Sentiment and Beta Herding

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Abstract

We propose a new non-parametric measure of herding, beta herding, based on linear factor models and apply it to investigate the nature of herd behaviour in the US, UK, and South Korean stock markets. Our measure is based on the cross-sectional variation of market betas hence we consider what might be called beta herding and herding towards the market index. We find clear evidence of beta herding when the market is evolving smoothly, either rising or falling, rather than when the market is in crisis. In fact we find that crises appear to lead investors to seek out fundamental value rather than herd. We examine the relationship between market wide sentiment and beta herding and show that there are separate forces at work. The evidence we find on herding provides an explanation for why we observe different impacts in cross-sectional asset returns after periods of negative and positive sentiment.

Keyword Herding, Sentiment, Non-central Chi Square Distribution, Market Crises.

JEL Code C12,C31,G12,G14
1 Introduction

Herding is widely believed to be an important element of behaviour in financial markets and yet the weight of empirical evidence is not conclusive. Most studies have failed to find strong evidence of herding except in a few particular cases, for example herding by market experts such as analysts and forecasters (see Hirshleifer and Teoh, 2001). One difficulty lies in the failure of statistical methods to differentiate between a rational reaction to changes in fundamentals and irrational herding behaviour\(^1\). It is critical to discriminate empirically between these two forces, since the former simply reflects an efficient reallocation of assets whereas the latter potentially leads to market inefficiency.

In this paper we investigate herding within a market and examine its impact on asset pricing. In order to do this we need to define herding in a slightly different way from the standard definition since herding needs to be understood in terms of risk and return relationship facing an investor. “Beta herding” measures the behaviour of investors who follow the performance of specific factors such as the market index or portfolio itself or particular sectors, styles, or macroeconomic signals and hence buy or sell individual assets at the same time disregarding the underlying risk-return relationship.

Although this measure can be easily applied to specific factors, say herding towards

\(^1\)The use of irrational here refers to market, as opposed to individual, irrationality. We recognise that there will be situations where it may be myopically rational for an individual to follow the herd and hence our use of irrational may seem inappropriate. However given that such behaviour may lead to inefficient asset prices and hence irrational behaviour for the market as a whole we will throughout this paper refer to herding simply as irrational.
the new technology sector, we focus here simply on (beta) herding towards the market portfolio\(^2\). The existence of this type of herding suggests that assets are mispriced while equilibrium beliefs are suppressed. For practitioners, using betas to form hedge portfolios may not be effective when there is herding towards the market portfolio as beta estimates will be biased as stocks move towards having similar betas (beta herding). Hedging strategies could work well when there is ‘adverse’ herding where the factor sensitivities (betas) are widely dispersed\(^3\). Based on this definition, Hwang and Salmon (2004) (from now on HS) proposed an approach based on a disequilibrium CAPM which leads to a measure that can empirically capture the extent of herding in a market.

The model proposed by HS is extended significantly by including an analysis which incorporates the interaction between sentiment and herding. Beta herding reflects cross sectional convergence within the stocks in a market in our model whereas we take sentiment as reflecting a market wide phenomenon that evolves over time. Our model shows that herding activity increases with market-wide sentiment. When there

\(^2\)Throughout this paper we implicitly assume that herding should be viewed in a relative sense rather than as an absolute and that no market can ever be completely free of some element of herding. Thus we argue that there is either more or less herding (including ‘adverse’ herding) in a market at some particular time and herding is a matter of degree. It seems to us conceptually difficult if not impossible to rigorously define a statistic which could measure an absolute level of herding. However, most herd measures that have been proposed, such as Lakonishok, Shleifer, and Vishny (1992), Wermers (1995), and Chang, Cheng and Khorana (2000), have apparently tried to identify herding in absolute terms.

\(^3\)Our term adverse herding is consistent with the use of “disperse” in Hirshleifer and Teoh (2001).
is market-wide positive sentiment, individual asset returns are expected to increase regardless of their systematic risks and thus herding increases. On the other hand negative sentiment is found to reduce herding. The empirical results of Baker and Wurgler (2006) can be explained by showing that it may be through downward biased betas from herding that market-wide positive sentiment affects relative (cross-sectional) returns for certain firms (i.e., newer, smaller, volatile, unprofitable, non-dividend paying, distressed firms), if these firms have higher betas. If the firm characteristics Baker and Wurgler (2006) use are closely related to the betas, then it is beta herding that will lead to the relative divergence in cross-sectional asset returns. For example, as in Fama and French (1992, 1993), size is closely related to beta and thus a size sorted portfolio would show a large divergence in cross-sectional returns after a period of negative sentiment (adverse herding). Our model also explains the findings in Welch (2000) in that we observe herding during bull markets. The empirical evidence from Brown and Cliff (2004) that there is a strong contemporaneous relationship between market returns and sentiment also suggests that herding activity is likely to increase in bull markets while it decreases during bear markets. We find that sentiment explains up to 25 percent of beta herding and thus the late 1990’s dot-com bubble could have been significantly affected by cross-sectionally biased betas.

We propose a new non-parametric method to measure herding based essentially on the cross-sectional variance of the betas. This approach is more flexible than the method introduced in HS as there is no dependence on any particular assumed parametric model for the dynamics of herding. We also develop a formal statistical framework for
testing the significance of movements in herding. We also study the dynamics of herding over an extended time horizon. Herding is generally perceived to be a phenomenon that arises rapidly and thus most studies of herding explicitly or implicitly examine herding over very short time intervals. However, Summers and Porterba (1988) and Fama and French (1988) show that noise in financial markets may be highly persistent and slow-moving over time. Shiller (2000) argues that if markets are not efficient at both the macro and micro levels and if the “conventional wisdom” given by experts only changes very slowly, then the short-run relationship may provide us with biased information about the level of stock prices. A dramatic case of slow-moving noise is a ‘bubble’ where the cycle of the bubble may not be completed within days, weeks or even months. It took years for these bubbles to develop and finally make their impact on the market. For example, bubbles such as the Tulip Bubble in seventeenth-century Holland, the real estate bubble in the late 1980s Japan, and the recent dot-com bubble were not formed over short time periods, see Shiller (2003) for example. If this argument is correct, then we should find evidence for slow moving herd behaviour and for this reason we use monthly data rather than higher frequency data.

We apply our non-parametric approach to the US, UK, and South Korean stock markets and find that beta herding does indeed move slowly, but is heavily affected by the advent of crises. Contrary to the common belief that herding is only significant

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4 Noise here is referred to any factor that makes asset prices deviate from fundamentals, and thus includes ‘anomalies’ such as sentiment and herding. See Black (1986) and DeLong et al (1990) for discussion on noise and asset pricing.
when the market is in stress, we find that herding can be much more apparent when
market rises slowly or when it becomes apparent that market is falling. Once a crisis
appears herding toward the market portfolio becomes much weaker, as individuals
become more concerned with fundamentals rather than overall market movements.

We also show that the herd measure we propose is robust to business cycle and
stock market movements. Our results confirm that herding occurs more readily when
investors’ expectations regarding the market are more homogeneous, or in other words
when the direction towards which the market is heading is relatively clear – whether
it be a bull or a bear market. The results suggest that herding is persistent and moves
slowly over time like stock prices around the intrinsic values as discussed in Shiller
(1981, 2000, 2003), but critically shows a different dynamic to stock prices.

The results from using portfolios formed on size and book to market are consistent
with the results we find from using individual stocks. However the difference in beta
herding between the portfolios formed on size and book-to-market and the individual
stocks reflects the relative distress of these portfolios (Fama and French, 1995).

In the next section we model beta herding in the presence of sentiment and consider
the implications for asset pricing. In section 3, the new measure of herding is defined
and a non-parametric method developed to estimate herd behaviour based on the cross-
sectional variance of the $t$-statistics of the estimated betas in a linear factor model. In
section 4, we apply this new method to the US, UK, and South Korean stock markets.
The empirical relationship between market sentiment and beta herding is examined in
section 5 and finally we draw some conclusions in section 6.
2 Herding, Sentiment, and Betas

In this section we develop a model in which betas are affected by cross-sectional herding and sentiment. Our concept of beta herding reflects the convergence of the betas of the individual stocks towards the beta associated with the market index and it is important to understand how beta herding affects asset prices. For example, suppose investors are for some reason optimistic and believe that the market as a whole is expected to increase by 20%. Then they would (beta) herd towards the market index by buying and selling individual assets until their individual prices increased by 20% disregarding the true risk return relationship and the equilibrium beta for the asset. Those assets whose equilibrium betas are less than 1 would rise less than 20% as the market rose and therefore might appear cheap relative to the expected market return and hence would be bought effectively raising their beta. Those assets whose betas were larger than 1 would rise by more than 20% as the market rose and therefore could appear relatively expensive compared to the market and hence might be sold reducing their betas towards the market beta. In each case this sort of herding behaviour would cause the betas on the individual assets to converge towards the market beta. Similar herding activity towards the market beta could arise if there was a negative view as to how the market was expected to move.
2.1 Cross-sectional Beta Herding

The ‘cross-sectional beta herding’ in individual assets is modelled in HS as follows;

\[
\frac{E_t^b(r_{it})}{E_t(r_{mt})} = \beta_{imt}^b = \beta_{imt} - h_{mt}(\beta_{imt} - 1),
\]  

(1)

where \(r_{it}\) and \(r_{mt}\) are the excess returns on asset \(i\) and the market at time \(t\), respectively, \(\beta_{imt}\) is the systematic risk, and \(E_t(.)\) is the conditional expectation based on information at time \(t\). Note that \(E_t^b(r_{it})\) and \(\beta_{imt}^b\) are the market’s biased conditional expectation of excess returns on asset \(i\) and its beta at time \(t\), because of the cross-sectional beta herding. The key parameter \(h_{mt}\) captures herding and how it changes over time. This is a generalized model that encompasses the equilibrium CAPM with \(h_{mt} = 0\), but allows for temporary disequilibrium.

Let us consider several cases in order to see how this type of herding affects individual asset prices given the evolution of the expected market return, \(E_t(r_{mt})\). First of all, when \(h_{mt} = 1\), \(\beta_{imt}^b = 1\) for all \(i\) and the expected excess returns on the individual assets will be the same as that on the market portfolio regardless of their systematic risks. Thus \(h_{mt} = 1\) can be interpreted as ‘perfect cross-sectional beta herding’. In general, when \(0 < h_{mt} < 1\), cross-sectional beta herding exists in the market, and the degree of herding depends on the magnitude of \(h_{mt}\). In terms of betas, when \(0 < h_{mt} < 1\), we have \(\beta_{imt} > \beta_{imt}^b > 1\) for an equity with \(\beta_{imt} > 1\), while \(\beta_{imt} < \beta_{imt}^b < 1\) for an equity with \(\beta_{imt} < 1\). Therefore when there is cross-sectional beta herding, the individual betas are biased towards 1. Given that the model is designed to return towards equilibrium betas over time and hence behaviour fluctuates around the equilibrium
CAPM, we also need to explain ‘adverse cross-sectional beta herding’ when \( h_{mt} < 0 \).

In this case an equity with \( \beta_{imt} < 1 \) will be less sensitive to movements in the market portfolio (i.e., \( \beta_{imt}^h < \beta_{imt} < 1 \)), while an equity with \( \beta_{imt} > 1 \) will be more sensitive to movements in the market portfolio (i.e., \( \beta_{imt}^h > \beta_{imt} > 1 \)).

It is worth emphasizing that \( E_t(r_{mt}) \) is treated as given in this framework and thus \( h_{mt} \) is conditional on market fundamentals. Therefore, the herd measure is not assumed to be affected by market-wide mispricing like bubbles, but is designed to capture cross-sectional herd behaviour within the market. Clearly however there is a link between the two and we extend the model by allowing the expected returns of the market portfolio and individual assets to be biased by investor sentiment. Sentiment will in this model drive the mispricing of the market as a whole while herding captures the cross sectional mispricing given the market position. As we will see below the model captures the interaction between these two forces, one essentially through time (sentiment) and the other cross-sectional (Beta herding).

