)\\
0 &= m_t^{dn*} - q_t^* - y_t^{n*}, && (21)\\
0 &= m_t^{dd*} - y_t^{d*}, && (22)\\
0 &= s_t^{n*} + o_t^{n*} - y_t^{n*}, && (23)\\
0 &= s_t^{d*} + o_t^{d*} - y_t^{d*} && (24)\\
q_t^* &= s_t^{d*} - s_t^{n*} && (25)\\
o_t &= \frac{O^n}{O} o_t^{n*} + \frac{O^d}{O} o_t^{d*} && (26)\\
z_t^n &= \rho^{z^n} z_{t-1}^n + u_t^{z^n}, \qquad u_t^{z^n} \overset{i.i.d.}{\sim} \left(0, \sigma^{z^n}\right) && (27)\\
z_t^d &= \rho^{z^d} z_{t-1}^d + u_t^{z^d}, \qquad u_t^{z^d} \overset{i.i.d.}{\sim} \left(0, \sigma^{z^d}\right) && (28)\\
o_t &= \rho^o o_{t-1} + u_t^o, \qquad u_t^o \overset{i.i.d.}{\sim} \left(0, \sigma^o\right) && (29)
\end{aligned}
$$

where $\vartheta \equiv \left(v - \frac{1}{\lambda}\right)$ and $\gamma \equiv (1-\sigma)\,\mu_n - 1$.

APPENDIX E: Log-linear Economy

Here we report the log-linear economy in extensive form:

$$\begin{aligned}
\widetilde{c}_t^n &= \frac{1}{\gamma}\left(\widehat{r}_t - E_t\pi_{t+1}^n - rr_t^*\right) + E_t\widetilde{c}_{t+1}^n + \frac{(1-\sigma)\,\mu_d}{\gamma}E_t\Delta\widetilde{d}_{t+1}, &(30)\\
\widetilde{c}_t^n &= \frac{1}{\mu_n\,(1-\sigma)}\left\{\left[1-\mu_d\,(1-\sigma)\right]\widetilde{d}_t + \frac{1}{1-\beta\,(1-\delta)}\left[\left(\mu_n\,(1-\sigma)-1\right)\widetilde{c}_t^n + \mu_d\,(1-\sigma)\,\widetilde{d}_t - \widetilde{q}_t\right] + \right.\\
&\quad -\frac{(1-\delta)\,\beta}{\left[1-\beta\,(1-\delta)\right]}\left[\left(\mu_n\,(1-\sigma)-1\right)E_t\widetilde{c}_{t+1}^n + \mu_d\,(1-\sigma)\,E_t\widetilde{d}_{t+1} - E_t\widetilde{q}_{t+1}\right] + &(31)\\
&\quad \left. +\Xi\left(\widetilde{d}_t - \widetilde{d}_{t-1}\right) - \beta\Xi\left(E_t\widetilde{d}_{t+1} - \widetilde{d}_t\right)\right\}\\
\widetilde{c}_t^d &= \frac{1}{\delta}\widetilde{d}_t - \frac{1-\delta}{\delta}\widetilde{d}_{t-1}, &(32)\\
\widetilde{rw}_t^n &= -\gamma\widetilde{c}_t^n - (1-\sigma)\,\mu_d\widetilde{d}_t + \vartheta\,(1-\phi)\,\widetilde{l}_t^d + \left(\vartheta\phi + \frac{1}{\lambda}\right)\widetilde{l}_t^n, &(33)\\
\widetilde{l}_t^n &= \lambda\left(\widetilde{rw}_t^n - \widetilde{rw}_t^d + \widetilde{q}_t\right) + \widetilde{l}_t^d, &(34)\\
\pi_t^n &= \beta E_t\pi_{t+1}^n + \frac{(1-\beta\theta_n)\,(1-\theta_n)}{\theta_n}\widetilde{rmc}_t^n + \eta_t^n, &(35)\\
\pi_t^d &= \beta E_t\pi_{t+1}^d + \frac{(1-\beta\theta_d)\,(1-\theta_d)}{\theta_d}\widetilde{rmc}_t^d + \eta_t^d, &(36)\\
\widetilde{y}_t^n &= \alpha_{Mn}\gamma_{nn}\widetilde{m}_t^{nn} + \alpha_{Mn}\gamma_{dn}\widetilde{m}_t^{dn} + \alpha_{Ln}\widetilde{l}_t^n + \alpha_{On}\widetilde{o}_t^n, &(37)\\
\widetilde{y}_t^d &= \alpha_{Md}\gamma_{nd}\widetilde{m}_t^{nd} + \alpha_{Md}\gamma_{dd}\widetilde{m}_t^{dd} + \alpha_{Ld}\widetilde{l}_t^d + \alpha_{Od}\widetilde{o}_t^d, &(38)\\
\widetilde{y}_t^n &= \frac{C^n}{Y^n}\widetilde{c}_t^n + \frac{M^{nn}}{Y^n}\widetilde{m}_t^{nn} + \frac{M^{nd}}{Y^n}\widetilde{m}_t^{nd} + \frac{S^nO^n}{Y^n}\left(\widetilde{s}_t^n + \widetilde{o}_t^n\right), &(39)\\
\widetilde{y}_t^d &= \frac{C^d}{Y^d}\widetilde{c}_t^d + \frac{M^{dn}}{Y^d}\widetilde{m}_t^{dn} + \frac{M^{dd}}{Y^d}\widetilde{m}_t^{dd} + \frac{S^dO^d}{Y^d}\left(\widetilde{s}_t^d + \widetilde{o}_t^d\right), &(40)\\
\widetilde{rmc}_t^n &= \widetilde{rw}_t^n + \widetilde{l}_t^n - \widetilde{y}_t^n, &(41)\\
\widetilde{rmc}_t^d &= \widetilde{rw}_t^d + \widetilde{l}_t^d - \widetilde{y}_t^d, &(42)\\
\widetilde{rmc}_t^n &= \widetilde{m}_t^{nn} - \widetilde{y}_t^n, &(43)\\
\widetilde{rmc}_t^d &= \widetilde{m}_t^{nd} + \widetilde{q}_t - \widetilde{y}_t^d, &(44)\\
\widetilde{rmc}_t^n &= \widetilde{m}_t^{dn} - \widetilde{q}_t - \widetilde{y}_t^n, &(45)\\
\widetilde{rmc}_t^d &= \widetilde{m}_t^{dd} - \widetilde{y}_t^d, &(46)\\
\widetilde{rmc}_t^n &= \widetilde{s}_t^n + \widetilde{o}_t^n - \widetilde{y}_t^n &(47)\\
\widetilde{rmc}^d &= \widetilde{s}_t^d + \widetilde{o}_t^d - \widetilde{y}_t^d &(48)\\
\widetilde{q}_t &= \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^* &(49)\\
0 &= \frac{O^n}{O}\widetilde{o}_t^n + \frac{O^d}{O}\widetilde{o}_t^d &(50)\\
\widetilde{q}_t &= \widetilde{s}_t^d - \widetilde{s}_t^n &(51)\\
&\eta_t^n \overset{i.i.d.}{\sim} \left(0, \sigma^{\eta^n}\right)\\
&\eta_t^d \overset{i.i.d.}{\sim} \left(0, \sigma^{\eta^d}\right)
\end{aligned}$$

