Leverage and Deepening Business Cycle Skewness*

Henrik Jensen† Ivan Petrella‡
Søren Hove Ravn§ Emiliano Santoro¶

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Abstract

We document that the U.S. economy has been characterized by an increasingly negative business cycle asymmetry over the last three decades. This finding can be explained by the concurrent increase in the financial leverage of households and firms. To support this view, we devise and estimate a dynamic general equilibrium model with collateralized borrowing and occasionally binding credit constraints. Higher leverage increases the likelihood that constraints become slack in the face of expansionary shocks, while contractionary shocks are further amplified due to binding constraints. As a result, booms become progressively smoother and more prolonged than busts. We are therefore able to reconcile a more negatively skewed business cycle with the Great Moderation in cyclical volatility. Finally, in line with recent empirical evidence, financially-driven expansions lead to deeper contractions, as compared with equally-sized non-financial expansions.

Keywords: Credit constraints, business cycles, skewness, deleveraging.
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†University of Copenhagen and CEPR. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Henrik.Jensen@econ.ku.dk.

‡University of Warwick and CEPR. Warwick Business School, University of Warwick, Scarman Rd, CV4 7AL Coventry, United Kingdom. *E-mail*: Ivan.Petrella@wbs.ac.uk.

§University of Copenhagen. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Soren.Hove.Ravn@econ.ku.dk.

¶University of Copenhagen. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Emiliano.Santoro@econ.ku.dk.

1 Introduction

Economic fluctuations across the industrialized world are typically characterized by asymmetries in the shape of expansions and contractions in aggregate activity. A prolific literature has extensively studied the statistical properties of this empirical regularity, reporting that the magnitude of contractions tends to be larger than that of expansions; see, among others, Neftci (1984), Hamilton (1989), Sichel (1993) and, more recently, Morley and Piger (2012). While these studies have generally indicated that business fluctuations are negatively skewed, the possibility that business cycle asymmetry has changed over time has been overlooked. Yet, the shape of the business cycle has evolved over the last three decades: For instance, since the mid-1980s the U.S. economy has displayed a marked decline in macroeconomic volatility, a phenomenon known as the Great Moderation (Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000). This paper documents that, over the same period, the skewness of the U.S. business cycle has become increasingly negative. Our key contribution is to show that occasionally binding financial constraints, combined with a sustained increase in financial leverage, allow us to account for several facts associated with the evolution of business cycle asymmetry.

Figure 1 reports the post-WWII rate of growth of U.S. real GDP, together with the 68% and 90% confidence intervals from a Gaussian density fitted on pre- and post-1984 data. Three facts stand out: First, as discussed above, the U.S. business cycle has become less volatile in the second part of the sample, even if we take into account the major turmoil induced by the Great Recession. Second, real GDP growth displays large swings in both directions during the first part of the sample, while in the post-1984 period the large downswings associated with the three recessionary episodes are not matched by similar-sized upswings. In fact, if we examine the size of economic contractions in conjunction with the drop in volatility occurring since the mid-1980s, it appears that recessions have become relatively more 'violent', whereas the ensuing recoveries have become smoother, as recently pointed out by Fatás and Mihov (2013). Finally, recessionary episodes have become less frequent, thus implying more prolonged expansions.

[Insert Figure 1]

These properties translate into the U.S. business cycle becoming more negatively skewed over the last three decades. Explaining this pattern represents a challenge for existing business cycle models. To meet this, a theory is needed that involves both non-linearities and a secular development of the underlying mechanism, so as to shape the evolution in the skewness of the business cycle. As for the first prerequisite, the importance of borrowing constraints as a source of business cycle asymmetries has long been recognized in the literature; see, e.g., the survey by Brunnermeier *et al.* (2013). In expansions, households and firms may find it optimal to borrow less than their available credit limit. Instead, financial constraints tend to be binding during recessions, so that borrowing is tied to the value of collateral assets. The resulting non-linearity translates into a negatively skewed business cycle. As for the second prerequisite, the past decades have witnessed a major deregulation of financial markets, with one result being a substantial increase in the degree of leverage of advanced economies. To see this, Figure 2 reports the credit-to-GDP and the loan-to-asset (LTA) ratios of both households and the corporate sector in the US.[1] This leveraging process is also confirmed, e.g., by Jordà *et al.* (2017) in a large cross-section of countries.

[Insert Figure 2]

Based on these insights, the objective of this paper is to propose a structural explanation of deepening business cycle skewness. To this end, we devise and estimate a dynamic stochastic general equilibrium (DSGE) model that allows for the collateral constraints faced by the firms and a fraction of the households not to bind at all points in time. We show that an increase in leverage raises the likelihood of financial constraints becoming slack in the face of expansionary shocks, dampening the magnitude of the resulting boom. By contrast, in the face of contractionary shocks borrowers tend to remain financially constrained, with debt reduction becoming more burdensome as leverage increases. In light of this mechanism, the skewness of the business cycle becomes increasingly negative. As in the data, the model also predicts that the duration of business cycle contractions does not change much as leverage increases, while the duration of expansions almost doubles.

We then juxtapose the drop in the skewness of the business cycle with the Great Moderation in macroeconomic volatility. While increasing LTV ratios cannot fully account for the Great Moderation, our analysis shows that the increase in the asymmetry of the business cycle is compatible with a drop in its volatility. Additionally, the decline in macroeconomic volatility mostly rests on the characteristics of the expansions, whose magnitude declines as an effect of collateral constraints becoming increasingly non-binding in the face of higher credit limits. This is in line with the recent empirical findings of Gadea-Rivas *et al.* (2014, 2015), who show that neither changes to the depth nor to the frequency of recessionary episodes account for the

[1] As we discuss in Appendix A, the aggregate loan-to-asset ratios reported in Figure 2 are likely to understate the actual LTV ratios requirements faced by the marginal borrower. While alternative measures may yield higher LTV ratios, they point to the same behavior of leverage over time (see also Graham *et al.*, 2014, and Jordà *et al.*, 2017).

stabilization of macroeconomic activity in the US.[2]

Recently, increasing attention has been devoted to the connection between the driving factors behind business cycle expansions and the extent of the subsequent contractions. Jordà *et al.* (2013) report that more credit-intensive expansions tend to be followed by deeper recessions—irrespective of whether the latter are accompanied by a financial crisis. Our model accounts for this feature along two dimensions. First, we show that contractions become increasingly deeper as the average LTV ratio increases, even though the boom-bust cycle is generated by the same combination of expansionary and contractionary shocks. Second, financially-driven expansions lead to deeper contractions, when compared to similar-sized expansions generated by non-financial shocks. Both exercises emphasize that, following a contractionary shock, the aggregate repercussions of constrained agents' deleveraging increases in the size of their debt. As a result, increasing leverage makes it harder for savers to compensate for the drop in consumption and investment of constrained agents. This narrative of the boom-bust cycle characterized by a debt overhang is consistent with the results of Mian and Sufi (2010), who identify a close connection at the county level in the US between pre-crisis household leverage and the severity of the Great Recession. Likewise, Giroud and Mueller (2017) document that, over the same period, counties with more highly leveraged firms suffered larger employment losses.

A key prediction of our model is that financial constraints on both households and firms have become less binding during the last three decades. This claim is consistent with existing accounts of the widespread financial liberalization that started in the US during the 1980s, which provide evidence of a relaxation of financial constraints over time (see, e.g., Justiniano and Primiceri, 2008). For households, Dynan *et al.* (2006) and Campbell and Hercowitz (2009) have discussed how the wave of financial deregulation taking place in the early 1980s paved the way for a substantial reduction in downpayment requirements and the rise of the subprime mortgage market. Combined with the boom in securitization some years later, this profoundly transformed household credit markets and gave rise to the leveraging process observed in Figure 2. Indeed, Guerrieri and Iacoviello (2017) report that non-binding credit constraints were prevalent among U.S. households from the late 1990s until the onset of the Great Recession.

[2] In this respect, downward wage rigidity has recently been pointed to as an alternative source of macroeconomic asymmetry (see Abbritti and Fahr, 2013). However, for this to act as a driver of deepening business cycle asymmetry, one would need to observe stronger rigidity over time, which does not seem to be the case. Most importantly, even if such a mechanism was at work, the resulting change in the skewness of the business cycle would primarily rest on the emergence of more dramatic recessionary episodes, without any major change in the key characteristics of expansions. However, this implication would stand in contrast with the evidence of Gadea-Rivas *et al.* (2014, 2015).

For businesses, the period since around 1980 has witnessed the emergence of a market for high-risk, high-yield bonds (Gertler and Lown, 1999) along with enhanced access to both equity markets and bank credit for especially small- and medium-sized firms (Jermann and Quadrini, 2009). Over the same period the investment-cash flow sensitivity in the US has declined substantially, a fact interpreted by several authors as an alleviation of firms' financial frictions (see, e.g., Agca and Mozumdar, 2008, and Brown and Petersen, 2009). Our findings point to these developments as an impetus of the deepening skewness of the U.S. business cycle observed during the same period.

The observation that occasionally binding credit constraints may give rise to macroeconomic asymmetries is not new. Mendoza (2010) explores this idea in the context of a small open economy facing a constraint on its access to foreign credit. As this constraint becomes binding, the economy enters a 'sudden stop' episode characterized by a sharp decline in consumption. In related work, Maffezzoli and Monacelli (2015) show that the aggregate implications of financial shocks are state-dependent, with the economy's response being greatly amplified in situations where agents switch from being financially unconstrained to being constrained. In a similar spirit, Guerrieri and Iacoviello (2017) report that house prices exerted a much larger effect on private consumption during the Great Recession—when credit constraints became binding—than in the preceding expansion. While all these studies focus on specific economic disturbances and/or historical episodes, a key insight of this paper is to show how different evolving traits of business cycle asymmetry may be accounted for by a secular process of financial liberalization, conditional on both financial and non-financial disturbances.

Our paper lends support to a recent empirical literature that focuses on the connection between leverage and business cycle asymmetry. Among various other business cycle facts, Jordà *et al.* (2017) report a positive correlation between the skewness of real GDP growth and the credit-to-GDP ratio for a large cross-section of countries observed over a long time-span. Popov (2014) exclusively focuses on business cycle asymmetry in a large panel of developed and developing countries, documenting two main results. First, the average business cycle skewness across all countries became markedly negative after 1991, consistent with our findings for the US. Second, this pattern is particularly distinct in countries that liberalized their financial markets. Also Bekaert and Popov (2015) examine a large cross-section of countries, reporting that more financially developed economies have more negatively skewed business cycles. Finally, Rancière *et al.* (2008) establish a negative cross-country relationship between real GDP growth and the skewness of credit growth in financially liberalized countries. While we focus on the asymmetry of output, we observe a similar pattern for credit, making our results comparable with their findings. On a more general note, all of these studies focus on the connection between business cycle skewness and financial factors in the cross-country dimension, whereas we examine how financial leverage may have shaped various dimensions of business cycle asymmetry over time.

The rest of the paper is organized as follows. In Section 2 we report evidence on the connection between leverage and changes in the shape of the business cycle in the US. Section 3 inspects the key mechanisms at play in our narrative within a simple two-period model. Section 4 presents our DSGE model, and Section 5 discusses the solution and estimation. Section 6 reports the main results. Section 7 shows that the model is capable of producing the type of debt overhang recession emphasized in recent empirical studies. Section 8 concludes. The Appendices contain supplementary material concerning the model solution and various empirical and computational details.

2 Empirical evidence

We first examine various aspects of business cycle asymmetry and how they changed over the last three decades. We then take advantage of cross-sectional variation across the U.S. States to document an empirical relationship between household leverage and the deepness of state-level contractions during the Great Recession.

2.1 Changing business cycle asymmetry

A number of empirical studies have documented a major reduction in the volatility of the U.S. business cycle since the mid-1980s. In this section we document changes in the asymmetry of the cycle that have occurred over the same timespan. Table 1 reports the skewness of the rate of growth of different macroeconomic aggregates in the pre- and post-1984 period.

[Insert Table 1]

The skewness is typically negative and not too distant from zero in the first part of the sample, but becomes more negative thereafter.[3,4] To supplement these findings, Figure 3 reports

[3] Appendix B1 reports measures of time-varying volatility and skewness of real GDP growth, based on a non-parametric estimator. The downward pattern in business cycle asymmetry emerges as a robust feature of the data, along with the widely documented decline in macroeconomic volatility.

[4] A drop in the skewness of GDP growth has also been pointed out in recent work by Garín *et al.* (2017). The present section expands on their finding in a number of directions, primarily by showing that the drop in the

the histogram of quarter-on-quarter GDP growth, as well as the corresponding fitted normal density over the two subsamples. Two features stand out: first, the histogram referring to the second subsample is much less dispersed—implying greater concentration of probability mass in the central part of the distribution—as compared with the one obtained from the first sample period. Second, as the probability density gets squeezed around its mean in the second part of the sample, more probability mass accumulates in the left tail, implying a more negative skewness coefficient. Formally, we employ the Kolmogorov-Smirnov test with estimated parameters (see Lilliefors, 1967), with the null hypothesis being that real GDP growth data in either of the two periods are drawn from a Normal distribution: This is strongly rejected for the second subsample (p-value=0.002), whereas it cannot be rejected in the first one (p-value=0.638).[5]

[Insert Figure 3]

Another way to highlight changes in the shape of the business cycle is to compare the upside and the downside semivariances over the two subsamples.[6] The overall volatility of the business cycle during the Great Moderation has dropped by more than 40% compared to the pre-Moderation period (1.75% vs. 3.07% when calculated on year-on-year GDP growth). However, the drop is not symmetric. In fact, whereas the upside and downside semivariance are roughly equal in the pre-Moderation sample, in the post-1984 sample the (square root of the) downside semivariance is more than 35% larger than its upside counterpart when calculated on year-on-year GDP growth. As highlighted in Figure 1, this implies an increase in the smoothness of the expansions, indicating that the emergence of the Great Moderation mostly rests on the characteristics of the upsides of the cycle, as recently argued by Gadea-Rivas *et al.* (2014, 2015).

All in all, our evidence suggests that the U.S. business cycle has become more asymmetric in the last three decades. While our focus in this paper is on the US, it is worth pointing

skewness of the business cycle is captured by an array of additional macroeconomic indicators. Moreover, we report that this drop is statistically significant and reflects into various traits of the shape of the business cycle, such as the relative duration of expansions and recessions, as well as their relative size. Finally, we show that the deepening in business cycle asymmetry is a feature shared by all major developed countries since around the mid-1980s.

[5]This result is confirmed by additional normality tests reported in Appendix B2. We also check that the drop in the skewness does not result from a moderate asymmetry in the first part of the sample being magnified by a fall in the volatility, such as the Great Moderation. The skewness of a random variable is defined as m_3/σ^3, where m_3 is the third central moment of the distribution and σ denotes its standard deviation: Therefore, an increase in the absolute size of the skewness could merely reflect a fall in σ, with m_3 remaining close to invariant. However, this is not the case, as $m_3 = -2.8169$ for the year-on-year growth rate of real GDP in the pre-1984 sample, while it equals -6.8755 afterwards.

[6]The upside (downside) semivariance is obtained as the average of the squared deviation from the mean of observations that are above (below) the mean. Semivariances are reported in Appendix B3.

out that a similar pattern emerges across the G7 economies, as we show in Appendix B4. Combined with the finding of Jordà *et al.* (2017) that secular increases in financial leverage are widespread across advanced economies, this suggests that our narrative may have wider relevance.

The next step in the analysis consists of translating changes in the business cycle asymmetry into some explicit measure of the deepness of economic contractions, while accounting for time-variation in the dispersion of the growth rate process. In line with Jordà *et al.* (2017), the first column of Table 2 reports the fall of real GDP during a given recession, divided by the duration of the recession itself: this measure is labelled as 'violence'.[7]

[Insert Table 2]

Comparing the violence of the contractionary episodes before and after 1984, we notice that the 1991 and 2001 recessions have not been very different from earlier contractions. However, to compare the relative magnitude of different recessions over a period that displays major changes in the volatility of the business cycle, it is appropriate to control for the average variability of the cycle around a given recessionary episode. To this end, the second column of Table 2 reports standardized violence, which is obtained by normalizing violence by a measure of the variability of real GDP growth.[8] Using this metric we get a rather different picture. The three recessionary episodes occurred during the Great Moderation are substantially deeper than the pre-1984 ones: averaging out the first seven recessionary episodes returns a standardized violence of 1.22%, against an average of 2.90% for the post-1984 period. Moreover, as highlighted in the last two columns of Table 2, the duration of business cycle contractions does not change much between the two samples, while the duration of the expansions doubles. This contributes to picturing the business cycle in the post-1984 sample as consisting of more smoothed and prolonged expansions, interrupted by shorter—yet, more dramatic—contractionary episodes.

2.2 Leverage and business cycle asymmetry: cross-state evidence

So far we have established that the post-1984 period is characterized by a smoother path of the expansionary periods and a stronger standardized violence of the recessionary episodes, as

[7] For earlier analyses on the violence and brevity of economic contractions see Mitchell (1927) and, more recently, McKay and Reis (2008).

[8] The volatility is calculated as the standard deviation of the year-on-year growth rate of real GDP over a 5-year window. We exclude the period running up to the recession by calculating the standard deviation up to a year before the recession begins. Weighting violence by various alternative mesures of business cycle volatility returns a qualitatively similar picture: Appendix B5 reports additional robustness evidence on the standardized violence of the recessions in the US.

compared with the pre-1984 period. In addition, over the same time window the process of financial deregulation has been associated with a sizeable increase in leverage of both households and firms. Relying on county-level US data, Mian and Sufi (2010) have identified a strong causal link between pre-crisis household leverage and the severity of the Great Recession. We now produce related evidence based on state-level data. Specifically, we take data on quarterly real Gross State Product (GSP) from the BEA Regional Economic Accounts and compute both the skewness of GSP growth and the violence of the Great Recession in the U.S. States.[9] Figure 4 correlates the resulting statistics to the average debt-to-income ratio prior to the recession. Notably, states where households were more leveraged not only have witnessed more severe GSP contractions during the last recession, but have also displayed a more negatively skewed GSP growth over the 2005-2016 time window. These findings echo those of Mian and Sufi (2010).

[Insert Figure 4]

To gain further insights into the cross-sectional connection between the magnitude of the Great Recession and business cycle dynamics, we order the U.S. states according to households' average pre-crisis debt-to-income ratio. We then construct two synthetic series, computed as the growth rates of the median real GSP of the top and the bottom ten states in terms of leverage, respectively. According to Figure 5, there are no noticeable differences in the performance of the two groups before and after the Great Recession, with both of them growing at a roughly similar pace. However, the drop in real activity has been much deeper for relatively more leveraged states. Altogether, this evidence points to a close link between leverage and business cycle asymmetries.

[Insert Figure 5]

3 A simple two-period model

Some preliminary insights into our main analysis can be offered through a simple two-period model of collateralized debt. The model shares many of the central aspects of our DSGE model, most notably an asset-based credit constraint. A representative household has utility

[9] To account for the possibility that the recession does not begin/end in the same period across the US, we define the start of the recession in a given state as the period with the highest level of real GSP in the window that goes from five quarters before the NBER peak date to one quarter after that. Similarly, the end of the recession is calculated as the period with the lowest real GSP in the window from one quarter before to five quarters after the NBER trough date.

