# Determinacy, Stock Market Dynamics and Monetary Policy Inertia*

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**Abstract**

We study equilibrium determinacy in a New-Keynesian model where the Central Bank responds to asset prices growth. Unlike Taylor-type rules reacting to asset prices, the proposed rule does not harm dynamic stability and may promote determinacy by inducing interest-rate inertia.

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## Introduction

The increasingly frequent episodes of financial turmoil in the last two decades have drawn considerable attention on stock markets developments and their interdependencies with the real economy. Both policy makers and researchers have debated around the opportunity to design policies capable to affect the dynamics of stock prices to improve the macroeconomic performance of both industrialized economies and emerging markets. At the same time, since the seminal work by Taylor (1993) it has become common practice to think about monetary policy in terms of interest rate rules whereby the monetary authority controls the nominal rate of interest in response to inflation and the output gap. These parallel developments have stimulated a long-standing debate on the role and scope of Central Banks to implement rules that involve adjusting the policy instrument in response to deviations of asset prices from their equilibrium level, along with reacting to changes in the economic conditions.[1] Among

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[1] Different views have been expressed in the literature and a clear consensus has not been reached so far. Bernanke and Gertler (1999, 2001) and Carlstrom and Fuerst (2007) conclude that there is no need for a direct response to asset

others, Bullard and Schaling (2002) show that responding to asset prices does not improve the economic performance, and might possibly harm real and financial stability. Specifically, including asset prices into a Taylor-type rule introduces a root of indeterminacy of the rational expectations equilibrium.

This paper shows that conditioning the policy instrument on asset prices growth may be beneficial from the vantage of equilibrium uniqueness. We first consider the case of a non-stochastic asset with maturity of one period and unitary payoff at maturity. We show that the original policy maker's reaction function is isomorphic to a Taylor rule featuring an interest rate smoothing term whose magnitude increases in the degree of aggressiveness towards asset prices growth. As shown by Woodford (2003) and Bullard and Mitra (2007) monetary policy inertia can help at alleviating problems of indeterminacy and non-existence of a stationary equilibrium observed for some commonly-studied monetary policy rules.[2] This principle also applies to more sophisticated economies where the payoff is explicitly linked to firms' profitability. In contrast to much of the existing theoretical literature, we show that the Central Bank can adjust the policy rate to control asset prices growth without necessarily incurring in problems of dynamic stability. In this respect, policy inertia may indeed reflect a certain concern in stock market developments from the policy maker's perspective. Rudebusch (2006), among others, suggests that policy gradualism could be justified upon some desire on the part of the Central Bank to reduce the volatility in interest rates and, more generally, in asset prices.

The remainder of the paper is laid out as follows: Section 1 introduces the theoretical setting and explores the conditions for equilibrium uniqueness under a rule that responds to asset prices growth and different assumptions about the nature of the underlying asset; Section 2 concludes.

# 1 Model

Bullard and Schaling (2002) develop their analysis on the general equilibrium sticky-price framework put forward by Rotemberg and Woodford (1999). The model, suitably linearized and simplified, produces equations (1) and (2) below. The first equation is:

$$x_t = E_t x_{t+1} - \sigma^{-1} \left( i_t - E_t \pi_{t+1} - i_t^n \right), \tag{1}$$

where $x_t$ denotes the output gap, $\pi_t$ is the inflation rate, $i_t$ is the nominal (risk free) interest rate, $i_t^n$ is a shock term that follows an AR(1) process.

Inflation is determined according to the standard New-Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{2}$$

prices. Conversely, Genberg et al. (2000), Di Giorgio and Nisticò (2007), and Pfajfar and Santoro (2008) show that there are conditions under which responding to asset prices misalignments may be beneficial for the sake of inducing dynamic stability.

[2]Inertia in interest rate setting is a well-documented feature of Central Banks' behavior in industrialized countries. Rudebusch (1995, 2006) provides insightful statistical analysis on this.

where $\kappa$ relates to the degree of price stickiness and $\beta$ denotes households' discount factor.