### 2.2 Sentiment and Beta Herding

While sentiment may affect the entire return distribution we will follow the majority of the literature and define sentiment with reference to the mean of noise traders’ subjective returns: if it is relatively high or low, we say that optimistic or pessimistic sentiment exists.\(^5\) Let \( \delta_{mt} \) and \( \delta_{it} \) represent the impact of sentiment on beliefs regarding

\(^5\)Several different approaches to defining sentiment have been proposed; see De Long, Shleifer, Summers, and Waldmann (1990), Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, Sub-
the returns on the market portfolio and asset $i$ respectively. Then the investors’ biased expectation in the presence of sentiment is the sum of two components; one due to fundamentals and the other sentiment,

$$E_t^s(r_{it}) = E_t(r_{it}) + \delta_{it}, \text{ and}$$

$$E_t^s(r_{mt}) = E_t(r_{mt}) + \delta_{mt},$$

where for consistency $\delta_{mt} = E_c(\delta_{it})$ and $E_c(.)$ represents cross-sectional expectation, and the superscript $s$ represents bias due to the sentiment.

The model in (1) is now generalised by allowing the expectation of the market return to be biased by sentiment as follows;

$$\beta_{imt}^s = \frac{E_t^s(r_{it})}{E_t^s(r_{mt})} = \frac{E_t(r_{it}) + \delta_{it}}{E_t(r_{mt}) + \delta_{mt}} = \beta_{imt} + \frac{s_{it}}{1 + s_{mt}},$$

where $s_{mt} = \frac{\delta_{mt}}{E_t(r_{mt})}$ and $s_{it} = \frac{\delta_{it}}{E_t(r_{mt})}$ represent the degree of optimism or pessimism by measuring the impact of sentiment on the market portfolio and asset $i$ relative to the expected market return in equilibrium. Several previous studies such as Neal and Wheatley (1998), Wang (2001), and Brown and Cliff (2004) report empirical evidence that asset returns are positively related to sentiment. Therefore positive values of $s_{mt}$ and $s_{it}$ are usually expected in bull markets, while negative market sentiment during bear markets.

rahmanyan (1998). Essentially we follow Shefrin (2005) and regard sentiment as an aggregate belief which affects the market as whole.
We can explore several specific cases of this structure that show how beta is biased in the presence of sentiment in individual assets and/or the market. Consider the following three situations:

\[
\beta_{int}^* = \begin{cases} 
\beta_{int} + s_{it} & \text{when } \delta_{it} \neq 0 \text{ and } \delta_{mt} = 0, \\
\frac{\beta_{int}}{1+s_{mt}} & \text{when } \delta_{it} = 0 \text{ and } \delta_{mt} \neq 0, \\
\frac{\beta_{int} + s_{it}}{1+s_{mt}} & \text{when } \delta_{it} \neq 0 \text{ and } \delta_{mt} \neq 0.
\end{cases}
\]  

(3)

The first case, where \( \delta_{mt} = 0 \), assumes that there is no aggregate market-wide sentiment although non-zero sentiment could exist for individual assets. Since \( \delta_{mt} = E_c(\delta_{it}) = 0 \) (or \( s_{mt} = E_c(s_{it}) = 0 \)), a special case arises by assuming \( s_{it} = 0 \). Even if there is no market-wide sentiment, the cross-sectional beta herding in equation (1) can be obtained with \( s_{it} = -h_{mt}(\beta_{int} - 1) \) conditional on \( E_t(r_{mt}) \). For a given equilibrium \( \beta_{int} \), it is a positive value of \( h_{mt} \) that creates cross-sectional beta herding, but the positive \( h_{mt} \) is not necessarily related with positive sentiment of that asset. For example, for an asset with \( \beta_{int} > 1 \), cross-sectional beta herding \( (h_{mt} > 0) \) is related with negative sentiment \( (s_{it} < 0) \). Cross-sectional beta herding can be observed when sentiment in individual assets appears in a systematic way as in (1).

The second case arises when there is a market-wide sentiment but no sentiment effect for the specific asset. Even if there is no sentiment for the specific asset its beta is biased because of the market-wide sentiment. For a positive market-wide sentiment the beta is biased downward and vice versa. However, for the market as a whole there should be other individual assets whose sentiment contributes to the non-zero market-wide sentiment.
The final case is when individual and market sentiments are both non-zero. Equation (2) suggests that only when \( s_{it} = \beta_{imt} s_{mt} \), we have \( \beta_{imt}^s = \beta_{imt} \). That is, when the market-wide sentiment affects the expected return of the individual asset through the equilibrium relationship will the beta in the presence of sentiment be equal to the equilibrium beta. However, it is hard to expect that the market-wide sentiment affects individual assets via the equilibrium relationship. When investors are over-confident (have positive sentiment), a similar level of sentiment is likely to be expected for individual assets regardless of the equilibrium relationship. In an extreme case, when sentiment is the same for all assets in the market, \( s_{mt} = s_{it} > 0 \) for all \( i \), \( \beta_{imt}^s \) moves towards 1; \( 1 > \beta_{imt}^s > \beta_{imt} \) for assets with \( \beta_{imt} < 1 \) and \( 1 < \beta_{imt}^s < \beta_{imt} \) for \( \beta_{imt} > 1 \). Similarly when \( s_{mt} = s_{it} < 0 \), \( 1 < \beta_{imt} < \beta_{imt}^s \) for assets with \( \beta_{imt} > 1 \) and \( 1 > \beta_{imt} > \beta_{imt}^s \) for \( \beta_{imt} < 1 \). Therefore the effects of sentiment on betas are similar to those of the cross-sectional beta herding explained above. All three cases suggest that the equilibrium beta would not be observable when there is sentiment in the individual assets or at the market level.

The second and the third cases could explain the empirical evidence of Baker and Wurgler (2006) on why certain firms – small, volatile, young, unprofitable, non-dividend paying, distressed – are likely to be more affected by changes in sentiment. For simplicity, take the second case where \( s_{it} = 0 \) and \( s_{mt} > 0 \). Then we have \( \beta_{imt}^s < \beta_{imt} \) and thus \( E^s(r_{it}) < E(r_{it}) \), which suggests that when sentiment is high, these assets are likely to show lower returns and vice versa. The impacts of sentiment on returns are higher for high beta assets rather than low beta stocks although the impacts are
the same in terms of proportion. The type of the firms that Baker and Wurgler (2006) investigate, e.g., small stocks, are likely to have higher betas, and thus the impact of sentiment on these firms is higher.

In order to model the impact of all three cases together with cross-sectional beta herding on asset pricing, we assume that the impact of sentiment on an individual asset is decomposed into three components, a common market-wide effect that evolves over time, $s_{mt}$, cross-sectional beta herding within the market, $h_{mt}(\beta_{int} - 1)$, and an idiosyncratic sentiment, $\omega_{it}$, such that:

$$s_{it} = s_{mt} - h_{mt}(\beta_{int} - 1) + \omega_{it},$$

where $\omega_{it}$ is an idiosyncratic sentiment of asset $i$. Alternative structures could be proposed, but equation (4) is both simple and general enough to capture the effects we need and the equation also satisfies the constraint that the cross-sectional expectation of all the individual assets’ sentiments is equal to the market-wide sentiment:

$$E_c(s_{it}) = E_c(s_{mt} - h_{mt}(\beta_{int} - 1) + \omega_{it}) = s_{mt},$$

since $E_c(\beta_{int} - 1) = E_c(\omega_{it}) = 0$.

By substituting $s_{it}$ into equation (2), we have beta in the presence of cross-sectional beta herding and sentiment:

$$\beta_{int}^* = 1 + \frac{1}{1 + s_{mt}} [(1 - h_{mt})(\beta_{int} - 1) + \omega_{it}].$$

(5)

Only when all three sentiment components are zero does (5) deliver the equilibrium beta, $\beta_{int}^* = \beta_{int}$. The sentiment process in (4) includes the two sources of herding
explained above; cross-sectional beta herding and sentiment. For given \( s_{mt} \) a positive \( h_{mt} \) (cross-sectional beta herding) will make \( \beta_{int}^s \) move towards 1 while a negative \( h_{mt} \) (adverse cross-sectional beta herding) will make \( \beta_{int}^s \) move away from 1. On the other hand, when \( s_{mt} \) increases for given \( h_{mt} \), \( \beta_{int}^s \) moves towards 1 and vice versa.

3 The Non-parametric Test of Herding

3.1 Measure of Beta Herding

When there is beta herding and sentiment, betas less than 1 tend to increases while betas larger than 1 tend to decrease. This tendency can be measured by calculating cross-sectional variance of individual (biased) betas. In particular, we make the natural assumption that the equilibrium \( \beta_{int} \) is not related to \( \omega_{it} \), and so

\[
V_{ar_c}(\beta_{int}^s) = \mathbb{E}_c \left[ \left( \frac{1}{1 + s_{mt}} [(1 - h_{mt})(\beta_{int} - 1) + \omega_{it}]^2 \right) \right]
\]

\[
= \frac{1}{(1 + s_{mt})^2} [(1 - h_{mt})^2 V_{ar_c}(\beta_{int}) + V_{ar_c}(\omega_{it})].
\]

Again for consistency with equilibrium CAPM we also make the following assumptions; that the cross-sectional variance of the true betas, \( V_{ar_c}(\beta_{int}) \), is constant over time and the cross-sectional variance of the idiosyncratic sentiments is constant over time.

The first assumption may appear strong but notice that are not claiming that the individual betas are constant over time which would in fact correspond more closely with equilibrium CAPM but we do require however that with a large number of assets the idiosyncratic movements in the \( \beta_{int} \)'s are expected to cancel out and thus
\( \text{Var}_c(\beta_{imt}) \) is not expected to change significantly over the short run. We show below that movements in the observed cross-sectional variance of the betas is not explained by either macroeconomic variables nor market variables. Likewise there appears to be no strong reason not to assume that the cross-sectional variance of the idiosyncratic sentiment \( \omega_{it} \) could be constant. In fact, as explained later this second assumption is not critical since we can create portfolios so that the idiosyncratic element of sentiment for the portfolio is negligible.

Therefore for given \( \text{Var}_c(\beta_{imt}) \) and \( \text{Var}_c(\omega_{it}) \) the left hand side of equation (6) decreases, ceteris paribus, when \( h_{mt} \) and \( s_{mt} \) increase. That is, we observe a reduction in \( \text{Var}_c(\beta^e_{imt}) \) when there is cross-sectional beta herding and positive market-wide sentiment. When there is no cross-sectional beta herding but market-wide sentiment exists, i.e., \( h_{mt} = 0 \) and \( s_{mt} \neq 0 \), changes in \( \text{Var}_c(\beta^e_{imt}) \) are due to market-wide movements in sentiment. For positive sentiment \( \text{Var}_c(\beta^e_{imt}) \) decreases suggesting that a bubble could reduce \( \text{Var}_c(\beta^e_{imt}) \).

Therefore under these assumptions, beta herding can be measured as follows.

**Definition 1**  The degree of beta herding is given by

\[
H_{mt} = \frac{1}{N_t} \sum_{i=1}^{N_t} (\beta^e_{imt} - 1)^2, \tag{7}
\]

where \( N_t \) is the number of stocks at time \( t \). Beta herding therefore decreases with \( H_{mt} \).

While equation (6) is useful for the investigation of herding in individual stocks we can also derive a similar equation for portfolios. There are several benefits from using portfolios. First, for a well diversified portfolio (with respect to \( \omega_{it} \)) the idiosyncratic
sentiment of the portfolio $s_{pt}$ is zero. i.e., $\omega_{pt} = 0$. Then the total sentiment effect on the portfolio will be decomposed into two components, market-wide sentiment and within market herding, such that:

$$s_{pt} = s_{mt} - h_{mt}(\beta_{pmt} - 1).$$ \hspace{1cm} (8)

Then we have

$$Var_c(\beta^s_{pmt}) = E_c \left[ \frac{1}{1 + s_{mt}} \left( 1 - h_{mt} \right)(\beta_{pmt} - 1)^2 \right]$$ \hspace{1cm} (9)

$$= \frac{(1 - h_{mt})^2}{(1 + s_{mt})^2} Var_c(\beta_{pmt}).$$

Under the assumption that $Var_c(\beta_{pmt})$ is invariant over time, we can observe beta herding by measuring $Var_c(\beta^s_{pmt})$. Second, using portfolio betas has an important empirical advantage in that the estimation error would be reduced. That is as the number of equities increases we have $p \lim \beta^s_{pmt} = \beta_{pmt}$.