$U = \mathrm{E}_0 \left\{ \sum_{t=1}^{2} \beta^{t-1} [a \log C_t + (1-a) \log H_t] \right\}$, $a \in (0,1)$, $\beta \in (0,1)$, where C_t and H_t denote the consumption of a nondurable good and (non-depreciating) land, respectively. In period 1, households' budget constraint is $C_1 + Q_1 (H_1 - H_0) - B_1 = Y_1 - RB_0$, where B_0 is initial debt, $R > 1$ is a constant gross real rate of interest, and Y_1 is a stochastic endowment, with $\mathcal{F}$ indicating its cumulative distribution function. We denote by Q_1 the price of land relative to that of nondurables. As in Kiyotaki and Moore (1997), the stock of debt in period 1 cannot exceed a fraction of the present value of land:

$$B_1 \leq s \frac{\mathrm{E}_1 \{Q_2\} H_1}{R}, \qquad s \in [0,1], \tag{1}$$

with s representing the loan-to-value ratio. In period 2, households are assumed to pay back, with interest, any acquired debt—irrespective of whether (1) was binding or not. Assuming a deterministic endowment Y in period 2, households therefore face the budget constraint $C_2 + Q_2 (H_2 - H_1) = Y - RB_1$. We assume that land is inelastically supplied in both periods.

Appendix C shows in detail the derivation of the model's competitive equilibrium, but here it suffices to consider the resulting nondurable consumption in period 1. When the constraint (1) is binding, we obtain

$$C_1 = Y_1 - RB_0 + \frac{s(1-a)}{a + s(1-a)} \frac{Y}{R}. \tag{2}$$

If (1) does not bind, instead, we retrieve the following solution:

$$C_1 = \frac{1}{1+\beta} (Y_1 - RB_0) + \frac{1}{R(1+\beta)} Y. \tag{3}$$

Several insights emerge from this simple set-up. A comparison of (2) and (3) reveals how negative skewness arises in connection with the tightness of the credit constraint. Variations in Y_1 affect consumption much stronger when the credit constraint binds, as compared to when it is slack. Not surprisingly, in financially-constrained states households behave according to a hand-to-mouth protocol, with a marginal propensity to consume out of current income equal to one. In financially-unconstrained states, on the other hand, households are able to smooth their lifetime resources across periods, implying a marginal propensity to consume of $1/(1+\beta)$. Now assume to start out at $Y_1 = \overline{Y}_1$, where $\overline{Y}_1$ is the income that equalizes C_1 given by (2) and (3), respectively. This 'trigger value' of income is the minimum value of income securing that (1) becomes slack; see Appendix C for further details. If a 'good' shock hits (i.e., $Y_1 = \overline{Y}_1 + \Delta$, $\Delta > 0$), consumption increases by $\Delta/(1+\beta)$, as (1) becomes non-binding. If a similar-sized 'bad' shock hits (i.e., $Y_1 = \overline{Y}_1 - \Delta$), consumption drops by $\Delta > \Delta/(1+\beta)$ since (1) becomes binding. Hence, consumption downturns are deeper than upturns.

From (2) we can see how the credit limit s, and thus financial leverage, plays a central role. Higher s means that more debt can be acquired in the constrained regime. *Ceteris paribus*, this implies that the household is less likely to become credit constrained. We formalize this argument by deriving $\overline{Y}_1$:

$$\overline{Y}_1 = RB_0 + \frac{a - \beta s\,(1-a)}{a + s\,(1-a)}\frac{Y}{\beta R}. \tag{4}$$

Since $Y_1 \leq \overline{Y}_1$ results in a binding constraint, the probability that the credit constraint binds is $\mathcal{F}\left(\overline{Y}_1\right)$. From (4), it follows that higher s, and thus higher leverage, decreases $\overline{Y}_1$ and the probability of the constraint being binding, as $\mathcal{F}' > 0$.

The next section introduces an estimated DSGE model where the mechanisms we have just described produce increasingly negative asymmetry, due to the financial constraints faced by different types of borrowers becoming more often slack in connection with a process of financial leveraging. Essentially, in such a model aggregate dynamics emerges as a mixture of the behavioral rules governing consumption and investment decisions under different regimes. A higher probability of non-binding financial constraints will be associated with more marked asymmetries, as those documented in Section 2.

4 A DSGE model

We adopt a standard real business cycle model augmented with collateral constraints, along the lines of Kiyotaki and Moore (1997), Iacoviello (2005), Liu *et al.* (2013), and Justiniano *et al.* (2015); *inter alia*. The economy is populated by three types of agents, each of mass one. These agents differ by their discount factors, with the so-called patient households displaying the highest degree of time preference, while impatient households and entrepreneurs have relatively lower discount factors. Moreover, patient and impatient households supply labor, consume nondurable goods and land services. Entrepreneurs only consume nondurable goods, and accumulate both land and physical capital, which they rent to firms. The latter are of unit mass and operate under perfect competition, taking labor inputs from both types of households, along with capital and land from the entrepreneurs. The resulting gross product may be used for investment and nondurable consumption.

4.1 Patient households

The utility function of patient households is given by:

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\log\left(C_t^P - \theta^P C_{t-1}^P\right) + \varepsilon_t \log\left(H_t^P\right) + \frac{\nu^P}{1-\varphi^P}\left(1 - N_t^P\right)^{1-\varphi^P} \right] \right\}, \tag{5}$$

$$0 < \beta^P < 1,\ \varphi^P \geq 0,\ \varphi^P \neq 1,\ \nu^P > 0,\ 0 \leq \theta^P < 1$$

where C_t^P denotes their nondurable consumption, H_t^P denotes land holdings, and N_t^P denotes the fraction of time devoted to labor. Moreover, β^P is the discount factor, θ^P measures the degree of habit formation in nondurable consumption and φ^P is the coefficient of relative risk aversion pertaining to leisure. Finally, ε_t is a land-preference shock satisfying

$$\log \varepsilon_t = \log \varepsilon + \rho_\varepsilon \left(\log \varepsilon_{t-1} - \log \varepsilon\right) + u_t, \qquad 0 < \rho_\varepsilon < 1, \tag{6}$$

where $\varepsilon > 0$ denotes the steady-state value and where $u_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$. Utility maximization is subject to the budget constraint

$$C_t^P + Q_t\left(H_t^P - H_{t-1}^P\right) + R_{t-1}B_{t-1}^P = B_t^P + W_t^P N_t^P, \tag{7}$$

where B_t^P denotes the stock of one-period debt held at the end of period t, R_t is the associated gross real interest rate, Q_t is the price of land in units of consumption goods, and W_t^P is the real wage.

4.2 Impatient households

The utility of impatient households takes the same form as that of patient households:

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\beta^I\right)^t \left[\log\left(C_t^I - \theta^I C_t^I\right) + \varepsilon_t \log\left(H_t^I\right) + \frac{\nu^I}{1-\varphi^I}\left(1 - N_t^I\right)^{1-\varphi^I} \right] \right\}, \tag{8}$$

$$0 < \beta^I < \beta^P,\ \varphi^I > 0,\ \varphi^I \neq 1,\ \nu^I > 0,\ 0 \leq \theta^I < 1$$

where, as for the patient households, C_t^I denotes nondurable consumption, H_t^I denotes land holdings, and N_t^I denotes the fraction of time devoted to labor. Households' difference in the degree of time preference is captured by imposing $\beta^P > \beta^I$. This ensures that, in the steady state, patient and impatient households act as lenders and borrowers, respectively. Impatient households are subject to the following budget constraint

$$C_t^I + Q_t\left(H_t^I - H_{t-1}^I\right) + R_{t-1}B_{t-1}^I = B_t^I + W_t^I N_t^I. \tag{9}$$

Moreover, impatient households are subject to a collateral constraint, according to which their borrowing B_t^I is bounded above by a fraction s_t^I of the expected present value of land holdings at the beginning of period $t+1$:

$$B_t^I \leq s_t^I \frac{\mathrm{E}_t\left\{Q_{t+1}\right\} H_t^I}{R_t}, \tag{10}$$

This constraint can be rationalized in terms of limited enforcement, as in Kiyotaki and Moore (1997). The loan-to-value (LTV) ratio (or credit limit), s_t^I, is stochastic and aims at capturing financial shocks (as in, e.g., Jermann and Quadrini, 2012 and Liu *et al.*, 2013):

$$\log s_t^I = \log s^I + \log s_t \tag{11}$$

$$\log s_t = \rho_s \log s_{t-1} + v_t, \qquad 0 < \rho_s < 1, \tag{12}$$

where $v_t \sim \mathcal{N}\left(0, \sigma_s^2\right)$ and s^I, the steady-state LTV ratio, is a proxy for the *average* stance of credit availability to the impatient households.

4.3 Entrepreneurs

Entrepreneurs have preferences over nondurables only (see Iacoviello, 2005; Liu *et al.*, 2013), and maximize

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\left(\beta^E\right)^t \log\left(C_t^E - \theta^E C_{t-1}^E\right)\right\}, \quad 0 < \beta^E < \beta^P,\ 0 \leq \theta^E < 1, \tag{13}$$

where C_t^E denotes entrepreneurial nondurable consumption. Utility maximization is subject to the following budget constraint

$$C_t^E + I_t + Q_t\left(H_t^E - H_{t-1}^E\right) + R_{t-1}B_{t-1}^E = B_t^E + r_{t-1}^K K_{t-1} + r_{t-1}^H H_{t-1}^E, \tag{14}$$

where I_t denotes investment in physical capital, K_{t-1} is the physical capital stock rented to firms at the end of period $t-1$, and H_{t-1}^E is the stock of land rented to firms. Finally, r_{t-1}^K and r_{t-1}^H are the rental rates on capital and land, respectively. Capital depreciates at the rate δ, and its accumulation is subject to investment adjustment costs determined by Ω, so that its law of motion reads as

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t, \qquad 1 > \delta > 0, \quad \Omega > 0. \tag{15}$$

Like impatient households, entrepreneurs are credit constrained, but they are able to use both capital and their holdings of land as collateral:[10]

$$B_t^E \leq s_t^E \mathrm{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\}, \tag{16}$$

where Q_t^K denotes the price of installed capital in consumption units and s_t^E behaves in accordance with

$$\log s_t^E = \log s^E + \log s_t, \tag{17}$$

where s^E denotes entrepreneurs' steady-state LTV ratio.[11] Together with households' average LTV ratio, this parameter will assume a key role in the analysis of the evolving connection between macroeconomic asymmetries and financial leverage.

4.4 Firms

Firms operate under perfect competition, employing a constant-returns-to-scale technology. They rent capital and land from the entrepreneurs and hire labor from both types of households in order to maximize their profits. The production technology for output, Y_t, is given by:

$$Y_t = A_t \left[\left(N_t^P\right)^{\alpha} \left(N_t^I\right)^{1-\alpha} \right]^{\gamma} \left[\left(H_{t-1}^E\right)^{\phi} K_{t-1}^{1-\phi} \right]^{1-\gamma}, \qquad 0 < \alpha,\ \phi,\ \gamma < 1, \tag{18}$$

with total factor productivity A_t evolving according to

$$\log A_t = \log A + \rho_A \left(\log A_{t-1} - \log A\right) + z_t, \qquad 0 < \rho_A < 1, \tag{19}$$

where $A > 0$ is the steady-state value of A_t, and $z_t \sim \mathcal{N}(0, \sigma_A^2)$.

[10] The importance of real estate as collateral for business loans has recently been emphasized by Chaney *et al.* (2012) and Liu *et al.* (2013).

[11] As we will discuss in Section 5.1.2, the LTA series are cointegrated and their deviations from the common trend are highly correlated, so we opt for a single financial shock.

4.5 Market clearing

Aggregate supply of land is fixed at H, implying that land-market clearing is given by

$$H = H_t^P + H_t^I + H_t^E. \tag{20}$$

The economy-wide net financial position is zero, such that

$$B_t^P + B_t^I + B_t^E = 0. \tag{21}$$

Finally, the aggregate resource constraint is

$$Y_t = C_t^P + C_t^I + C_t^E + I_t. \tag{22}$$

5 Equilibrium, solution and estimation

An equilibrium is defined as a sequence of prices and quantities which, conditional on the sequence of shocks $\{A_t, \varepsilon_t, s_t\}_{t=0}^{\infty}$ and initial conditions, satisfy the agents' optimality conditions, the budget and credit constraints, as well as the technological constraints and the market-clearing conditions. The optimality conditions are reported in Appendix D. Due to the assumptions about the discount factors, $\beta^I < \beta^P$ and $\beta^E < \beta^P$, both collateral constraints are binding in the steady state. However, the optimal level of debt of one or both agents may fall short of the credit limit when the model is not at its steady state, in which case the collateral constraints will be non-binding.

To account for the occasionally binding nature of the collateral constraints, our solution method follows Laséen and Svensson (2011) and Holden and Paetz (2012). The idea is to introduce a set of (anticipated) 'shadow value shocks' to ensure that the shadow values associated with each of the two collateral constraints remain non-negative at all times.[12] We present the technical details of the method in Appendix E.

5.1 Calibration and estimation

In the remainder we aim at assessing the extent to which a relaxation of the credit limits faced by the borrowers can account for the evolution of the asymmetry of the business cycle. With

[12]For first-order perturbations, we have verified that our solution produces similar simulated moments as using the method of Guerrieri and Iacoviello (2015); see also Holden and Paetz (2012).

this in mind, we assign parameter values that allow us to match a set of characteristics of the U.S. business cycle in the pre-1984 sample. We do this by calibrating a subset of the parameters, while estimating the remaining ones using the simulated method of moments (SMM). Next, we simulate the model for progressively higher average LTV ratios faced by households and firms, and track the implied changes in the skewness of output and other macroeconomic variables, as well as other business cycle statistics.

5.1.1 Calibrated parameters

The calibrated parameters are summarized in Panel A of Table 3. We choose to calibrate a subset of the model parameters that can be pinned down using a combination of existing studies and first moments of U.S. data. We interpret one period as a quarter. We therefore set $\beta^P = 0.99$, implying an annualized steady-state rate of interest of about 4%. Moreover, we set $\beta^I = \beta^E = 0.96$, in the ballpark of the available estimates for relatively more impatient agents; see, e.g., Iacoviello (2005) and references therein. The utility weight of leisure is set to ensure that both types of households work 1/4 of their time in the steady state. This implies a value of $\nu^i = 0.27$ for $i = \{P, I\}$. The Frisch elasticity of labor supply is given by the inverse of φ^i, multiplied by the steady-state ratio of leisure to labor hours. Having pinned down the latter to 3, we set $\varphi^i = 9$, $i = \{P, I\}$, implying a Frisch elasticity of 1/3, a value which is broadly in line with the available estimates (see, e.g., Herbst and Schorfheide, 2014). In line with Iacoviello (2005) and Iacoviello and Neri (2010), we set the share of labor income pertaining to patient households, α, to 0.7. To pin down the labor income share we follow Elsby *et al.* (2013) and use the official estimate of the Bureau of Labor Statistics: The average value for the years 1948-1983 implies $\gamma = 0.6355$.

We set $\delta, \varepsilon, \phi$, and s^I to jointly match the following four ratios (all at the annual frequency) for the period from World War II until 1984: A ratio of residential land to output of 1.10, a ratio of commercial land to output of 0.63, an average capital to output ratio of 1.11, and an average ratio of private nonresidential investment to output of 0.23.[13] The depreciation rate of capital consistent with these figures is 0.0518, somewhat higher than standard values, as it

[13] Our computations of these ratios largely follow those of Liu *et al.* (2013). For residential land, we use owner-occupied real estate from the Flow of Funds tables. For commercial land, Liu *et al.* (2013) use Bureau of Labor Statistics data on land inputs in production, which are not available for the sample period we consider. Instead, we compute the sum of the real estate holdings of nonfinancial corporate and nonfinancial noncorporate businesses from the Flow of Funds, and then follow Liu *et al.* (2013) in multiplying this number by a factor of 0.5 to impute the value of land. For capital, we compute the sum of the annual stocks of equipment and intellectual property products of the private sector and consumer durables. We use the corresponding flow variables to measure investment. Finally, we measure output as the sum of investment (as just defined) and private consumption expenditures on nondurable goods and services.

reflects that our measure of capital excludes residential capital and structures, which feature lower depreciation rates than, e.g., intellectual properties. We obtain a value of $\phi = 0.1340$, which, multiplied by $(1-\gamma)$, measures land's share of inputs, and a weight of land in the utility function of $\varepsilon = 0.0763$. The implied value for impatient households' average LTV ratio is 0.62. Finally, cointegration tests reveal that the loan-to-asset ratios of households and firms reported in the right panel of Figure 2 share a common trend. Thus, we pin down the average LTV ratio of the entrepreneurs by calibrating $s^E - s^I$ to the sample average of the difference between these two series. The resulting difference amounts to 0.09, implying $s^E = 0.71$.[14]

5.1.2 Estimated parameters

We rely on the Simulated Method of Moments (SMM) to estimate the remaining model parameters, as this method is particularly well-suited for DSGE models involving non-binding constraints or other non-linearities. Ruge-Murcia (2012) studies the properties of SMM estimation of non-linear DSGE models, and finds that this method is computationally efficient and delivers accurate parameter estimates. Moreover, Ruge-Murcia (2007) performs a comparison of the SMM with other widely used estimation techniques applied to a basic RBC model, showing it fares quite well in terms of accuracy and computing efficiency, along with being less prone to misspecification issues than Likelihood-based methods.

We estimate the following parameters: The investment adjustment cost parameter (Ω), the parameters measuring habit formation in consumption (θ^P, θ^I, and θ^E), and the parameters governing the persistence and volatility of the shocks ($\rho_A, \rho_s, \rho_\varepsilon, \sigma_A, \sigma_s, \sigma_\varepsilon$).[15] In the estimation, we use five macroeconomic time series for the U.S. economy spanning the sample period 1952:I–1984:II: The growth rates of real GDP, real private consumption, real non-residential investment, real house prices, and the average of the deviations from trend of the two LTA series reported in the right panel of Figure 2, where the trend is computed using a multivariate Beveridge-Nelson decomposition (Robertson *et al.*, 2006). The beginning of the sample is dictated by the availability of quarterly Flow of Funds data, while the end of the sample coincides with the onset of the Great Moderation.[16] In the estimation, we match the following empirical moments: The standard deviations and first-order autoregressive parameters of

[14] These values for the average LTV ratios are lower than those typically employed in models calibrated over the Great Moderation sample (see, e.g., Calza *et al.*, 2013, Liu *et al.*, 2013, and Justiniano *et al.*, 2014), as our calibration covers the period before the subsequent wave of financial liberalization.

[15] In the estimation we impose that $\theta^I = \theta^E$, as initial attempts to identify these two parameters separately proved unsuccessful.

[16] In fact, house prices are only available starting in 1963:I. We choose not to delay the beginning of other data series to this date.

each of the five variables, the correlation of consumption, investment, and house prices with output, and the skewness of output, consumption, and investment. This gives a total of 16 moment conditions to estimate nine parameters. We provide more details about the data and our estimation strategy in Appendix F.

The estimated parameters are reported in Panel B of Table 3.[17] The estimate of Ω is in line with existing results from estimated DSGE models; see, e.g., Justiniano *et al.* (2013). Likewise, the degree of habit formation of impatient households and entrepreneurs is close to the estimates of Justiniano *et al.* (2013) and Guerrieri and Iacoviello (2017), whereas the estimated habit parameter for patient households is virtually zero. The volatility and persistence parameters of the technology shock are in line with those typically found in the real business cycle literature; see, e.g., Mandelman *et al.*, 2011. The finding of quite large and persistent land-demand shocks is consistent with the results of Iacoviello and Neri (2010) and Liu *et al.* (2013). Finally, the financial shocks in our model are more volatile than found by Jermann and Quadrini (2012) and Liu *et al.* (2013), but less persistent.