## 1.1 Asset Prices

In this framework arbitrage relationships can be used to price any asset, provided that financial markets are complete. This means that a financial claim on a random nominal quantity $X_T$ has value $E_t\left[\delta_{t,T} X_T\right]$ at time $t$, where $\delta_{t,T}$ is the stochastic discount factor:

$$\delta_{t,T} = \frac{\beta U'\left(C_T\right)}{U'\left(C_t\right)}$$

and $U'\left(C_t\right)$ is the marginal utility derived from consumption at time $t$. The gross nominal interest rate on a nominal one-period bond is then given by $R_t = E_t\left[\delta_{t,t+1}\right]^{-1}$. In Section 1.3 we relax this assumption and consider a financial asset whose payoff is explicitly related to firms' profitability.

Since the stochastic discount factor prices all assets in this model, let us denote the asset price by $Q_t$ and note that $Q_t = 1/R_t$. As in Rotemberg and Woodford (1998), the short-term nominal interest rate is defined as $i_t = \ln R_t$. Therefore, as $\ln R_t = -\ln Q_t$, we conclude that:

$$i_t = -q_t, \tag{3}$$

where $q_t = \ln Q_t$.

## 1.2 Monetary Policy and Asset Prices

We close the model with a Taylor-type rule, whereby the nominal rate of interest reacts to the lagged values of inflation and the output gap. We also assume that the policy maker wishes to include an explicit response to the growth rate of asset prices, $\Delta q_t (= q_t - q_{t-1})$.[3]

The form of the policy rule we wish to study is:

$$i_t = \gamma_\pi \pi_{t-1} + \gamma_x x_{t-1} + \gamma_q \Delta q_t, \tag{4}$$

with $\gamma_q > 0$. Given (3), this rule can be re-parameterized:

$$i_t = \phi_i i_{t-1} + \phi_\pi \pi_{t-1} + \phi_x x_{t-1}, \tag{5}$$

where

$$\phi_i = \frac{\gamma_q}{1+\gamma_q}, \quad \phi_\pi = \frac{\gamma_\pi}{1+\gamma_q}, \quad \phi_x = \frac{\gamma_x}{1+\gamma_q}.$$

[3] This specification is considered operational by McCallum (1999), as it does not call for the Central Bank to react to contemporaneous or expected future data on output gap and inflation. We select this rule over other alternatives as it requires information that is plausibly in possess of the Central Bank at time $t$. Notice that asset prices are assumed to be observable at the time the policy rate is set, as stock market statistics are available at a higher frequency, compared to macroeconomic data.

Thus, the policy instrument is set as a convex combination between the lagged rate of interest and a component reflecting the original responses to $\pi_{t-1}$ and $x_{t-1}$. In alternative, if the Central Bank responds to a term $(q_t - q^*)$ we obtain an instrumental rule equivalent to that explored by Bullard and Schaling (2002):

$$i_t = \phi_\pi \pi_{t-1} + \phi_x x_{t-1}.$$

Ceteris paribus, the responses to inflation and the output gap decrease in the degree of aggressiveness towards asset prices, which is indexed by $\gamma_q$. As the response to equity prices misalignments increases, it drives $\phi_\pi$ and $\phi_x$ to zero. Bullard and Schaling (2002) refer to the analysis in Bullard and Mitra (2002) to discuss this implication and show that, as $\gamma_q \to \infty$, indeterminacy is inevitable.

When implementing the policy rule (5), the response to both $\pi_{t-1}$ and $x_{t-1}$ decreases in $\gamma_q$ (ceteris paribus), but the monetary authority attaches a higher weight to the smoothing coefficient $(\phi_i)$:

$$\frac{\partial \phi_i}{\partial \gamma_q} > 0; \quad \frac{\partial \phi_\pi}{\partial \gamma_q} < 0; \quad \frac{\partial \phi_x}{\partial \gamma_q} < 0.$$

This feature of rule (5) turns out to be crucial to our analytical results, although the baseline intuition holds for more sophisticated models of equity claim, as that used in Section 1.3.