When the impacts of the idiosyncratic sentiment (i.e., $Var_c(\omega_{it})$) and the estimation error are disregarded, beta herding measured with individual stocks (6) is equivalent to that measured with portfolios (9) only when the $\beta_{imt}$’s are not correlated with each other;

$$H^p_{mt} = \frac{1}{N_p} H^i_{mt} + \frac{(N_p - 1)}{N_p} Cov_c(\beta_{imt}, \beta_{jmt}),$$

where $H^p_{mt}$ and $H^i_{mt}$ are herd measures for portfolios and individual assets respectively, $N_p$ is the number of stocks in a portfolio and $Cov_c(\beta_{imt}, \beta_{jmt})$ is the covariance between $\beta_{imt}$ and $\beta_{jmt}$, $i \neq j$. However in general the betas are correlated and thus the results obtained with individual stocks and portfolios are expected to be different.

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Finally, a question we could ask is what is the contribution of sentiment to beta herding. To answer this question, we take logs of equation (9) to give

\[
\ln H_{mt} = \ln [(1 - h_{mt})^2 \text{Var}(\beta_{pm})] - 2 \ln(1 + s_{mt}),
\]  

suggesting a negative relationship between \(\ln H_{mt}\) and \(\ln(1 + s_{mt})\), which provides an obvious hypothesis to test. In addition the \(R^2\) from the estimation of this equation will indicate how much sentiment contributes to beta herding.

### 3.2 Estimating and Testing Beta Herding

The major obstacle in calculating the herd measure is that \(\beta_{imt}\) is unknown and needs to be estimated. It is well documented that observed betas are not constant but time-varying. (See Harvey, 1989; Ferson and Harvey, 1991, 1993; and Ferson and Korajczyk, 1995) and several methods have been proposed to estimate time-varying betas; see for instance Gomes, Kogan and Zhang (2003), Santos and Veronesi (2004), and Ang and Chen (2006).

In what follows, we use rolling windows to capture the time variation in betas for several reasons. First, as in Jagannathan and Wang (1996) multi-factor unconditional models can capture the same effects as a single factor conditional model. We estimate the betas using multi-factor models in our empirical tests below and as argued by Fama and French (1996) if size, book-to-market and momentum factors are used, then this can minimise the problems with estimating the betas by least squares. Second we avoid the potentially spurious parametric restrictions used in Ang and Chen (2006).
where the latent processes – the betas, market risk premia, and volatility are assumed to follow first order autoregressive processes. In addition, as pointed out by Fama and French (2005), this model could be over-parameterised. Ghysels (1998) argues that it is difficult to obtain time-varying betas unless the true model for the betas is known. Third, betas are found to be extremely persistent; for example the monthly autocorrelations of conditional betas reported by Gomes, Kogan and Zhang (2003) and Ang and Chen (2006) are 0.98 and 0.99 respectively. Therefore for a highly persistent process choosing shorter windows (i.e., 24 months rather than 60 months) could minimise the problems that arise from time-variation in the betas. Moreover we can easily monitor any unfavourable effects of using long windows on the herd measure. Finally using the OLS estimates of betas we can investigate how estimation error affects the herd measure. The state space model estimated using Gibbs sampling and Markov Chain Monte Carlo methods used by Ang and Chen (2006) is not convenient for evaluating how estimation error affects the herd measure ($H_{mt}$), and the cost of calculating thousands of stocks using the Bayesian method would be too high.

A simple market model is used to show the difficulty that arises by using OLS estimates of betas directly in the herd measure and why using the $t$-statistics of the OLS estimates of the betas provides a better way to measure herding\(^6\). Given $\tau$ (window size) observations, the market model is represented as

$$r_{it} = \alpha_{it} + \beta_{it} r_{mt} + \varepsilon_{it}, \ t = 1, 2, \ldots, \tau,$$  \hspace{1cm} (11)

where $\varepsilon_{it}$ is the idiosyncratic error for which we assume $\varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon_it})$. The OLS

\(^6\)The same argument applies in multi-factor models if the factors are orthogonal to each other.
estimator of $\beta_{imt}^*$ for asset $i$ at time $t$, $b_{imt}^*$, is then simply

$$b_{imt}^* = \frac{\hat{\sigma}_{imt}^2}{\hat{\sigma}_{mt}^2},$$

(12)

$$\text{Var}(b_{imt}^*) = \frac{\hat{\sigma}_{\epsilon_{imt}}^2}{\hat{\sigma}_{mt}^2},$$

(13)

where $\hat{\sigma}_{imt}^2$ is the sample covariance between $r_{it}$ and $r_{mt}$, $\hat{\sigma}_{mt}^2$ is the sample variance of $r_{mt}$, and $\hat{\sigma}_{\epsilon_{imt}}^2$ is the sample variance of the OLS residuals. Using the OLS betas, we could then estimate the measure of herding as

$$H_{mt}^O = \frac{1}{N_t} \sum_{i=1}^{N_t} (b_{imt}^* - 1)^2.$$  

(14)

However, $H_{mt}^O$ will also be numerically affected by estimates of $\beta_{imt}^*$’s that are in fact statistically insignificant. The significance of the OLS estimates of the betas could also change over time, affecting $H_{mt}^O$ even if $\beta_{imt}^*$ was constant. In addition, the OLS estimates in equations (12) and (13) have several undesirable properties. Suppose that the market model in (11) were multiplied by a non-zero $\kappa$. Then we would have

$$r_{it}^* = \alpha_{it}^* + \beta_{imt}^* r_{mt}^* + \epsilon_{it}^*,$$

(15)

where $r_{it}^* = \kappa r_{it}$, $r_{mt}^* = \kappa r_{mt}$, $\alpha_{it}^* = \kappa \alpha_{it}^*$, and $\epsilon_{it}^* = \kappa \epsilon_{it}$, leaving $b_{imt}^*$ unchanged. Only when $r_{it}$, $r_{mt}$, and $\epsilon_{it}$ move at the same rate, would the market model hold with the same beta and the OLS estimator will be unaffected; however, in general this is unlikely. A similar econometric problem has been discussed when measuring contagion during market crises. When the volatility in one country increases dramatically during international financial crises, the volatility of the returns in its neighbour country may
not move in proportion, and the correlation coefficient between the two countries may not reflect the true relationship.\(^7\)

When \( r_{it}, r_{mt}, \) and \( \varepsilon_{it} \) do not move at the same rate, \( \text{Var}(b_{imt}^s) \) is affected by heteroskedasticity in \( \varepsilon_{it} \) or \( r_{mt} \). To evaluate the impact of the heteroskedasticity on \( H_{mt}^o \), we first note that

\[
E_c[b_{imt}^s] = E_c[\beta_{imt}^s + \eta_{imt}] = 1,
\]

where \( \eta_{imt} \) is the OLS estimation error, \( \eta_{imt} \sim N(0, \sigma_{\varepsilon mt}^2/\sigma_{mt}^2) \). So using the estimated parameters in the herd measure, \( H_{mt}^o \), is given by

\[
E_c[H_{mt}^o] = E \left[ \frac{1}{N_t} \sum_{i=1}^{N_t} (b_{imt}^s - 1)^2 \right] \tag{16}
\]

\[
= E \left[ \frac{1}{N_t} \sum_{i=1}^{N_t} (\beta_{imt}^s + \eta_{imt} - 1)^2 \right]
\]

\[
= E \left[ \frac{1}{N_t} \sum_{i=1}^{N_t} (\beta_{imt}^s - 1)^2 + \frac{1}{N_t} \sum_{i=1}^{N_t} \eta_{imt}^2 \right]
\]

\[
= H_{mt} + \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{\sigma_{\varepsilon mt}^2}{\sigma_{mt}^2},
\]

since \( E \left[ \frac{1}{N_t} \sum_{i=1}^{N_t} (\beta_{imt}^s - 1) \eta_{imt} \right] = 0 \). When \( r_{it}, r_{mt}, \) and \( \varepsilon_{it} \) all move in unison, \( \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{\sigma_{\varepsilon mt}^2}{\sigma_{mt}^2} \) (the cross-sectional average of estimation errors, from now on we call it CAEE) is constant over time and any movement in \( H_{mt} \) can be captured by \( H_{mt}^o \). However, if either

\(^7\)For example Forbes and Rigobon (2002) show that the correlation coefficient between the two countries increases during market crises when the volatility of idiosyncratic errors remains unchanged. Therefore they conclude that increased correlations between two countries may not necessarily be the evidence of contagion. However, many studies point out that the assumption is not appropriate. See Corsetti, Pericoli, and Sbracia (2003), Bae, Karolyi and Stulz (2003), Pesaran and Pick (2004), Dungey, Fry, Gonzalez-Hermosillo, and Martin (2003, 2004) among others.
the cross-sectional average of idiosyncratic variances (i.e., $\frac{1}{N_t} \sum_{i=1}^{N_t} \sigma_{iit}^2$) or the market variance (i.e., $\sigma_{mt}^2$) is heteroskedastic, then changes in $H_{mt}^O$ do not necessarily arise from herd behaviour, but also come from the changes in the ratio of firm level variance to market variance.

To avoid this unpleasant property of $H_{mt}^O$, we standardize $b_{imt}^*$ with its standard error; in other words we use the $t$ statistic which will have a homoskedastic distribution and thus will not be affected by any heteroskedastic behaviour in CAEE. Using $t$ statistics will also reduce the influence of the impact of changes in market volatility in particular during market crises.

**Definition 2** The standardised measure of beta herding is now defined as

$$H_{mt}^* = \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \frac{b_{imt}^* - 1}{\hat{\sigma}_{iit} / \hat{\sigma}_{mt}} \right)^2,$$

where $b_{imt}^*$ are the observed estimates of betas for the market portfolio for stock $i$ at time $t$, and $\hat{\sigma}_{iit}$ and $\hat{\sigma}_{mt}$ are as defined in equations (12) and (13). Standardised beta herding increases with decreasing $H_{mt}^*$. We call expression (14) as the beta-based herd measure while $H_{mt}^*$ in (17) is the standardised herd measure. The following distributional result applies to (17).\(^8\)

**Theorem 1** Let $B_{mt}^* = (B_{1mt}^* \ B_{2mt}^* \ B_{N_i,mt}^*)'$, where $B_{imt}^* = \frac{b_{imt} - 1}{\hat{\sigma}_{iit} / \hat{\sigma}_{mt}}$. Then with the classical OLS assumptions,

$$B_{mt}^* \sim N \left( \delta_{mt}^* \ ; \ V_{mt}^* \right),$$

\(^8\)A similar result can be obtained for the beta-based herd measure in (14).
where \( \delta^*_m = \left( \delta^*_{1m} \, \delta^*_{2m} \, \ldots \, \delta^*_{Nm} \right)' \), \( \delta^*_{imt} = \frac{\beta^*_{imt} - 1}{\sigma_{ximt}} \), and \( V^*_m \) is covariance matrix of \( B^*_m \). Then

\[
H^*_m = \frac{1}{N_t} B^*_m B^*_m
\]

\[
\sim \frac{1}{N_t} \left[ \chi^2(R; \delta^*_m) + c^* \right],
\]

where \( R \) is the rank of \( V^*_m \), \( \delta^*_m = \sum_{j=1}^{R} (\delta^*_m)^2 / \lambda^*_j \), and \( c^* = \sum_{j=R+1}^{N} (\delta^*_m)^2 \), where \( \delta^*_j \) is the jth element of the vector \( C^*_m B^*_m \), where \( C^*_m \) is the \((N_t \times N_t)\) matrix of eigenvectors of \( V^*_m \), i.e., \( V^*_m = C^*_m \Lambda^*_m C^*_m \), where \( \Lambda^*_m \) is the \((N_t \times N_t)\) diagonal matrix of eigenvalues. The eigenvalues are sorted in descending order.

**Proof.** See the Appendix.

This measure can be easily calculated using any standard estimation program since it is based on the cross-sectional variance of the \( t \) statistics of the estimated coefficients on the market portfolio. Theorem 1 shows that this new measure of herding is distributed as \( 1/N_t \) times the sum of non-central \( \chi^2 \) distributions with degrees of freedom \( R \) and with non-centrality parameters \( \delta^*_m \) and a constant. Therefore the variance of \( H^*_m \) is given by;

\[
Var[H^*_m] = \frac{2}{N_t} \left[ R + 2\delta^*_m \right].
\]

It is important to note that this distributional result depends on the assumption that the number of observations to estimate \( \beta^*_{imt} \) is sufficiently large and \( B^*_m \) is multivariate normal. With too few observations, the confidence level implied in the theorem above would be smaller than it would be asymptotically and we will reject the null hypothesis
too frequently. In practice, the non-centrality parameter would be replaced with its sample estimate.