[Insert Table 3]

6 Asymmetric business cycles and collateral constraints

We can now examine how our model generates stronger business cycle asymmetries when average financial leverage increases. We do so in three steps. First, we inspect a set of impulse responses to build intuition around the non-linear transmission of different shocks. Next, we present various business cycle statistics obtained from simulating the model at different degrees of leverage. Finally, we examine the behavior of business cycle asymmetry in conjunction with lower macroeconomic volatility. Our ensuing quantitative exercises primarily aim at assessing the model's ability to reproduce various dimensions of changing business cycle asymmetry by relying exclusively on an increase in financial leverage, which we engineer by raising the average LTV ratios faced by households (s^I) and entrepreneurs (s^E).[18]

[17] The implied business cycle moments and their empirical counterparts are reported in Appendix F.

[18] The aim of the exercise is not to account for the process of financial innovation and liberalization lying behind the increase in leverage in the last decades—a task the model is not suitable for. Instead, we take this increase for granted and examine how it has affected the shape of the business cycle.

6.1 Impulse-response functions

To gain a preliminary insight into the nature of our framework, and how this evolves under different LTV ratios, we study the propagation of different shocks. Figure 6 displays the response of output to a set of positive shocks, as well as the mirror image of the response to equally-sized negative shocks, under different credit limits.[19] Looking at the first row of the figure, technology shocks of either sign produce symmetric responses under the calibrated LTV ratios for impatient households and entrepreneurs. By contrast, at higher credit limits a positive technology shock renders the borrowing constraint of the entrepreneurs slack for three quarters, while impatient households remain constrained throughout.[20] Entrepreneurs optimally choose to borrow less than they are able to. This attenuates the expansionary effect on their demand for land and capital, dampening the boom in aggregate economic activity. On the contrary, following a negative technology shock, the borrowing constraints remain binding throughout. As a result, impatient households and entrepreneurs are forced to cut back on their borrowing in response to the drop in the value of their collateral assets. This produces a stronger output response. In other words, under relatively high LTV ratios a negative technology shock has a larger impact on output than a similar-sized positive shock.

[Insert Figure 6]

As for the stochastic shifts in household preferences, the second row of Figure 6 indicates that entrepreneurs' collateral constraint becomes non-binding for two quarters after a positive land demand shock in the scenario with high LTV ratios, while impatient households remain constrained throughout. Therefore, entrepreneurs have no incentive to expand their borrowing capacity by increasing their stock of land. By contrast, there is no attenuation of negative shocks to the economy. In that case, both collateral constraints remain binding, giving rise to a large output drop.

Similar observations apply to the transmission of the financial shock, with the main difference being that upward shifts in the credit limits bear a greater potential of rendering the financial constraints non-binding, as they exert a direct impact on the borrowing limit. In fact, under high average LTV ratios the entrepreneurs are unconstrained during the first five periods following a positive shock. For the reasons discussed above, this leads to a smooth response of output, as compared with what happens following a negative shock. In this case

[19] Appendix G reports the corresponding impulse-responses for total consumption, investment, and total debt.

[20] In our stochastic simulations, instead, combinations of all the shocks will generate episodes of non-binding constraints for both types of borrowers.

entrepreneurs are forced into a sizeable deleveraging, reducing the stock of land available for production. Simultaneously, also impatient households deleverage and bring down their stock of land, which further depresses the land price, and thus the borrowing capacity of both types of constrained agents. The result is a large drop in output.

The impulse-response analysis offers a clear message: As leverage increases, economic expansions tend to become smoother than contractions, paving the way to a negatively skewed business cycle. This is broadly consistent with the observation of lower volatility of the upside of the business cycle, as compared with its downside. Moreover, the three types of shock we consider exert similar effects on business cycle asymmetry, so that their relative contribution is not crucial to our qualitative findings.

6.2 Leverage and asymmetries

To deepen our understanding of the properties of the model in connection with the degree of leverage, we report a number of statistics from dynamic simulations of the model, in which we progressively increase the average LTV ratios.[21] In line with the two-period economy of Section 3, Figure 7 shows that the frequency of episodes of non-binding constraints increases with the degree of leverage. This is the case for both types of agents, with impatient households always being less often unconstrained than entrepreneurs, as the borrowing capacity of the former is affected by a lower steady-state LTV ratio and only one type of collateral asset. Given these properties, in light of the impulse-response analysis of the previous section we should expect the increasing prevalence of periods of lax credit constraints to be associated with an increasingly negative asymmetry of the resulting macroeconomic aggregates.

[Insert Figure 7]

The left panel of Figure 8 confirms this intuition, displaying the skewness of the year-on-year growth rates of output, aggregate consumption and investment: All statistics start from being negative at our calibrated average LTV ratios, and decline thereafter.[22] Therefore, the model

[21] Specifically, we retrieve each statistic as the median from 501 simulations each running for 2000 periods. Unless stated otherwise, from now on we report the variable of interest for different average LTV ratios faced by the impatient households. In each simulation the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limit, in line with the baseline calibration of the model.

[22] In our dynamic simulations, impatient households and entrepreneurs may sometimes find themselves unconstrained even during economic downturns. This situation may result, for instance, when a positive credit limit shock coincides with a negative non-financial shock. In such cases—which are most likely to occur at high LTV ratios—even recessions may be dampened, thereby mitigating business cycle skewness. This explains the small reversal of the skewness of the growth rate of consumption and investment at high LTV ratios.

generates an increasingly negatively skewed business cycle in connection with an increase in financial leverage. In fact, relying exclusively on this mechanism allows our model to account for about half of the fall in the skewness of real GDP growth in the US. This property has major implications for the size of the recessions in our artificial economy, as indicated by the right panel of Figure 8. At the baseline calibration, the standardized violence of the recessions computed from the simulated time series of gross output is quantitatively in line with its data analogue reported in Table 2 for the pre-1984 sample. As leverage rises, the standardized violence increases, up to the point it doubles at the upper end of the interval of average LTV ratios, being broadly in line with what is observed in the post-1984 sample.

[Insert Figure 8]

It is also important to highlight that the model is capable of reproducing relative changes in the duration of contractions and expansions similar to those documented in Table 2. As leverage increases, expansions tend to last much longer—as indicated by the left panel of Figure 9—while the duration of the contractions displays a pattern that is virtually unchanged between the pre- and post-financial leveraging scenario. An increase in the average LTV ratios allows households and firms to take advantage of non-binding credit constraints to smooth consumption and investment during expansions, which therefore become smoother and more prolonged. By contrast, financial constraints tend to remain binding in recessions, so that higher LTV ratios do not enhance consumption and investment smoothing during these phases. As a result, little difference can be observed in the duration of contractions as leverage increases.

[Insert Figure 9]

6.3 Skewness and volatility

Recent statistical evidence has demonstrated that the Great Moderation was never associated with smaller or less frequent downturns, but has been driven exclusively by the characteristics of the expansions, whose magnitude has declined over time (Gadea-Rivas *et al.*, 2014, 2015). We now examine this finding in conjunction with the change in the skewness of the business cycle, which has largely occurred over the same time span.

[Insert Figure 10]

The left panel of Figure 10 reports the standard deviation of output growth as a function of the average LTV ratios. As shown by Jensen *et al.* (2016) in a similar model, macroeconomic volatility displays a hump-shaped pattern: Starting from low credit limits, higher availability of credit allows financially constrained agents to engage in debt-financed consumption and investment, as dictated by their relative impatience, thus reinforcing the macroeconomic repercussions of shocks that affect their borrowing capacity. This pattern eventually reverts, as higher LTV ratios increase the likelihood that credit constraints become non-binding. In such cases, the consumption and investment decisions of households and entrepreneurs delink from changes in the value of their collateral assets, dampening the volatility of aggregate economic activity. In fact, at the upper end of the range of average LTV ratios we consider, volatility drops below the value we match under the baseline calibration.

A key property of a model with occasionally binding constraints is that the volatility reversal is much stronger for positive than for negative shocks, in the face of which financial constraints tend to remain binding. This inherent property of our framework indicates that the drop in output volatility observed beyond $s^I \approx 0.75$ is mostly connected with expansionary periods. The right panel of Figure 10 confirms this view: Here we compare the volatility of expansionary and contractionary episodes, respectively, as a function of the average LTV ratios. The volatility of expansions is always lower than that of contractions, and declines over most of the range of average credit limits. The volatility of contractions, on the other hand, initially increases and then reverts at a relatively high degree of leverage: This drop is due to financial constraints being potentially non-binding even during economic contractions (see Footnote 22).

While our framework points to a hump-shaped relationship between credit limits and macroeconomic volatility, the key driver of business cycle asymmetry—endogenous shifts between binding and non-binding collateral constraints—in itself works as an impetus of lower macroeconomic volatility, *ceteris paribus*. Thus, despite our analysis not warranting the claim that the empirical developments in the volatility and skewness of the business cycle necessarily have the same origin, higher credit limits do eventually lead to a drop in the overall volatility of our model economy by making financial constraints increasingly slack.[23]

Notably, the increasing prevalence of non-binding credit constraints allows the model to account for different correlations between the volatility and the skewness of output growth, conditional on different credit limits. Based on the comparison between Figure 8 and the left panel of Figure 10, this correlation is increasingly negative until $s^I \approx 0.75$, thus becoming

[23] In fact, several authors have pointed to financial liberalization and the associated easing of financial constraints of both households and firms as a contributor to the Great Moderation (see, e.g., Justiniano and Primiceri, 2008 and, for a review of the literature, Den Haan and Sterk, 2010). A related question is whether our main finding of increasingly negative business cycle skewness would survive in the presence of an exogenous reduction in macroeconomic volatility of the magnitude observed during the Great Moderation. Appendix H documents that this is indeed the case.

positive as financial deepening reaches very advanced stages. These results are reminiscent of the evidence reported by Bekaert and Popov (2015), who document a positive long-run correlation between the volatility and skewness of output growth in a large cross-section of countries, but also a negative short-run relationship: As financial leverage reaches a certain level across advanced economies, our results predict that skewness and volatility will eventually decline in conjunction.

7 Debt overhang and business cycle asymmetries

Several authors have recently pointed to the nature of the boom phase of the business cycle as a key determinant of the subsequent recession. Using data for 14 advanced economies for the period 1870–2008, Jordà *et al.* (2013) find that more credit-intensive expansions tend to be followed by deeper recessions, whether or not the recession is accompanied by a financial crisis. This evidence is consistent with our cross-state evidence, as well as with the results of Mian and Sufi (2010) and Giroud and Mueller (2017), who document a strong connection between the severity of the Great Recession and the pre-crisis leverage of households and firms at the county level, respectively.

In this section we demonstrate that our model is also capable of reproducing these empirical facts. Figure 11 reports the results of the following experiment: Starting in the steady state, we generate a boom-bust cycle for different average LTV ratios. We first feed the economy with a series of positive shocks of all three types in the first five periods (up to period 0 in the figure). During the boom phase, we calibrate the size of the expansionary shocks hitting the economy so as to make sure that the boom in output is identical across all the experiments. Hereafter, starting in period 1 in the figure, we shock the economy with contractionary shocks of all three types for two periods, after which the negative shocks are 'phased out' over the next three periods. Crucially, the contractionary shocks are identical across calibrations. This ensures that the severity of the recession is solely determined by the endogenous response of the model at each different LTV ratio.[24] As the figure illustrates, the deepness of the contraction increases with the steady-state LTV ratios. A boom of a given size is followed by a more severe recession when debt is relatively high, as compared with the case of more scarce credit availability. At

[24] During both the boom and the bust we keep the relative size of the three shocks fixed and equal to their estimated standard deviations. However, we set their persistence parameters to zero, in order to avoid that the shape of the recession may be determined by lagged values of the shocks during the boom. Finally, we make sure that impatient households and entrepreneurs remain constrained in all periods of each of the cases, so as to enhance comparability.

higher average LTV ratios, households and entrepreneurs are more leveraged during the boom, and they therefore need to face a more severe process of deleveraging when the recession hits. By contrast, when credit levels are relatively low, financially constrained agents face lower credit availability to shift consumption and investment forward in time during booms, and are therefore less vulnerable to contractionary shocks.

[Insert Figure 11]

We next focus on the nature of the boom and how this spills over to the ensuing contraction. The left panel of Figure 12 compares the path of output in two different boom-bust cycles, while the right panel shows the corresponding paths for aggregate debt. In each panel, the dashed line represents a non-financial boom generated by a combination of technology and land-demand shocks, while the solid line denotes a financial boom generated by credit limit shocks.[25] We calibrate the size of the expansionary shocks so as to deliver an identical increase in output during each type of boom (which lasts for five periods, up until period 0 in the figure). As in the previous experiment, we then subject the economy to identical sets of contractionary shocks of all three types, so as to isolate the role played by the specific type of boom in shaping the subsequent recession. The contractionary shocks hit in periods 1 and 2 in the figure, and are then 'phased out' over the next three periods. While the size of the expansion in output is identical in each type of boom, the same is not the case for total debt, which increases by more than twice as much during the financial boom. The consequences of this build-up of credit show up during the subsequent contraction, which is much deeper following the financially fueled expansion, in line with the empirical findings of Jordà *et al.* (2013). As in Mian and Sufi (2010), this exercise confirms that the macroeconomic repercussions of constrained agents' deleveraging is increasing in the size of their debt.[26]

[Insert Figure 12]

[25] In the non-financial boom we keep the relative size of the technology and land-demand shocks in line with the values estimated in Section 5.1.2. As in the previous experiment, we set the persistence parameters of all the shock processes to zero.

[26] Addressing the endogeneity of credit and business cycle dynamics, Gadea-Rivas and Perez-Quiros (2015) stress that growing credit is not a predictor of future contractions. Our model simulations are consistent with this view. In fact, as displayed by Figure 12, output and credit growth are strongly correlated, regardless of whether the boom is driven by financial shocks. At the same time, the model predicts that a boom driven by financial shocks is associated with a stronger increase in debt and a deeper contraction, as compared with an equally-sized non-financial boom.

8 Concluding comments

We have documented how different dimensions of business cycle asymmetry in the US have changed over the last few decades, and pointed to the concurrent increase in private debt as a potential driver of these phenomena. We have presented a dynamic general equilibrium model with credit-constrained households and firms, in which increasing leverage translates into a more negatively skewed business cycle. This finding relies on the occasionally binding nature of financial constraints: As their credit limits increase, households and firms are more likely to become unconstrained during booms, while credit constraints tend to remain binding during downturns.

These insights shed new light on the analysis of the business cycle and its developments. The Great Moderation is widely regarded as the main development in the statistical properties of the U.S. business cycle since the 1980s. We point to a simultaneous change in the shape of the business cycle closely connected with financial factors. Enhanced credit access as observed over the last few decades implies both a prolonging and a smoothing of expansionary periods as well as less frequent—yet, relatively more dramatic—economic contractions, exacerbated by deeper deleveraging episodes. As for the first part of this story, several contributions have pointed to the attenuation of the upside of the business cycle as the main statistical trait of the Great Moderation. Nevertheless, insofar as financial liberalization and enhanced credit access can be pointed to as key drivers of an increasingly skewed business cycle, the second insight implies that large contractionary episodes, albeit less frequent, might represent a 'new normal'.

Our results are also of interest to macroprudential policymakers, as we complement a recent empirical literature emphasizing that the seeds of the recession are sown during the boom (see, e.g., Mian *et al.*, 2017). The nature of the expansionary phase, as much as its size, is an important determinant of the ensuing downturn, and policymakers should pay close attention to the build-up of credit during expansions in macroeconomic activity.

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Tables and Figures

Table 1. The skewness of the U.S. business cycle

	QoQ		YoY	
	1947:I-1984:II	1984:III-2016:II	1947:I-1984:II	1984:III-2016:II
GDP	-0.1178	-1.2122	-0.0983	-1.3037
	[-0.685 ; 0.450]	[-1.831 ; -0.594]	[-0.719 ; 0.522]	[-2.000 ; -0.608]
Consumption	-0.5064	-0.4678	-0.2023	-1.0005
	[-1.073 ; 0.060]	[-1.094 ; 0.159]	[-0.817 ; 0.413]	[-1.734 ; -0.267]
Investment	-0.2100	-0.8271	-0.0067	-1.3994
	[-0.773 ; 0.353]	[-1.440 ; -0.214]	[-0.604 ; 0.590]	[-2.066 ; -0.732]

Notes: In the 'QoQ' ('YoY') column we report, for different macroeconomic aggregates, the coefficient of skewness computed on the quarter-on-quarter (year-on-year) growth rate over the 1947:I-1984:II and 1984:III-2016:II samples. We report 68% confidence intervals in brackets. Data source: Federal Reserve Economic Data.

Table 2. The violence of recessions in the US

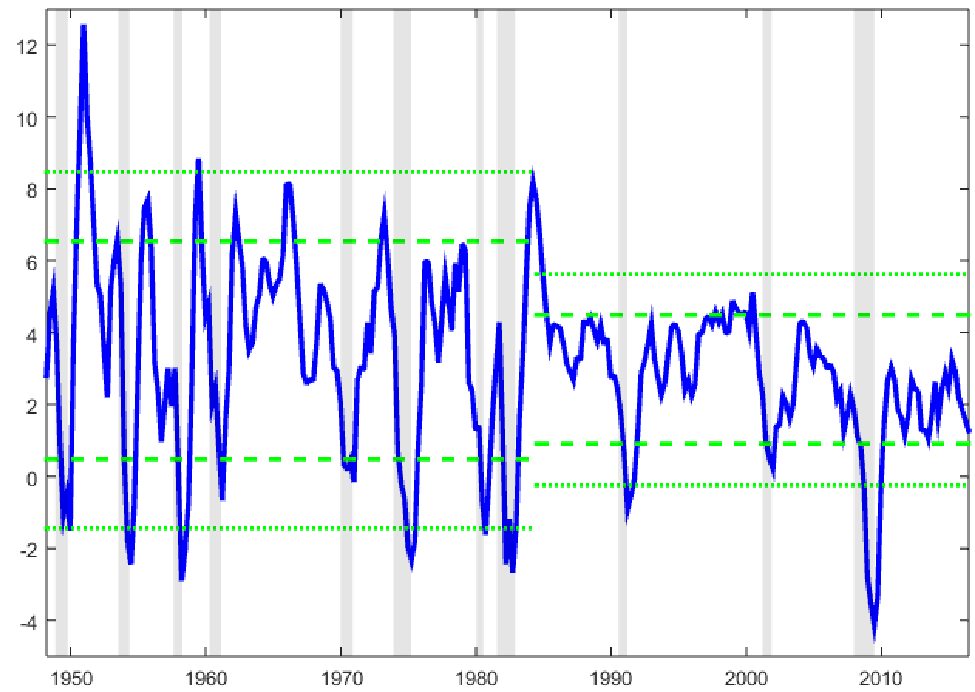
	Violence	Std. Violence	Duration (quarters)	
			Contractions	Expansions
1953:II – 1954:II	3.4106	0.9488	4	–
1957:III – 1958:II	7.3088	2.5419	3	13
1960:II – 1961:I	1.8009	0.5718	3	8
1969:IV – 1970:IV	0.4710	0.2674	4	35
1973:IV – 1975:I	2.5293	1.2275	5	12
1980:I – 1980:III	4.4006	1.7747	2	20
1981:III – 1982:IV	2.6785	1.1896	5	4
1990:III – 1991:I	2.6511	3.6238	2	31
2001:I – 2001:IV	1.2671	1.7850	3	40
2007:IV – 2009:II	2.8909	3.2966	6	24
Average				
Pre-1984	3.2285	1.2174	3.7143	15.3333
Post-1984	2.2697	2.9018	3.6667	31.6667

Notes: For every recession we calculate 'Violence' as the annualized fall of real GDP from the peak to the trough of the contractionary episode, divided by the length of the recession; 'Std. Violence' standardizes the violence of the recession by the average business cycle volatility prior to the recession itself. The latter is calculated as the standard deviation of the year-on-year growth rate of real GDP over a 5-year window. We exclude the period running up to the recession by calculating the standard deviation up to a year before the recession begins. Data source: NBER.

