The effect of credit risk transfer on financial stability

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Abstract

This paper shows under which conditions debt securitization of banks can increase the systemic risks in the banking sector. We use a simple model to show how securitization can reduce the individual banks’ economic capital requirements by transferring risks to other market participants. This can increase systemic risks and impact financial stability in two ways. First, if the risks are transferred to unregulated market participants there is less capital in the economy to cover these risks. And second, if banks invest in asset-backed securities, the transferred risk causes interbank linkages to grow. This results in an increasing systemic risk for which the economic capital put aside is insufficient. We develop a modified version of the infectious defaults model of Davis and Lo (2001) and use this model to quantify the augmented systemic risk of increased bank linkages in the banking sector.

JEL Classification: G21, G28
Keywords: Systemic risk, financial stability, debt securitization, economic capital, expected loss, infectious defaults model

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1 Introduction

Asset-Backed Securities (ABS) issuance, especially Collateral Debt Obligations (CDO) and Collateral Loan Obligations (CLO), shows a remarkable growth during recent years and can be expected to continue over the next years.

What makes these ABS bonds so attractive for banks? Several factors play a role. First, there is an economic risk transfer, that is, banks can transfer part of their risks to the market by selling fractions of their debt. Secondly, funding a loan can become significantly cheaper since the expected loss and economic capital are reduced. Third, securitization offers arbitrage spread opportunities and finally, banks can obtain regulatory capital relief. Using the 8%-rule this can easily be shown (e.g. see Bluhm et al. (2003), and Basel Committee on Banking Supervision (2003)). In addition, banks increasingly invest in ABS bonds since this can offer new investment opportunities.

This paper focusses on the economic risk transfer and shows under which conditions this risk transfer of an individual bank can increase the systemic risks of the whole banking sector. It is also an attempt to present a systemic risk model called for by Goodhart (2004) who states that “...we need to construct models of systemic stability, not just of individual bank probability of default [...]” (page 3)

We propose an extended version of the ”infectious default” model of Davis and Lo (2001) to build a model of the banking system that explicitly includes interbank linkages through securitization while also modelling potential contagious effects (e.g. see Giesecke and Weber, 2004).

The paper is structured as follows. First, we present a simple model to demonstrate the economic risk transfer through securitization. In the next section a model of the banking system including contagious effects is proposed. Third, we present simulation results for different structures of interbank linkages caused by an increased securitization of banks
and show under which conditions the amount of capital in the banking system is insuffi-
cient given the linkages between banks. Finally, we summarize our findings and conclude.

2 Risk transfer through securitization

Banks engage in the securitization business due to different reasons. The most important
ones are economic risk transfer, different funding, spread arbitrage opportunities and reg-
ulatory capital relief. In this section we focus on economic risk transfer and use a simple
model to quantify the amount of risk which is transferred from a bank to the market.

Assume we have an index set, $I = \{1, \ldots, m\}$, referring to loans of a portfolio. The
easiest case possible is to assume that the complete portfolio is selected for securitization.
Based on this collateral portfolio, an equity piece and one or more mezzanine and senior
pieces are sold to different investors. The equity piece, often called the first loss piece
(FLP), receives interest and principal payments only if all other investors received their
promised payments.

In the literature different ways to model the cash flows are proposed. The most common
ones are the simple BET (e.g. Moody’s Investors Service (1996)), double BET (e.g. Moody’s
Investors Service (1998)), the lognormal model (e.g. Moody’s Investors Service (2000))
and methods using Monte Carlo simulations or fourier transforms (e.g. Moody’s Investors
Service (2003)). In the BET and double BET a new portfolio of equal and independent
loans is constructed so that it mimics the original portfolio. The number of defaults is
modelled by use of the binomomial distribution. The lognormal method uses a lognormal
distribution for the cumulative number of default instead of the binomial distribution. In
this way there is no need to assume that all the loans are equal and independent. In the
Monte Carlo method and fourier transform method we use the information of all the loans
in the portfolio directly and also account for the correlation between the loans to find the
cumulative probability of default. A drawback of this last two methods is that they are more time consuming.