where $\gamma = (1-\sigma)\,\mu_n - 1$ and

$$\begin{aligned}
\frac{O^n}{O} &= \frac{\alpha_{On}\alpha_{Ld}\phi}{\alpha_{Ld}\phi\alpha_{On} + \alpha_{Ln}\,(1-\phi)\,\alpha_{Od}},\\
\frac{O^d}{O} &= \frac{\alpha_{Ln}\,(1-\phi)\,\alpha_{Od}}{\alpha_{Ld}\phi\alpha_{On} + \alpha_{Ln}\,(1-\phi)\,\alpha_{Od}}.
\end{aligned}$$

APPENDIX F: Second-order Approximation of the Utility Function

Following Woodford (2003), we derive a well-defined welfare function from the utility function of the representative household:

$$\mathcal{W}_t = U\left(C_t^n, D_t\right) - V\left(L_t\right).$$

We start from a second-order approximation of the utility from consumption of durable and non-durable goods:

$$\begin{aligned} U\left(C_t^n, D_t\right) \approx\ & U\left(C^n, D\right) + U_{C^n}\left(C^n, D\right)\left(C_t^n - C^n\right) + \frac{1}{2}U_{C^nC^n}\left(C^n, D\right)\left(C_t^n - C^n\right)^2 && (52)\\ & + U_D\left(C^n, D\right)\left(D_t - D\right) + \frac{1}{2}U_{DD}\left(C^n, D\right)\left(D_t - D\right)^2 + \frac{1}{2}\Xi U_D\left(C^n, D\right)\left(D_t - D_{t-1}\right)^2 \\ & + U_{C^nD}\left(C^n, D\right)\left(C_t^n - C^n\right)\left(D_t - D\right) + O\left(\|\xi\|^3\right), && (53) \end{aligned}$$

where $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher. Notice that:

$$\begin{aligned} U_D\left(C^n, D\right) &= \left(\mu_d C^n / \mu_n D\right) U_{C^n}\left(C^n, D\right), \\ U_{C^nC^n}\left(C^n, D\right) &= \left[\mu_n\left(1-\sigma\right) - 1\right]\left(C^n\right)^{-1} U_{C^n}\left(C^n, D\right), \\ U_{DD}\left(C^n, D\right) &= \left[\mu_d\left(1-\sigma\right) - 1\right]\left(\mu_d C^n / \mu_n D\right) U_{C^n}\left(C^n, D\right), \\ U_{C^nD}\left(C^n, D\right) &= \mu_d\left(1-\sigma\right) D^{-1} U_{C^n}\left(C^n, D\right). \end{aligned}$$

As $\frac{C_t^n - C^n}{C^n} = \widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2$, where $\widehat{c}_t^n = \log\left(\frac{C_t^n}{C^n}\right)$ is the log-deviation from steady state under sticky prices, we obtain:

$$\begin{aligned} U\left(C_t^n, D_t\right) \approx\ & U\left(C^n, D\right) + U_{C^n}\left(C^n, D\right) C^n \left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right] + \\ & + \frac{1}{2}\left[\mu_n\left(1-\sigma\right) - 1\right] U_{C^n}\left(C^n, D\right) C \left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right]^2 + \\ & + U_D\left(C^n, D\right) D \left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right) + \frac{1}{2}\left[\mu_d\left(1-\sigma\right) - 1\right] U_D\left(C^n, D\right) D \left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right)^2 + \\ & + \frac{1}{2}\Xi U_D\left(C^n, D\right) D \left(\widehat{d}_t - \widehat{d}_{t-1}\right)^2 + \\ & + \mu_d\left(1-\sigma\right) U_{C^n}\left(C^n, D\right) C^n \left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right]\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right) + \text{t.i.p.} + O\left(\|\xi\|^3\right), \end{aligned}$$

where t.i.p. collects terms independent of policy stabilization.