4 Empirical Results

One straightforward approach to testing herding towards the market is then to calculate the measure given in (18) and its confidence level using Theorem 1 for particular sample periods of interest. If there is any significant difference between any two periods, we could conclude that one period shows relatively more herding than the other. Many studies on herding and contagion have taken this approach, especially when examining behaviour around and during market crises; see Bikhchandani and Sharma (2000) for a survey of empirical studies. An alternative approach that we adopt below is to calculate the statistics recursively using rolling windows and in this way we can investigate whether the degree of herding has changed significantly over time.

In the empirical study, we use two different datasets; individual stocks and portfolios. We first present our results using individual stocks in the US, UK and South Korean markets, and then compare herd behaviour across these different markets. We then apply the method to the Fama-French 25 and 100 portfolios formed on size and book-to-market from January 1927 to December 2003.
4.1 Beta Herding in the US Market

4.1.1 Data

We use the Center for Research in Security Prices (CRSP) monthly data file to investigate herding in the US stock market. Ordinary common stocks listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and NASDAQ markets are included, and thus ADRs, REITs, closed-end-funds, units of beneficial interest, and foreign stocks are excluded from our sample. Our sample period consists of 488 monthly observations from July 1963 to December 2003. For excess market returns the CRSP value weighted market portfolio returns and 1 month treasury bills are used. For the other factors we use Fama-French’s (1993) size (Small minus Big, SMB) and book-to-market (High Minus Low, HML), and momentum from Kenneth French’s data library.

An appropriate number of monthly observations, i.e., $\tau$, needs to be chosen to obtain the OLS estimate. We have chosen a shorter window, i.e., $\tau=24$, but we have also tried a range of values, i.e., $\tau=36$, 48, and 60, and found that the results are not effectively different from one other. (See the results in the Appendix.) The procedure by which we calculated each herd measure is as follows. We use the first 24 observations up to June 1965 to obtain the OLS estimates of betas and their $t$ statistics for each stock (or portfolio) and then calculate $H_{mt}^*$ and its test statistic for June 1965. We then add one observation at the end of the sample and drop the first and so use the next 24 observations up to July 1965 to calculate the herd measure and its statistic for July.
An important issue in the estimation of the betas will arise from the lack of liquidity in particular assets. When prices do not reflect investors’ expectations because of illiquidity, our measure using observed prices may not fully reflect what it intends to show. One problem is nonsynchronous trading problem for illiquid stocks, which was first recognized by Fisher (1966). Scholes and Williams (1977) show that the OLS estimates of betas of infrequently traded stocks are negatively biased while those of frequently traded stocks are positively biased. They derive consistent estimators of the market model, but the estimators are consistent only for returns that suffer from nonsynchronous trading. In general the effects of illiquidity on asset returns are multifaceted and difficult to summarise in a single explanatory variable and so we filter out small illiquid stocks in our empirical work by controlling for the following three liquidity proxies; volatility, size, and turnover rate.\(^9\)

- (Volatility) When the true betas are not known, the market-adjust-return model of Campbell et al (2001) is useful;

\[
r_{it} = r_{mt} + \varepsilon_{it},
\]

which is a restricted version of the market model in (11) with \(\alpha_{jt} = 0\) and \(\beta^s_{imt} = 1\).

Here it is easy to show that \(\sigma_{jt} \geq \sigma_{mt}\). We remove any stocks whose volatility is

\(^9\)There are many studies on liquidity. Liquidity is a function of 1) the cost of liquidating a portfolio quickly, 2) the ability to sell without affecting prices, 3) the ability of prices to recover from shocks, and 4) costs associated with selling now, not waiting. See Kyle (1981) and Grossman and Miller (1998) for example.
less than half of the market volatility, i.e., $\sigma_{it} < 0.5\sigma_{mt}$.

We find that there are less than 5 percent of stocks (19 percent in market value) whose volatility is less than half of the market volatility.

- (Turnover Rate) Annual turnover rates on the NYSE from 1963 to 2003 range from 14 (1964) to 105 (2002) percents and are increasing (www.nysedata.com). In our study we remove stocks from our sample whose average monthly turnover rates (average over $\tau$ period) are less than 0.5 percent (6 percent a year). The proportion of stocks removed by this process is less than 45 percent in both numbers and capitalisation values.

- (Size) We also remove small stocks whose market values are less than 0.01 percent of the total market capitalisation. The proportion of stocks removed by this process is less than 14 percent in values, but as large as 88 percent in numbers.

Applying these filters leaves the number of stocks ranging from 570 to 1185 for our sample period. Compared with the total number of stocks, the method seems to be strict and looks arbitrary. However the filtering should not matter for discovering whether the measure changes over time significantly in order to detect herding empirically. In the Appendix we try several different values of cut-off size and show that our

\footnote{At each time $t$ we estimate $\sigma_{it}$ and $\sigma_{mt}$ using past $\tau$ observations.}

\footnote{We find a small number of stocks that have very small standard deviations of regression residuals, i.e., less than 0.1 percent a month. The low standard deviations make the standard errors of estimated betas very small, and thus their $t$-statistics becomes unusually large. These outliers are excluded from our sample. The number of these stocks is less than 10.}
choice does not affect the results significantly. Controlling firm sizes is also important in our study for another purpose. The statistics proposed in Theorem 1 are for equally weighted herd measure rather than value weighted measure since value weights are hard to include in the principal component analysis. By controlling for firm size we could also investigate if there is any difference in herding between big and small firms.

4.1.2 Empirical Properties of Various Beta Herd Measures

The beta is estimated with the Fama-French three factor model with momentum. The estimated betas and $t$-statistics are used to calculate the herd measures as in (14) and (17). We also calculate the herd measures with the market model and with the Fama-French three factor model for comparison purpose.

Table 1 reports some basic statistical properties of the herd measures. The four herd measures we calculate are not normally distributed. In particular when calculated with the estimated betas they are positively skewed and leptokurtic. On the other hand the non-normality of the standardised herd measure is much less pronounced. Because of the non-normality rank correlations are calculated to investigate the relationship between the four measures.

There are noticeable differences in beta herding between the market model and the four factor model, in particular in the standardised beta measure. The difference between the market model and the four factor model in the beta-based measure (first two columns of Table 1) suggests that beta needs to be estimated with the other factors. As in Fama and French (1992, 1993, 1996) if SMB and HML are important factors
to explain returns, betas estimated without these factors (and momentum) would not represent systematic risk appropriately. Another issue is that the standardised measure is more sensitive to these additional factors, suggesting that standard errors of the estimated betas are also affected significantly by the additional factors.

The last row in Table 1 reports a considerable difference between the beta-based and standardised herd measures; the rank correlations between $H_{mt}^{O}$ and $H_{mt}^{s}$ are significantly negative. As explained earlier, the CAEE in equation (16) (i.e., $\frac{1}{N_t} \sum_{i=1}^{N_t} \eta_{mt}^2$) could affect these herd measures in an opposite direction. To examine the effects of heteroskedasticity of the estimation errors, we regress the beta-based and standardised herd measures on the CAEE. The herd measures are calculated using rolling windows (overlapping samples) and are highly persistent. Although overlapping information provides efficiency, it causes moving average effects. Therefore we report the Newey and West (1987) heteroskedasticity consistent standard errors for the regressions.

Table 2 shows that the coefficients on the CAEE are large and positive for the beta-based herd measures and the $R^2$ values are 0.836 for the four factor model. On the other hand the coefficients for the standardised herd measures are negative significant, but the $R^2$ values are far less than those of the beta-based herd measure. Therefore the dynamics of the beta-based herd measures are dominated by the heteroskedasticity in the estimation errors, while the dynamics of the standardised herd measures are only marginally explained by the heteroskedasticity of the estimation errors. These negative and positive coefficients on the CAEE for the standardised and beta-based herd measures respectively explain the negative correlation between the standardised and
beta-based herd measures in Table 1. Both measures have the undesirable property that the estimation errors affect the dynamics of the herd measures, but the standardised herd measures are far less affected by the errors.

One may concern that standardising estimated betas (t-statistics) may not represent beta herding since it could distort the cross-sectional risk-return relationship. One way to answer the question is to show that portfolios formed on t-statistics have the same cross-sectional pattern in betas, and vice versa. Every month we regress individual stock returns on the Fama-French factors and momentum using past 24 monthly observations as in the above, and then use the t-statistics on the betas to form decile portfolios and calculate the average values of t-statistics and betas in each of the ten portfolios. We also increase the number of portfolios to 100 to see if larger estimation errors in 100 portfolios could affect cross-sectional relationship between betas and their t-statistics.

The procedure is repeated every month and the results are summarised in Figure 1. First, cross-sectionally t-statistics and betas show little difference; the betas and t-statistics move in the same direction. The correlations between the two we calculated every month are on average 0.96 with the minimum value of 0.9. Thus using t-statistics does not distort the cross-sectional risk-relationship, but can reduce the estimation errors significantly.

We conclude that the results in Tables 1 and 2 and Figure 1 support the standardised herd measure with the four factors. Several other cases reported in the Appendix do not show significant difference. The cases we considered are if the standardised herd
measure is robust to different capitalisation cutoff points (a large number of stocks vs a small number of stocks), different groups of betas (high beta vs low beta stocks), different models (market model, Fama-French’s three factor model, and Fama-French’s three factor with momentum), and various sizes of windows ($\tau$) for the estimation of betas.

4.1.3 Macroeconomic Variables

We test if the proposed measure can be explained by stock market movements or macroeconomic activity which are known to affect stock returns. When a herd measure is explained by these variables, the dynamics of herding merely reflect changes in fundamentals and thus efficient reallocations of assets in the stock market. Irrational herding would not be explained by changes in these fundamentals.

The herd measures are regressed on a number of market and macroeconomic variables. Four macroeconomic variables are used; the dividend-price ratio ($DP_t$), the relative treasury bill rate ($RTB_t$), the term spread ($TS_t$), and the default spread ($CS_t$). The choice of these four macroeconomic variables follows from previous studies such as those of Chen, Roll, Ross (1986), Fama and French (1988, 1989), Ferson and Harvey (1991), Goyal and Santa-Clara (2003), and Petkov and Zhang (2005). We also add market returns and market volatility to investigate how our herd measure is related to the movements in the mean and variance of the market portfolio.\footnote{Market volatility is calculated by summing squared daily returns as in Schwert (1989).} We use the log-dividend-price ratio of S&P500 index for $DP_t$, the difference between the US 3 month
treasury bill rate and its 12 month moving average for $RTB_t$, the difference between
the US 10 year treasury bond rate and the US 3 month treasury bill rate for $TS_t$, and
the difference between Moody’s AAA and BAA rated corporate bonds for $CS_t$. The
dividend-price ratio of S&P500 index is obtained from Robert Shiller’s homepage and
the other data are from the Federal Reserve Board of the US.

The results of the linear regression for the standardised and beta-based herd mea-
sures are reported in Table 3. Panel A shows that four variables – market volatility,
dividend-price ratio, term spread, and credit spread, explain the standardised herd
measure calculated with the four factor model. However the value of $R^2$ is only 0.09.
The explanatory power of these variables become much weaker when the CAEE is
added in the regression. For the beta-based herd measure the dividend-price ratio is
the only significant variable, but with the CAEE, the significance of the dividend-price
ratio disappears. These results suggest that although several market related variables
appear to be significant in explaining the standardised herd measure, the proportion
that these variables explain is very limited and vast majority of the dynamics in the
standardised herd measure arise from herding.

4.1.4 Beta Herding and Economic Events

Figure 2 shows the evolution of beta herding. With hundreds or thousands of stocks,
the confidence level calculated by equation (19) becomes very small indeed and thus is
not clearly visible in the figure. As expected, the dynamic behaviour of $H^Q_{mt}$ is almost
the same as that of the CAEE in equation (16). This confirms our preference towards
measuring herding based on \( t \)-statistics (or standardised betas). In the following we focus on our standardised herd measure.

There are a number of significant sudden changes in \( H_{mt}^* \) in the sense that these changes are far above or below the previous upper and lower boundaries at the 95% confidence level. Several sharp positive jumps can be found in 1970 (Recession), 1973 (Oil Shock), 1982 (Mexican Crisis), 1987 (Market Crash), 1991 (Dissolution of USSR), 1998 (Russian Crisis), 2001 (September the Eleventh). When these shocks happened, the herding level decreased significantly. On the other hand there are three sharp declines in the measure during 1980, 1989, and 1993, of which the last two simply reflect the reversals from the positive increase of 1987 and 1991 after 24 months \((\tau)\). The large drop in the herd measure during 1980 comes from sudden increase in the interest rate at the beginning of the recession of 1980. We might infer that the sudden increase in the interest rate increased herding through an increased expectation of a future bear market.