Table 3. Parameter values		
Panel A: Calibrated parameters		
Parameter	Description	Value
β^P	Discount factor, patient households	0.99
β^i, $i = \{I, E\}$	Discount factor, impatient households + entrepreneurs	0.96
φ^i, $i = \{P, I\}$	Curvature of utility of leisure	9
ν^i, $i = \{P, I\}$	Weight of labor disutility	0.27
ε	Weight of housing services utility	0.0763
ϕ	Non-labor input share of land	0.1340
γ	Labor share of production	0.6355
δ	Capital depreciation rate	0.0518
α	Income share of patient households	0.7
s^I	Initial loan-to-value ratio, impatient households	0.6239
s^E	Initial loan-to-value ratio, entrepreneurs	0.7139
Panel B: Estimated parameters		
Parameter	Description	Value
Ω	Investment adjustment cost parameter	2.9827 (0.9585)
θ^P	Habit formation, patient households	0.0031 (0.0526)
θ^I	Habit formation, impatient households + entrepreneurs	0.7723 (0.1380)
ρ_A	Persistence of technology shock	0.9894 (0.0326)
ρ_s	Persistence of credit-limit shock	0.8873 (0.0658)
ρ_ε	Persistence of land-demand shock	0.9893 (0.0527)
σ_A	Std. dev. of technology shock	0.0080 (0.0012)
σ_s	Std. dev. of credit-limit shock	0.0345 (0.0011)
σ_ε	Std. dev. of land-demand shock	0.0361 (0.1156)

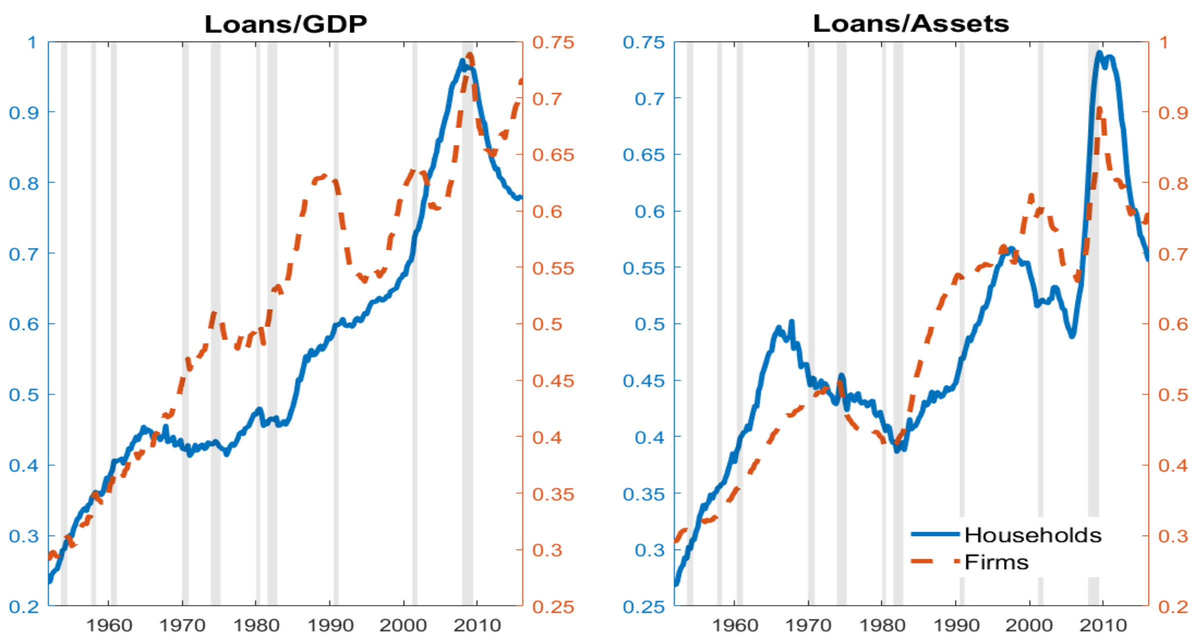
Note: The standard errors of the estimated parameters are reported in brackets.

Figure 1. Growth rates of U.S. real GDP



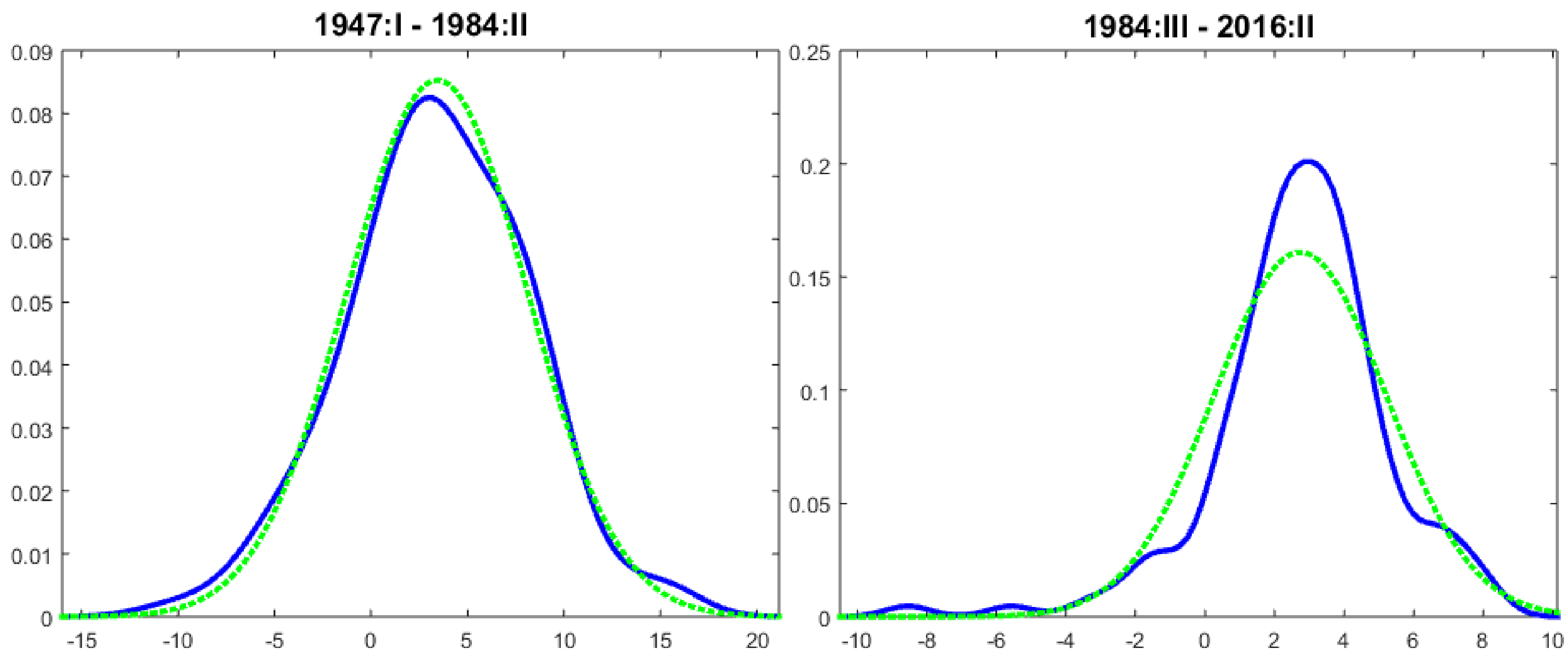
Notes: Figure 1 reports the year-on-year rate of growth of U.S. real GDP over the 1947:I-2016:II sample. The green bands correspond to the 68% and 90% confidence intervals from a Gaussian density fitted on the 1947:I-1984:II and 1984:III-2016:II samples. The vertical shadowed bands denote the NBER recession episodes. Data source: Federal Reserve Economic Data.

Figure 2. Household and corporate leverage in the US



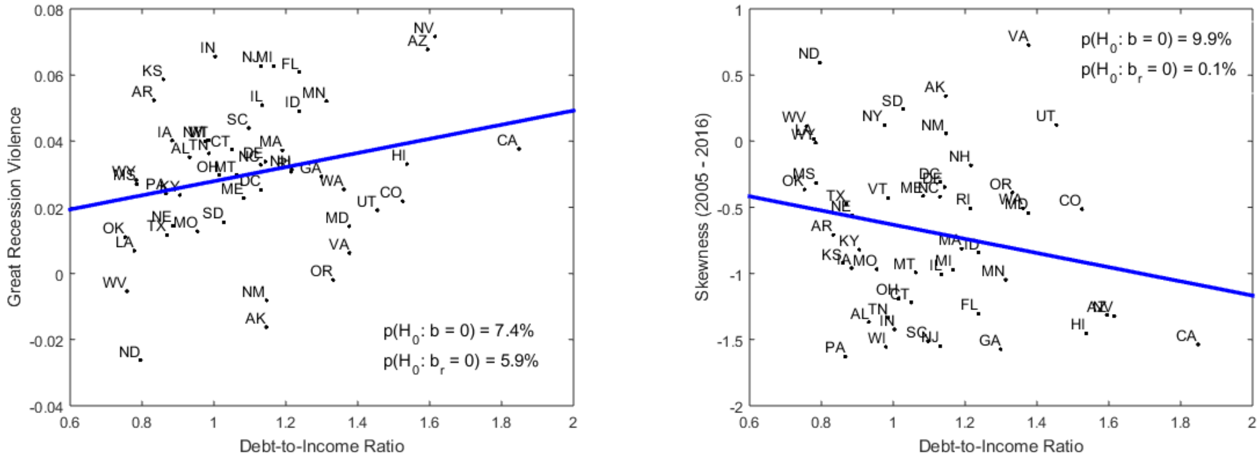
Notes: Left panel: the solid-blue line graphs the ratio between loans to households and GDP, while the dashed-red line reports the same variable at the corporate level. Right panel: the solid-blue line graphs the ratio between households' liabilities and assets, while the dashed-red line reports the same variable at the corporate level. The vertical shadowed bands denote the NBER recession episodes. Data source: Flow of Funds data, Financial Accounts of the US. See Appendix A for details.

Figure 3. GDP growth: 1947:I-1984:II vs. 1984:III-2016:II



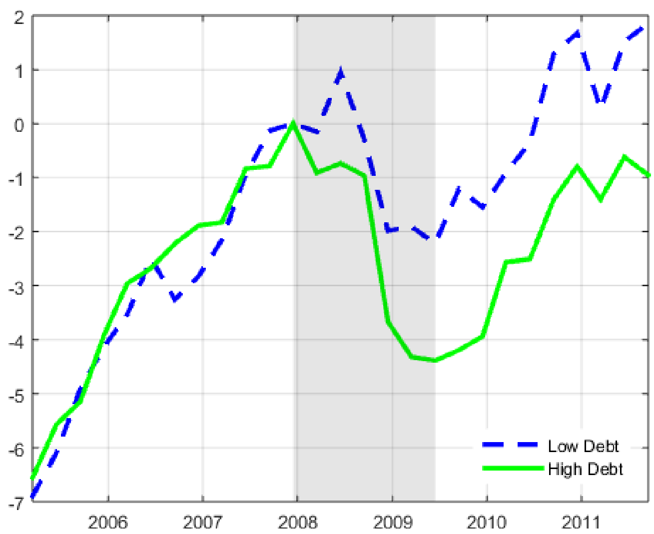
Notes: Figure 3 reports the histogram of the quarter-on-quarter growth rate of real GDP (solid-blue line), as well as the corresponding fitted normal density (dotted-green line) over the 1947:I-1984:II and 1984:III-2016:II samples. Data source: Federal Reserve Economic Data.

Figure 4. Leverage and asymmetry across U.S. States



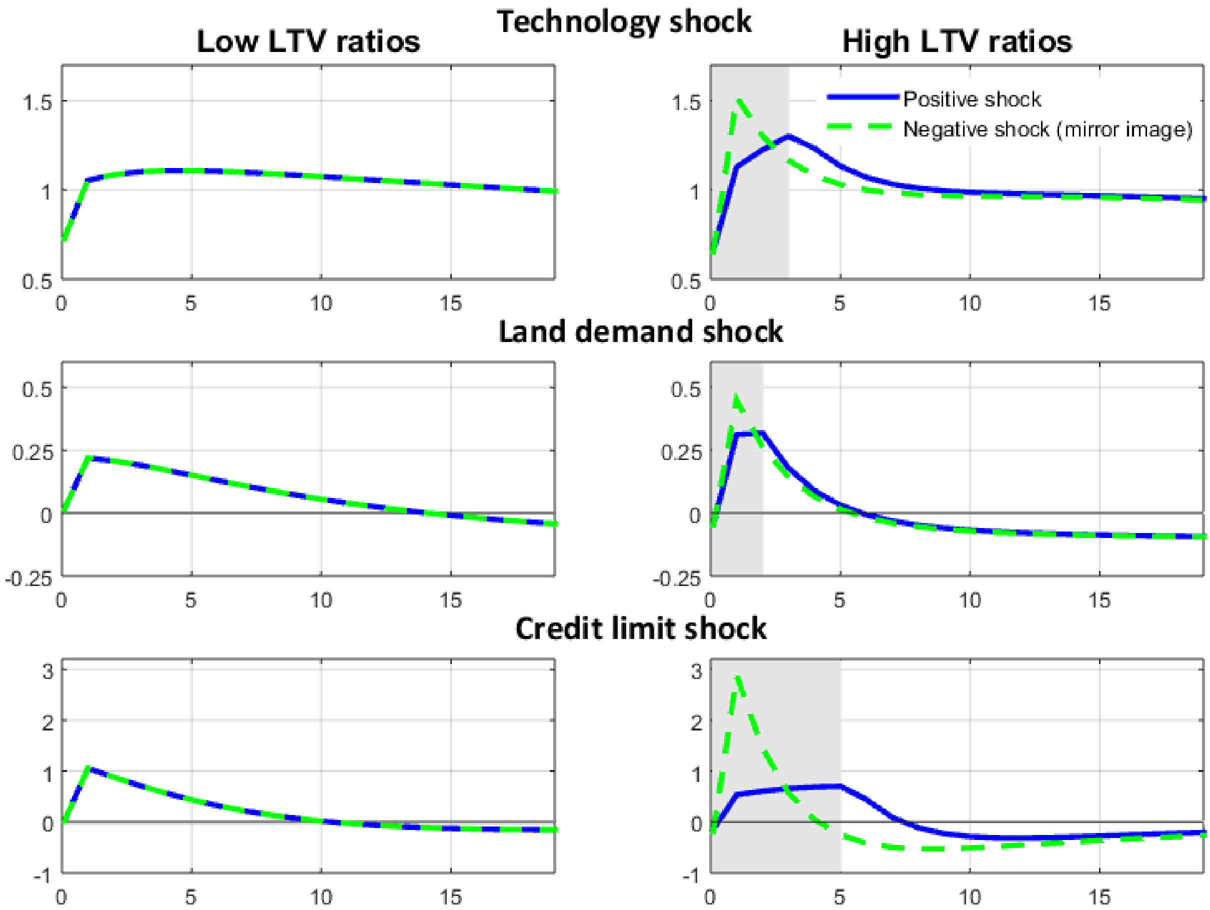
Notes: The left panel plots the violence of the Great Recession in each U.S. State against the average debt-to-income ratio at the household level over the period 2003:I-2007:I. To allow for the fact that the recession does not begin/end at the same time throughout the US, we calculate the start (end) of the recession in a given state as the period with the highest (lowest) level of real Gross State Product (GSP) in a window that goes from 5 quarters before (after) to one quarter after (before) the NBER dates. The right-hand panel plots the skewness of year-on-year real GSP growth over the 2005:I-2016:I period against the average debt-to-income ratio. In each panel we report the p-values associated with the slope coefficient: the first p-value is calculated on the slope coefficient estimated by OLS, while the second p-value refers to the slope estimated by excluding outliers (i.e., the observations whose standardized residuals from a first stage OLS regression are classified as being out of the 5/95% Gaussian confidence interval). In both cases we compute White (1980) heteroskedasticity-robust standard errors. Data sources: State Level Household Debt Statistics produced by the New York Fed and BEA Regional Economic Accounts.

Figure 5. GSP dynamics and household leverage



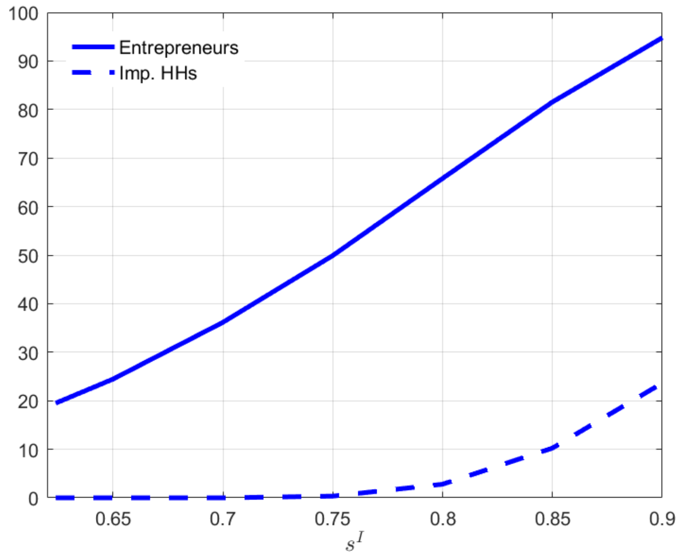
Notes: Figure 5 reports the growth rates of two synthetic GSP series obtained by ranking the U.S. States according to their average debt-to-income ratio in the 5 years before the Great Recession. The dashed-blue line is calculated from the median real GSP of the top 10 states, while the solid-green line is obtained from the median for the bottom 10 states. The resulting statistics have been normalized to zero at the beginning of the Great Recession (i.e., 2007:IV). The vertical shadowed band denotes the 2007:IV-2009:II recession episode. Data sources: State Level Household Debt Statistics produced by the New York Fed and BEA Regional Economic Accounts.