Next, we introduce a second-order approximation to the transition law for the stock of durables. This will substitute out the linear term for durables in the expression above (see Erceg and Levin, 2006). The law of motion reads as:

$$D_t = \left(1-\delta\right) D_{t-1} + C_t^d.$$

For a general function $F\left(Y, X\right)$ the second-order Taylor approximation can be written as:

$$\begin{aligned} F\left(Y, X\right) \approx\ & F_Y\left(Y, X\right) Y y + F_X\left(Y, X\right) X x + \frac{1}{2}\left(F_X\left(Y, X\right) X + F_{XX}\left(Y, X\right) X^2\right) x^2 \\ & + \frac{1}{2}\left(F_Y\left(Y, X\right) Y + F_{YY}\left(Y, X\right) Y^2\right) y^2 + F_{YX}\left(Y, X\right) Y X x y. \end{aligned}$$

Now, we can rewrite the accumulation equation as:

$$F(D_{t-1}, C_t^d) = \log\left[(1-\delta)\, D_{t-1} + C_t^d\right].$$

Therefore:

$$\begin{aligned}
F_D &= \frac{(1-\delta)}{(1-\delta)\, D + C^d} = \frac{(1-\delta)}{(1-\delta)\, D + \delta D} = \frac{(1-\delta)}{D}, \\
F_{C^d} &= \frac{1}{(1-\delta)\, D + C^d} = \frac{1}{D}, \\
F_{DD} &= -\frac{(1-\delta)^2}{\left[(1-\delta)\, D + C^d\right]^2} = -\frac{(1-\delta)^2}{D^2}, \\
F_{C^dC^d} &= -\frac{1}{\left[(1-\delta)\, D + C^d\right]^2} = -\frac{1}{D^2}, \\
F_{DC^d} &= -\frac{1-\delta}{\left[(1-\delta)\, D + C^d\right]^2} = -\frac{1-\delta}{D^2}.
\end{aligned}$$

Considering that in the steady state $C^d = \delta D$:

$$\begin{aligned}
\widehat{d}_t &\approx \frac{(1-\delta)}{D} D\widehat{d}_{t-1} + \frac{1}{D}\delta D\widehat{c}_t^d + \\
&\quad + \frac{1}{2}\left[\frac{(1-\delta)}{D} D - \frac{(1-\delta)^2}{D^2} D^2\right]\widehat{d}_{t-1}^2 + \\
&\quad + \frac{1}{2}\left(\frac{1}{D} D - \frac{1}{D^2} D^2\right)\left(\widehat{c}_t^d\right)^2 - \frac{1-\delta}{D^2}\widehat{d}_{t-1}\widehat{x}_t \\
&\approx (1-\delta)\,\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \frac{(1-\delta)\,\delta}{2}\widehat{d}_{t-1}^2 + \frac{(1-\delta)\,\delta}{2}\left(\widehat{c}_t^d\right)^2 - \frac{(1-\delta)\,\delta}{2}\widehat{c}_t^d\widehat{d}_{t-1} \\
&\approx (1-\delta)\,\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \frac{(1-\delta)\,\delta}{2}\left(\widehat{d}_{t-1} - \widehat{c}_t^d\right)^2.
\end{aligned}$$

Thus:

$$\widehat{d}_t \approx (1-\delta)\,\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \psi_t, \tag{54}$$

where:

$$\begin{aligned}
\widehat{\psi}_t &= \frac{(1-\delta)\,\delta}{2}\left(\widehat{c}_t^d - \widetilde{d}_{t-1}\right)^2 \\
&= \frac{(1-\delta)}{2\delta}\left(\widehat{d}_t - \widetilde{d}_{t-1}\right)^2.
\end{aligned}$$

Now, let us iterate backward (54), to obtain:

$$\textstyle\sum_{t=0}^{\infty}\beta^t\widehat{d}_t = \frac{1}{1-\beta\,(1-\delta)}d_0 + \sum_{t=0}^{\infty}\beta^t\left[\frac{\delta}{1-\beta\,(1-\delta)}\widehat{c}_t^d + \frac{1}{1-\beta\,(1-\delta)}\widehat{\psi}_t\right].$$

In turn, the term on the RHS will replace the one on the LHS into the intertemporal loss function.

The next step is to derive a second-order approximation for labor disutility. Recall that:

$$\widehat{l}_t = \phi\widehat{l}_t^n + (1-\phi)\,\widehat{l}_t^d.$$

Therefore the second-order approximation reads:

$$\begin{aligned} V(L_t) &\approx V_L(L)\,L\left[\phi\widehat{l}_t^n + (1-\phi)\,\widehat{l}_t^d + \frac{\phi(1+2v\phi)}{2}\left(\widehat{l}_t^n\right)^2 + \frac{(1-\phi)\,[1+2v(1-\phi)]}{2}\left(\widehat{l}_t^d\right)^2\right] + \\ &\quad + \text{t.i.p.} + O\left(\|\xi\|^3\right). \end{aligned}$$

After these preliminary steps, we need to find an expression for $\widehat{l}_t^n$ and $\widehat{l}_t^d$. Given the definition of the marginal cost, in equilibrium we get:

$$\begin{aligned} L_t^n &= \frac{\alpha_{Ln} MC_t^n}{W_t^n}\int_0^1 Y_{jt}^n dj = \alpha_{Ln}\frac{Q_t^{-\alpha_{Mn}\gamma_{dn}}\,(RW_t^n)^{\alpha_{Ln}-1}\,(S_t^n)^{\alpha_{On}}}{Z_t^n} Y_t^n \int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj, \\ L_t^d &= \frac{\alpha_{Ld} MC_t^d}{W_t^d}\int_0^1 Y_{kt}^d dk = \alpha_{Ld}\frac{Q_t^{\alpha_{Md}\gamma_{nd}}\,\left(RW_t^d\right)^{\alpha_{Ld}-1}\,\left(S_t^d\right)^{\alpha_{Od}}}{Z_t^d} Y_t^d \int_0^1 \left(\frac{P_{kt}^d}{P_t^d}\right)^{-\varepsilon_t^d} dk. \end{aligned}$$

Thus, we can report the linear approximation of the expressions above:

$$\begin{aligned} \widehat{l}_t^n &= -\alpha_{Mn}\gamma_{dn}\widehat{q}_t + (\alpha_{Ln}-1)\,\widehat{rw}_t^n + \alpha_{On}\widehat{s}_t^n - z_t^n + \widehat{y}_t^n + X_{nt}, \\ \widehat{l}_t^d &= \alpha_{Md}\gamma_{nd}\widehat{q}_t + (\alpha_{Ld}-1)\,\widehat{rw}_t^d + \alpha_{Od}\widehat{s}_t^d - z_t^d + \widehat{y}_t^d + X_{dt}, \end{aligned}$$

where:

$$X_{nt} = \log\left[\int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj\right] \qquad X_{dt} = \log\left[\int_0^1 \left(\frac{P_{kt}^d}{P_t^d}\right)^{-\varepsilon_t^d} dk\right] \tag{55}$$