Herding is also clear before the Russian crisis in 1998 and between late 1999 and August 2001. The first herding period could be characterized by a bull market, and the second period by a bear market. When there were shocks such as Russian crisis or ‘September the Eleventh’, herding disappears. Interestingly the US market does not seem to be affected significantly by the Asian crisis despite the sudden jump in market volatility. This could be interpreted as despite the crisis, the US market was dominated by a strong positive herding, and the Asian crisis in 1997 was not strong enough to remove the positive mood.
Two implications can be drawn from the results. The first is that herding happens in both bull or bear markets. When the economy is in a recession during 2001, we observe a high level of herding and on the other hand we can also observe herding when economy is booming, for example in the late 1990’s or mid-1980’s. The second result is that when there are crises or unexpected shocks, herding disappears. See for example 1973 Oil Shock, 1987 crash, 1988 Russian crisis, and ‘September the Eleventh’.

Our findings are not necessarily inconsistent with previous studies. We note that many empirical studies on herding in advanced markets find little concrete evidence of herd behaviour, see Bikhchandani and Sharma (2000). However, in the South Korean case, Kim and Wei (1999) and Choe, Kho, and Stulz (1999) study herd behaviour around the Asian Crisis in 1997 and find some evidence during the Crisis. These studies use the Lakonishok, Shleifer, and Vishny (1992) measure which focuses a subset of market participants. Therefore, we cannot conclude that their results are inconsistent with ours since our measure considers beta herding in the whole market rather than a subset of participants. Chang, Cheng and Khorana (2000), using a variant of the method of Christie and Huang (1995), suggest the presence of herding in emerging markets such as South Korea and Taiwan, but failed to find evidence in the US, Hong Kong and Japanese markets.

However, our evidence is in sharp contrast with the view that herding happens when financial markets are in stress (or in crisis). Figure 2 clearly shows that it is cross-sectional estimation error that derives sharp decreases in beta-based herd measure. On the contrary, standardised herd measure shows that herd behaviour can be clearly
detected when the markets are not in stress and thus investors are confident on the outlook of the future stock market. If the direction towards which the market is heading is assured, herding begins to occur regardless of whether it is a bull or a bear market; these periods are the late 1960’s, before the Oil Shock in 1973, early 1980’s and before the 1987 crash, the 1990’s and early 2000’s. We could interpret this as suggesting that it is the investors’ over-confidence or consensus that induces herding. This also explains how sentiment contributes to beta herding. When market is in stress, however, investors lose confidence and begin to focus more on fundamentals. In this sense market stress is beneficial to the market rather than harmful, although it may create stress for market participants.

From this point of view, we do not agree with the view that herding only arises when financial markets are in stress. When a market is in crisis, we can observe large negative returns in the market index and the majority of the individual assets will also show negative returns, which could be interpreted as herding in whole market. However, as far as individual asset returns move following their systematic risks, the argument may be different. Instead in popular linear factor models we could claim herding only arises when the factor loadings of individual assets are systematically biased by the crisis and thus the long-run relationship between individual asset returns and factor returns no longer holds. So the fact that the majority of assets show negative returns during a market crisis is not sufficient evidence of herding itself.

13 The regression residuals from the six variables and CAEE in panel A in Table 3 are not different from the dynamics of the standardised herd measure in Figure 2.
4.2 International Comparison with UK and South Korean Stock Markets

4.2.1 Data

We use monthly data from January 1993 to November 2002 to compare beta herding in the UK and South Korean stock markets. The inclusion of the South Korean market is useful as we can compare herding in both advanced markets and an emerging market and also to examine how the South Korean market reacted during the 1997 Asian crisis. In general investigating herding in emerging markets is interesting given their structural and institutional differences, see Bekaert, Erb, Harvey, and Viskanta (1997) for example, for some key discussions of what are the important factors in emerging markets. Our sample period covers several important crises, the 1997 Asian crisis and the 1998 Russian crisis, as well as the bull market during the 1990’s and the bear market in the early 2000’s.

The herd measures are calculated using the constituents of the FTSE350 index for the UK market (247 stocks), and the KOSPI index for the South Korean market (454 ordinary stocks). We use FTSE350 index and KOSPI index monthly returns as our market portfolio returns. To calculate the excess returns, 3 month treasury bills are used for the UK market, whereas for the South Korean market, the 1 year Korea Industrial Financial Debentures are used.\textsuperscript{14} For these two markets SMB and HML using the same methods as described in Fama and French (1993) are calculated with

\textsuperscript{14}There is no treasury bill available in South Korea during our sample period.
the constituents in the FTSE350 index and the KOSPI index.\textsuperscript{15}

4.2.2 Beta Herding in the UK and South Korea

Figures 3 and 4 show the evolution of beta herd measures for the two countries. As in the US market, beta herding disappeared in the UK during the Russian crisis. However, the UK market does not show any significant movement in herding around ‘September the Eleventh’. Certainly this event had a huge impact on the US investors, but not much impact on UK investors despite the sharp increase in the UK volatility. Another difference between the US and UK markets is that in the UK we do not observe herding during the recent bear market. As in the US the UK market does not seem to be affected significantly by the Asian crisis. On the other hand the South Korean market which directly suffered from the Asian crisis shows that the high level of herding disappeared during the crisis. In the late 1999 and during the 2000, when investors began to regain confidence in the market, the level of herding in the South Korean market returned to the pre-Asian crisis level, but then began to decrease with global economic uncertainty.

As in the US case, market volatility does not explain herd behaviour. We also plot the CAEE in Figures 3 and 4. As in the US, these are highly correlated with the beta-based herd measures (the rank correlations between the CAEE and beta-based herding are 0.79 and 0.71 for the UK and South Korea).

\textsuperscript{15}See Hwang and Salmon (2004) for detailed explanations on how we calculate these factors in the UK and South Korea.
4.3 Beta Herding in Portfolios

An application of our herd measure to portfolios provides several important insights, both in terms of methodology and the implications. First, we can minimise the estimation error of beta (i.e., the CAEE), and thus the beta-based and standardised herd measures should become similar for portfolios. Second, herding in portfolios may not be the same as herding in individual stocks because of the idiosyncratic sentiments of individual stocks. In addition herding in portfolios could be influenced by changes in the cross-correlation of $\beta_{mt}$’s. Therefore the dynamics of herding measured by portfolios and individual stocks should be close to each other, but are not expected to be the same.

When too many portfolios are formed out of a given number of stocks, the idiosyncratic sentiment and the estimation error may not be minimised because of the smaller number of stocks in each portfolio. On the other hand when the number of portfolios is too small, the herd measure has large standard errors because of the small number of portfolios (See equation (19)). In our study we use Fama-French 25 and 100 equally weighted portfolios formed on size and book-to-market, which have been used in many finance literature.\textsuperscript{16} A total number of 924 monthly observations from January 1927 to December 2003 is used. Betas are estimated in the Fama-French three factor model with momentum by rolling windows of 60 monthly observations. As explained in the Appendix using 60 monthly observations increases the moving average effects, but the

\textsuperscript{16}The minimum numbers of portfolios used for the calculation of the herd measures are 24 and 74 for the 25 and 100 portfolios respectively because of missing data in the early sample period.
effects on the herd measures are not significant (see the Appendix). In addition the results are comparable with many other previous studies such as Fama and French (1992, 1993). The monthly data are obtained from Kenneth French’s data library.

The herd measures are plotted in Figure 5. First, the beta-based and standardised herd measures appear to have similar dynamics. The rank correlations between the beta-based and standardised herd measures are 0.54 and 0.26 for the 25 and 100 portfolios respectively. Note that the rank correlations between the two measures were negative for individual stocks because of the estimation error (see Table 1). Forming a small number of portfolios for given number of stocks (or a larger number of stocks in each portfolio) reduces the estimation error and thus we have large positive correlations, in particular for the 25 portfolio case. In order to further investigate if the increase in the correlations is attributed to the reduction in the estimation error in beta, we calculate the rank correlation of 0.67 between the 25 and 100 portfolios for the beta-based measure and the rank correlation of 0.94 between the 25 and 100 portfolios for the standardised herd measure. Using a larger number of stocks in each portfolio has little impact on the standardised herd measure while it creates a large difference in the beta-based herd measure, most of which comes from the reduction in estimation error. Second, as expected when the number of portfolios is small, the confidence level in the herd measures increases dramatically. For example for the 25 portfolio case, we could not plot the 95% confidence level for the beta-based herd measure since it is too large to be plotted on the same scale. The confidence level for the standardised herd measure is relatively small, but still much larger than that we obtained for the
individual stocks.

Since the dynamics of the standardised herd measures from the 25 and 100 portfolios are not different, we explain herding with the results of the 100 portfolios given their tight confidence bands and robustness to estimation error. There was herding in the early 1930’s and before the beginning of World War II. At the outset of the war herding decreased but it started to increase as the war continued. This supports the view that regardless of bull or bear markets, a clear homogeneity of view in the direction in which the market is likely to move creates herding. In the early 1930’s the US economy was obviously in deep trouble and we empirically observe herding with our measure. The outbreak of the Second World War however and the resultant uncertainty brought about adverse herding. During the 1950’s and 1960’s there was herding except for a few years after 1957 when a sharp recession began.

Herding is clearly observed between 1974 and 1978 (after the first Oil Shock in 1973) which was not clearly seen with the results from individual stocks. We could interpret that during this period the Oil shock in 1973 affected individual stocks differently and thus there were significant differences in the idiosyncratic elements of sentiment in individual stocks. Figure 5 reports that during the 1980’s herding was either at an average level or low in particular just before the 1987 Crash. The herding measure calculated with individual stocks on the other hand showed strong herding during the same period. The difference in herding level between the portfolios and individual stocks during the early 1980’s can be interpreted as follows. If the firm characteristics that are used to form the 100 portfolios, in particular book-to-market, are related to
distress (Fama and French, 1995; Zhang, 2005; Petkova and Zhang, 2005; Hwang and Rubesam, 2006), our herd measures calculated with the 100 portfolios should show heterogeneity between high and low book-to-market firms when market is distressed. When interest rates are unusually high in the early 1980’s and thus the difference in distress between high and low book-to-market firms increases, we do not expect herding among these portfolios. Therefore strong herding in individual level does not necessarily suggests similar level of herding in portfolios. In fact herding in portfolios depends on how the portfolios are formed.

In order to investigate if using portfolios with different formation methods results in different herding activity, we also plot herd measure using 49 industry portfolios in Figure 5.\textsuperscript{17} Generally speaking the dynamics of the herding activity calculated with the 49 industry portfolios looks similar to those calculated with 25 and 100 portfolios formed on size and book-to-market. The rank correlation between the 100 portfolios formed on size and book-to-market and the 49 industry portfolios is 0.2 for the period for the period between December 1931 to December 2003, and 0.4 for the period between July 1963 to December 2003. Although these are significant, the difference between the two is not negligible. The results indicate that although beta herding follows similar dynamics, investors could feel differently if their interests are not the same, i.e., some are interested in forming portfolios based on size, and others try to form portfolios based on industry.

Recall that we interpret the level of dispersion in the betas as a measure of herding

\textsuperscript{17}The data comes from Kenneth French’s data library.
when the market-wide dispersion changes without changes in fundamentals. So the measure gives an insight into why CAPM does not work after 1963. Since Fama and French (1992, 1993) showed beta does not explain cross-sectional average returns, the failure of CAPM has been investigated by many academics. Our beta-based herd measure provides some basic insight into this question. As we can see before 1963 there was a relatively large dispersion in the betas, but after 1963 the level of dispersion was generally low except for the recent decade. For CAPM to work a necessary condition needs to be satisfied; the betas of portfolios should be sufficiently different from each other. Otherwise even if the equity premium is large, cross-sectional returns calculated with betas are not differentiable.

5 Market Sentiment and Beta Herding

Our model proposes that beta herding is related to market-wide sentiment. First, beta herding increases with sentiment ceteris paribus, and thus cross-sectional variance of betas is negatively related to sentiment; see equation (??). It is also our concern how much market-wide sentiment contributes to beta herding. If beta herding is significantly affected by sentiment, asset prices are seriously biased when sentiment is extremely strong or during bubble periods. Second, changes in beta herding affect cross-sectional asset returns. For example, as in Fama and French (1992, 1993, 1996), since size is closely related to beta, then beta herding explains why size sorted portfolios show large cross-sectional difference during negative sentiment, which is found by
5.1 The Relationship between Market Sentiment and Beta Herding

We first investigate whether or not sentiment explains beta herding, and if this is indeed the case, what is the contribution of sentiment to the beta herding. In order to answer these questions, we run the following regression

\[
\ln \text{Var}_c(\beta_{pm}) = \alpha + \beta S_{mt} + \nu_t,
\]  

(21)

where \( S_{mt} \) is a sentiment index.