Bullard and Mitra (2007) study the effect of policy inertia on the conditions for equilibrium uniqueness. They consider a rule similar to (5):

$$i_t = \psi_i i_{t-1} + \psi_\pi \pi_{t-1} + \psi_x x_{t-1}, \tag{6}$$

where $\psi_i$, $\psi_\pi$, $\psi_x$ are non-negative parameters. We can re-write the system (1), (2) and (6) under its state-space representation:

$$E_t y_{t+1} = B y_t + C i_t^n,$$

where $y_t = [x_t, \pi_t, i_t]'$ and $B$ is a $3 \times 3$ matrix of structural parameters. Since $i_t$ is predetermined, while $x_t$ and $\pi_t$ are free variables, according to Blanchard and Kahn (1980) the equilibrium is determinate if and only if exactly one eigenvalue of $B$ lies within the unit circle. Woodford (2003) provides necessary and sufficient conditions for a determinate equilibrium.[4] The following two conditions are shown to be jointly necessary for determinacy:

$$\kappa(\psi_\pi + \psi_i - 1) + (1-\beta)\psi_x > 0, \tag{7}$$

$$[\kappa\sigma + (2+\beta)]\psi_i + 2(1+\beta) > \sigma[\kappa(\psi_\pi - 1) + (1+\beta)\psi_x]. \tag{8}$$

Condition (7) is precisely what Woodford (2001, 2003) refers to as the Taylor principle, whereby in the event of a permanent one percent rise in inflation, the cumulative increase in the nominal interest rate is more than one percent. However, if the Central Bank merely responds to inflation and the output

[4] More details of these calculations are provided in Bullard and Mitra (2007), Appendix A.

gap without a sufficient degree of inertia, the conditions for determinacy may be violated. It is also shown that a set of necessary and sufficient conditions required for determinacy reduce to (7), (8) and:

$$\psi_i > 2 - (1 + \kappa\sigma)\, \beta^{-1}. \tag{9}$$

Bullard and Mitra (2007) show that these analytical results provide intuition for a number of results obtained in more complicated models, such as those explored by Rotemberg and Woodford (1999) and McCallum and Nelson (1999). These studies generally confirm that large values of $\psi_i$ tend to be associated with a unique equilibrium, provided that other conditions on the structural parameters are satisfied.

Let us now transpose this analysis to our context. If we replace (6) with (5), conditions (7) and (8) can be expressed as:

$$\begin{aligned} \kappa\,(\gamma_\pi - 1) + (1-\beta)\,\gamma_x &> 0, & (10)\\ [2\kappa\sigma + (2+\beta)]\,\gamma_q + 2\,(1+\beta)\left(1+\gamma_q\right) &> \sigma\,[\kappa\,(\gamma_\pi - 1) + (1+\beta)\,\gamma_x]\,. & (11) \end{aligned}$$

Again, the first condition corresponds to the Taylor principle. Notice that introducing an explicit response to asset prices growth only affects the second condition. Thus, we can formulate the following proposition.

**Proposition 1** *Assume that $\kappa\,(\gamma_\pi - 1) + (1-\beta)\,\gamma_x > 0$ for the inertial lagged data interest rule (5). Then a necessary condition for determinacy is:*

$$[2\kappa\sigma + (2+\beta)]\,\gamma_q + 2\,(1+\beta)\left(1+\gamma_q\right) > \sigma\,[\kappa\,(\gamma_\pi - 1) + (1+\beta)\,\gamma_x]\,.$$

The left hand expression in (11) increases in $\gamma_q$. Therefore, provided that the Taylor principle holds, an increase in the degree of responsiveness to asset prices growth relaxes the constraint. Moreover, to account for the full set of sufficient and necessary conditions for determinacy, the following constraint should be added to (10) and (11):

$$\gamma_q > \frac{2-\chi}{\chi - 1}, \tag{12}$$

where $\chi = (1+\kappa\sigma)\,\beta^{-1}$. These conditions show that a large enough value of $\gamma_q$ will always result in determinacy since it contributes to satisfy conditions (10), (11), and (12).