Applying one of the methods above we can model the cash flows in case of securitization given a value for the first loss piece. For simplicity the loss given default (LGD) is taken equal to 100% which is equal to putting the recovery rate equal to zero. The loss statistic for the portfolio $I$ is given by $(L_1, \ldots, L_m)$. Hence the total loss in case there is no securitization is equal to

$$L = \sum_{i=1}^{m} L_i.$$ 

For the securitization we assume that no Interest Coverage tests (IC) or Overcollateralization tests (OC) are used, also no cash reserve account is available and the model horizon is one year. We also suppose that the bank manages to sell all the senior tranches and keeps the equity piece itself. The securitized portfolio is hence protected against losses exceeding the first loss piece. The loss of the securitized portfolio is equal to the loss of the equity piece and will be denoted with $L_{eq}$. Since the equity piece absorbs all the losses up to a certain level FLP its loss is given by

$$L_{eq} = \min(\sum_{i=1}^{m} L_i, FLP).$$

The change (denoted by $\Delta$) in expected loss $E(L)$ from the bank that securitizes its debt is given by

$$\Delta EL = E(L) - E(L_{eq}).$$

The required economic capital (denoted with $EC_\alpha$) is defined as the difference of the $\alpha$%-quantile ($q_\alpha$) and the expected loss. Hence the difference in economic capital due to the
securitization is

\[
\Delta EC_{\alpha} = EC_{\alpha}(L) - EC_{\alpha}(L_{eq})
\]

\[
= q_{\alpha}(L) - E(L) - (q_{\alpha}(L_{eq}) - E(L_{eq}))
\]

\[
= \Delta q_{\alpha} - \Delta EL
\]  

(1)

with \(\Delta q_{\alpha} = q_{\alpha}(L) - q_{\alpha}(L_{eq})\), the change of the \(\alpha\)\%-quantile. In this case without tests and without a cash reserve account the losses of the portfolio with and without securitization will be increasing functions of percentages of defaults in the portfolio. The mezzanine and senior pieces that are sold to the market have a total loss which is denoted with \(L_{sp}\). Since no extra money is transferred, the sum of the loss of the equity piece \(L_{eq}\) and the loss of the senior pieces \(L_{sp}\) will always be equal to the loss in case there is no securitization \(L\).

This leads to

\[
E(L) = E(L_{eq}) + E(L_{sp}).
\]  

(2)

For the quantile functions the sum is more complicated. In the considered case the three losses are all nondecreasing functions of the percentages of default. It also holds the equality \(L = L_{eq} + L_{sp}\). However, for quantiles the relationship is less clear. In general, sub-additivity does not hold for quantiles. The standard counterexample for sub-additivity is a portfolio with two identical bonds \(A\) and \(B\). Each defaults with probability 0.04 with a loss of 100 and no loss otherwise. Hence, the 95%-quantile of the loss for each bond is zero: \(q_{95}(A) = q_{95}(B) = 0\). In case \(A\) and \(B\) are independent the result for \(A + B\) is given in table 1.

The table shows that a loss of 0 occurs with probability \(0.96^2 = 0.9216\), a loss of 200 happens with probability \(0.04^2 = 0.0016\) and a loss of 100 with probability \(1 - 0.9216 - 0.0016 = 0.0768\).

It follows that \(q_{95}(A + B) = 100\). Hence, sub-additivity does not hold since \(q_{95}(A + B) >\)
Table 1: **Example sub-additivity does not hold**

<table>
<thead>
<tr>
<th>loss of B</th>
<th>0</th>
<th>100</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>loss of A</td>
<td>0</td>
<td>0.9216</td>
<td>0.0384</td>
</tr>
<tr>
<td>100</td>
<td>0.0384</td>
<td>0.0016</td>
<td>0.0400</td>
</tr>
<tr>
<td>Σ</td>
<td>0.9600</td>
<td>0.0400</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: **Example for securitization**

<table>
<thead>
<tr>
<th>loss of B</th>
<th>0</th>
<th>100</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>loss of A</td>
<td>0</td>
<td>1 - π₁</td>
<td>π₁ - π₂</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>π₂</td>
<td>π₂</td>
</tr>
<tr>
<td>Σ</td>
<td>1 - π₁</td>
<td>π₂</td>
<td>1</td>
</tr>
</tbody>
</table>

\(q_{95}(A) + q_{95}(B)\).

