Discretion vs. Timeless Perspective under Model-consistent Stabilization Objectives
Technical Appendix

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1 The Model

We embed an input-output production structure into an otherwise standard dynamic general equilibrium New Keynesian model. Firms operate within a monopolistically competitive setting. Their production technology embodies both labor and intermediate goods, so that the gross product of each firm in the economy is both consumed and used in the production of all other goods in the economy.

1.1 Consumers

Households derive income from working in firms, investing in bonds, and from the stream of profits generated by firms in the economy. They have preferences defined over a composite of goods (C_t) and labor (L_t). They maximize the expected present discounted value of their utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \varrho \frac{L_t^{1+\nu}}{1+\nu} \right], \ \varrho > 0 \tag{1}$$

where β is the discount factor, σ is the inverse of the intertemporal elasticity of substitution, ν is the inverse of the Frisch elasticity of labor supply.

The following sequence of (nominal) budget constraints applies:

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$$P_t C_t + B_t = R_{t-1} B_{t-1} + P_t W_t L_t - T_t + \Psi_t, \tag{2}$$

where P_t is the price of the composite good, B_t denotes a one-period risk-free nominal bond remunerated at the gross risk-free rate $R_t \equiv 1 + i_t$, W_t is the real wage rate, T_t is a lump-sum tax paid to the government and Ψ_t is the aggregate nominal flow of firm dividends.

1.2 Producers

The production side of the economy consists of one sector producing a continuum of differentiated goods $i \in [0, 1]$. We assume that the consumption composite takes the form of a Dixit-Stiglitz aggregator:

$$C_t = \left[\int_0^1 (C_{it})^{\frac{\varepsilon_t - 1}{\varepsilon_t}} \, di \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}, \tag{3}$$

where ε_t denotes the time-varying elasticity of substitution between differentiated goods in the consumption composite. It is possible to show that a generic firm i faces the following demand schedule:

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon_t} C_t, \tag{4}$$

where P_{it} is the price of the generic good i.

As in Basu (1995), Bergin and Feenstra (2000) and Moro (2009) we assume a Cobb-Douglas production technology for a generic firm i:[1]

$$Y_{it} = Z_t M_{it}^{\alpha} L_{it}^{1-\alpha}, \tag{5}$$

where Z_t is a productivity shifter, L_{it} denotes the number of hours worked in the ith firm and M_{it} denotes the amount of material inputs employed by firm i. Material inputs are combined according to a CES aggregator:

$$M_{it} = \left[\int_0^1 (M_{kit})^{(\varepsilon_t - 1)/\varepsilon_t} \, dk \right]^{\varepsilon_t/(\varepsilon_t - 1)}, \tag{6}$$

where M_{kit} is the intermediate input produced by firm k and employed in the production process of firm i. This specification implies the following demand function for the kth intermediate good:

[1] The key insights reported in the remainder of this paper are valid under more general production technologies, such as the CES specification of Dotsey and King (2006).

$$M_{kit} = \left(\frac{P_{kt}}{P_t}\right)^{-\varepsilon_t} M_{it}. \tag{7}$$

The gross product of the ith firm may be sold on the market for final consumption goods or used as an intermediate good by all firms in the economy, so that $Y_{it} = C_{it} + M_{it}$.

Firms are assumed to adjust their price with probability $1 - \theta$ in each period. When they are able to do so, they set the price that maximizes expected profits:

$$\max_{P_{it}} E_t \sum_{n=0}^{\infty} (\beta\theta)^n \Omega_{t+n} \left[(1+\tau) P_{it} - MC_{it+n}\right] \frac{Y_{it+n}}{P_t} \tag{8}$$

where Ω_t is the stochastic discount factor consistent with households' maximizing behavior, τ is a steady state subsidy to producers[2] and MC_{it} denotes firm's i nominal marginal cost of production. In every period each firm solves a cost minimization problem to meet demand at its stated price, so that:

$$MC_{it} = \frac{P_t W_t L_{it}}{(1-\alpha) Y_{it}} = \frac{P_{it} M_{it}}{\alpha Y_{it}}. \tag{9}$$

1.3 The Government and the Monetary Authority

The government serves two purposes in the economy. First, it delegates monetary policy to an independent Central Bank. The second task of the government consists of taxing households and providing subsidies to firms to eliminate distortions arising from monopolistic competition in the markets for both classes of consumption goods. This task is pursued via lump-sum taxes that maintain a balanced fiscal budget.

[2]The subsidy will be set so as to neutralize the monopolistic competition inefficiency in the steady state. In fact, for the sake of making a correct welfare ranking between discretion and timeless perspective within our linear-quadratic framework, steady state efficiency is mandatory (Woodford, 2003). Otherwise, in the presence of no subsidy the steady state would be distorted, leading to a spurious welfare analysis.

References

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