We have shown that $\phi_i \left[= \gamma_q / \left(1-\gamma_q\right)\right]$ can be mapped from a Taylor rule according to which the monetary authority responds to $\Delta q_t$, along with reacting to $\pi_{t-1}$ and $x_{t-1}$. In turn, this function is isomorphic to a rule featuring policy inertia. This suggests that the reaction parameters in the inertial rule could indeed reflect a certain concern in stock market developments from the policy maker's perspective. Rudebusch (2006) expresses arguments in line with this view, suggesting that an obvious rationale for policy gradualism would be some desire on the part of the Central Bank to reduce the volatility in interest rates and, more generally, in asset prices.

### 1.3 Equity Prices

So far we have considered a class of financial assets consistent with the no-arbitrage condition $Q_t = E_t[\delta_{t,T} X_T]$ and with two hypotheses: (i) maturity of one period (so that the relevant stochastic discount factor is $\delta_{t,T} = \delta_{t,t+1}$); (ii) a non-stochastic, unitary payoff at maturity, i.e. $X_T = 1$. Both assumptions, however, might appear unreasonable for equity claims, which are commonly regarded as assets with payoffs that depend on stochastic future dividends. To characterize further implications of responding to stock prices (or their rate of growth), the latter can be distinguished from riskless assets, and linked to real activity. The most natural avenue to pursue this scope in our small-scale model is to link the payoff of the equity claim to the stream of dividends paid by monopolistically competitive firms. The implications of this assumption have been explored by Carlstrom and Fuerst (2007) within a simple model setup.

We start by replacing (3) with a linear stock price equation analogous to Equation 10 in Carlstrom and Fuerst (2007):

$$q_t = (1-\beta) E_t d_{t+1} + \beta E_t q_{t+1} - (i_t - E_t \pi_{t+1} - i_t^n), \tag{13}$$

where $d_t (= -\vartheta x_t)$ denotes the dividend payments and $\vartheta = (z(1+\sigma+\eta)-1)(1-z)^{-1}$. Moreover, $\eta$ is the inverse of the Frisch elasticity of labour supply and $z$ is the inverse of the steady-state mark-up. As discussed by Carlstrom and Fuerst (2007), $\vartheta$ is positive for a wide range of empirically relevant calibrations and represents a well-known feature of sticky-price models. This implies that an output increase translates into an increase in the real marginal cost that contracts firms' profitability and stock prices.

We assume that the model can alternatively be closed by the following rules:

$$i_t = \chi_\pi \pi_t + \chi_q q_t, \tag{14}$$

$$i_t = \chi_\pi \pi_t + \chi_{\Delta q} \Delta q_t. \tag{15}$$

According to (14) and (15), along with reacting to inflation, the monetary authority responds to either stock prices ($q_t$) or their growth rate ($\Delta q_t$). Carlstrom and Fuerst (2007) explore the first option in analytical detail. Conversely, if we assume that the Central Bank implements (15), retrieving analytical conditions loses much of the usual appeal in terms of the power to draw clear conclusions. We find more intuitive to plot the region of determinacy through a numerical simulation of the model over a sub-space defined by the reaction coefficients in the policy rule. We use the parameterization employed by Carlstrom and Fuerst (2007): $\beta = 0.99$, $\sigma = \eta = 2$, $z = 0.85$, $\kappa = 0.076$.[5]

[5] The results reported in this section are also robust to alternative calibrations, such as those proposed by Woodford (1999) and Clarida et al. (1999).