In the case of securitization, there is a strict relationship between the loss of the securitized portfolio and the loss of the senior pieces. If \(A\) is the equity piece and \(B\) is the whole group of senior pieces, \(B\) can only default if \(A\) has defaulted as well. Equal default probabilities for \(A\) and \(B\) will lead to either a total loss of 200 or no loss, the intermediate case of a loss of 100 is no longer possible. Then the additivity certainly holds. More general, if we have a probability \(π₁\) for a loss of 100 for the equity piece \(A\) and a probability \(π₂\) for a loss of 100 for \(B\) we get the following cases shown in table 2.

Naturally, the condition \(π₁ > π₂\) has to be satisfied to guarantee non-negative probabilities. Given this constraint, sub-additivity always holds. There are three cases of interest: (i) no defaults, (ii) only \(A\) defaults and (iii) \(A\) and \(B\) defaults. Sub-additivity always holds since the individual number of defaults is always equal to the total number of defaults. For example, for \(π₁ = 0.06\) and \(π₂ = 0.02\), we get \(q_{95}(A) = 0, q_{95}(B) = 0\) and \(q_{95}(A + B) = 0.\)
Assume alternatively that \( \pi_1 = 0.08 \) and \( \pi_2 = 0.06 \), we get \( q_{95}(A) = 100 \), \( q_{95}(B) = 100 \) and \( q_{95}(A + B) = 200 \).

Obviously this cannot be considered to be a proof since not the whole range of losses is analyzed in the above example but only two values, that is, no loss or a loss of 100.

We will show in the following that the strict constraint that losses on the senior pieces only occur when the equity piece is totally lost will force the quantiles to be sub-additive:

\[
q_\alpha(L) \leq q_\alpha(L_{\text{eq}}) + q_\alpha(L_{\text{sp}}).
\] (3)

To illustrate this, we use an example with a collateral pool of total size 100. It consists of 50 loans with variable default probabilities. Using a first loss piece of size 30 and only one senior piece of size 70, a Monte Carlo simulation constructs empirical distribution functions for the loss functions and finds the corresponding quantiles. A visualization of them is given in Figure 1.

In this particular case, recovery rates are chosen randomly between 0\% and 50\% for the collateral pool and the maturity is 7 years. The 99\%-quantiles are: \( q_{99}(L) = 53.48 \),
\[ q_{99}(L_{eq}) = 25.77 \quad \text{and} \quad q_{99}(L_{sp}) = 29.60. \] For a one year maturity we find the 99%-quantiles to be: \( q_{99}(L) = 49.03 \), \( q_{99}(L_{eq}) = 30 \) and \( q_{99}(L_{sp}) = 19.03 \).

If we only consider the one year period from the launch of the deal until one year later (and assume that in case of default no recovery takes place) it is possible to proof:

\[ q_{\alpha}(L) = q_{\alpha}(L_{eq}) + q_{\alpha}(L_{sp}). \] (4)

In this case we have a similar situation as in a reinsurance company. All claims up to a certain level \( d \) are covered by the insurance company. The reinsurance company will only play a role in the case that the claim sizes exceed the given level. Then the payout of the insurance company will be \( d \) and the reinsurer will pay the part to which the claim exceeds the level. More formally, let \( X \) denote the random variable for the claim size with a distribution function \( f_X \). The part the reinsurer takes for his account \( Y \) will be equal to zero in case \( X \leq d \) and will be \( X - d \) otherwise. For the distribution function we get

\[ f_Y(y) = \begin{cases} 
F_X(d) & \text{if } y = 0 \quad \text{or} \quad x \leq d, \\
 f_X(y + d) & \text{if } y > 0 \quad \text{or} \quad x > d. 
\end{cases} \] (5)

Similar here we could say that \( X \) is the loss in case we have no securitization. If the boundary value \( d \) is equal to the first loss piece, the senior piece will only be affected by the losses if they exceed a certain level \( d \) and the senior piece will have a distribution function like \( Y \). The loss of the securitized portfolio can be seen as the part the insurance company has to take for his account. It is denoted by \( Z \) and has a distribution function

\[ f_Z(z) = \begin{cases} 
f_X(z) & \text{if } z < d \quad \text{or} \quad x < d, \\
1 - F_X(d) & \text{if } z = d \quad \text{or} \quad x \geq d.
\end{cases} \] (6)

Using the above distribution function the \( \alpha \)-quantiles can be calculated to prove the additivity.