Figure 6. Impulse responses



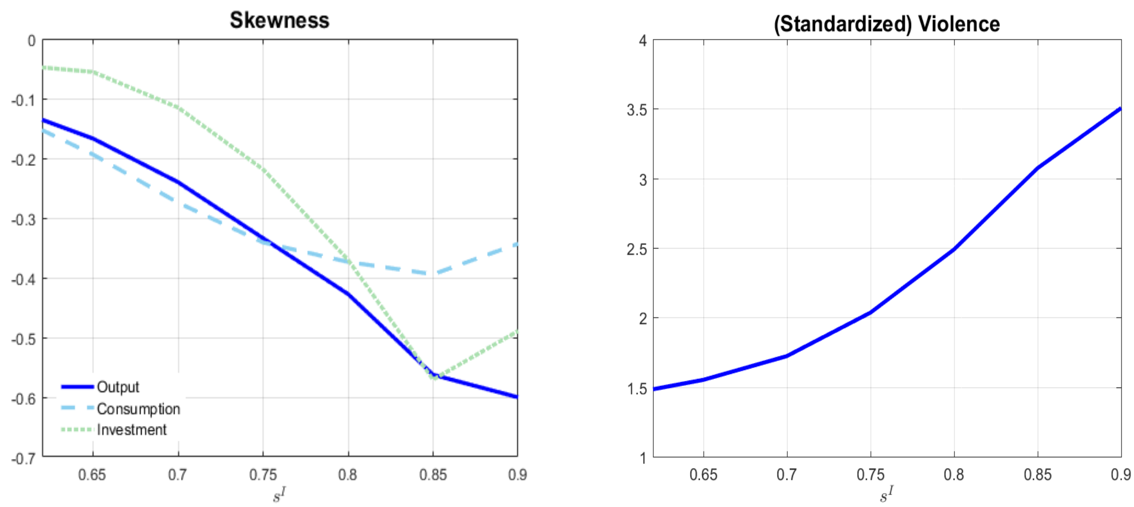
Notes: Impulse responses of gross output (% deviation from the steady state) to a one-standard deviation shock to technology (row 1), land demand (row 2), and credit limits (row 3). Left column: $s^I = 0.62$, $s^E = 0.71$; right column: $s^I = 0.85$, $s^E = 0.94$. The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

Figure 7. Leverage and frequency of non-binding collateral constraints



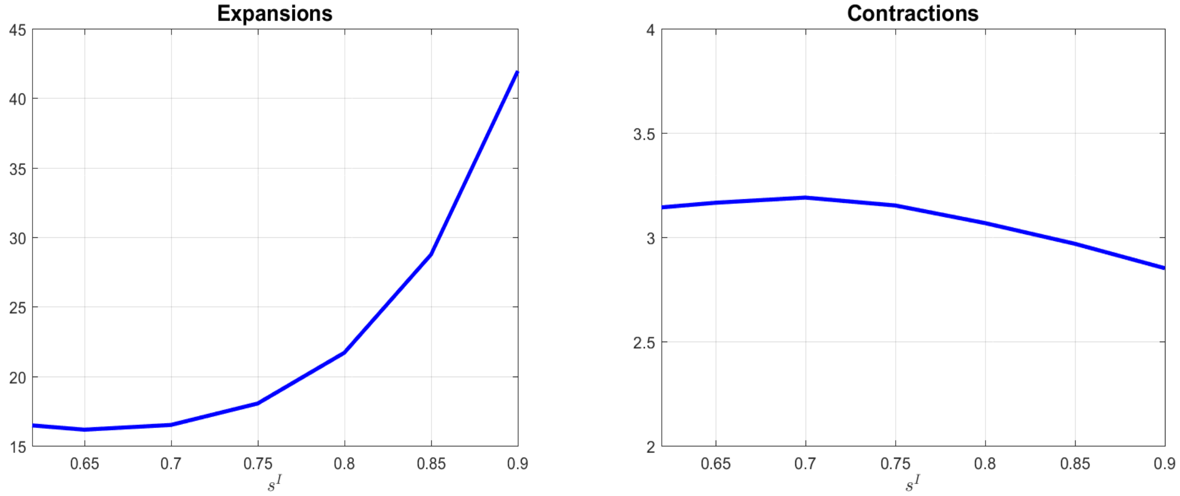
Notes: Frequency of non-binding constraints for entrepreneurs (solid line) and impatient households (dashed line). Both statistics are graphed for different average LTV ratios faced by the impatient household. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 8. Business cycle asymmetries



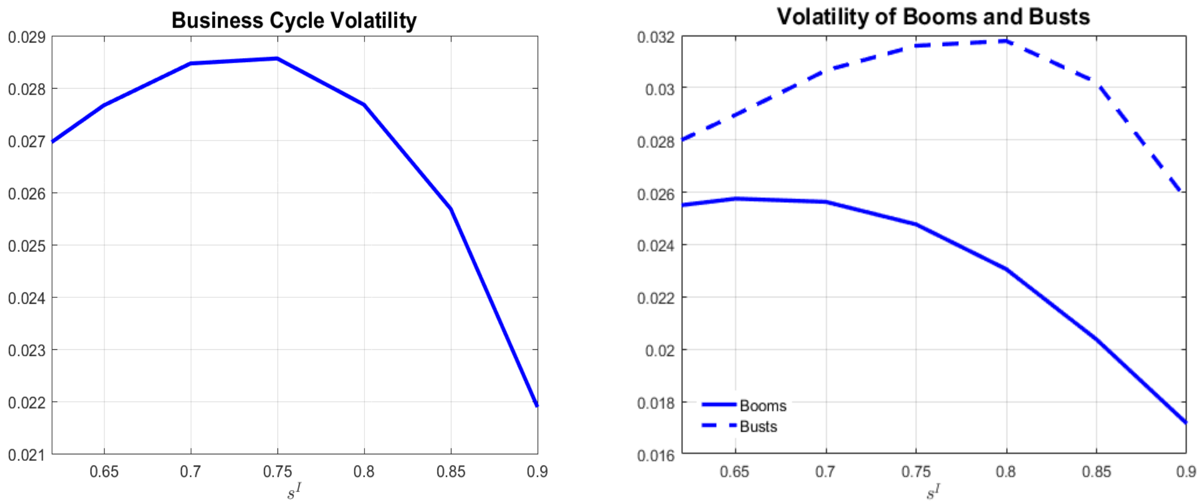
Notes: Skewness of the year-on-year growth rate of output, consumption and investment (left panel), and the standardized violence of the recessions (right panel), for different average LTV ratios faced by the impatient household. To identify the recessionary episodes in our simulated gross output series, we use the Harding and Pagan (2002) algorithm. We then compute violence as the average fall of output over a given recession, divided by the length of the recession itself. Finally, we standardize violence by means of the volatility of year-on-year output growth over the five years prior to the recession. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 9. Average duration of expansions and contractions



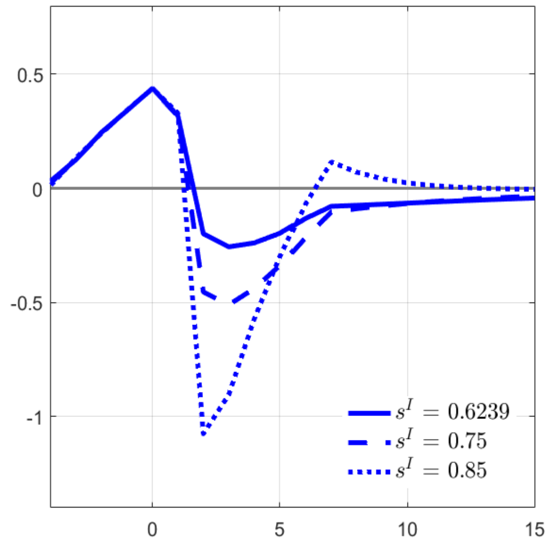
Notes: Duration of expansions and contractions (in quarters). To identify expansions and contractions in our simulated gross output series, we use the Harding and Pagan (2002) algorithm. The duration of both cyclical phases is graphed for different average LTV ratios faced by the impatient household. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 10. Leverage and volatility



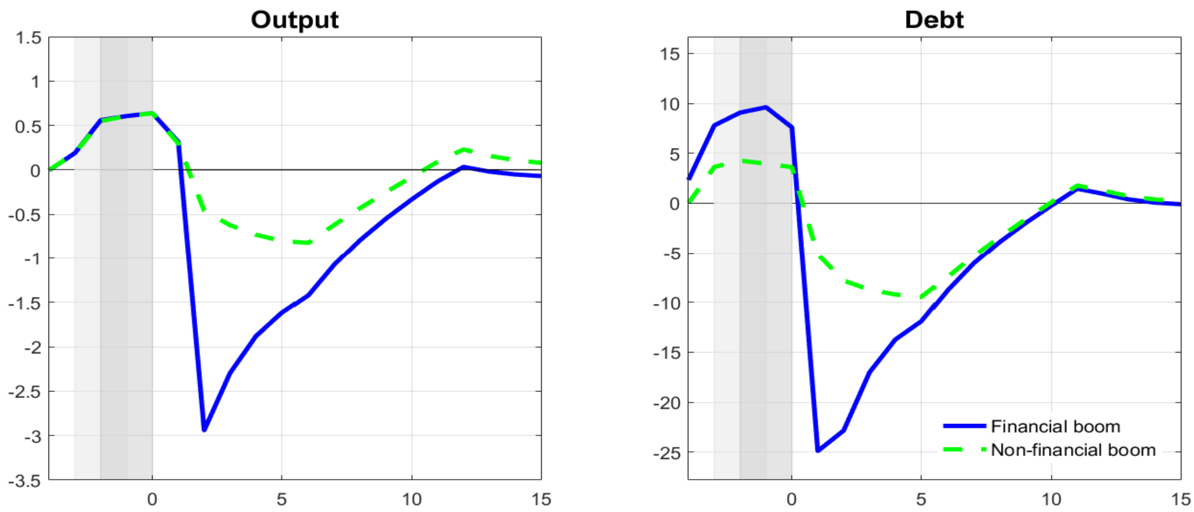
Notes: The left panel reports the standard deviation of output growth, while the right panel reports the standard deviation of expansions (solid line) and contractions (dashed line) in economic activity. These are determined based on whether output is above or below its steady-state level. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 11. Boom-bust cycles and leverage



Notes: The figure shows the path of output (in % deviations from the steady state). Starting in steady state, we generate a boom-bust cycle for different steady-state debt levels, as implied by different average LTV ratios. We first feed the economy with a series of positive shocks during the first five periods, up until period 0. The size of the expansionary shocks is set so as to make sure that the boom is identical across all the calibrations. Thus, we shock the economy with identical contractionary shocks for two periods, after which the negative shocks are 'phased out' over the next three periods, i.e., their size is reduced successively and linearly. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 12. Financial vs. non-financial recessions



Notes: The figure shows the path of output (left panel) and aggregate debt (right panel, both in % deviations from the steady state). The solid-blue line represents a financial boom, while the dashed-green line represents a non-financial boom. The light-grey area denotes periods in which the entrepreneur becomes financially unconstrained in the financial boom, while the darker grey area denotes periods in which the entrepreneur becomes unconstrained in the non-financial boom. The darkest grey area thus represents periods in which the entrepreneur becomes unconstrained in both types of boom (and the areas overlap). Impatient households remain constrained throughout in both types of booms. In this experiment, we set the average LTV ratios to $s^I = 0.85$ and $s^E = 0.94$. We calibrate the size of the expansionary shocks so as to deliver an identical increase in output during each type of boom (which lasts for five periods, up until period 0). We then subject the economy to identical sets of contractionary shocks of all three types. The contractionary shocks hit in periods 1 and 2, and are then 'phased out' over the next three periods, i.e., their size is reduced successively and linearly.

Appendix A: Assets and liabilities in the US

Figure 2 shows the ratio of liabilities to assets for households and firms in the United States, respectively. All data are taken from FRED (Federal Reserve Economic Data), Federal Reserve Bank of St. Louis. The primary source is Flow of Funds data from the Board of Governors of the Federal Reserve System. For business liabilities we use the sum of debt securities and loans of nonfinancial corporate and noncorporate businesses. For assets we follow Liu *et al.* (2013) and use data on both sectors' equipment and software as well as real estate at market value. For households and nonprofit organizations, we again use the sum of debt securities and loans as data for liabilities and use as assets both groups' real estate at market value and equipment and software of nonprofit organizations.

The ratios reported in Figure 2 are aggregate measures, and may therefore not reflect actual loan-to-value (LTV) requirements for the marginal borrower. Nonetheless, we report these figures since the flow of funds data deliver a continuous measure of LTV ratios covering the entire period 1952–2016. For households, the aggregate ratio of credit to assets in the economy is likely to understate the actual downpayment requirements faced by households applying for a mortgage loan, since loans and assets are not evenly distributed across households. In our model we distinguish between patient and impatient households, and we assume that only the latter group is faced with a collateral constraint. In the data we do not make such a distinction, so that the LTV ratio for households reported in Figure 2 represents an average of the LTV of patient households (savers), who are likely to have many assets and small loans, and that of impatient households (borrowers), who on average have larger loans and fewer assets. Justiniano *et al.* (2014) use the Survey of Consumer Finances and identify borrowers as households with liquid assets of a value less than two months of their income. Based on the surveys from 1992, 1995, and 1998, they arrive at an average LTV ratio for this group of around 0.8, while our measure fluctuates around 0.5 during the 1990s. Following Duca *et al.* (2011), an alternative approach is to focus on first-time home-buyers, who are likely to fully exploit their borrowing capacity. Using data from the American Housing Survey, these authors report LTV ratios approaching 0.9 towards the end of the 1990s; reaching a peak of almost 0.95 before the onset of the recent crisis. While these alternative approaches are likely to result in higher *levels* of LTV ratios, we are especially interested in the development of these ratios over a rather long time span. While we believe the Flow of Funds data provide the most comprehensive and consistent time series evidence in this respect, substantial increases over time in the LTV ratios faced by households have been extensively documented; see, e.g., Campbell and Hercowitz (2009), Duca *et al.* (2011), Favilukis *et al.* (2017), and Boz and Mendoza (2014). It should be noted that for households, various government-sponsored programs directed at lowering the down-payment requirements faced by low-income or first-time home buyers have been enacted by different administrations (Chambers *et al.*, 2009). These are likely to have contributed to the increase in the ratio of loans to assets illustrated in the left panel of Figure 2.

Likewise, the aggregate ratio of business loans to assets in the data may cover for a disparate distribution of credit and assets across firms. In general, the borrowing patterns and conditions of firms are more difficult to characterize than those of households, as their credit demand is more volatile, and their assets are less uniform and often more difficult to assess. Liu *et al.* (2013) also use Flow of Funds data to calibrate the LTV ratio of the entrepreneurs, and arrive at a value of 0.75. This ratio is based on the assumption that commercial real estate enters with a weight of 0.5 in the asset composition of firms. The secular increase in firm leverage over the second half of the 20th century has also been documented by Graham *et al.* (2014)

using data from the Compustat database.[27,28] These authors report loan-to-asset ratios that are broadly in line with those we present. More generally, an enhanced access of firms to credit markets over time has been extensively documented in the literature, as also discussed in the main text.

Appendix B: Additional empirical evidence

B1. Time-varying volatility and skewness

In the main text we report evidence on the skewness of real GDP growth being different before and during the Great Moderation. The choice of a cut-off date is inspired by a large literature that has documented a drop in the volatility over the two samples. This exercise entails a possible drawback: The estimates of the skewness can be biased by the first and second moment of the business cycle changing over time. In particular: i) There is now ample evidence that the volatility of the business cycle displays a cyclical behavior (see, e.g., Kim and Nelson, 1999; and McConnell and Perez-Quiros, 2000) and ii) the long-run growth rate of the economy since around 2000 is substantially lower than the average for the entire sample (see, e.g., Antolin-Diaz *et al.*, 2017). To account for these issues we report a measure of time-varying skewness of real GDP growth for the entire sample, relying on a nonparametric estimator. To this end, take a generic time series, y_t, so that its variance and skewness can be respectively calculated as

$$\begin{aligned}
\sigma^2 &= Var\left(y_t\right) = \frac{1}{T}\sum_{t=1}^{T}\left(y_t-\mu\right)^2, \\
\varrho &= Skew\left(y_t\right) = \left\{\frac{1}{T}\sum_{t=1}^{T}\left(y_t-\mu\right)^2\right\}^{-3/2}\left\{\frac{1}{T}\sum_{t=1}^{T}\left(y_t-\mu\right)^3\right\},
\end{aligned}$$

where T denotes the number of observations in the sample and $\mu = E\left(y_t\right) = T^{-1}\sum_{t=1}^{T} y_t$ is the sample average. Define the sample autocovariance and autocorrelation as

$$\begin{aligned}
\gamma_\tau &= \frac{1}{T}\sum_{t=1}^{T-|\tau|}\left(y_{t-|\tau|}-\mu\right)\left(y_t-\mu\right), \\
\rho_\tau &= \frac{\gamma_\tau}{\sigma^2}.
\end{aligned}$$

[27]It should be mentioned that they also show a Flow of Funds-based measure of debt to total assets at historical cost (or book value) for firms. The increase over time in this measure is smaller. However, we believe that the ratio of debt to *pledgeable* assets at market values (as shown in Figure 2) is the relevant measure for firms' access to collateralized loans, and hence more appropriate for our purposes.

[28]We emphasize that Figure 2 reports a *gross* measure of firm leverage. Bates *et al.* (2009) report that firm leverage *net of* cash holdings has been declining since 1980, but that this decline is entirely due to a large increase in cash holdings.

When y_t is a Gaussian process with absolutely summable autocovariances, it can be shown that the standard errors associated with the two measures are:[29]

$$\begin{aligned} Var\left(\sigma^2\right) &= \frac{2}{T}\left(\sum_{\tau=-\infty}^{\infty}\gamma_\tau\right)^2, \\ Var\left(\varrho\right) &= \frac{6}{T}\sum_{\tau=-\infty}^{\infty}\rho_\tau^3. \end{aligned}$$

In practice the two summations are truncated at some appropriate (finite) lag k.

The framework we follow in order to account for time-variation in the variance and skewness has a long pedigree in statistics, starting with the work of Priestley (1965), who introduced the concept of slowly varying process. This work suggests that time series may have time-varying spectral densities which change slowly over time, and proposed to describe those changes as the result of a non-parametric process. This work has more recently been followed up by Dahlhaus (1996), as well as Kapetanios (2007) and Giraitis *et al.* (2014) in the context of time-varying regression models and economic forecasting, respectively. Specifically, the time-varying variance and skewness are calculated as

$$\begin{aligned} \sigma_t^2 &= Var_t\left(y_t\right) = \sum_{j=1}^{t}\omega_{j,t}\left(y_j-\mu_t\right)^2, \\ \varrho_t &= Skew_t\left(y_t\right) = \left\{\sum_{j=1}^{t}\omega_{j,t}\left(y_j-\mu_t\right)^2\right\}^{-3/2}\left\{\sum_{j=1}^{t}\omega_{j,t}\left(y_j-\mu_t\right)^3\right\}, \end{aligned}$$

where $\mu_t = \sum_{j=1}^{t}\omega_{j,t}y_j$. Thus, the sample moments are discounted by the function $\omega_{t,T}$:

$$\omega_{j,t} = cK\left(\frac{t-j}{H}\right),$$

where c is an integration constant and $K\left(\frac{T-t}{H}\right)$ is the kernel function determining the weight of each observation j in the estimation at time t. This weight depends on the distance to t normalized by the bandwidth H. Giraitis *et al.* (2014) show that the estimator has desirable frequentist properties. They suggest using Gaussian kernels with the optimal bandwidth value $H = T^{1/2}$.