There are more than a dozen sentiment measures proposed by many authors. In the recent study on sentiment Brown and Cliff (2004) investigated various sentiment indices and concluded that direct sentiment measures (surveys) are closely related to indirect measures. In this study we have taken several sentiment indices. The first proxy sentiment index is the direct sentiment (Direct) index constructed by Investors Intelligence for the period of November 1968 to December 2003.\(^{18}\) Each week newsletter opinions on the future market movements are grouped as bullish, bearish, or neutral and we use the bull-bear ratio as a proxy of sentiment as in Brown and Cliff (2004). The second index is an index on business conditions for the next 12 months, a component in the

\(^{18}\)By counting the last week’s sentiment index in each month we construct monthly sentiment index. Thus the monthly sentiment index could suffer measurement error. However because of the nature of the sentiment index, in particular smoothness, the impact should not seriously devalue our results.
Index of Consumer Sentiment, Michigan university (Michigan). The Survey Research Center at the University of Michigan conducts at least 500 telephone interviews, asking several questions regarding the respondents current and future perception of business and financial conditions. Among the many components we have chosen an index on business conditions for the next 12 months, which reflects the investors’ sentiment in financial markets. For the third index we obtain monthly sentiment index by interpolating the annual sentiment of Baker and Wurgler (2006) (Interpolated BW), which we use for comparison purpose. Finally for robustness, we calculate a moving average direct sentiment index (MV Direct) by rolling windows of 60 months for the direct sentiment index (the first index above) in order to match the dependent and independent variables with the same moving average method. As in the previous section, these sentiment indices are highly persistent and thus Newey and West (1987) heteroskedasticity consistent standard errors are reported. We tested various cases; beta-based and standardised measures, various values of $\tau$, herd measures from individual stocks and portfolios. We find that beta-based herd measures calculated for the portfolios and the individual stocks are affected significantly by estimation errors. (Not reported) However, in general these results are not different from each other, and thus we report the results obtained from the Fama-French 100 portfolios for brevity.

Table 4 supports that sentiment index is negatively related with the herd measures. Most cases show significance at least at the 5 percent level even in the presence of the market and macroeconomic variables and the CAEE. The negative relationship is stronger for the standardised herd measure in panel A. Beta-based herd measure in
panel B is under serious influence of the estimation error, suggesting that forming 100 portfolios does not remove the estimation error. In an unreported table we also find that the beta-based herd measure with the Fama-French’s 25 portfolios still shows significance to the estimation error although the level of significance is reduced. Therefore in the following we explain our results with the standardised herd measure.

The values of $R^2$ vary from 2.6 percent (Direct) to 25.6 percent (Michigan). Some of the difference comes from different sample periods. For the same period as Michigan, i.e., From January 1978 to December 2003, we find that the estimated coefficient on Direct is -0.325 with the standard error of 0.108 ($R^2$ 11.7 percent). It is worth noting that in early sample period the Direct index suffers measurement errors (for example, surveys conducted fortnight), and thus is not entirely reliable. For herding measured with individual stocks we obtained stronger relationship between the sentiment indices and beta herding despite the idiosyncratic sentiments and estimation error. Thus $\text{Var}_c(\beta_{pmt}^s)$ decreases when sentiment increases, which is more likely during bull markets rather than bear markets because of the contemporaneous relationship between returns and sentiment. On the other hand, a decrease in $\text{Var}_c(\beta_{pmt}^s)$ by increased $h_{mt}$ is possible at any time.

The relationship between herding and sentiment is plotted in Figure 6. In most periods sentiment and beta herding move in opposite directions. There are some periods that the two move in the same direction. During 1998 to 1999 we expect decrease in $\text{Var}_c(\beta_{pmt}^s)$ because of increasing sentiment, but the result shows that both increase. This suggests that during this time $h_{mt}$ decreases far more than sentiment increases,
and thus $\text{Var}_c(\beta_{pmt}^{s})$ increases.

### 5.2 Cross-sectional Asset Returns and Beta Herding

In order to investigate if it is beta herding that derives the results of Baker and Wurgler (2006), we hypothesize 1) sentiment affects future beta herding, and 2) beta herding in turn affects cross-sectional asset returns by biased betas. For the first hypothesis, we directly investigate the impact of sentiment on the disperse of future betas by regressing difference in the t-statistics of the top and bottom quintile portfolios formed on t-statistics on past or current market-wide sentiment index. T-statistics are calculated for 100 Fama-French portfolios formed on size and book-to-market in the Fama-French three factors and momentum model, and then are sorted into five groups according to the t-statistics in order to create quintile portfolios. The difference in average t-statistics between the top and bottom portfolios is regressed on the lagged sentiment index with other control variables. For the sentiment index we use Direct and Michigan. Table 5 shows that for considerable future periods sentiment affects the beta herd measure negatively; Michigan is still significant for 12 month ahead beta herding. The results support that sentiment biases current and future betas; when sentiment is high, the difference in betas is smaller.\(^{19}\)

\(^{19}\)The cross-sectional variance of betas was regressed on current or past sentiment with other control variables, and we found the same results. (not reported)

In order to test the second hypothesis, we select two widely used firm characteristics; size and book-to-market. At every month we use the t-statistics of individual
stocks (using 60 monthly observations in the Fama-French three factor model with momentum) to form decile portfolios and then calculate the average value of size and book-to-market in each of the ten portfolios. The book values are from Compustat data files. The results in Figure 7 show that size decreases with t-statistic, while book-to-market is larger for high or low beta stocks. Therefore positive sentiment reduces beta disperse, which in turn reduces cross-sectional return difference in size sorted portfolios. On the other hand negative sentiment makes beta disperse more widely, and thus the cross-sectional difference in returns between large and small size portfolios increases. For book-to-market and beta, the impact of beta herding is expected to be smaller since high and low betas belong to high book-to-market portfolios and the impacts of sentiment on betas are cancelled out. Thus for book-to-market sentiment is not expected to show as clear picture as that for size. These are what Baker and Wurgler (2006) find; cross-sectional return difference in size sorted portfolios after positive and negative sentiment is large but that in book-to-market sorted portfolios are much smaller.

6 Conclusions

Herding is widely believed to be an important element of behaviour in financial markets and particularly when the market is in stress, such as during the Asian and Russian Crises of 1997 and 1998. In this study, we have proposed an alternative method of measuring and testing for slow moving herd behaviour in the market. We have applied our
measure to the US, UK, and South Korean stock market and found that beta herding disappeared during the Russian crisis in 1998 in the US and UK markets while herding in the South Korean market disappeared during the Asian crisis in 1997. Contrary to a common belief that beta herding is significant when the market is in stress, we find that beta herding can be more apparent when investors feel confident regarding the future direction of the market. Once a crisis appears beta herding becomes much weaker as a concern for fundamentals takes over.

We also find that the proposed herd measure is robust to business cycle and stock market movements. This is consistent with our underlying assumption on market-wide herding. That is, herding occurs when investors’ expectations on the market is homogeneous, or in other words when the direction towards which the market is heading is clear regardless of whether it is a bull or a bear market. Then investors are obsessed by the prospects or the market outlook rather than the equilibrium relationship between individual asset returns and factors.

The herd measure calculated with Fama-French portfolios supports our main findings. However there were several time periods in which this measure provides a different explanation from the herd measure calculated with individual stocks. We argue that the difference can be explained by changes in the idiosyncratic sentiments of individual stocks. Methodologically it has been shown that the estimation error can be far more significant in cross-sectional asset pricing; even Fama-French’s 25 portfolios formed on size and book-to-market show non-negligible estimation error in beta.

Finally sentiment explains beta herding up to 25 percent, suggesting the late 1990’s
dot-com bubble would herd betas significantly. In addition, the different patterns in cross-sectional asset returns reported by Baker and Wurgler (2006) following negative and positive sentiment appears to be due to biased betas. If the certain characteristics they use are related to betas, then it is beta herding that explains the cross-sectional difference in asset returns.

Clearly our empirical work has just scratched the surface of the potential applications of the approach we have developed here and more detailed analyses of herding attractors in different phases of market development now seem possible. This study has applied the new measure of herding to the market as a whole. However, the approach can also be applied at a sector (industry) level and different herding behaviour may well be found in different sectors such as IT or old economy stocks or on a geographical basis.
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Appendix

Proof of Theorem 1

With the assumption of \( b_{int}^s \sim N(\beta_{int}^s, \sigma_{it}^2/\sigma_{mt}^2) \) and \( \tau \) observations, we obtain the following non-central \( t \) distribution with the degrees of freedom \( \tau - K - 1 \):

\[
\frac{b_{int}^s - E_c(\beta_{int}^s)}{\hat{\sigma}_{it}/\hat{\sigma}_{mt}} \sim t(\tau - K - 1; \delta_{int}^*) ,
\]

where \( \delta_{int}^* \) is a non-centrality parameter, i.e., \( \delta_{int}^* = (\beta_{int}^s - E_c(\beta_{int}^s))/\sqrt{\sigma_{it}^2/\sigma_{mt}^2} \). Let \( B_{int}^s \equiv (b_{int}^s - E_c(\beta_{int}^s))/\sqrt{\sigma_{it}^2/\sigma_{mt}^2} \), then for a large \( \tau - K - 1 \),

\[
B_{int}^s \sim N(\delta_{int}^*, 1) .
\]

Let \( B_{mt}^* = \left( B_{1mt}^* \quad B_{2mt}^* \quad \ldots \quad B_{Nmt}^* \right)' \). Then with the classical OLS assumption, for a large \( \tau - K - 1 \),

\[
B_{mt}^* \sim N\left(\delta_{mt}^*, \mathbf{V}_{mt}^*\right),
\]

where \( \delta_{mt}^* = \left( \delta_{1mt}^* \quad \delta_{2mt}^* \quad \ldots \quad \delta_{Nmt}^* \right)' \), and \( \mathbf{V}_{mt}^* \) is covariance matrix of \( B_{mt}^* \).

In general, we may not assume that the matrix \( \mathbf{V}_{mt}^* \) is fully ranked, since a large number of equities could mean \( \tau - K - 1 < N \), suggesting the \((N \times N)\) variance-covariance matrix \( \mathbf{V}_{mt}^* \) being singular. Let \( \mathbf{Z} = \mathbf{C}'\mathbf{B}_{mt}^* \), where \( \mathbf{C} \) is the \((N \times N)\) matrix of eigenvectors of the symmetric matrix of \( \mathbf{V}_{mt}^* \), i.e., \( \mathbf{V}_{mt}^* = \mathbf{C}\mathbf{A}\mathbf{C}' \) and \( \mathbf{A} \) is the \((N \times N)\) diagonal matrix of eigenvalues. Note that the eigenvalues are sorted in descending order and the eigenvectors are also sorted according to the sorted eigenvalues. Then using
$C'C = I$ and

$$E(Z) = C'E(B^*_{mt}) = C'\delta^*_{mt}$$

$$\text{Var}(Z) = E \left[ (C'B^*_{mt} - C'\delta^*_{mt}) (C'B^*_{mt} - C'\delta^*_{mt})' \right]$$

$$= C'V^*_{mt}C$$

$$= \Lambda,$$

we have $Z \sim (\delta^*_{mt}, \Lambda)$, where $\delta^*_{mt} = C'\delta^*_{mt}$. When the rank $(R)$ of the matrix $V$ is less than $N$, i.e., $R \leq N$, the first $R$ variables in the vector $Z$ are normally distributed, $z_i \sim N(\delta^*_{int}, \lambda_i)$, where $z_i$ is the $i$th variable of $Z$, $\delta^*_{int}$ is the $i$th element of vector $\delta^*_{mt}$, and $\lambda_i$ is the $i$th eigenvalue of the diagonal matrix $\Lambda$. On the other hand, the remaining $N - R$ variables of $z_i$, $i = R + 1, \ldots, N$, are just constants since $\lambda_i = 0$ for $i = R + 1, \ldots, N$. Thus we have

$$B^*_{mt}B^*_{mt} = (CZ)'CZ$$

$$= CZ'C'Z$$

$$= Z'Z$$

$$= \sum_{i=1}^{R} z_i^2 + \sum_{i=R+1}^{N} z_i^2.$$ 

Since $z_i \sim N(\delta^*_{int}, \lambda_i)$ is independent (orthogonal) of $z_j$ for all $i \neq j$ for $i, j \leq R$, the first component is

$$\sum_{i=1}^{R} z_i^2 \sim \chi^2(R; \delta^R_k),$$

where $\delta^R_k$ is the non-centrality parameter, i.e., $\delta^R_k = \sum_{i=1}^{R} (\delta^*_{int})^2 / \lambda_i$. The second com-
ponent is a constant, i.e.,
\[ c \equiv \sum_{i=R+1}^{N} z_i^2 = \sum_{i=R+1}^{N} (\delta_{int}^A)^2. \]
Thus
\[ B_{mt}' B_{mt}^* \sim \chi^2 (R; \delta_k^R) + c. \]

Therefore, our herd measure follows
\[ h_{kt} = \frac{1}{N_t} B_{mt}' B_{mt}^* \]
\[ \sim \frac{1}{N_t} \left[ \chi^2 (R; \delta_k^R) + c \right]. \quad QED. \]

**Robustness of the Herd Measure**

We have proposed a measure of herding and argued that the measure is designed to capture investor sentiment and biased pricing. The empirical results we reported are obtained with some assumptions which could be relaxed to see how robust are our arguments. Using the US data, we investigate several extensions.