We need to distinguish two cases. The case where the probability to have losses greater or equal then \( d \) is smaller than \( 1 - \alpha \% \) or the case where this probability is greater or
equal than \(1 - \alpha\%\). The construction of the proof is the same for both cases. Hence we only consider the second case \(\Pr(X \geq d) > 1 - \alpha\%\) which can be written as \(\Pr(X < d) < \alpha\%\).

For the total loss \(X\) we can only say that the \(\alpha\)-quantile is equal to an unknown value \(q_{\alpha}(L)\) such that \(q_{\alpha}(L)\) is the smallest possible value for which

\[
F_X(q_{\alpha}(L)) \geq \alpha\%
\]  
(7)

holds. For the securitized portfolio or the equity piece we look for the smallest value which is not exceeded with a probability greater or equal than \(\alpha\%\). Given only the second case is considered, we have

\[
\Pr(Z < d) < \alpha\%
\]

and \(\Pr(Z \leq d) = 1\). Since \(d\) is the smallest possible value for which the loss of the equity piece is not exceeded with a probability greater or equal than \(\alpha\%\), \(q_{\alpha}(L_{eq}) = d\).

For the senior piece we need to use the definition of the quantile function once again. The quantile is so that \(F_Y(q_{\alpha}(L_{sp})) \geq \alpha\%\). Given that \(Y\) has a distribution function as in (5) this can be written as

\[
F_Y(q_{\alpha}(L_{sp})) = \Pr(Y \leq q_{\alpha}(L_{sp})) = \Pr(Y = 0) + \Pr(0 < Y \leq q_{\alpha}(L_{sp}))
\]

\[
= F_X(d) + \int_{0+}^{q_{\alpha}(L_{sp})} f_Y(y) \, dy = F_X(d) + \int_{0+}^{q_{\alpha}(L_{sp})} f_X(y + d) \, dy
\]

\[
= F_X(d) + \int_{d}^{q_{\alpha}(L_{sp})+d} f_X(t) \, dt = F_X(d + q_{\alpha}(L_{sp})) \geq \alpha\%.
\]  
(8)

The values \(q_{\alpha}(L)\) in equation (7) and \(d + q_{\alpha}(L_{sp})\) in equation (8) are both the smallest possible values for which \(F_X(*) \geq \alpha\%\) holds. Since \(F_X\) is an increasing function, it can be concluded that \(q_{\alpha}(L) = d + q_{\alpha}(L_{sp})\). This proves that \(q_{\alpha}(L) = q_{\alpha}(L_{eq}) + q_{\alpha}(L_{sp})\).

Using this finding, the reduction of economic capital \(\Delta EC_{\alpha}\) can be calculated. In other words, it is the regulatory capital relief obtained by securitization. The proof also shows that securitization does not automatically augment or lower the economic capital in the financial system. Due to the additivity of the risks of the tranches, the economic capital
remains unchanged. If additivity did not hold, economic capital could decrease or increase thereby compensating or amplifying the risk transfer effect. The question is now whether the new structure, that is, the distribution of credit risks in the market requires a higher or lower amount of economic capital. For this we analyze two cases of credit risk transfer.

First, the tranches are sold to unregulated market participants or entities outside the financial system and second, the tranches are sold to other, regulated, financial intermediaries within the financial system.

In the first case, the overall amount of capital put aside can be lower than before the securitization since market participants are not part of the financial system or put an amount of capital aside inferior to the amount required for regulated market participants. Hence, the positive effect of the risk transfer, that is, the spread of risk, can be compensated to a certain degree.\textsuperscript{1} In the second case, the amount of capital put aside stays constant but can be insufficient from a system-wide perspective due to the increased interbank linkages. We will show in the next section that these stronger linkages increase systemic risks and therefore require an augmented amount of capital.