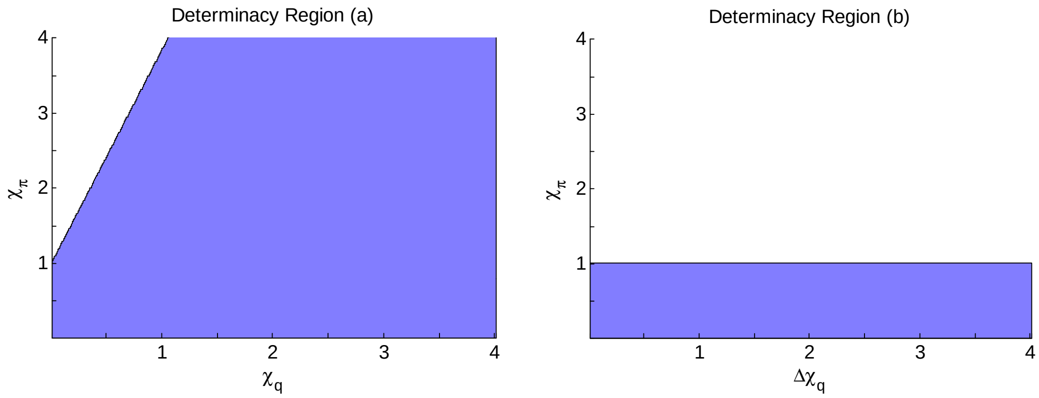


Figure 1: Determinacy (white region) under (14) and (15).

Recall that the sign of $\vartheta$ implies a negative relationship between dividends and the output gap, for a wide range of plausible parameterizations. Price stickiness is of key importance for this result. Carlstrom and Fuerst (2007) show that, as the real marginal cost is proportional to the output gap, an interest rate rule featuring a positive response to (expected or current) stock price deviations from their frictionless level is a rule that responds positively to firms' profitability. This means that the Central Bank reacts negatively to the underlying distortion in the economy, the real marginal cost. This situation is clearly reflected in the left-hand panel of Figure 1, where the shaded area denotes the space of indeterminacy. As expected, the possibility to induce multiple equilibria increases in $\chi_q$.

The right-hand panel of Figure 1 illustrates how the situation improves when (15) is implemented. Conditioning the policy rate to stock prices growth enhances the prospects of attaining a determinate equilibrium, compared to what is otherwise observed when the Central Bank reacts to $q_t$. In fact, whereas under (14) a maximum bound to $\chi_q$ arises beyond which no determinate outcome is attained, under (15) no indeterminacy bound can be detected apart from that related to inflation responses (the so-called Taylor principle). In this case, assuming that the Central Bank has an interest in adjusting the policy rate in response to asset prices movements, no trade-off exists (from the perspective of determinacy) when balancing the relative response to inflation and stock prices growth in the instrumental rule.

As in the case of a riskless asset, responding to changes in stock prices implies a certain degree of policy inertia, induced by the negative relationship between $q_t$ and $i_t$. In turn, interest rate smoothing exerts a beneficial impact from the vantage of equilibrium uniqueness. The negative correlation between asset prices and the nominal rate of interest is crucial to this mechanism, as it avoids inducing a counterproductive negative response to the real marginal cost. As a result, the monetary authority can set the policy rate to control asset prices developments without incurring in problems of dynamic stability.

## 2 Concluding Remarks

In the last decade a number of contributions have explored the role and the scope of monetary authorities to enhance financial stability, along with ensuring price stability. The general wisdom is that reacting to equity prices does not improve the economic performance, and might possibly harm both real and financial stability, by inducing rational expectations equilibrium indeterminacy.

This paper shows that allowing for an explicit response to asset prices growth translates into a certain policy gradualism that may be beneficial from the vantage of equilibrium uniqueness. The degree of interest rate smoothing increases in the policy maker's aggressiveness towards the growth rate of asset prices, under different assumptions on the nature of the underlying asset. As a result, responding to asset prices growth does not harm dynamic stability and in some cases may promote determinacy.

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