**The Effects of Small Stocks**

By removing small stocks whose market values are less than 0.01 percent of the total market capitalisation, the analysis above focused on herding among larger stocks. To investigate the effects of small stocks on our herd measure we calculate herd measure with stocks whose market values are 0.1, 0.01, 0.001, 0.0001 percents of the total market capitalisation, and stocks whose returns are available for the entire sample period (243 stocks). In particular in the last case we could expect survivalship bias in the data.
Figure A1 shows that the herd measures calculated with these four different sets of stocks are close to each other. There is little noticeable difference in our herd measure from including small stocks whose market values are less than 0.01 percent of the total market capitalisation. In particular the result we obtain with 243 stocks that are available for the entire sample period indicates that our measure is robust against survivorship bias. This result is important in our study since we use some sets of stocks that may suffer survivorship bias in the UK and South Korean stock markets.

**The Effects of Betas**

Since our sample is a subgroup, our results may also be exposed to selection bias. For example we have remove certain stocks which we believe are less liquid and thus may cause bias in the \( t \)-statistic. Since our measure is based on the estimates of betas and their standard deviations, we need to investigate how robust is the herd measure for different subsets of betas. Using the estimated betas to rank the stocks, we make four subgroups; large beta stocks (top 70%), small beta stocks (bottom 70%), middle beta stocks (middle 70%) and high-low beta stocks (except middle 30%). Then for each of these sub-samples we apply the same procedure outlined above. The calculated herd measures for the four subgroups are plotted in Figure A2. The figure shows that there little difference in the herd measures between these groups.
Factors in Linear Factor Models

We use factor mimicking portfolios such as SMB, HML and momentum as control variables. Some correlation between the factors within the sample is inevitable given that firm specific characteristics are used to construct the factors. During our sample period positive correlations are found between the excess market return, SMB, and HML, whereas SMB is negatively correlated with momentum (not reported). Another impact from using additional factors could come from changes in the standard deviation of estimated beta. Adding factors can change the idiosyncratic errors and thus the standard deviation of the estimated beta even if there is no multicollinearity problem.

To evaluate the effects of using additional factors, we calculate the herd measures using the simple market model, the Fama-French three factor model, and the four factor model. We find that there is no significant difference in herding towards the market portfolio between the Fama-French three factor model and the four factor model except for the early 1980’s and 2003. However, the herd measure based on the simple market model is generally larger than the two models, and is more volatile than the two. For example the large increase of the herd measure with the market model during the late 1990’s and 2000’s is attributed to the SMB, HML and momentum. Therefore without considering these factors, our measure may be biased.

Number of Observations Used When Estimating the Betas

As mentioned earlier, when the number of observations for the estimation of beta (i.e., \( \tau \)) is small, we could capture the time variation in betas, but small sample effects may
be significant. For example in many academic studies 60 months is common, and in practice 5 to 7 years are used. To evaluate the effects of different periods of $\tau$ on herd measure, we also use 36, 48, and 60 monthly observations to estimate betas.

Figure A4 shows that the standardised herd measures move in similar directions. As expected the longer the time period to calculate betas, the smoother the herd measure becomes. An unfavourable moving average effect increases when $\tau$ becomes larger, but all cases we report in Figure A4 keep similar dynamics. These results suggest that the negative effects of using 24 months are not serious, and we could get more dynamics from using a short window.
Table 1 Properties of Beta Herd Measure in the US Market

The Beta-based herd measure is calculated with the cross-sectional variance of OLS estimates of betas while the standardised herd measure is calculated with the cross-sectional variance of t-statistics of OLS estimates of betas. We use 24 past monthly returns to estimate betas in the market model and in the Fama-French three factor model with momentum. Using 486 monthly observations from July 1963 to December 2003 and rolling windows of 24 months, we obtain 463 monthly herd measures from June 1965 to December 2003. In order to reduce possible bias from illiquid stocks we choose ordinary stocks whose market values are larger than 0.01% of the total market capitalisation, turnovers are larger than 6% a year, and volatilities are larger than half of the market portfolio’s volatility. ** represents significance at the 1% level.

<table>
<thead>
<tr>
<th></th>
<th>Beta-Based Herd Measure</th>
<th>Standardised Herd Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Market Model (A)</td>
<td>Fama-French Three Factor with Momentum (B)</td>
</tr>
<tr>
<td>Mean</td>
<td>0.322</td>
<td>0.416</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.216</td>
<td>0.188</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.744</td>
<td>1.084</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>9.154</td>
<td>1.062</td>
</tr>
<tr>
<td>Jarque-Bera Statistics</td>
<td>2197.756 **</td>
<td>112.484 **</td>
</tr>
<tr>
<td>Spearman Rank Correlations between A and B, and C and D</td>
<td>0.856 **</td>
<td></td>
</tr>
<tr>
<td>Spearman Rank Correlation between Cross-sectional Standard Deviations of Betas and t-Statistics</td>
<td>Between A and C</td>
<td>Between B and D</td>
</tr>
<tr>
<td></td>
<td>-0.200 **</td>
<td>-0.361 **</td>
</tr>
</tbody>
</table>
The herd measures are regressed on the cross-sectional average of variances of the estimation errors of betas (CAEE). A total number of 463 monthly observations from June 1965 to December 2003 is used. The numbers in brackets are Newey and West (1987) heteroskedasticity consistent standard errors. ** represents significance at the 1% level.

### Table 2: Regression of Herd Measures on the Cross-sectional Average of the Variances of Estimation Errors of Betas

<table>
<thead>
<tr>
<th>Beta-Based Herd Measure</th>
<th>Constant</th>
<th>Average of the Variances of Estimation Errors of Betas</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Model</td>
<td>0.096**</td>
<td>1.554**</td>
<td>0.577</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.228)</td>
<td></td>
</tr>
<tr>
<td>Fama-French Three Factor with Momentum</td>
<td>0.105**</td>
<td>0.973**</td>
<td>0.836</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.050)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standardised Herd Measure</th>
<th>Constant</th>
<th>Average of the Variances of Estimation Errors of Betas</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Model</td>
<td>4.290**</td>
<td>-5.313**</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>(0.189)</td>
<td>(1.045)</td>
<td></td>
</tr>
<tr>
<td>Fama-French Three Factor with Momentum</td>
<td>2.444**</td>
<td>-1.717**</td>
<td>0.329</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.172)</td>
<td></td>
</tr>
</tbody>
</table>
Table 3  Regression of Beta Herding in Individual Stocks on Various Variables

Herd measures calculated with Fama-French three factor model with momentum are regressed on various explanatory variables using 463 monthly observations from June 1965 to December 2003. Rm and Vm represent market return and volatility. DP and RTB represent the dividend price ratio and the relative treasury bill rate, while TS and CS show the term spread and the default spread. The numbers in the brackets are Newey-West heteroskedasticity robust standard errors. ** represents significance at the 1% level and * represents significance at the 5% level.

A. standardised herd Measure

<table>
<thead>
<tr>
<th>Constant</th>
<th>Rm</th>
<th>Vm</th>
<th>DP</th>
<th>RTB</th>
<th>TS</th>
<th>CS</th>
<th>CAEE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.596**</td>
<td>-0.002</td>
<td>0.176*</td>
<td>12.242**</td>
<td>-0.010</td>
<td>0.112*</td>
<td>-0.390**</td>
<td></td>
<td>0.092</td>
</tr>
<tr>
<td>(0.172)</td>
<td>(0.006)</td>
<td>(0.075)</td>
<td>(4.439)</td>
<td>(0.056)</td>
<td>(0.052)</td>
<td>(0.142)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.919**</td>
<td>-0.001</td>
<td>0.090</td>
<td>-1.394</td>
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<td>0.088*</td>
<td>-0.474**</td>
<td>-2.138**</td>
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<tr>
<td>(0.176)</td>
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<td>(0.039)</td>
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B. Beta-Based Herd Measure

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<tr>
<th>Constant</th>
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<th>Vm</th>
<th>DP</th>
<th>RTB</th>
<th>TS</th>
<th>CS</th>
<th>CAEE</th>
<th>R²</th>
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<td>0.685**</td>
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<td>-0.015</td>
<td>-7.100**</td>
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<td>(0.069)</td>
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<td>(0.030)</td>
<td>(1.844)</td>
<td>(0.018)</td>
<td>(0.016)</td>
<td>(0.045)</td>
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<tr>
<td>0.095**</td>
<td>-0.001</td>
<td>0.023</td>
<td>-1.011</td>
<td>-0.002</td>
<td>0.018</td>
<td>0.004</td>
<td>0.955**</td>
<td>0.860</td>
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<td>(0.034)</td>
<td>(0.001)</td>
<td>(0.018)</td>
<td>(0.971)</td>
<td>(0.008)</td>
<td>(0.007)</td>
<td>(0.024)</td>
<td>(0.048)</td>
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</tr>
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</table>
Table 4  Regression of Beta Herding in Portfolios on Various Variables

The herd measures calculated with the Fama-French three factor model with momentum are regressed on log sentiment index and other control variables. For the sentiment index we use bull-bear ratio of Investors Intelligence as in Brown and Cliff (2004), the 12 month expected business condition in the Michigan consumer confidence index, monthly sentiment index interpolated from Baker and Wurgler (2006), and finally 60 monthly average of the direct sentiment index. Rm and Vm represent market return and volatility. DP and RTB represent the dividend price ratio and the relative treasury bill rate, while TS and CS show the term spread and the default spread. The numbers in the brackets are Newey-West heteroskedasticity robust standard errors. The bold numbers represent significance at the 5% level.

