Prudential Liquidity Regulation and the Insurance Aspect of Lender of Last Resort

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This article considers prudential liquidity regulation as quid pro quo for emergency liquidity assistance by the central bank. In the presence of information-induced bank runs and an objective by the central bank to maintain a zero expected cost of the lender of last resort (LOLR) safety net, it is argued that the more debt-constrained the banking sector is and the higher its potential profitability, the more prudential liquidity regulation becomes socially desirable. Otherwise, liquidity regulation is too costly from a welfare perspective, even after taking into account the social value of LOLR insurance.

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Central banks have been deeply involved in the design of banking regulation even in cases where responsibilities for supervision of banking institutions have been transferred to other authorities. Basel 2 on capital adequacy of internationally active banks and recent efforts to overhaul prudential liquidity standards in a number of jurisdictions are examples of central banks’ involvement in the design of prudential standards for banks. That could raise the question of whether central banks have an interest in the design of prudential banking regulation that is possibly distinct from that of supervisors.

There are at least two obvious answers to that. First, regulatory design may affect financial stability in which central banks have a vested interest due to possible implications for monetary policy. Bank failures, for example, are often associated with systemic externalities and may have macro implications (e.g. Japan in 1990s), but so does regulatory intervention, depending on design. Second, regulation may have implications for the likelihood of banks needing lender of last resort (henceforth LOLR) assistance, in which case a central bank may have an interest in preempting unintended consequences of the LOLR facility for its balance sheet and, as a consequence of that, for the fiscal burden imposed on tax payers. Moreover, the LOLR facility conflicts with the best principles that a central bank would wish to operate in money markets, such as predictability in its actions and lack of favouritism in the choice of counterparties. Thus, among central

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bankers, emergency liquidity assistance has been perceived as literally a last resort policy tool, rather than an alternative to banking regulation.

In view of recent proposals in the UK for changing the liquidity regime for banks1, this paper focuses on the optimal design of liquidity regulation in the presence of bank runs and an explicit objective by the central bank to maintain a zero expected cost of LOLR intervention. By liquidity regulation we mean here a prudential standard specifying an appropriate level of highly liquid assets that a bank needs to hold in relation to its liabilities. We have in mind a quantitative (stock) liquidity requirement similar to the Sterling Stock Liquidity Ratio (SSLR) requirement of the UK’s Financial Services Authority (FSA)2, which requires banks to hold enough liquid assets to meet outflows over a given horizon. That requirement is distinct from reserve balances that banks need to maintain with the central banks in certain jurisdictions – including the U.S. – which have been typically used by governments as tax on bank profits3 and to enhance central banks’ ability to implement monetary policy4.

In the UK, Sterling Stock Liquidity policy has been justified as a means of buying time in case of crisis. According to UK’s FSA rules5, the SSLR requirement for banks is to hold liquid assets to meet outflows over a period of five days – also called the survival period – in order to enable a bank in crisis to continue business for some time and have the opportunity to arrange more permanent funding solutions. Conventional wisdom also suggests that the basic presumption underlying such a five-day horizon is the need to cover the worst case scenario where a bank faces a liquidity crisis on a Monday, while authorities are able to reach the next weekend and assess the situation better. That would possibly provide some comfort to the authorities, especially to the central bank, in considering the possibility of LOLR intervention without undue pressure due to pending market developments.

Official sector’s involvement to deal with unintended consequences of financial intermediation has been discussed in the literature both in terms of crisis prevention and crisis management. Diamond and Dybvig (1983) argue in favour of a deposit insurance scheme to resolve coordination problems among depositors that could lead to bank runs, inefficient liquidation of bank’s assets and bankruptcy. Bhattacharya and Gale (1987) offer a rationale for official monitoring of banks’ reserves of liquid assets due to possible liquidity shortages arising from banks’ incentives to free-ride on the common pool of liquidity in the interbank market, rather than holding costly reserves themselves. Dewatripont and Tirole (1994) argue that capital requirements provide an instrument for allocating control rights to the deposit insurance fund if things go badly. Holmström and Tirole (1998, 2000)

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2According to a recent survey by the Bank of England on the prudential regulation of banks’ liquidity (see Financial Stability Review, 15) there is currently no harmonisation of supervisory liquidity requirements at either a G10 or EU level. In February 2000, the Basel Committee on Banking Supervision published “Sound Practices for Managing Liquidity in Banking Organizations”, which sets out broad qualitative guidelines for how banks should manage liquidity risk.
3According to Feinman (1993), the U.S. Congress and many other governments are well aware that banks are a source of general revenue via central-bank profits that are remitted to the Treasury. Together with zero-interest currency, required reserves can generate significant seigniorage revenue, which, according to Greenbaum (1983), is the only reason that legal reserve requirements exist.
4For example, the U.S. Monetary Control Act of 1980 aimed at broadening the base of reservable deposits and enhancing the Fed’s ability to implement monetary policy by making all depository institutions subject to the Fed’s reserve requirements. Feinman (1993) offers a detailed discussion of reserve balances with the central bank and their historical evolution.
5See FSA Interim Prudential sourcebook, pp. 533-534.
suggest that the official sector can improve welfare by managing appropriately the supply of government debt on the grounds that banks may fail to cross-insure firms if liquidity shocks are correlated. Last but not least, Hellmann, Murdock and Stiglitz (2000) argue in favour of an interest rate ceiling on deposits to complement capital requirements in mitigating bank moral hazard by increasing the franchise value of banks.