If we set $\widehat{p}_{jt}^n$ to be the log-deviation of $\frac{P_{jt}^n}{P_t^n}$ from its steady state, which means that a second-order Taylor expansion of $\int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj$ reads as:

$$\begin{aligned} \int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj &\approx \int_0^1 \left[1 - \varepsilon^n\widehat{p}_{jt}^n - \varepsilon^n\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n + \frac{1}{2}(\varepsilon^n)^2\left(\widehat{p}_{jt}^n\right)^2\right] dj + O\left(\|\xi\|^3\right) \\ &= 1 - \varepsilon^n\mathbf{E}_i\widehat{p}_{jt}^n - \varepsilon^n\mathbf{E}_i\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n + \frac{1}{2}(\varepsilon^n)^2\,\mathbf{E}_i\left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right), \end{aligned}$$

where $\mathbf{E}_i\widehat{p}_{jt}^n \equiv \int_0^1 \widehat{p}_{jt}^n dj$ and $\mathbf{E}_i\left(\widehat{p}_{jt}^n\right)^2 \equiv \int_0^1 \left(\widehat{p}_{jt}^n\right)^2 dj$. At this stage, we need an expression for $\mathbf{E}_i\widehat{p}_{jt}^n$. Let us start from

$$P_t^n = \left[\int_0^1 \left(P_{jt}^n\right)^{1-\varepsilon_t^n} dj\right]^{\frac{1}{1-\varepsilon_t^n}},$$

which can be re-arranged as:

$$1 \equiv \int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n} dj.$$

Following the procedure above, it can be shown that:

$$\left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n} \approx 1 + (1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2}(1-\varepsilon^n)^2 \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Substituting this into the preceding equations yields:

$$0 = \int_0^1 \left[(1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2}(1-\varepsilon^n)^2 \left(\widehat{p}_{jt}^n\right)^2\right] dj + O\left(\|\xi\|^3\right),$$

which reduces to:

$$\mathbf{E}_i \widehat{p}_{jt}^n = \frac{\varepsilon^n - 1}{2} \mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Thus:

$$\int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj = 1 + \frac{\varepsilon^n}{2} \mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Now, notice that:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 = \mathbf{E}_i \left[\left(p_{jt}^n\right)^2 - 2 p_{jt}^n p_t^n + \left(p_t^n\right)^2\right] + O\left(\|\xi\|^3\right),$$

where lower case letters denote the log-value of the capital letters. Here we can use a first-order approximation of $p_t^n = \int_0^1 p_{jt}^n dj$, as this term is multiplied by other first-order terms each time it appears. With this, we have a second-order approximation:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 \equiv var_j p_{jt}^n.$$

Therefore, the second-order approximation can be represented as:

$$X_{nt} = \frac{\varepsilon^n}{2} var_j p_{jt}^n + O\left(\|\xi\|^3\right).$$

Analogous steps in the sector producing durable goods lead us to:

$$X_{dt} = \frac{\varepsilon^d}{2} var_k p_{kt}^d + O\left(\|\xi\|^3\right).$$

Following Woodford (2003, Ch. 6, Proposition 6.3), we can obtain a correspondence between cross-sectional price dispersions in the two sectors and their inflation rates:

$$\begin{aligned} var_j p_{jt}^n &= \theta_n var_j p_{jt-1}^n + \frac{\theta_n}{1-\theta_n} \left(\pi_t^n\right)^2 + O\left(\|\xi\|^3\right), \\ var_k p_{kt}^d &= \theta_d var_k p_{kt-1}^d + \frac{\theta_d}{1-\theta_d} \left(\pi_t^d\right)^2 + O\left(\|\xi\|^3\right). \end{aligned}$$

Iterating these expressions forward leads to:

$$\sum_{t=0}^{\infty}\beta^t var_j p_{jt}^n = (\kappa_n)^{-1}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^n\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{56}$$

$$\sum_{t=0}^{\infty}\beta^t var_k p_{kt}^d = (\kappa_d)^{-1}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^d\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{57}$$

where

$$\begin{aligned}\kappa_n &= \frac{(1-\beta\theta_n)(1-\theta_n)}{\theta_n},\\ \kappa_d &= \frac{(1-\beta\theta_d)(1-\theta_d)}{\theta_d}.\end{aligned}$$

After these preliminary steps, we can write $\mathcal{W}_t$ as:

$$\begin{aligned}\mathcal{W}_t \approx\ & U_{C^n}(C^n,D)\,C^n\left\{\widehat{c}_t^n + \frac{1}{2}\left[\mu_n(1-\sigma)\right]\left(\widehat{c}_t^n\right)^2 + (\mu_d/\mu_n)\,\widehat{d}_t + \right.\\ & \left. + \frac{1}{2}\left[\mu_d(1-\sigma)\right](\mu_d/\mu_n)\,\widehat{d}_t^2 + \mu_d(1-\sigma)\,\widehat{c}_t^n\widehat{d}_t + \frac{1}{2}\Xi\,(\mu_d/\mu_n)\left(\widehat{d}_t - \widehat{d}_{t-1}\right)^2\right\} + \\ & - V_L(L)\,L\left\{\phi\widehat{l}_t^n + (1-\phi)\,\widehat{l}_t^d + \right.\\ & \left. + \left(\frac{1+v}{2}\right)\left[\phi^2\left(\widehat{l}_t^n\right)^2 + (1-\phi)\left(\widehat{l}_t^d\right)^2 + 2\phi(1-\phi)\,\widehat{l}_t^n\widehat{l}_t^d\right]\right\} + \\ & + \text{t.i.p.} + O\left(\|\xi\|^3\right).\end{aligned}$$