A. Standardised Herd Measure Calculated with Fama-French 100 Portfolios Based on Size and Book-to-Market

<table>
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<tr>
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<th>Vm</th>
<th>DP</th>
<th>RTB</th>
<th>TS</th>
<th>CS</th>
<th>CAEE</th>
<th>$R^2$</th>
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<tr>
<td>Direct Sentiment Index (November 1968 - December 2003)</td>
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<td>-0.002</td>
<td>-0.045</td>
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<td>-0.009</td>
<td>0.035</td>
<td>-0.069</td>
<td>-22.688</td>
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<tr>
<td></td>
<td>(0.032)</td>
<td>(0.106)</td>
<td>(0.140)</td>
<td>(0.002)</td>
<td>(0.043)</td>
<td>(2.728)</td>
<td>(0.024)</td>
<td>(0.023)</td>
<td>(0.064)</td>
<td>(11.693)</td>
</tr>
<tr>
<td>Michigan Consumer Index (January 1978 - December 2003)</td>
<td>1.316</td>
<td>-0.353</td>
<td>1.410</td>
<td>-0.003</td>
<td>-0.068</td>
<td>3.056</td>
<td>-0.037</td>
<td>0.050</td>
<td>-0.152</td>
<td>2.954</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.061)</td>
<td>(0.119)</td>
<td>(0.002)</td>
<td>(0.030)</td>
<td>(1.940)</td>
<td>(0.023)</td>
<td>(0.019)</td>
<td>(0.056)</td>
<td>(6.942)</td>
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<td>Monthly Sentiment Index Interpolated from Baker and Wurgler (2006) (November 1968 - December 2003)</td>
<td>0.959</td>
<td>-0.077</td>
<td>0.884</td>
<td>-0.001</td>
<td>-0.033</td>
<td>5.135</td>
<td>-0.021</td>
<td>0.031</td>
<td>-0.051</td>
<td>-7.749</td>
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<tr>
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<td>(0.026)</td>
<td>(0.034)</td>
<td>(0.137)</td>
<td>(0.002)</td>
<td>(0.035)</td>
<td>(2.510)</td>
<td>(0.025)</td>
<td>(0.022)</td>
<td>(0.071)</td>
<td>(9.649)</td>
</tr>
<tr>
<td>60 Monthly Average Direct Sentiment Index (November 1968 - December 2003)</td>
<td>0.993</td>
<td>-0.644</td>
<td>1.200</td>
<td>0.007</td>
<td>1.966</td>
<td>-0.027</td>
<td>0.033</td>
<td>-0.093</td>
<td>27.703</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.266)</td>
<td>(0.160)</td>
<td>(0.002)</td>
<td>(0.039)</td>
<td>(2.887)</td>
<td>(0.026)</td>
<td>(0.023)</td>
<td>(0.064)</td>
<td>(11.505)</td>
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### B. Beta-Based herd Measure Calculated with Fama-French 100 Portfolios Based on Size and Book-to-Market

<table>
<thead>
<tr>
<th>Measure</th>
<th>Constant</th>
<th>$S_m$</th>
<th>$R_m$</th>
<th>$V_m$</th>
<th>$D_P$</th>
<th>$R_{TB}$</th>
<th>$T_S$</th>
<th>$C_S$</th>
<th>$C_AEE$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Sentiment Index (November 1968 - December 2003)</td>
<td>-4.096</td>
<td>0.121</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>-4.883</td>
<td>-0.070</td>
<td>0.000</td>
<td>-0.034</td>
<td>0.814</td>
<td>-0.028</td>
<td>0.044</td>
<td>-0.061</td>
<td>108.130</td>
<td>0.529</td>
</tr>
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<td>Michigan Consumer Index (January 1978 - December 2003)</td>
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<td>0.180</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.015</td>
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<tr>
<td></td>
<td>-4.464</td>
<td>-0.500</td>
<td>0.000</td>
<td>-0.088</td>
<td>-0.084</td>
<td>-0.004</td>
<td>0.000</td>
<td>0.059</td>
<td>-0.144</td>
<td>0.704</td>
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<tr>
<td>Monthly Sentiment Index Interpolated from Baker and Wurgler (2006) (November 1968 - December 2003)</td>
<td>-4.081</td>
<td>-0.006</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>-5.196</td>
<td>-0.136</td>
<td>0.000</td>
<td>-0.047</td>
<td>4.362</td>
<td>-0.043</td>
<td>0.042</td>
<td>0.013</td>
<td>131.202</td>
<td>0.575</td>
</tr>
<tr>
<td>60 Monthly Average Direct Sentiment Index (November 1968 - December 2003)</td>
<td>-3.951</td>
<td>-1.391</td>
<td>0.001</td>
<td>0.006</td>
<td>-0.191</td>
<td>-0.045</td>
<td>0.044</td>
<td>-0.064</td>
<td>100.971</td>
<td>0.552</td>
</tr>
</tbody>
</table>

| Direct Sentiment Index (November 1968 - December 2003) | -4.741   | -0.960 | 0.001 | 0.006 | -0.191| -0.045   | 0.044 | -0.064| 100.971 | 0.552  |
### C. Standardised Herd Measure Calculated with Fama-French 25 Portfolios Based on Size and Book-to-Market

<table>
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<tr>
<th></th>
<th>Constant</th>
<th>( S_{mt} )</th>
<th>( R_m )</th>
<th>( V_m )</th>
<th>( D_P )</th>
<th>( R_{TB} )</th>
<th>( T_S )</th>
<th>( C_S )</th>
<th>( C_{AE} )</th>
<th>( R^2 )</th>
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<td></td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>(November 1968 - December 2003)</td>
<td>(0.044)</td>
<td>(0.149)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>1.468</td>
<td>-0.146</td>
<td>-0.001</td>
<td>-0.016</td>
<td>1.552</td>
<td>-0.017</td>
<td>0.046</td>
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<td>0.105</td>
</tr>
<tr>
<td></td>
<td>(0.221)</td>
<td>(0.158)</td>
<td>(0.003)</td>
<td>(0.071)</td>
<td>(4.458)</td>
<td>(0.037)</td>
<td>(0.036)</td>
<td>(0.093)</td>
<td>(32.909)</td>
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</tr>
<tr>
<td>Michigan Consumer Index</td>
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<td>-0.561</td>
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<td>0.313</td>
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<tr>
<td>(January 1978 - December 2003)</td>
<td>(0.084)</td>
<td>(0.084)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>2.163</td>
<td>-0.702</td>
<td>-0.002</td>
<td>-0.083</td>
<td>0.134</td>
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<td>(0.178)</td>
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<td>(0.044)</td>
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<td>(0.027)</td>
<td>(0.081)</td>
<td>(23.869)</td>
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<td></td>
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<td>0.079</td>
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<td>Interpolated from Baker and</td>
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<td>(0.044)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wurgler (2006) (November 1968</td>
<td>1.245</td>
<td>-0.135</td>
<td>-0.001</td>
<td>-0.037</td>
<td>5.391</td>
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<td>0.036</td>
<td>0.000</td>
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<tr>
<td>- December 2003)</td>
<td>(0.229)</td>
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<td>(0.053)</td>
<td>(4.81)</td>
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<td>(0.032)</td>
<td>(0.103)</td>
<td>(33.504)</td>
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<td>(0.401)</td>
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<td>1968 - December 2003)</td>
<td>1.516</td>
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<td>1.947</td>
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<td>(0.036)</td>
<td>(0.094)</td>
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D. Beta-Based herd Measure Calculated with Fama-French 25 Portfolios Based on Size and Book-to-Market

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<th>$S_m$</th>
<th>$R_m$</th>
<th>$V_m$</th>
<th>DP</th>
<th>RTB</th>
<th>TS</th>
<th>CS</th>
<th>CAEE</th>
<th>$R^2$</th>
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<tr>
<td>(November 1968 - December 2003)</td>
<td>(0.066)</td>
<td>(0.220)</td>
<td></td>
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<td>(0.047)</td>
<td>(0.041)</td>
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<td>(0.051)</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
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<td>Wurgler (2006) (November 1968</td>
<td>-5.622</td>
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<td>6.195</td>
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<td></td>
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<td>Sentiment Index (November</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>1968 - December 2003)</td>
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<td>(0.071)</td>
<td>(4.903)</td>
<td>(0.050)</td>
<td>(0.044)</td>
<td>(0.107)</td>
<td>(36.772)</td>
<td></td>
</tr>
</tbody>
</table>
Table 5  Regression of Difference in t-statistics between Top and Bottom Quintile Portfolios Formed on t-statistics on Past and Current Market-wide Sentiment and Various Variables

We first estimate the t-statistics of beta for each of the 100 Fama-French portfolios formed on size and book-to-market using the Fama-French three factors and momentum. As in Table 4 past 60 monthly returns are used. We then sort the t-statistics into five groups according to the t-statistics in order to create quintile portfolios. The difference in average t-statistics between the top and bottom portfolios is calculated. The procedure is repeated every month to obtain a time series from November 1968 to December 2003. The difference in t-statistics is then regressed on the lagged sentiment index and other control variables. For the sentiment index we use bull-bear ratio of Investors Intelligence as in Brown and Cliff (2004) and the 12 month expected business condition in the Michigan consumer confidence index. Rm and Vm represent market return and volatility. DP and RTB represent the dividend price ratio and the relative treasury bill rate, while TS and CS show the term spread and the default spread. The numbers in the brackets are Newey-West heteroskedasticity robust standard errors. The bold numbers represent significance at the 5% level.

<table>
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<tr>
<th>Lags in $S_m$</th>
<th>Constant</th>
<th>$S_m$</th>
<th>Rm</th>
<th>Vm</th>
<th>DP</th>
<th>RTB</th>
<th>TS</th>
<th>CS</th>
<th>CAEE</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td>0</td>
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Every month we regress individual stock returns on the Fama-French factors and momentum using past 24 monthly observations, then use the t-statistics on beta to form decile portfolios and calculate the average values of t-statistics and betas in each of the ten portfolios. Using the same method we also form decile portfolios based on estimated betas and then calculate average values of betas and t-statistics in each of the decile portfolios. The procedure is repeated 463 times from June 1965 to December 2003. Figure 1C represents correlations between betas and t-statistics of the decile portfolios that are calculated each month. The three figures in the right hand are obtained with the same method but with 100 portfolios formed on betas or t-statistics.
The Beta-based herd measure is calculated with the cross-sectional variance of OLS estimates of betas while the standardised herd measure is calculated with the cross-sectional variance of t-statistics of OLS estimates of betas. We use 24 past monthly returns to estimate betas in the market model and in the Fama-French three factor model with momentum. In order to reduce possible bias from illiquid stocks we choose ordinary stocks whose market values are larger than 0.01% of the total market capitalisation, turnovers are larger than 6% a year, and volatilities are larger than half of the market portfolio's volatility. CAEE represents cross-sectional average of variances of the estimation errors of betas.

Figure 2 Beta Herding in the US Market
The herd measure is calculated using the constituents of the FTSE350 index (247 stocks), and FTSE350 index and 3 month treasury bills are used to calculate the excess returns. The standardised herd measure is calculated with the cross-sectional variance of t-statistics of OLS estimates of betas. The sample period is from January 1993 to November 2002, and 24 past monthly returns are used to estimate betas in the Fama-French three factor model. CAEE represents cross-sectional average of variances of the estimation errors of betas.
The herd measure is calculated using the constituents of the KOSPI index (454 ordinary stocks), and the KOSPI index and 1 year Korea Industrial Financial Debentures are used to calculate the excess returns. The standardised herd measure is calculated with the cross-sectional variance of t-statistics of OLS estimates of betas. The sample period is from January 1993 to November 2002, and 24 past monthly returns are used to estimate betas in the Fama-French three factor model. CAEE represents cross-sectional average of variances of the estimation errors of betas.
Figure 5A  Standardised Beta Herding Calculated with Fama-French 25 Portfolios Formed on Size and Book-to-Market
The Beta-based herd measure is calculated with the cross-sectional variance of OLS estimates of betas while the standardised herd measure is calculated with the cross-sectional variance of t-statistics of OLS estimates of betas. We use 60 past monthly returns to estimate betas in the market model and in the Fama-French three factor model with momentum. The portfolio returns come from Kenneth French's data library.
Herd measure is performed with t-statistics of the OLS estimates of betas in the Fama-French three factor model with momentum for the Fama-French 100 portfolios formed on size and book-to-market. Rolling windows of past 60 months observations are used for the OLS estimation. The direct sentiment index is the bull-bear ratio of Investors Intelligence, Michigan Consumer Confidence index represents index on business conditions for the next 12 months, and the interpolated Baker and Wurgler index is monthly index we interpolated from the annual index of Baker and Wurgler (2006).
At every month we use the t-statistics of individual stocks using 60 monthly observations in the Fama-French three factor model with momentum, and then form decile portfolios according to the t-statistics. Sizes of each portfolio is calculated by equally weighting sizes of individual stocks in the portfolio. The same procedure is repeated for the book-to-market ratio. The figures show average sizes and book-to-market ratios over 427 months from June 1968 to December 2003.
A1. Herd Measure towards the Market Portfolio with Different Size Firms

- 0.1% of Market Capitalisation
- 0.01% of Market Capitalisation
- 0.0001% of Market Capitalisation

- Number of Stocks

A2. A. Herd Measure towards the Market Portfolio in the Presence of Systematic Bias in Betas

- Large Beta Stocks (Top 70%)
- Small Beta Stocks (Bottom 70%)
- Large-Small Beta Stocks (Top 35% and Bottom 35%)
- Middle Beta Stocks (Middle 70%)
A3. Herd Measure towards the Market Portfolio with Different Pricing Models

- Market Model
- Fama-French Three Factor Model
- Fama-French Three Factor with Momentum

A4. Herd Measure with Different Past Observations

- 24 Monthly Observations
- 36 Monthly Observations
- 48 Monthly Observations
- 60 Monthly Observations