There is also a third possibility of increased risks. If banks use the new capital obtained by securitization to expand their loan business they incur more systematic risk (see Franke and Krahnen (2004)). Since systematic risk is different to systemic risk we will not investigate this case in the paper.\textsuperscript{2}

A survey from the European Central Bank from May 2004 (European Central Bank, 2004) studies the transfer of risk between several European banks. A summary of the transfer of the structured products (Asset-backed securities and synthetic collateral debt obligations) is given in Table 3 and shows that this last risk is non negligible since up to

\textsuperscript{1}Krahnen (2005) argues that senior tranches placed outside the financial system reduces the risk of contagion among banks.

\textsuperscript{2}Here, systematic risk means the market risk and systemic risk means a risk due to the system per se.
Table 3: **Summary of the transfer of structured products (ABS and synthetic CDO’s)**

<table>
<thead>
<tr>
<th>Country</th>
<th>Protection buying</th>
<th>Protection selling</th>
<th>Number of institutions surveyed</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>0.2 – 1.5%</td>
<td>0.1 – 1.8%</td>
<td>3</td>
</tr>
<tr>
<td>Spain</td>
<td>3 – 15%</td>
<td>n.a.</td>
<td>4</td>
</tr>
<tr>
<td>Ireland</td>
<td>1 – 10%</td>
<td>0.2 – 0.6%</td>
<td>6-9</td>
</tr>
<tr>
<td>Italy</td>
<td>0 – 6.5%</td>
<td>0.2 – 7.5%</td>
<td>4</td>
</tr>
<tr>
<td>Portugal</td>
<td>5 – 30%</td>
<td>n.a.</td>
<td>4</td>
</tr>
</tbody>
</table>

30% (see Portugal) of a banks capital can be invested in CDO’s from other banks. In the next section we will concentrate on the effect of this change in linkages.

Based on the table above, we will model the effect of securitization on interbank linkages. We will assume that an economy consists of $N$ banks with equal interbank exposures $\alpha$ for all banks. We then assume that $n$ banks ($n \in N$) securitize a percentage $\gamma$ of their debt and that $m$ banks ($m \in N$) buy tranches of this securitized debt accounting for some percentage of their book capital. This increase in capital from other banks will increase interbank linkages and therefore the systemic risk due to the fact that the loss distribution gets heavier tails with higher correlations.

In section 4 a Monte-Carlo simulation is performed with different linkage matrices representing different interbank linkages resulting from the risk transfer described above. It is shown how a small increase in such linkages and thus in correlations can require an increased amount of capital as a cushion for systemic risks. In doing this, we will also differentiate between complete and incomplete bank structures as introduced by Allen and Gale (2002).
3 The Model

In this section we present a latent variables version of the "infectious default" model by Davis and Lo (2001) to analyze systemic risk and financial stability. In the original version of Davis and Lo it is assumed that a given portfolio of $n$ bonds may either default directly or as a result of infection, i.e. due to the default of some other bond. The static version of this model is

$$Z_i = X_i + (1 - X_i) \left( 1 - \prod_{j \neq i} (1 - X_j Y_{ji}) \right)$$

(9)

where $Z_i = 1$ if the $i$th bond defaults, and $Z_i = 0$ otherwise, $X_i = 1$ if the $i$th bond defaults directly and $X_i = 0$ otherwise, if $X_j = 1$ and $Y_{ji} = 1$ the infection occurs, for $Y_{ji} = 0$ no infection occurs.

We extend this model in two respects: First, we relax the assumption of homogeneous entities in the portfolio and second, we modify the "infection matrix" $Y$ to capture the true interbank linkages. Hence, the matrix $Y$ is not restricted to have values of 0 and 1 but can contain all possible values in the interval $[0, 1]$.

In order to relax the assumption of homogeneous bonds, we use a Monte-Carlo simulation to obtain the default and loss distribution. Since we aim to present a model of systemic risk and not of an individual bank's portfolio, we model bank defaults and interbank linkages. In other words, we are modelling a countries' "bank portfolio" and not an individual banks' portfolio.

We assume that each banks' default probability $\pi_i$ ($i = 1, ..., n$) can be modelled with a latent factor $S_i$ that follows some distribution with mean zero and variance one. All $S_i$ are independent and identically distributed.