We now consider the linear terms in $\mathcal{W}_t$, which are collected under $\mathcal{LW}_t$:

$$\begin{aligned}\mathcal{LW}_t =\ & \frac{U_{C^n}(C^n,D)\,C^n}{\mu_n}\left\{\mu_n\widehat{c}_t^n + \mu_d\widehat{d}_t\right\} + \\ & - V_L(L)\,L\left\{\phi\left(-\alpha_{Mn}\gamma_{dn}\widehat{q}_t + (\alpha_{Ln}-1)\,\widehat{rw}_t^n + \alpha_{On}\widehat{s}_t^n + \widehat{y}_t^n\right) + \right.\\ & \left. + (1-\phi)\left(\alpha_{Md}\gamma_{nd}\widehat{q}_t + (\alpha_{Ld}-1)\,\widehat{rw}_t^d + \alpha_{Od}\widehat{s}_t^d + \widehat{y}_t^d\right)\right\} + \\ & + \text{t.i.p.} + O\left(\|\xi\|^2\right).\end{aligned}$$

We then substitute for the real wages from marginal cost expressions, so as to get:

$$\begin{aligned}\mathcal{LW}_t =\ & \frac{U_{C^n}(C^n,D)\,C^n}{\mu_n}\left\{\mu_n\widehat{c}_t^n + \mu_d\widehat{d}_t\right\} + \\ & - \frac{V_L(L)\,L\phi}{\alpha_{Ln}}\left(\widehat{y}_t^n - \alpha_{Mn}\gamma_{nn}\widehat{m}_t^{nn} - \alpha_{Mn}\gamma_{dn}\widehat{m}_t^{dn} - \alpha_{On}\widehat{o}_t^n\right) + \\ & - \frac{V_L(L)\,L(1-\phi)}{\alpha_{Ld}}\left(\widehat{y}_t^d - \alpha_{Md}\gamma_{nd}\widehat{m}_t^{nd} - \alpha_{Md}\gamma_{dd}\widehat{m}_t^{dd} - \alpha_{Od}\widehat{o}_t^d\right) + \\ & + \text{t.i.p.} + O\left(\|\xi\|^2\right).\end{aligned}$$

After iterating forward and substituting the second-order approximation for the accumulation equation of durables we get:

$$\begin{aligned}\mathcal{LW}_t &= U_{C^n}(C^n,D)\,C^n\left\{\widehat{c}_t^n+\frac{\delta}{1-\beta(1-\delta)}\frac{\mu_d}{\mu_n}\widehat{c}_t^d+\frac{1}{1-\beta(1-\delta)}\mu_d\psi_t\right\}+ \\ &\quad -V_L(L)\,L\left\{\frac{\phi}{\alpha_{Ln}}\left(\widehat{y}_t^n-\alpha_{Mn}\gamma_{nn}\widehat{m}_t^{nn}-\alpha_{Mn}\gamma_{dn}\widehat{m}_t^{dn}-\alpha_{On}\widehat{o}_t^n\right)+\right. \\ &\quad \left.+\frac{(1-\phi)}{\alpha_{Ld}}\left(\widehat{y}_t^d-\alpha_{Md}\gamma_{nd}\widehat{m}_t^{nd}-\alpha_{Md}\gamma_{dd}\widehat{m}_t^{dd}-\alpha_{Od}\widehat{o}_t^d\right)\right\}+ \\ &\quad +\text{t.i.p.}+O\left(\|\xi\|^2\right).\end{aligned}\tag{58}$$

We now employ the following steady-state relationships:

$$\begin{aligned}V_{L^n}(L^n)\,L^n &= \phi V_L(L)\,L \\ V_{L^d}\left(L^d\right)L^d &= (1-\phi)\,V_L L(L) \\ \frac{-V_{L^n}(L^n)}{U_{C^n}(C^n)} &= \frac{Y^n\alpha_{Ln}}{L^n} \\ \frac{-V_{L^d}\left(L^d\right)}{U_{C^n}(C^n)} &= \frac{Y^d\alpha_{Ld}}{L^dQ} \\ U_H(H)\,H &= \frac{U_{C^n}(C^n,D)\,C^n}{\mu_n}\end{aligned}$$

So as to get:

$$\begin{aligned}\mathcal{LW}_t &= U_H(H)\,H\left\{\begin{array}{c}\mu_n\widehat{c}_t^n+\frac{\delta\mu_d}{1-\beta(1-\delta)}\widehat{c}_t^d \\ -\frac{Y^n}{C^n}\mu_n\left(\widehat{y}_t^n-\alpha_{Mn}\gamma_{nn}\widehat{m}_t^{nn}-\alpha_{Mn}\gamma_{dn}\widehat{m}_t^{dn}-\alpha_{On}\widehat{o}_t^n\right)+ \\ -\frac{Y^d}{C^d}\frac{C^c}{C^n}\mu_n\left(\widehat{y}_t^d-\alpha_{Md}\gamma_{nd}\widehat{m}_t^{nd}-\alpha_{Md}\gamma_{dd}\widehat{m}_t^{dd}-\alpha_{Od}\widehat{o}_t^d\right)+\end{array}\right\} \\ &\quad +\text{t.i.p.}+O\left(\|\xi\|^2\right).\end{aligned}$$

where the term $\frac{1}{1-\beta(1-\delta)}\mu_d\psi_t$ has been included among the non-linear elements of $\mathcal{W}_t$. It is now possible to show that $\mathcal{LW}_t=0$, using the following steady-state relationships:

$$\begin{aligned}C^n &= (1-\beta(1-\delta))\frac{\mu_n}{\mu_d}\frac{1}{\delta}C^d, \\ Y^n &= \frac{\alpha_{Ld}\phi}{\alpha_{Ln}(1-\phi)}Y^d,\end{aligned}$$

as well as the linearized conditions that express the gross production in the two sectors.