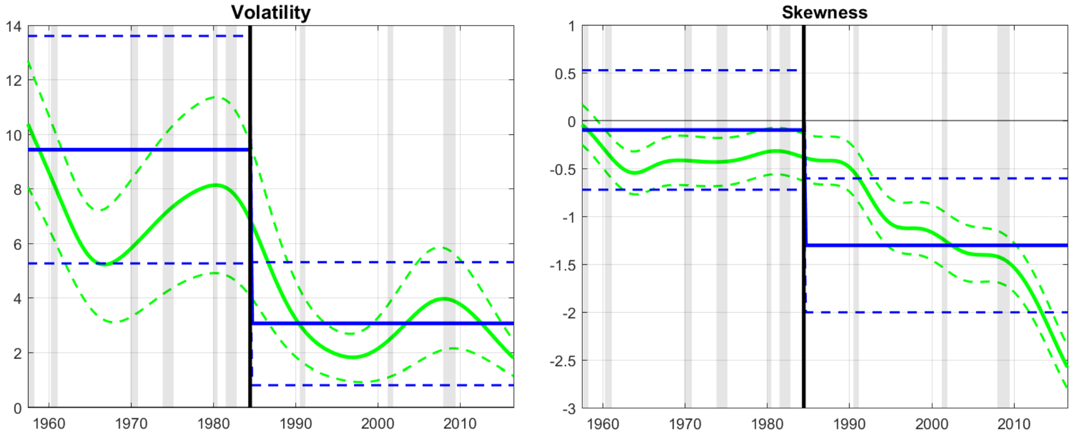
Similarly, we can compute the time-varying standard deviation of variance and skewness estimates using time-varying estimates of the sample autocovariance and autocorrelations:

$$\begin{aligned} \gamma_{\tau,t} &= \sum_{j=1}^{t-|\tau|}\omega_{j,t}\left(y_{j-|\tau|}-\mu_t\right)\left(y_j-\mu_t\right), \\ \rho_{\tau,t} &= \frac{\gamma_{\tau,t}}{\sigma_t^2}. \end{aligned}$$

Based on this, Figure B2 reports time-varying measures of volatility and skewness of GDP growth. The left panel confirms the widely documented decline in volatility. From the right panel, it is clear that skewness drops in the second subsample, with a first drop being identified after the 1991 recession and a further one after the Great Recession.

[29] The first expression computes the variance as the Newey-West variance of the squared residuals, in order to account for the autocorrelation of the errors. The second equality follows from Gasser (1975) and Psaradakis and Sola (2003).

Figure B1. Time-varying volatility and skewness



Notes. Figure B1 reports the time-varying variance and skewness of year-on-year growth of real GDP (green-continuous lines)—obtained by using a nonparametric estimator in the spirit of Giraitis *et al.* (2014)—as well as the associated 68% confidence interval (green-dashed lines). We also report the variance and skewness of real GDP growth computed over the pre- and post-Great Moderation sample (blue-continuous lines), as well as the associated 68% confidence interval (blues-dashed lines). The vertical shadowed bands denote the NBER recession episodes. Sample: 1947:I-2016:II. The first 10 years of data are dropped to initialize the algorithm.

B2. Normality tests

Table B2. Normality tests

	GDP growth (QoQ)	
	1947:I-1984:II	1984:III-2016:II
KS	0.638	0.002
AD	0.534	0.000
SW	0.507	0.000
JB	≫0.50	≪0.001

	GDP growth (YoY)	
	1947:I-1984:II	1984:III-2016:II
KS	0.289	0.004
AD	0.060	0.000
SW	0.091	0.000
JB	≫0.50	≪0.001

Notes. Table B2 reports the p-values of a battery of tests assuming the null hypothesis that real GDP growth is normally distributed in a given sample. KS refers to Kolmogorov-Smirnov test with estimated parameters (see Liliefors, 1967); AD refers to the test of Anderson and Darling (1954); SW refers to the Shapiro-Wilk test (Shapiro and Wilk, 1965) with p-values calculated as outlined by Royston (1992); JB refers to the Jarque-Bera test for normality (Jarque and Bera, 1987).

B3. Semivariances and asymmetry

Table B3. Semivariances

	GDP growth (QoQ)	
	1947:I-1984:II	1984:III-2016:II
σ	4.701	2.358
σ^-/σ^+	1.035	1.290
	GDP growth (YoY)	
	1947:I-1984:II	1984:III-2016:II
σ	3.070	1.747
σ^-/σ^+	1.062	1.366

Notes: Table B3 reports the volatility of different business cycle indicators and its decomposition into upside and downside semivariance. Specifically $\sigma = \sqrt{\sum_{t=1}^{T}(x_t - \overline{x})^2/T} = \sqrt{\sigma_U^2 + \sigma_D^2}$, with the upside and downside semivariance defined as $\sigma^+ = \sqrt{\sum_{t=1}^{T}(x_t - \overline{x})^2 \mathbb{1}(x_t \geq \overline{x})/T}$ and $\sigma^- = \sqrt{\sum_{t=1}^{T}(x_t - \overline{x})^2 \mathbb{1}(x_t < \overline{x})/T}$ respectively, where $1(z)$ is an indicator function taking value 1 when condition z is true and 0 otherwise.

B4. Business cycle asymmetries across the G7 countries

In this appendix we extend the empirical analysis on evolving asymmetries in the business cycle to the remaining G7 countries.[30] To calculate asymmetry statistics for these countries, it is crucial that the underlying long-run growth is appropriately removed, as the country-specific growth rates display large changes over the sample under investigation (see, e.g., Antolin-Diaz *et al.*, 2017).[31] To this end, we estimate long-run growth as the first difference in the smooth trend of the real GDP series. The latter is retrieved through the modified HP filter of Rotemberg (1999).[32] We use data from 1961:II to 2016:II and split the sample in the second quarter of 1984, so as to be consistent with the analysis in the main body of the paper.[33] Table B4 reports the skewness statistics and the ratio between the (square root of the) negative and the positive semivariance of the two subsamples for real GDP growth. For all countries we detect a substantial fall in skewness, along with a relative increase in the negative semivariance in the post-1984 sample.

[30] Stock and Watson (2005) have shown that, for these countries, the mid-1980s are associated with a sharp reduction in macroeconomic volatility.

[31] The results reported in Table 1 are robust to subtracting the underlying long-run growth rate in the US.

[32] The non-parametric method of Rotemberg (1999) ensures that changes in trend growth are not associated with the current stage of the cycle, thus obtaining a modicum of independence between the two series.

[33] The results are robust to delaying the cut-off date.

Table B4. Changing asymmetry for the remaining G7 countries

	QoQ				YoY			
	Skewness		σ^{-}/σ^{+}		Skewness		σ^{-}/σ^{+}	
	Pre-84	Post-84	Pre-84	Post-84	Pre-84	Post-84	Pre-84	Post-84
Canada	-0.1976	-0.9456	1.0401	1.2273	-0.9558	-1.0225	1.2835	1.3129
	[-0.901 ; 0.506]	[-1.566 ; -0.326]			[-1.704 ; -0.208]	[-1.717 ; -0.329]		
France	0.3276	-0.9127	0.9680	1.1915	0.2755	-0.9043	1.0287	1.2406
	[-0.367 ; 1.022]	[-1.541 ; -0.284]			[-0.427 ; 0.978]	[-1.611 ; -0.198]		
Germany	-0.3563	-1.4303	1.0786	1.2936	-0.5077	-1.0742	1.1722	1.2213
	[-1.06 ; 0.347]	[-2.043 ; -0.817]			[-1.257 ; 0.241]	[-1.757 ; -0.391]		
Italy	0.2474	-1.1005	0.9765	1.2750	-0.4313	-1.4200	1.1425	1.4398
	[-0.456 ; 0.951]	[-1.719 ; -0.482]			[-1.179 ; 0.317]	[-2.107 ; -0.733]		
Japan	-0.5166	-0.9817	1.0854	1.2054	-0.5267	-1.0209	1.1663	1.2473
	[-1.221 ; 0.188]	[-1.594 ; -0.369]			[-1.299 ; 0.246]	[-1.683 ; -0.358]		
United Kingdom	0.6330	-0.9726	0.8667	1.2197	-0.4359	-1.4920	1.1484	1.4685
	[-0.071 ; 1.337]	[-1.605 ; -0.34]			[-1.19 ; 0.318]	[-2.203 ; -0.781]		

Notes: For each country the table reports the skewness (68% confidence intervals in brackets) and the ratio of the (square root of the) negative over positive semivariance for detrended GDP growth, 1961:I-2016:II. The data are taken from the OECD quarterly database.

B5. Additional evidence on the standardized violence of the US business cycle

Table B5. Standardized violence of U.S. recessions (Robustness)

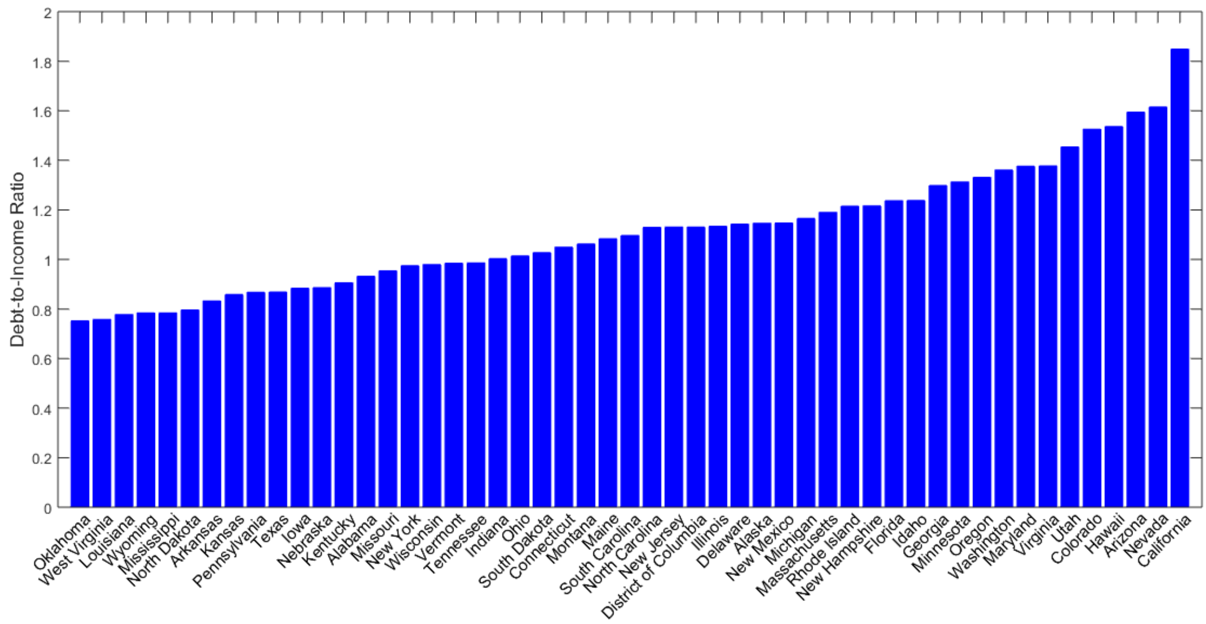
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1953:II – 1954:II	0.6353	1.1270	0.7337	0.7313	0.5989	0.6644	0.5835
1957:III – 1958:II	1.6292	2.4152	1.5723	1.7671	1.7163	1.6875	1.7846
1960:II – 1961:I	0.3512	0.5951	0.3874	0.4288	0.3074	0.4636	0.3038
1969:IV – 1970:IV	0.1631	0.1556	0.1013	0.2476	0.1548	0.2398	0.1584
1973:IV – 1975:I	0.6618	0.8358	0.5441	0.8468	0.5417	0.8329	0.5086
1980:I – 1980:III	0.9991	1.4542	0.9467	1.2388	0.9719	1.2342	0.8633
1981:III – 1982:IV	0.5977	0.8851	0.5762	0.6452	0.4481	0.6211	0.3995
1990:III – 1991:I	1.9098	1.5271	1.1319	1.3633	1.2550	1.2294	1.3007
2001:I – 2001:IV	0.7295	0.7299	0.5410	0.7551	0.4630	0.7010	0.4191
2007:IV – 2009:II	1.8467	1.6652	1.2343	2.0200	1.6071	1.9154	1.5710
Average							
Pre-84	0.7196	1.0669	0.6945	0.8437	0.6770	0.8205	0.6574
Post-84	1.4953	1.3074	0.9691	1.3795	1.1083	1.2819	1.0969

Notes: Table B5 reports different measures of standardized violence that change depending on the business cycle volatility employed in the denominator. Column (1) follows the same procedure employed to obtain standardized violence in Table 2, though the volatility measure is retrieved from quarter-on-quarter growth rates of real GDP. In the remaining computations, even column numbers report violence statistics that are standardized by volatility measures retrieved from quarter-on-quarter growth rates or real GDP, while in odd column numbers the standardization is operated through volatility measures obtained from year-on-year growth rates. Columns (2) and (3) calculate the volatility by splitting the data between pre- and post-Great Moderation. In columns (4) and (5) the standardization is operated by considering the following stochastic volatility model for real GDP growth: $y_t = \rho_0 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \sigma_t \varepsilon_t$, where $\sigma_t^2 = \sigma_{t-1}^2 + \kappa \sigma_t^2 \left(\varepsilon_t^2 - 1\right)$ and $\varepsilon_t \sim N(0,1)$. In columns (6) and (7) the standardization is operated by considering a time-varying AR model for real GDP growth with stochastic volatility similar to that of Stock and Watson (2005), where all the time-varying parameters follow random walk laws of motion (as in Delle Monache and Petrella, 2017).

B6. Household leverage in the US

Figure B6 displays the ranking of most highly leveraged US states used in Section 2.2.

Figure B6. U.S. States ordered by households' average debt-to-income ratio



Notes. U.S. States ordered by the average debt-to-income ratio in the household sector, over the period 2003-2007.

Appendix C: Details on the solution of the two-period model

Here, we provide details on the computation of the competitive equilibrium of the two-period model discussed in Section 3. The notation is explained in the main text. The utility of the representative household is given by

$$\mathrm{E}_0 \left\{ \sum_{t=1}^{2} \beta^{t-1} \left[a \log C_t + (1-a) \log H_t \right] \right\}. \tag{23}$$

The budget constraints in periods 1 and 2 are

$$C_1 + Q_1 (H_1 - H_0) - B_1 = Y_1 - RB_0, \tag{24}$$
$$C_2 + Q_2 (H_2 - H_1) = Y - RB_1, \tag{25}$$

respectively. The collateral constraint on debt is

$$B_1 \leq s \frac{\mathrm{E}_1 \{Q_2\} H_1}{R}. \tag{26}$$

The model is solved by backwards induction. In period 2 the household solves

$$\max_{C_2} \quad \beta \left\{ a \log C_2 + (1-a) \log \left(H_1 + \frac{Y - RB_1 - C_2}{Q_2} \right) \right\},$$

by taking as given Q_2. The first-order condition is

$$aH_2Q_2 = (1-a)\,C_2. \tag{27}$$

Combined with (25), we obtain two policy functions for nondurable consumption and land:

$$\begin{aligned} C_2 &= a\,(Y - RB_1 + Q_2H_1)\,, &(28)\\ H_2Q_2 &= (1-a)\,(Y - RB_1 + Q_2H_1)\,. &(29)\end{aligned}$$

These provide period 2's demand schedules as functions of the state (H_1, B_1), and the price Q_2. Note that (28) and (29) hold for *any* B_1, and therefore irrespective of whether the household was constrained or not in period 1 in its debt choice.

In period 1, the individual solves

$$\begin{aligned} \max_{H_1,B_1} \quad & [a\log[Y_1 - RB_0 + B_1 - Q_1(H_1 - H_0)] + (1-a)\log H_1] \\ & +\beta\,[a\log C_2 + (1-a)\log H_2]\,, \\ & \text{s.t. } (26) \text{ and } (28)\text{–}(29), \end{aligned}$$

taking prices Q_1 and Q_2 as given. The first-order conditions are

$$\begin{aligned} -\frac{a}{C_1}Q_1 + \frac{1-a}{H_1} + \beta\left(\frac{a}{C_2}\frac{\partial C_2}{\partial H_1} + \frac{1-a}{H_2}\frac{\partial H_2}{\partial H_1}\right) + \mu s\frac{Q_2}{R} &= 0, &(30)\\ \frac{a}{C_1} + \beta\left(\frac{a}{C_2}\frac{\partial C_2}{\partial B_1} + \frac{1-a}{H_2}\frac{\partial H_2}{\partial B_1}\right) - \mu &= 0, &(31)\end{aligned}$$

where $\mu \geq 0$ is the multiplier on (26) and the partial derivatives can be recovered from (28) and (29), i.e.:

$$\frac{\partial C_2}{\partial H_1} = aQ_2, \qquad \frac{\partial H_2}{\partial H_1} = 1-a, \qquad \frac{\partial C_2}{\partial B_1} = -Ra, \qquad \frac{\partial H_2}{\partial B_1} = -\frac{R\,(1-a)}{Q_2}. \tag{32}$$

For future reference, it is convenient to state a consolidated first-order condition found by adding (30) and (31), and applying (32):

$$\frac{1-a}{H_1} + \frac{\beta\,(Q_2 - Q_1R)}{Y - RB_1 + Q_2H_1} = \mu\left(Q_1 - s\frac{Q_2}{R}\right). \tag{33}$$

Definition. *A competitive equilibrium is a vector* $\{C_1, C_2, H_1, H_2, Q_1, Q_2, B_1, \mu\}$, *which given* H_0, B_0, Y_1, and Y *satisfies*

(i) *the first-order conditions (27), (30), (31),*

(ii) *the budget constraints (24), (25),*

(iii) *the land market-clearing conditions*

$$\begin{aligned} H_1 &= H, &(34)\\ H_2 &= H, &(35)\end{aligned}$$

where $H > 0$ *is the exogenous stock of land,*

(iv) *the complementary slackness condition associated with (26):*

$$\left(B_1 - s\frac{\mathrm{E}_1\{Q_2\}H_1}{R}\right)\mu = 0. \tag{36}$$

In the following, we provide closed-form solutions for the variables most relevant to our analysis in the main text. We divide the exposition into the 'regime' where the credit constraint (26) does not bind, $\mu = 0$, and the one where it does, $\mu > 0$. Then we show that it is period-1 income that determines which regime prevails. We then prove that the critical income value (below which the credit constraint becomes binding) is identical to the income level that exactly makes consumption under either regime identical. For simplicity, we let the initial stock of land equal its value in periods 1 and 2, i.e., we assume $H_0 = H$.