A bank $i$ defaults directly ($X_i = 1$) if the realization of the latent variable $S_i$ is below
some threshold \(D_i\) as follows

\[X_i = 1 \iff S_i \leq D_i\]

so that \(\pi_i = P(S_i \leq D_i)\). Hence, \(X_i\) denotes a direct default. An indirect default, denoted as \(X_i^* = 1\), is given if there is a linkage between \(X_i\) and one or more other banks \(X_j\) that defaulted directly. Here, we extend the original model and allow \(Y_{ji}\) to contain any value in \([0, 1]\) representing the percentage of assets hold by another bank. If the percentages of assets deposited with other banks that directly defaulted exceeds a bank-specific threshold \(d_i\), bank \(i\) defaults (indirectly).

\[X_i^* = 1 \iff \{YX\}_i \geq d_i\]

Hence, obligor \(i\) defaults either directly or indirectly indicated by \(Z_i\) as follows \(Z_i = X_i + (1 - X_i)X_i^*\) which implies

\[Z_i = 1 \iff X_i = 1 \lor X_i^* = 1.\]

The important feature of this model is that linkages between bonds or banks can be modelled via a dependence matrix \(Y\) that can incorporate direct linkages that are asymmetric or symmetric. We believe that this approach is superior to the use of correlation matrices since these implicitly assume symmetric linkages. Indirect linkages are not taken into account for simplicity.\(^3\) We will also differentiate between complete and incomplete linkages (i.e. structures) as in Allen and Gale (2001) and in Upper and Worms (2002).

Assume the following infection matrix \(Y\) with 4 banks, called A, B, C and D:

\(^3\)The inclusion of indirect linkages would increase the computational time of the Monte-Carlo simulation.
This matrix represents the values of the linkages, i.e. \( \alpha \) deposits of bank \( j \) with bank \( i \). More precisely, bank \( j \) has deposits at bank \( i \). Hence, the matrix states that bank A has deposits at bank D, that bank B has deposits at bank A, bank C has deposits at bank B and bank D has no deposits with any bank. If we assume further that only bank B defaults directly, the vector \( X \) is given as follows

\[
X = \begin{pmatrix}
0 \\
1 \\
0 \\
0
\end{pmatrix}
\]

Then, \( YX \) yields

\[
YX = \alpha \begin{pmatrix}
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
0 \\
1 \\
0 \\
0
\end{pmatrix} = \alpha \begin{pmatrix}
0 \\
0 \\
1 \\
0
\end{pmatrix}.
\]

Hence, bank C defaults by infection, i.e. indirectly under the assumption that the loss of \( \alpha \) deposits leads to a default of bank C.

We now present four different matrices \( Y \) (i.e. structures) of direct linkages. First, a complete and symmetric structure. Second, a complete and asymmetric structure. Third, an incomplete and symmetric structure and finally, an incomplete and asymmetric struc-
ture. We still assume for simplicity that the values of the linkages, i.e. the percentages of capital of bank $i$ held by bank $j$ is equal among all linkages and denoted by $\alpha$. This simplification is just for presentation purposes and will be relaxed later.

The four matrices $Y$ are presented below. The complete and symmetric structure for an equal linkage denoted by $\alpha$ is given by the following matrix

$$Y^1 = \alpha \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$ \hfill (13)

Evidently, this structure is characterized by symmetric one-to-one relationships between banks. Hence, to match these linkages with real data, the $\alpha$s have to be smaller in a complete structure than in an incomplete structure since the sum of linkages in a symmetric structure can easily exceed realistic values. Note that different percentages of linkages ($\alpha_{ji} \neq \alpha \land \alpha_{ji} > 0$, $\forall j, i$) would lead to a complete but asymmetric structure.

An example of such a matrix is given as follows:

$$Y = \begin{pmatrix} 0 & 0.1 & 0.1 & 0.2 \\ 0.05 & 0 & 0.15 & 0.01 \\ 0.02 & 0.05 & 0 & 0.1 \\ 0.025 & 0.05 & 0.05 & 0 \end{pmatrix}. \hfill (14)$$

An example of a complete and asymmetric structure for $\alpha_{ji} = \alpha$ $\forall j, i$ is represented by

$$Y^2 = \alpha \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \hfill (15)$$
This structure is complete since the linkages could also be represented by an upper-
triangular matrix.

Finally, we present two examples for incomplete structures. An incomplete and sym-
metric structure is given by
\[
Y^3 = \alpha \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0
\end{pmatrix}
\]
where the matrix entries could be random but must follow \( Y_{ji} = Y_{ij} \forall i, j \).