After dropping the linear terms in $\mathcal{W}_t$ we are left with:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{W}_t &\approx U_H(H)\,H\sum_{t=0}^{\infty}\beta^t\left\{\frac{1-\sigma}{2}\left(\mu_n\widehat{c}_t^n+\mu_d\widehat{d}_t\right)^2+\frac{1}{1-\beta(1-\delta)}\mu_d\widehat{\psi}_t+\frac{\mu_d}{2}\Xi\left(\widehat{d}_t-\widehat{d}_{t-1}\right)^2+\right. \\ &\quad -\frac{\Theta}{2}\left[\phi\varepsilon^n(\kappa_n)^{-1}\left(\pi_t^n\right)^2+(1-\phi)\,\varepsilon^d(\kappa_d)^{-1}\left(\pi_t^d\right)^2\right]+ \\ &\quad \left.-\left(\frac{1+v}{2}\right)\Theta^{-1}\left[\mu_n\widehat{c}_t^n+\frac{\delta\mu_d}{1-\beta(1-\delta)}\widehat{c}_t^d\right]^2\right\}+ \\ &\quad +\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

where

$$\Theta = \left(\frac{C^n}{Y^n}\right)^{-1} \frac{\alpha_{Ln}\mu_n}{\phi} = \frac{\mu_n \left[1 - \beta\left(1-\delta\right)\right] + \mu_d \delta}{1 - \beta\left(1-\delta\right)}.$$

We next consider the deviation of social welfare from its Pareto-optimal level:

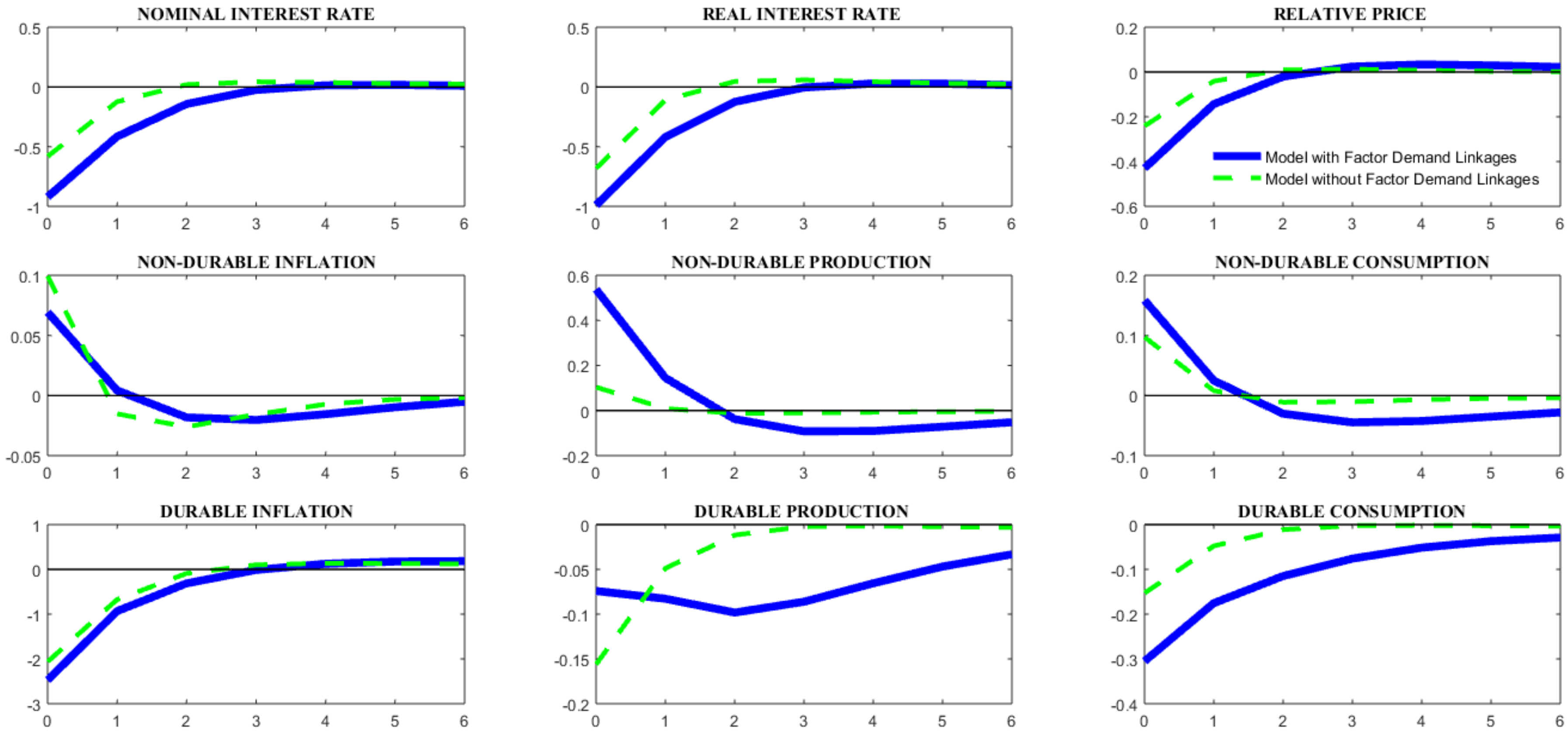
$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \widetilde{\mathcal{W}}_t &= \sum_{t=0}^{\infty} \beta^t \left(\mathcal{W}_t - \mathcal{W}_t^*\right) \approx \\
& -\frac{U_H\left(H\right) H}{2} \Theta \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma - 1}{\Theta} \left(\mu_n \widetilde{c}_t^n + \mu_d \widetilde{d}_t\right)^2 + \right. \\
& + \left[\mu_d \Theta^{-1} \Xi + \left(1-\delta\right)\left(1-\omega\right)\delta^{-2}\right] \left(\widetilde{d}_t - \widetilde{d}_{t-1}\right)^2 + \\
& \left. + \varsigma \left[\varpi \left(\pi_t^n\right)^2 + \left(1-\varpi\right)\left(\pi_t^d\right)^2\right] + \left(1+v\right)\left[\omega \widetilde{c}_t^n + \left(1-\omega\right)\widetilde{c}_t^d\right]^2 \right\} + \text{t.i.p.} + O\left(\|\xi\|^3\right),
\end{aligned}$$