The case of a non-binding credit constraint

In case of a non-binding credit constraint, we have $\mu = 0$, and therefore (33) reduces to

$$\frac{1-a}{H_1} = -\frac{\beta(Q_2 - Q_1R)}{Y - RB_1 + Q_2H_1}.$$

Using (34) this becomes

$$(1-a)(Y - RB_1) + (1-a)Q_2H = -\beta HQ_2 + \beta RQ_1H.$$

Combining (29) with (35) we recover

$$Q_2 = \frac{1-a}{aH}(Y - RB_1). \tag{37}$$

which inserted into the previous expression yields

$$(Y - RB_1)\frac{(1-a)(1+\beta)}{a} = \beta RQ_1H. \tag{38}$$

Then use (31) along with with $\mu = 0$:

$$\frac{a}{C_1} = -\beta\left(\frac{a}{C_2}\frac{\partial C_2}{\partial B_1} + \frac{1-a}{H_2}\frac{\partial H_2}{\partial B_1}\right),$$

which by (32) gives

$$\frac{a}{Y_1 - RB_0 + B_1 - Q_1(H_1 - H_0)} = \beta R\left(\frac{a^2}{C_2} + \frac{(1-a)^2}{H_2Q_2}\right).$$

Applying (34), (35) and (37) we then obtain the solution for B_1:

$$B_1 = \frac{\beta}{1+\beta}(RB_0 - Y_1) + \frac{1}{R(1+\beta)}Y. \tag{39}$$

Combining (39) with (38) we can recover Q_1 as

$$Q_1 = \frac{(1-a)}{aH}\left(Y_1 - RB_0 + \frac{Y}{R}\right).$$

Consumption in period 1, in the absence of a binding credit constraint, then follows from (24) as

$$C_1 = \frac{1}{1+\beta}(Y_1 - RB_0) + \frac{1}{R(1+\beta)}Y, \tag{40}$$

which is (3).

The case of a binding credit constraint

In the case where (26) binds, we have $\mu > 0$ and

$$B_1 = s\frac{Q_2 H_1}{R}. \tag{41}$$

By use of (34) and (37), we obtain $B_1 = [s(1-a)/(aR)](Y - RB_1)$ from which we recover period-1 debt as

$$B_1 = \frac{s(1-a)}{a+s(1-a)}\frac{Y}{R}. \tag{42}$$

Using (42) together with the budget constraint (24), immediately gives

$$C_1 = Y_1 - RB_0 + \frac{s(1-a)}{a+s(1-a)}\frac{Y}{R}, \tag{43}$$

which is (2).

The multiplier μ and the role of $\overline{Y}_1$

It is straightforward to find the period-1 income level that equalizes nondurable consumption under either regime, (40) and (43). This is $\overline{Y}_1$ as given by (4). To formally relate this value to the question of whether the credit constraint binds or not, we derive the value of the multiplier $\mu \geq 0$. We have from (31) that

$$\mu = \frac{a}{C_1} + \beta\left(\frac{a}{C_2}\frac{\partial C_2}{\partial B_1} + \frac{1-a}{H_2}\frac{\partial H_2}{\partial B_1}\right).$$

Using (34), (35), (24), and applying (32), this becomes

$$\mu = \frac{a}{Y_1 - RB_0 + B_1} - \frac{\beta R}{Y - RB_1 + Q_2 H}.$$

By (37) and (39), we finally get

$$\mu = \frac{a}{Y_1 - RB_0 + \frac{s(1-a)}{a+s(1-a)}\frac{Y}{R}} - \frac{a\beta R}{\left(1 - \frac{s(1-a)}{a+s(1-a)}\right)Y}. \tag{44}$$

Since the denominator in the first expression on the right-hand side of (44) is positive (otherwise $C_1 \leq 0$), μ is strictly decreasing in Y_1. We can therefore find the minimum value of Y_1 that leads to $\mu = 0$. Call this value $\widetilde{Y}_1$. It then follows that for all $Y_1 < \widetilde{Y}_1$ we have $\mu > 0$, and for all $Y_1 \geq \widetilde{Y}_1$, we have $\mu = 0$. From (44) we see that $\widetilde{Y}_1$ satisfies

$$\frac{1}{\widetilde{Y}_1 - RB_0 + \dfrac{s\,(1-a)}{a+s\,(1-a)}\dfrac{Y}{R}} = \frac{\beta R}{\left(1 - \dfrac{s\,(1-a)}{a+s\,(1-a)}\right) Y},$$

from which we recover

$$\widetilde{Y}_1 = RB_0 + \frac{a - \beta s\,(1-a)}{a+s\,(1-a)}\frac{Y}{\beta R}. \tag{45}$$

A comparison between (45) and (4) reveals $\widetilde{Y}_1 = \overline{Y}_1$. Hence, as stated in the main text, $Y_1 \geq \overline{Y}_1$ involves an unconstrained regime implying consumption given by (3). Similarly, $Y_1 < \overline{Y}_1$ characterizes the constrained regime implying that nondurable consumption is given by (2).

Appendix D: First-order conditions of the DSGE model

This appendix reports the first-order conditions for each agent in the model.

Patient households

Patient households' optimal behavior is described by the following first-order conditions:

$$\frac{1}{C_t^P - \theta^P C_{t-1}^P} - \frac{\beta\theta^P}{\mathrm{E}_t\left\{C_{t+1}^P\right\} - \theta^P C_t^P} = \lambda_t^P, \tag{46}$$

$$\nu^P\left(1 - N_t^P\right)^{-\sigma_N^P} = \lambda_t^P W_t^P, \tag{47}$$

$$\lambda_t^P = \beta^P R_t \mathrm{E}_t\left\{\lambda_{t+1}^P\right\}, \tag{48}$$

$$Q_t = \frac{\varepsilon_t}{\lambda_t^P H_t^P} + \beta^P \mathrm{E}_t\left\{\frac{\lambda_{t+1}^P}{\lambda_t^P} Q_{t+1}\right\}, \tag{49}$$

where λ_t^P is the multiplier associated with (7).

Impatient households

The first-order conditions of the impatient households are given by:

$$\frac{1}{C_t^I - \theta^I C_{t-1}^I} - \frac{\beta\theta^I}{\mathrm{E}_t\left\{C_{t+1}^I\right\} - \theta^I C_t^I} = \lambda_t^I, \tag{50}$$

$$\nu^I\left(1 - N_t^I\right)^{-\sigma_N^I} = \lambda_t^I W_t^I, \tag{51}$$

$$\lambda_t^I - \mu_t^I = \beta^I R_t \mathrm{E}_t\left\{\lambda_{t+1}^I\right\}, \tag{52}$$

$$Q_t = \frac{\varepsilon_t}{\lambda_t^I H_t^I} + \beta^I \mathrm{E}_t\left\{\frac{\lambda_{t+1}^I}{\lambda_t^I} Q_{t+1}\right\} + s_t^I \frac{\mu_t^I}{\lambda_t^I}\frac{\mathrm{E}_t\left\{Q_{t+1}\right\}}{R_t}, \tag{53}$$

where λ_t^I is the multiplier associated with (9), and μ_t^I is the multiplier associated with (10). Additionally, the complementary slackness condition

$$\mu_t^I\left(B_t^I - s_t^I \frac{\mathrm{E}_t\left\{Q_{t+1}\right\} H_t^I}{R_t}\right) = 0, \tag{54}$$

must hold along with $\mu_t^I \geq 0$ and (10).

Entrepreneurs

The optimal behavior of the entrepreneurs is characterized by:

$$\begin{aligned}
\frac{1}{C_t^E - \theta^E C_{t-1}^E} - \frac{\beta\theta^E}{\mathrm{E}_t\left\{C_{t+1}^E\right\} - \theta^E C_t^E} &= \lambda_t^E, && (55)\\
\lambda_t^E - \mu_t^E &= \beta^E R_t \mathrm{E}_t\left\{\lambda_{t+1}^E\right\}, && (56)
\end{aligned}$$

$$\lambda_t^E = \psi_t^E\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E\Omega\mathrm{E}_t\left\{\psi_{t+1}^E\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right\}, \tag{57}$$

$$\begin{aligned}
\psi_t^E &= \beta^E r_t^K \mathrm{E}_t\left\{\lambda_{t+1}^E\right\} + \beta^E(1-\delta)\mathrm{E}_t\left\{\psi_{t+1}^E\right\} + \mu_t^E s_t^E \frac{\mathrm{E}_t\left\{Q_{t+1}^K\right\}}{R_t}, && (58)\\
Q_t &= \beta^E r_t^H \mathrm{E}_t\left\{\frac{\lambda_{t+1}^E}{\lambda_t^E}\right\} + \beta^E \mathrm{E}_t\left\{\frac{\lambda_{t+1}^E}{\lambda_t^E} Q_{t+1}\right\} + s_t^E \frac{\mu_t^E}{\lambda_t^E}\frac{\mathrm{E}_t\left\{Q_{t+1}\right\}}{R_t}, && (59)
\end{aligned}$$

where λ_t^E, μ_t^E and ψ_t^E are the multipliers associated with (14), (15), and (16), respectively. Moreover,

$$\mu_t^E\left(B_t^E - s_t^E \mathrm{E}_t\left\{\frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t}\right\}\right) = 0, \tag{60}$$

holds along with $\mu_t^E \geq 0$ and (16). Finally, the definition of Q_t^K implies that

$$Q_t^K = \psi_t^E / \lambda_t^E. \tag{61}$$

Firms

The first-order conditions for the firms determine the optimal demand for the input factors:

$$\begin{aligned}
\alpha\gamma Y_t / N_t^P &= W_t^P, && (62)\\
(1-\alpha)\gamma Y_t / N_t^I &= W_t^I, && (63)\\
(1-\gamma)(1-\phi)\mathrm{E}_t\left\{Y_{t+1}\right\}/K_t &= r_t^K, && (64)\\
(1-\gamma)\phi\mathrm{E}_t\left\{Y_{t+1}\right\}/H_t^E &= r_t^H. && (65)
\end{aligned}$$

Appendix E: Solution method

We log-linearize the model around its non-stochastic steady state, and then solve it numerically as described in the following. When solving the model, we treat the collateral constraints as inequalities, accounting for two complementary slackness conditions (54) and (60). We then adopt the solution method of Holden and Paetz (2012), on which this appendix builds. In turn, Holden and Paetz (2012) expand on previous work by Laséen and Svensson (2011). With first-order perturbations, this solution method is equivalent to the piecewise linear approach discussed by Guerrieri and Iacoviello (2015). We have verified that their proposed solution method does indeed produce identical results. Furthermore, Holden and Paetz (2012) and Guerrieri and Iacoviello (2015) evaluate the accuracy of their respective methods against a global solution based on projection methods. This is done for a very simple model with a borrowing constraint, for which a highly accurate global solution can be obtained and used as a benchmark. They find that the local approximations are very accurate. For the model used in this paper, the large number of state variables (14 endogenous state variables and three shocks) renders the use of global solution methods impractical due to the curse of dimensionality typically associated with such methods.

The collateral constraints put an upper bound on the borrowing of each of the two constrained agents. While the constraints are binding in the steady state, this may not be the case outside the steady state, where the constraints may not bind. Observe that we can reformulate the collateral constraints in terms of restrictions on each agent's shadow value of borrowing; μ_t^j, $j = \{I, E\}$: We know that $\mu_t^j \geq 0$ if and only if the optimal debt level of agent j is exactly at or above the collateral value. In other words, we need to ensure that $\mu_t^j \geq 0$. If this restriction is satisfied with inequality, the constraint is binding, so the slackness condition is satisfied. If it holds with equality, the collateral constraint becomes non-binding, but the slackness condition is still satisfied. If instead $\mu_t^j < 0$, agent j's optimal level of debt is lower than the credit limit, so that treating his collateral constraint as an equality implies that we are forcing him to borrow 'too much'. In this case, the slackness condition is violated. We then need to add shadow price shocks so as to 'push' μ_t^j back up until it exactly equals its lower limit of zero and the slackness condition is satisfied. To ensure compatibility with rational expectations, these shocks are added to the model as 'news shocks'. The idea of adding such shocks to the model derives from Laséen and Svensson (2011), who use such an approach to deal with pre-announced paths for the interest rate setting of a central bank. The contribution of Holden and Paetz (2012) is to develop a numerical method to compute the size of these shocks that are required to obtain the desired level for a given variable in each period, and to make this method applicable to a general class of potentially more complicated problems than the relatively simple experiments conducted by Laséen and Svensson (2011).

We first describe how to compute impulse responses to a single generic shock, e.g., a technology shock. The first step is to add independent sets of shadow price shocks to each of the two log-linearized collateral constraints. To this end, we need to determine the number of periods T in which we conjecture that the collateral constraints may be non-binding. This number may be smaller than or equal to the number of periods for which we compute impulse responses; $T \leq T^{IRF}$. For each period $t \leq T$, we then add shadow price shocks which hit the economy in period t but become known at period 0, that is, at the same time the economy is hit by the technology shock.

Let $\widehat{X_t}$ denote the log-deviation of a generic variable X_t from its steady-state value X, except for the following variables: For the interest rates, $\widehat{R}_t \equiv R_t - R$, $\widehat{r}_t^H \equiv r_t^H - r^H$ and $\widehat{r}_t^K \equiv r_t^K - r^K$, and for debt, $\widehat{B}_t^i \equiv (B_t^i - B^i)/Y$, $i = P, I, E$. We can then write the log-linearized collateral constraints, augmented with the shadow price shocks, as follows:

$$\frac{Y}{B^I}\widehat{B}_t^I = \widehat{s}_t^I + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \widehat{H}_t^I - \beta^P\widehat{R}_t - \sum_{s=0}^{T-1}\varepsilon_{s,t-s}^{SP,I},$$

$$\frac{Y}{B^E}\widehat{B}_t^E = \widehat{s}_t^E - \beta^P\widehat{R}_t + \frac{K}{K+QH^E}\left(\mathrm{E}_t\left\{\widehat{Q}_{t+1}^K\right\} + \widehat{K}_t\right) + \frac{QH^E}{K+QH^E}\left(\mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \widehat{H}_t^E\right) - \sum_{s=0}^{T-1}\varepsilon_{s,t-s}^{SP,E},$$

where $\varepsilon_{s,t-s}^{SP,j}$ is the shadow price shock that hits agent j in period $t = s$, and is anticipated by all agents in period $t = t - s = 0$ ensuring consistency with rational expectations. We let all shadow price shocks be of unit magnitude. We then need to compute two sets of weights α_{μ_I} and α_{μ_E} to control the impact of each shock on μ_t^I and μ_t^E. The 'optimal' sets of weights ensure that μ_t^I and μ_t^E are bounded below at exactly zero. The weights are computed by solving the following quadratic programming problem:

$$\begin{aligned}\alpha^* &\equiv \begin{bmatrix}\alpha^{*\prime}_{\mu_I} & \alpha^{*\prime}_{\mu_E}\end{bmatrix}' \\ &= \arg\min \begin{bmatrix}\alpha'_{\mu_I} & \alpha'_{\mu_E}\end{bmatrix} \left[\begin{bmatrix}\mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A}\end{bmatrix} + \begin{bmatrix}\widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}}\end{bmatrix}\begin{bmatrix}\alpha_{\mu_I} \\ \alpha_{\mu_E}\end{bmatrix}\right],\end{aligned}$$

subject to

$$\alpha'_{\mu_j} \geq 0,$$

$$\mu^j + \widetilde{\mu}^{j,A} + \widetilde{\mu}^{j,\varepsilon^{SP,j}}\alpha_{\mu_j} + \widetilde{\mu}^{j,\varepsilon^{SP,k}}\alpha_{\mu_k} \geq 0,$$

$j = \{I, E\}$. Here, μ^j and $\widetilde{\mu}^{j,A}$ denote, respectively, the steady-state value and the unrestricted relative impulse response of μ^j to a technology shock, that is, the impulse-response of μ^j when the collateral constraints are assumed to always bind. In this respect, the vector $\begin{bmatrix}\mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A}\end{bmatrix}$ contains the absolute, unrestricted impulse responses of the two shadow values stacked. Further, each matrix $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ contains the relative impulse responses of μ^j to shadow price shocks to agent k's constraint for $j, k = \{I, E\}$, in the sense that column s in $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ represents the response of the shadow value to a shock $\varepsilon^{SP,j}_{s,t-s}$, i.e. to a shadow price shock that hits in period s but is anticipated at time 0, as described above.[34] The off-diagonal elements of the matrix $\begin{bmatrix}\widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}}\end{bmatrix}$ take into account that the impatient household may be affected if the collateral constraint of the entrepreneur becomes non-binding, and *vice versa*. Following the discussion in Holden and Paetz (2012), a sufficient condition for the existence of a unique solution to the optimization problem is that the matrix $\begin{bmatrix}\widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}}\end{bmatrix} + \begin{bmatrix}\widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}}\end{bmatrix}'$ is positive definite. We have checked and verified that this condition is in fact always satisfied.

We can explain the nature of the optimization problem as follows. First, note that $\mu^j + \widetilde{\mu}^{j,A} + \widetilde{\mu}^{j,\varepsilon^{SP,j}}\alpha_{\mu_j} + \widetilde{\mu}^{j,\varepsilon^{SP,k}}\alpha_{\mu_k}$ denotes the combined response of μ^j_t to a given shock (here, a technology shock) *and* a simultaneous announcement of a set of future shadow price shocks for a given set of weights. Given the constraints of the problem, the objective is to find a set of optimal weights so that the impact of the (non-negative) shadow-price shocks is exactly large enough to make sure that the response of μ^j_t is never negative. The minimization ensures that the impact of the shadow price shocks will never be larger than necessary to obtain this. Finally, we only allow for solutions for which the value of the objective function is zero. This ensures that at any given horizon, positive shadow price shocks occur if and only if at least one of the two constrained variables, μ^I_t and μ^E_t, are at their lower bound of zero in that period. As pointed out by Holden and Paetz (2012), this can be thought of as a complementary slackness condition on the two inequality constraints of the optimization problem. Once we have solved the minimization problem, it is straightforward to compute the bounded impulse responses of all endogenous variables by simply adding the optimally weighted shadow price shocks to the unconstrained impulse responses of the model in each period.

We rely on the same method to compute dynamic simulations. In this case, however, we need to allow for more than one type of shock. For each period t, we first generate the shocks hitting the economy. We then compute the unrestricted path of the endogenous variables given those shocks and given the simulated values in $t-1$. The unrestricted paths of the bounded

[34] Each matrix $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ needs to be a square matrix, so if the number of periods in which we guess the constraints may be non-binding is smaller than the number of periods for which we compute impulse responses, $T < T^{IRF}$, we use only the first T rows of the matrix, i.e., the upper square matrix.

variables (μ_t^I and μ_t^E) then take the place of the impulse responses in the optimization problem. If the unrestricted paths of μ_t^I and μ_t^E never hit the bounds in future periods, our simulation for period t is fine. If the bounds are hit, we follow the method above and add anticipated shadow price shocks for a sufficient number of future periods. We then compute restricted values for all endogenous variables, and use these as our simulation for period t. Note that, unlike the case for impulse responses, in our dynamic simulations not all anticipated future shadow price shocks will eventually hit the economy, as other shocks may occur before the realization of the expected shadow price shocks and push the restricted variables away from their bounds.

Appendix F: Data description and estimation strategy

As described in the main text, we use data for the following five macroeconomic variables of the U.S. economy spanning the period 1952:I–1984:II: The year-on-year growth rates (in log-differences) of real GDP, real private consumption, real non-residential investment, and real house prices, and the average of the deviations from trend of the two LTA series reported in the right panel of Figure 2. All data series are taken from the Federal Reserve's FRED database, with the exception of the house price, which is provided by the US Census Bureau. The series are the following:

- Growth rate of *Real Gross Domestic Product*, billions of chained 2009 dollars, seasonally adjusted, annual rate (FRED series name: GDPC1).
- Growth rate of *Real Personal Consumption Expenditures*, billions of chained 2009 dollars, seasonally adjusted, annual rate (FRED series name: PCECC96).
- Growth rate of *Real private fixed investment: Nonresidential* (chain-type quantity index), index 2009=100, seasonally adjusted (FRED series name: B008RA3Q086SBEA).
- Growth rate of *Price Index of New Single-Family Houses Sold Including Lot Value*, index 2005=100, not seasonally adjusted. This series is available only from 1963:Q1 onwards.
  - To obtain the house price in real terms, this series is deflated using the GDP deflator (*Gross Domestic Product: Implicit Price Deflator*, index 2009=100, seasonally adjusted, FRED series name: GDPDEF).
- LTA data: See Appendix A. We use the average of the cyclical components—obtained through a multivariate Beveridge-Nelson decomposition—of the series in the right panel of Figure 2 for the period up until 1984:II.