An incomplete and asymmetric structure is represented by
\[
Y^4 = \alpha \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]
where the off-diagonal matrix entries could be random without further restrictions.

The next section uses these four different structures for simulations with different val-
ues for \( \alpha \) for all banks \( N \). We will also investigate the effect, the number of originating
banks \( N \) and the number of buying banks \( M \) has on the market-wide required economic
capital. The number of buying banks \( M \) is equal to the total number of linkages. For ex-
ample, \( M = 3 \) in the last example representing an incomplete and asymmetric structure
for \( N = 4 \). This value \( M \) can be viewed as a measure of the concentration (diversification)
of the risk transfer.
4 Simulations

We now perform a Monte-Carlo simulation of the model presented in section 3 in order to quantify the change in required economic capital resulting from increased interbank linkages.

The default probability \( dp \) is set to \( dp = 1\% \) for all banks. For each simulation run samples of \( S_i \) are drawn from a normal distribution. These draws determine the number of direct defaults and the linkage matrix \( Y \) decides about the number of indirect or contagious defaults. This is repeated \( n = 10,000 \) times in order to obtain the joint default distribution of all banks.

We are aware of the fact that changing the copula has a considerable effect on the shape of the loss distribution and hence the EC (e.g. see Duffie and Singleton (2003)).

The MC simulation is performed for different number of banks \( N \) in the economy, different linkage structures and different numbers of interbank linkages \( M \) within the assumed structures.

The aim of the simulation is to evaluate the change in economic capital for different values of \( \alpha \) that represent the percentages of deposits a bank holds of another bank. The threshold value \( d_i \) that determines a default of bank \( i \) is set to \( d_i = 0.2 \). We compute the 99.99\% quantile of the resulting default distribution and compute the economic capital for this quantile.\(^4\) This economic capital is computed under the assumption that there are no linkages \( (\alpha = 0) \) and different degrees of linkages, i.e. \( \alpha = 0.050, 0.075, 0.1, 0.2 \) for each assumed structure of linkages. Results are presented in table 4 and show that the additional economic capital necessary increases with increasing \( \alpha \) and increasing number of linkages (diversification) \( M \). The additional economic capital decreases with increasing

\(^4\)We abstract from absolute losses and recovery rates. However, this is just for presentation purposes since the Monte Carlo simulation can also be used to compute a loss distribution of the banking sector.
number of banks $N$.

For the complete and symmetric structure (top panel) and no linkages $\alpha = 0$, the EC is 49%, 19%, 14% and 5.5% for $N = 4$, $N = 10$, $N = 20$ and $N = 100$, respectively. For example, the EC for $N = 10$ increases from 19% ($\alpha = 0$) to 29% ($\alpha = 0.05$). Since it is a complete structure, linkages for higher $\alpha$ are not plausible since the sum of deposits held by each bank would well exceed any realistic level. This is especially true for $N = 100$ where we only report results for $\alpha = 0$.

For the complete and asymmetric structure, the EC is smaller for the largest reported values of $\alpha$ for $N = 4$ and $N = 10$ than for the complete and symmetric structure.

We now focus on the incomplete structures. Here, we assume different values of $M$. Columns three and four show that the number of linkages $M$ for $N = 20$ banks matters for $\alpha > 0$. For example, for $\alpha = 0.2$ and $M = 10$, the EC is 29% while for $\alpha = 0.1$ and $M = 20$, the EC is 19%.

We can conclude that a larger number of banks and a larger number of linkages yield lower values of EC. This diversification effect has an important implication. The higher the number of (equal) linkages between banks, the lower is the resulting difference between the sum of economic capital put aside by each bank and the optimal economy-wide economic capital required as a cushion against extreme (systemic) risks.