where the following notation has been introduced:

$$\begin{aligned}
\omega &= \frac{\mu_n \left[1 - \beta\left(1-\delta\right)\right]}{\mu_n \left[1 - \beta\left(1-\delta\right)\right] + \mu_d \delta}, \\
\varpi &= \frac{\phi \varepsilon^n \left(\kappa_n\right)^{-1}}{\varsigma}, \\
\varsigma &= \phi \frac{\varepsilon^n}{\kappa_n} + \left(1-\phi\right) \frac{\varepsilon^d}{\kappa_d}.
\end{aligned}$$

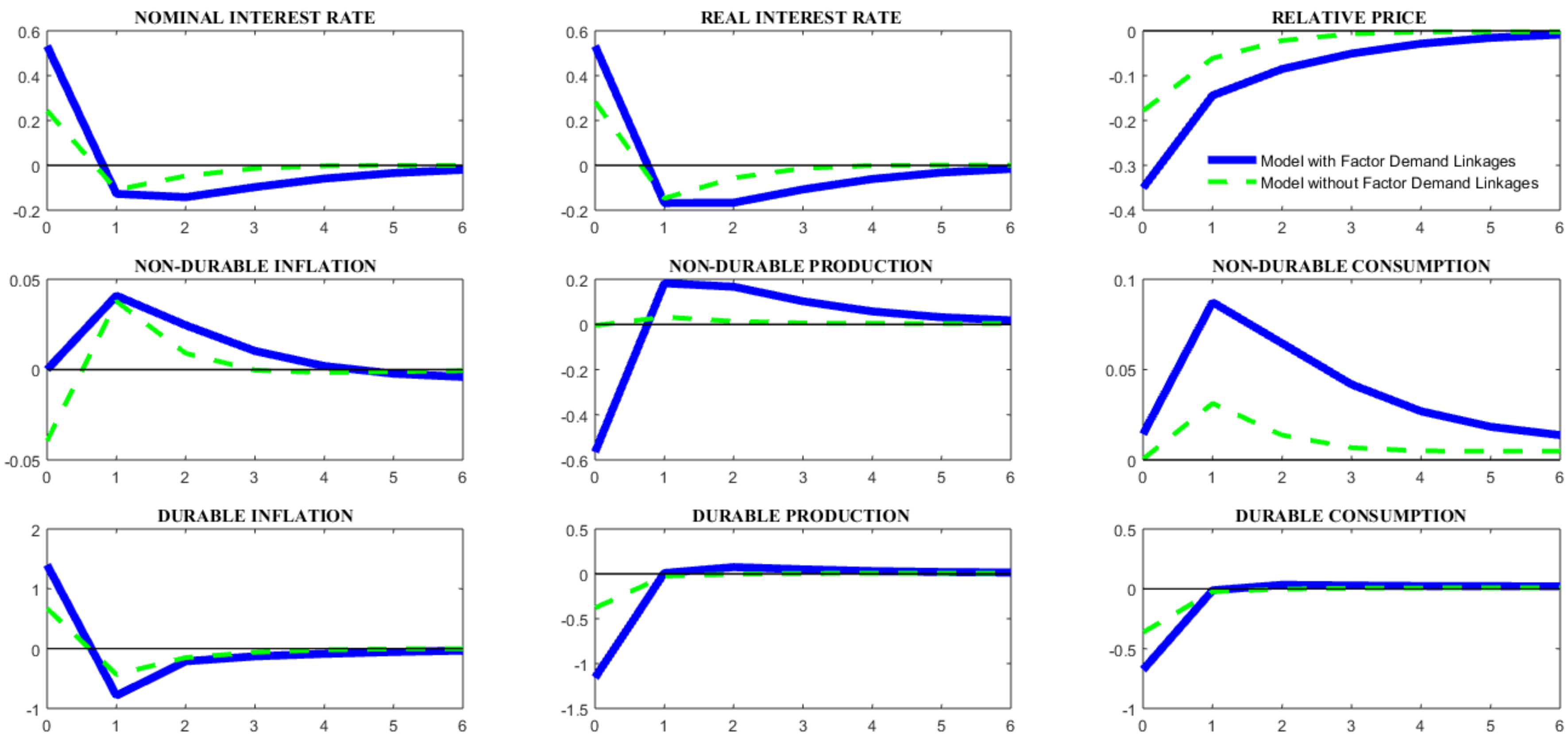
APPENDIX G: Impulse-responses to Shocks in the Durable Goods Sector

FIGURE 1.G: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE DURABLE GOODS SECTOR



Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.

FIGURE 2.G: IMPULSE RESPONSES TO A COST-PUSH SHOCK IN THE DURABLE GOODS SECTOR



Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.

APPENDIX H: Domestically Generated Inflation

In policy environments it is customary to build measures of core inflation that remove fluctuations in the oil price from changes in the general price level (typically CPI/PCE inflation rates). However, in our model oil only enters as an imported production input. Therefore, to obtain equation (31) in the manuscript we appeal to the concept of domestically generated inflation, which represents a viable option to exclude the price of imported goods from a given measure of aggregate inflation. Specifically, domestically generated inflation, π_t^{dg}, is defined as the contribution of domestic factor price inflation to an overall rate of inflation. As a first accounting identity, we define domestically-generated gross output inflation as a weighted average of domestically generated inflation at the sectoral level:

$$\pi_t^{dg} = \varkappa \pi_t^{n,dg} + (1-\varkappa)\,\pi_t^{d,dg} \tag{59}$$

where $\pi_t^{i,dg}$ $(i = n, d)$ denotes domestically generated inflation in sector and $\varkappa \equiv Y^n / \left(Y^n + Y^d\right)$ accounts for the relative size of the non-durable goods sector. In turn, to disentangle the effect of the price inflation of imported and domestic input materials, one can define the inflation rate the ith sector as