Estimation

We use 16 empirical moments in the SMM estimation: The standard deviations and first-order autoregressive parameters of each of the five variables described above, the correlation of consumption, investment, and house prices with output, and the skewness of output, consumption, and investment. These moments are matched to their simulated counterparts from the theoretical model. Our estimation procedure seeks to minimize the sum of squared deviations between empirical and simulated moments. As some of the moments are measured in different units (e.g., standard deviations vs. correlations), we use the percentage deviation from the empirical moment in each case. In order for the minimization procedure to converge, it is crucial to use the same set of shocks repeatedly, making sure that the only change in the simulated moments from one iteration to the next is that arising from updating the parameter values. In practice, since the list of parameter values to be estimated includes the variance of the shocks in the model, we draw from the standard normal distribution with zero mean and unit variance, and then scale the shocks by the variance of each of the three shock processes, allowing us to estimate the latter. We use a draw of 2000 realizations of each of the three shocks in the model, thus obtaining simulated moments for 2000 periods.[35] To make sure that the draw of shocks used is representative of the underlying distribution, we make 501 draws of potential shock matrices, rank these in terms of the standard deviations of each of the three shocks, and select the shock matrix closest to the median along all three dimensions. This matrix of shocks is then used in the estimation. In the estimation, we impose only very general bounds on parameter values: All parameters are bounded below at zero, and the habit formation parameters along with all AR(1)-coefficients are bounded above at 0.99—a bound that is never attained.

To initiate the estimation procedure a set of initial values for the estimated parameters are needed. These are chosen based on values reported in the existing literature. It is important to state that the estimation results proved robust to changes in the set of initial values, as long as these remain within the range of available estimates. Based on the empirical estimates of Justiniano *et al.* (2013), we set the initial values of the investment adjustment cost parameter (Ω) and the parameters governing habit formation in consumption for the three agents to 4 and 0.7, respectively.[36] For the technology shock, we choose values similar to those used in most of the real business cycle literature, $\rho_A = 0.97$ and $\sigma_A = 0.005$ (see, e.g., Mandelman *et al.*, 2011). For the credit limit shock, we set the persistence parameter $\rho_s = 0.98$, while the standard deviation is set to $\sigma_s = 0.01$, consistent with the values estimated by Jermann and Quadrini (2012) and Liu *et al.* (2013). Finally, for the land-demand shock, we set $\rho_\varepsilon = 0.96$ and $\sigma_\varepsilon = 0.06$, in line with Iacoviello and Neri (2010) and Liu *et al.* (2013).

We abstain from using an optimal weighting matrix in the estimation. This choice is based on the findings of Altonji and Segal (1996), who show that when GMM is used to estimate covariance structures and, potentially, higher-order moments such as variances, as in our case, the use of an optimal weighting matrix causes a severe downward bias in estimated parameter values. Similar concerns apply to SMM as to GMM. The bias arises because the moments used to fit the model itself are correlated with the weighting matrix, and may thus be avoided by the use of fixed weights in the minimization. Altonji and Segal (1996) demonstrate that minimization schemes with fixed weights clearly dominate optimally weighted ones in such circumstances. Ruge-Murcia (2012) points out that parameter estimates remain consistent for any positive-definite weighting matrix, and finds that the accuracy and efficiency gains associated with an optimal weighting matrix are not overwhelming. The empirical moments and their model counterparts upon estimation are reported in Table F1.

When computing standard errors, we rely on a version of the delta method, as described, e.g., in Hamilton (1994). We approximate the numerical derivative of the moments with respect to the estimated parameters using the secant that can be computed by adding and subtracting ϵ to/from the estimates, where ϵ is a very small number. The covariance (or spectral density) matrix is estimated using the Newey-West estimator.

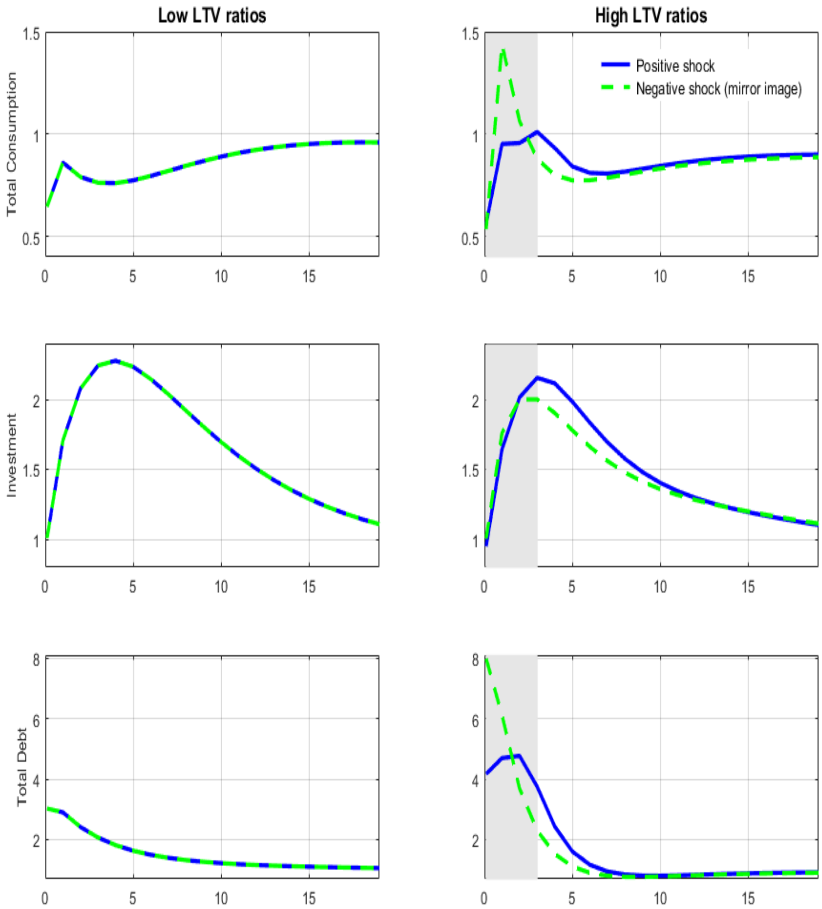
[35] Our simulated sample is thus more than 15 times longer than the actual dataset (which spans 130 quarters). Ruge-Murcia (2012) finds that SMM is already quite accurate when the simulated sample is five or ten times longer than the actual data.

[36] Unlike the other estimated parameters, θ^P and θ^I also affect the steady state of the model. To account for this, we rely on the following iterative procedure: We first calibrate the model based on the starting value for θ^P and θ^I. Upon estimation, but before simulating the model, we recalibrate it for the estimated values of the habit parameters. This leads only to a small change in the value of ε, while the remaining parameters are unaffected.

Table F1. Empirical and simulated moments		
	Model simulations	U.S. data (1952:I–1984:II)
	Standard deviations (percent)	
Output	2.75	2.84
Consumption	2.00	2.23
Investment	6.25	6.91
House price	3.71	3.05
LTV ratio	7.46	6.96
	Skewness	
Output	−0.16	−0.38
Consumption	−0.17	−0.31
Investment	−0.07	−0.41
	Autocorrelations	
Output	0.78	0.82
Consumption	0.58	0.81
Investment	0.89	0.84
House price	0.61	0.79
LTV ratio	0.89	0.92
	Correlations with output	
Consumption	0.93	0.85
Investment	0.92	0.75
House price	0.66	0.38

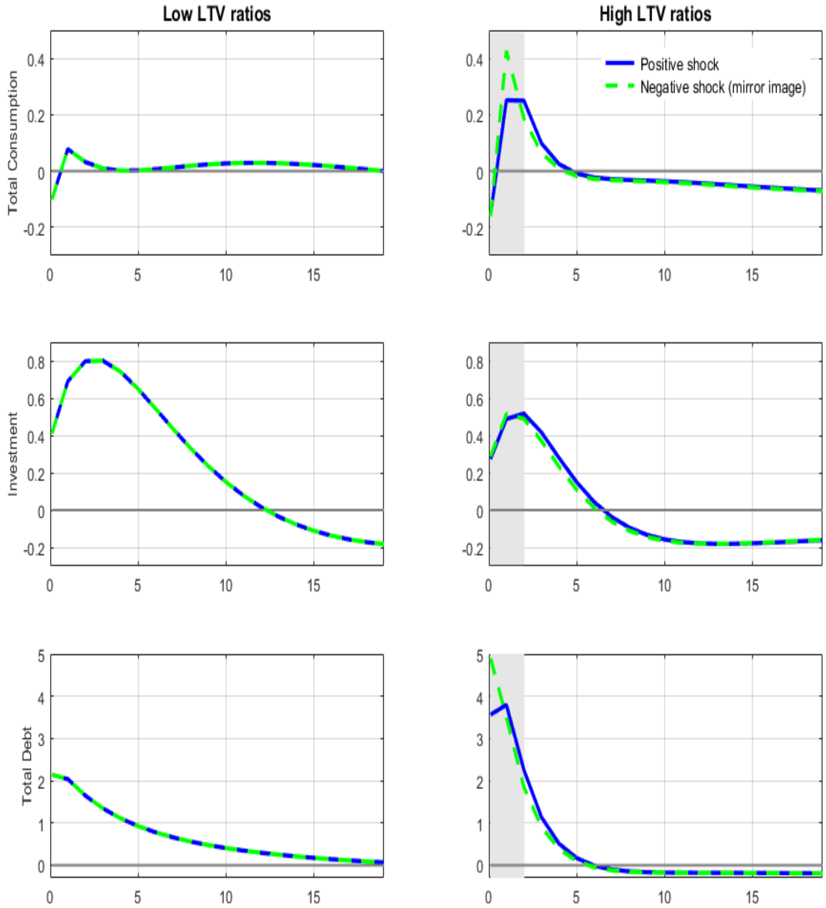
Appendix G: Impulse-responses

Figure G1. Impulse responses to a Technology Shock



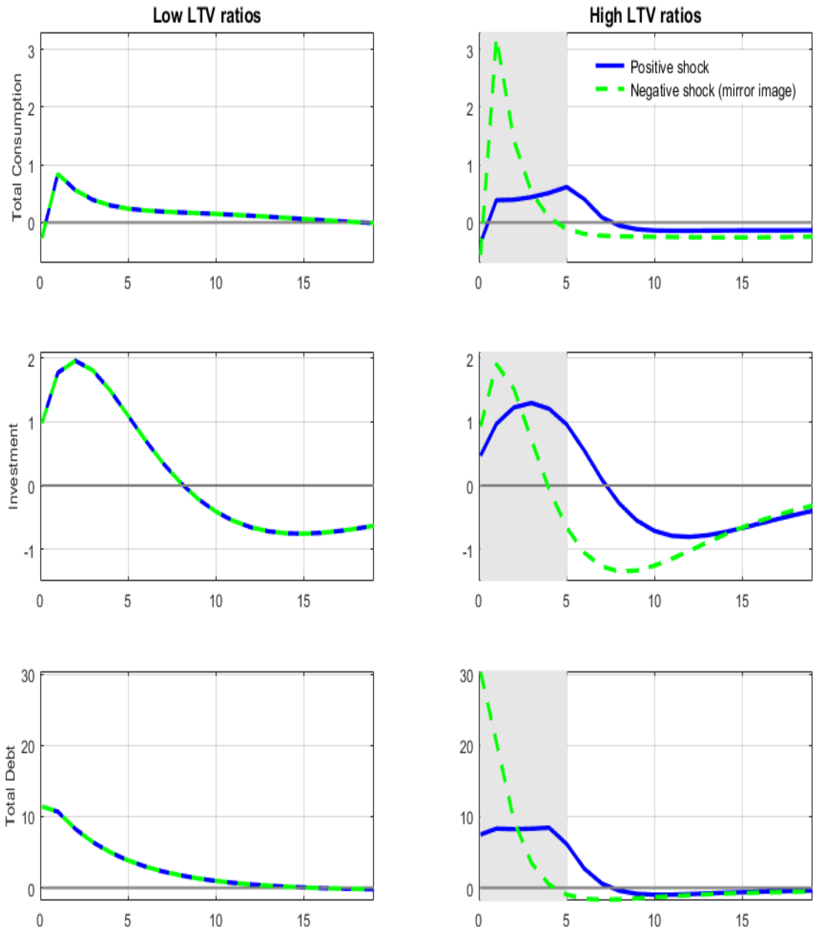
Notes: Impulse responses of key macroeconomic variables (% deviation from the steady state) to a one-standard deviation shock to technology. Left column: $s^I = 0.62$, $s^E = 0.71$; right column: $s^I = 0.85$, $s^E = 0.94$. The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

Figure G2. Impulse responses to a Land Demand Shock



Notes: Impulse responses of key macroeconomic variables (% deviation from the steady state) to a one-standard deviation shock to land demand. Left column: $s^I = 0.62$, $s^E = 0.71$; right column: $s^I = 0.85$, $s^E = 0.94$. The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

Figure G3. Impulse responses to a Credit Limit Shock



Notes: Impulse responses of key macroeconomic variables (% deviation from the steady state) to a one-standard deviation shock to credit limit. Left column: $s^I = 0.62$, $s^E = 0.71$; right column: $s^I = 0.85$, $s^E = 0.94$. The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

Appendix H: Skewness and the Great Moderation

In the main text, we demonstrated that our model was able to generate a more negatively skewed business cycle along with a drop in macroeconomic volatility when we raise the steady-state LTV ratios. However, the drop in the standard deviation of GDP growth documented in Figure 10 falls short of the decline observed in US data during the Great Moderation period. In this respect, it is important to recognize that none of the factors to which the Great Moderation is typically ascribed are featured in our model. One widely cited explanation for the Great Moderation is the so-called 'Good Luck' hypothesis, according to which the Great Moderation was simply a result of smaller shocks hitting the US economy (see, e.g., Stock and Watson, 2003).[37] The goal of this appendix is to demonstrate that our main finding of an increasingly negatively skewed business cycle holds up in an environment where increasing LTV ratios are combined with smaller macroeconomic shocks to obtain a drop in output volatility similar to

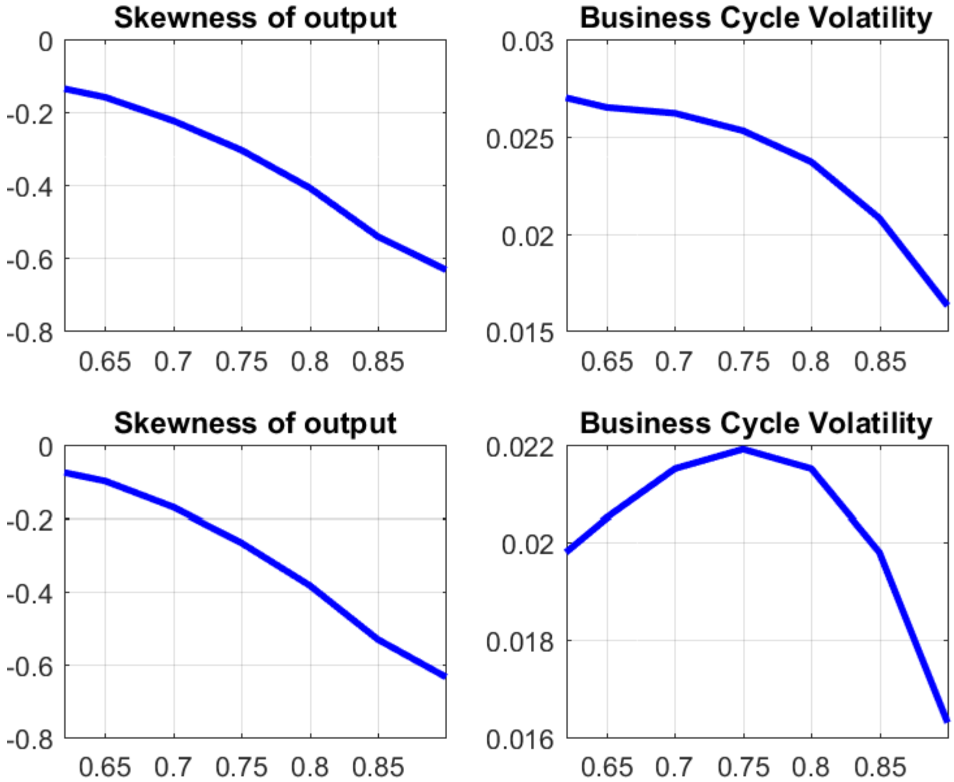
[37] Other popular explanations include better monetary policy (Boivin and Giannoni, 2006) and smaller dependence on oil (Nakov and Pescatori, 2010).

that observed in the data. It is important to stress that this coexistence is not trivial: All else equal, reducing the size of the shocks hitting the economy lowers the probability that collateral constraints become non-binding, thus potentially weakening the key driver of business cycle skewness in our model.

The Great Moderation entailed a decline in the volatility of GDP growth in the US economy of around 40%, see Section 2.1. In the following, we engineer a similar decline in output volatility in our model simulations by reducing the standard deviations of all three shocks in the model, keeping their relative size fixed in accordance with the estimation in Section 5.1.2. To obtain the desired drop in the volatility of GDP growth at $s^I = 0.90$, we need to reduce the standard deviation of each shock by 30%. We first assume that the decline in the size of the shocks occurred gradually along with the increase in LTV ratios. The top row of Figure H1 shows the pattern of the standard deviation and skewness of GDP growth in this experiment. The right panel illustrates that in this case, the drop in macroeconomic volatility is similar to that observed in the data during the Great Moderation, as desired. In addition, in contrast to the results reported in Section 6.3, the standard deviation of output growth now declines monotonically. The left panel shows that the decline in skewness of GDP growth survives in this environment. In fact, the magnitude of the drop in skewness is almost identical to the baseline results presented in Section 6.2 of the main text. This demonstrates that the mechanism giving rise to business cycle skewness in our model is compatible with the Good Luck hypothesis of the decline in macroeconomic volatility observed during the Great Moderation.

Some may argue that the Great Moderation entailed a discrete, downward shift in the size of macroeconomic shocks hitting the US economy rather than the gradual decline assumed above. The bottom row of Figure H1 shows the results from an experiment in which the standard deviation of each shock is reduced by 30% starting at $s^I = 0.6239$, and then kept at this new level as the LTV ratios are increased. By design, the standard deviation of GDP growth reaches the same level as in the top row of Figure H1 when $s^I = 0.90$. The hump-shaped pattern of output volatility presented in Section 6.3 is preserved in this experiment, though at a lower level. Importantly, skewness of GDP growth displays roughly the same pattern as in the top row, confirming again the robustness of our main finding.

Figure H1. Skewness and the Great Moderation



Notes. The top row displays the skewness and standard deviation of output growth in the experiment where the size of shocks is reduced gradually, while the bottom row displays the same statistics in the case where the size of shocks is reduced at once. In each case, the numbers reported are median values from 501 stochastic model simulations of 2000 periods each.

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