In other words, if banks do not explicitly account for increased linkages with other banks, the risks associated with this are minimized the larger the number of banks is and the more these banks are linked to each other. However, this last assumption only holds for equal or similar linkages. Extreme asymmetries, i.e. extreme heterogeneity of the values $\alpha_i$ would lead to different results.
Table 4: Simulation Results, EC($q = 99.99\%$)

<table>
<thead>
<tr>
<th>structures</th>
<th>$N = 4$</th>
<th>$N = 10$</th>
<th>$N = 20$</th>
<th>$N = 20$</th>
<th>$N = 100$</th>
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<tr>
<td><strong>complete symmetric</strong></td>
<td>$M = 12$</td>
<td>$M = 90$</td>
<td>$M = 380$</td>
<td>$M = 9900$</td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0$</td>
<td>0.49</td>
<td>0.19</td>
<td>0.14</td>
<td></td>
<td>0.055</td>
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<tr>
<td>$\alpha = 0.050$</td>
<td>0.49</td>
<td>0.29</td>
<td>0.14</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>$\alpha = 0.075$</td>
<td>0.49</td>
<td>0.59</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>$\alpha = 0.100$</td>
<td>0.99</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>$\alpha = 0.200$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td><strong>complete asymmetric</strong></td>
<td>$M = 6$</td>
<td>$M = 45$</td>
<td>$M = 190$</td>
<td>$M = 4950$</td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0$</td>
<td>0.49</td>
<td>0.19</td>
<td>0.14</td>
<td></td>
<td>0.055</td>
</tr>
<tr>
<td>$\alpha = 0.05$</td>
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<td>0.29</td>
<td>0.14</td>
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<td>-</td>
</tr>
<tr>
<td>$\alpha = 0.075$</td>
<td>0.49</td>
<td>0.34</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>$\alpha = 0.10$</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>$\alpha = 0.20$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td><strong>incomplete symmetric</strong></td>
<td>$M = 6$</td>
<td>$M = 10$</td>
<td>$M = 10$</td>
<td>$M = 20$</td>
<td>$M = 20$</td>
</tr>
<tr>
<td>$\alpha = 0$</td>
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<td>0.19</td>
<td>0.14</td>
<td>0.14</td>
<td>0.055</td>
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<tr>
<td>$\alpha = 0.05$</td>
<td>0.49</td>
<td>0.24</td>
<td>0.14</td>
<td>0.16</td>
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</tr>
<tr>
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<td>0.14</td>
<td>0.16</td>
<td>0.055</td>
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<td>0.14</td>
<td>0.19</td>
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</tr>
<tr>
<td>$\alpha = 0.20$</td>
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<td>0.59</td>
<td>0.29</td>
<td>0.34</td>
<td>0.075</td>
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<tr>
<td><strong>incomplete asymmetric</strong></td>
<td>$M = 3$</td>
<td>$M = 5$</td>
<td>$M = 5$</td>
<td>$M = 10$</td>
<td>$M = 10$</td>
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<td>0.14</td>
<td>0.14</td>
<td>0.055</td>
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<tr>
<td>$\alpha = 0.05$</td>
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<td>0.14</td>
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<td>0.055</td>
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<tr>
<td>$\alpha = 0.075$</td>
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<td>0.29</td>
<td>0.14</td>
<td>0.14</td>
<td>0.055</td>
</tr>
<tr>
<td>$\alpha = 0.10$</td>
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<td>0.29</td>
<td>0.14</td>
<td>0.14</td>
<td>0.055</td>
</tr>
<tr>
<td>$\alpha = 0.20$</td>
<td>0.99</td>
<td>0.39</td>
<td>0.24</td>
<td>0.24</td>
<td>0.060</td>
</tr>
</tbody>
</table>

$d_i = 0.2$

$M$ is equal to the number of interbank linkages due to the CDO issuance. For example, in the complete symmetric structure $M = N^2 - N$. For $N = 4$, $M$ is equal to 12.
5 Conclusions

This paper shows how banks can reduce their capital requirements by transferring risks to other market participants. We focus on the case that these risks are transferred to other banks thereby increasing the interbank linkages. We develop a model for the whole banking sector that accounts for these linkages and shows how these linkages can increase extreme or systemic risks and thus pose a threat to the stability of the financial system. We analyze this effect for different linkage structures of the banking sector and find that risks can increase significantly especially if the linkages are complete and symmetric rather than incomplete and asymmetric. In addition, the larger the number of banks, the lower is the increase of systemic risks.

This paper is a first step to develop global models of systemic risk and financial stability.

Future research could also calibrate this model to real data of the banking sector of the European Union or different individual countries.
References


21