$$\pi_t^i = \gamma_i \pi_t^{i,m} + (1-\gamma_i)\,\pi_t^{i,dg}, \quad i = n, d. \tag{60}$$

where $\pi_t^{i,m}$ $(i = n, d)$ denotes the price inflation rate of imported input materials by sector i and γ_i is the cost share of imported input materials in the production of sector i. Given that oil is the only imported good by both sectors (i.e., $\pi_t^{n,m} = \pi_t^{d,m}$), it is natural to set $\gamma_i = \alpha_{O_i}$ and $\pi_t^{i,m}$ can be replaced with π_t^O. Thus, rearranging terms in the Equation (1b) one obtains the following definition for the domestically generated inflation rate at the sectoral level:

$$\pi_t^{i,dg} = \frac{1}{1-\alpha_{O_i}}\pi_t^i - \frac{\alpha_{O_i}}{1-\alpha_{O_i}}\pi_t^O, \quad i = n, d. \tag{61}$$

Once we isolate the domestically generated part of sectoral inflation in each of the two sectors, we obtain π_t^{dg} as

$$\pi_t^{dg} = \varkappa\left(\frac{1}{1-\alpha_{O_n}}\pi_t^n - \frac{\alpha_{O_n}}{1-\alpha_{O_n}}\pi_t^O\right) + (1-\varkappa)\left(\frac{1}{1-\alpha_{O_d}}\pi_t^d - \frac{\alpha_{O_d}}{1-\alpha_{O_d}}\pi_t^O\right) \tag{62}$$

APPENDIX I: Optimal Rules in Models without Factor Demand Linkages

The present appendix reports further results from the implementation of a contemporaneous data rule and an inertial rule. Table I1 reports the coefficients of the optimal rule and the associated loss within a model economy without factor demand linkages. Moreover, as we restrict our focus to the comparison between aggregate and sticky-price inflation, we also eliminate oil from the production technology, without loss of generality. As a preliminary check, we can note that the standard result reported by Woodford (2003), according to which targeting sticky-price inflation is preferable to focusing on aggregate inflation, is confirmed in the model without factor demand linkages, conditional on the economy being perturbated by technology shocks only.

Notably, in a model where input materials are disregarded but sectors remain fundamentally asymmetric, policy inertia is only accepted when the policy maker targets sticky-price inflation and the economy is perturbated by all the three types of shock. This is due to various structural and exogenous factors inducing sectoral asymmetry and contributing to shift the relative price, even in the absence of factor demand linkages. Under these circumstances, when targeting aggregate inflation the policy maker seeks to attach the highest possible response to general price changes, while featuring no reaction to the output gap. By contrast, sticky-price inflation targeting needs to be complemented by a certain degree of inertia. How to explain this finding? When the policy maker targets π_t^{sticky}, inflation in the durable goods sector receives a rather low weight, both because this is relatively smaller and because it features lower price rigidity: this allows the central bank to adopt a certain degree of policy inertia, so as deal with the intersectoral stabilization trade-off that emerges due to various factors inducing sectors to co-move negatively. By contrast, when targeting a measure of general price inflation that merely considers the relative size of each sector, durable goods inflation is necessarily overweighed: to balance this bias, interest rate inertia is rejected, so as to avoid attributing too much importance to the durable sector, whose reaction in the face of both cost-push and technology shocks typically calls for a persistent response of the policy instrument (see Erceg and Levin, 2006). This intuition is confirmed by comparing Table I1 with the welfare outcomes of a perfectly symmetric model where neither input materials nor oil are employed (i.e., $\alpha_{Li} = \alpha_{Li} = 1$, $i = \{n, d\}$), consumption goods feature the same expenditure share (i.e., $\mu_n = \mu_d$), both sectors produce a non-durable good (i.e., $\delta = 1$), labor is perfectly mobile across sectors (i.e., $\lambda \to \infty$) and sectoral shocks are perfectly correlated. As predicted by Schmitt-Grohe and Uribe (2007), interest rate inertia is accepted in this case. In fact, $\rho = 0.73$ and $\phi_\pi = \phi_y = 5$ conditional on a technology shock only, while $\rho = 0.25$ and $\phi_\pi = 5$, $\phi_y = 0$, conditional on all shocks.

TABLE I1: OPTIMAL INTEREST RATE RULES - NO FACTOR DEMAND LINKAGES

	Technology shock				Technology shock				
	ϕ_π	ϕ_y	L^R	L^T	ρ	ϕ_π	ϕ_y	L^R	L^T
Aggregate	3.2940	5	0.0022	0.0072	0	3.2940	5	0.0022	0.0072
Sticky-price	5	5	0.0020	0.0072	0	5	5	0.0020	0.0072
	All shocks				All shocks				
	ϕ_π	ϕ_y	L^R	L^T	ρ	ϕ_π	ϕ_y	L^R	L^T
Aggregate	5	0	0.1676	0.5095	0	5	0	0.1676	0.5095
Sticky-price	5	0	0.2353	0.5095	0.5416	4.9919	0	0.1196	0.5095

Notes: Table I1 reports – conditional on different shock configurations – the reaction coefficients under the contemporaneous data rule and the inertial rule in the absence of factor demand linkages and oil (i.e., $\alpha_{Mi} = \alpha_{Oi} = 0$, $i = \{n, d\}$). The parameters ρ, ϕ_π and ϕ_y are computed so as to minimize the loss of social welfare (Eq. 18). The table also reports the loss under timeless perspective, L^T, as well as L^R, which denotes the log-deviation of the loss under the optimal rule and the loss under timeless perspective. All losses are expressed as a percentage of steady state consumption. The average duration of the price of non-durables is set at 4 quarters, while durable prices are re-set every 1.3 quarters.

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