As far as the role of LOLR in dealing with banking crises is concerned, Goodfriend and King (1988) argue that solvent banks could perfectly insure against the possibility of a bank run via a sophisticated interbank market, suggesting that a central bank’s role should be confined to maintain a sufficient amount of liquidity in the banking system, rather than to provide the LOLR facility. However, Donaldson (1992) finds evidence supporting the view that liquidity-rich banks may act strategically and abuse the market by charging higher than the competitive rate in case of crisis, which could justify LOLR intervention. Rochet and Vives (2002) argue that the LOLR may prevent inefficient liquidation of bank’s assets and improve welfare if the central bank has perfect foresight of bank’s fundamentals. Repullo (2003) discusses the effect of LOLR existence on holdings of liquid assets by banks, showing that a bank may end up keeping a lower level of liquid assets, which is consistent with empirical evidence by Gonzalez-Eiras (2003). Repullo (2003) also argues that the existence of the LOLR may lead to more efficient outcomes since holding liquid assets is typically costly. Yet Naqvi (2003) suggests that if the supervisory process of the LOLR is subject to noise, then the gains from ex-post efficiency, from holding a lower stock of liquid assets, may be outweighed by ex ante inefficiencies induced by moral hazard which is conducive to lower lending rates in the economy. Goodhart and Huang (2004) favour the existence of an LOLR arguing that the interbank market cannot provide sufficient liquidity when the amount needed to bail out a bank is too large to be accommodated by a single bank and concerted action by a group of banks may be inhibited by coordination problems. They also suggest that the interbank market might not be able to provide insurance against liquidity shocks if those shocks happen to be systemic, affecting the whole banking sector.

In this paper we search for an economic rationale for prudential liquidity regulation, considering the costs associated with liquidity regulation as an implicit insurance premium paid by a bank, quid pro quo for (partial) liquidity insurance by the official sector under the LOLR facility. To keep the analysis tractable, we assume away the role of liquidity requirements as a potential medium to buy time in case of crisis, yet with no intention to suggest that potential information asymmetries in case of banking crisis are trivial. Instead, we investigate how liquidity regulation for banks could be optimally combined with a LOLR policy in order to maximise the expected surplus from financial intermediation and we search for conditions under which a combination of prudential liquidity regulation and LOLR insurance could be welfare improving of a laissez-faire regime without prudential liquidity requirements and a LOLR safety net.

6In December 1996, the central bank of Argentina received access to contingent credit lines from a group of international banks, which enhanced its ability to act as a LOLR. Banks have indicated their reliance on the enhanced ability of the central bank to provide liquidity by reducing their liquid asset holdings by approximately 6.7 %.

7According to FSA’s Interim Prudential sourcebook, p. 495, banks are reluctant to hold a large stock of immediately available cash or marketable assets, as these generate no return (in the case of cash) or a comparatively low yield (in the case of easily marketable assets, e.g. government bonds).
Nevertheless, the idea of considering regulation as a form of \textit{implicit insurance}, and regulatory costs as insurance premia — or \textit{taxes} — against the implicit subsidy that such an insurance would imply, is not new. It dates back to Posner (1971) who explains a number of phenomena of regulated industries through the prism of \textit{taxation by regulation}. Buser, Chen and Kane (1981), for example, consider banking regulations by the Federal Deposit Insurance Corporation (FDIC) as a condition for banks receiving deposit insurance. They interpret the deadweight costs of such regulations as \textit{implicit insurance} premia which develop over and above the explicit fees that are charged by the FDIC\footnote{According to Buser et al. (1981), in order for the FDIC to induce voluntary participation of banks in its regulatory jurisdiction, its sets its explicit insurance premium below market value. As a result, the FDIC insurance fund amounts to only about 0.80 per cent of total deposits in insured banks and FDIC regulations aim at protecting a bank’s charter value, serving as a first line of defence against bank losses.}. They also argue that the FDIC, through regulatory interference, effectively employs a risk-based — as opposed to a flat — structure of insurance premia. Yet Buser et al. discuss the interactive process between FDIC regulation and deposit insurance, stopping short of modelling optimal FDIC response. Consequently, they offer no insights into the welfare implications of FDIC insurance/regulation and whether such an official intervention is warranted from a welfare perspective.

In an influential paper on the fair pricing of deposit insurance, Chan, Greenbaum and Thakor (1992) focus on information problems between deposit insurers and banks, showing that the mere notion of a competitive banking industry contradicts the possibility of fairly priced deposit insurance, i.e. when the deposit insurer breaks even on every insured bank. Along the lines of Buser et al. (1981), they show that deposit-linked subsidies — such as underpriced deposit insurance — are necessary in resolving moral hazard problems associated with bankers’ risk-shifting motives in the presence of information asymmetries between deposit insurers and insured banks. However, Chan et al. (1992) abstract from welfare implications of full deposit insurance which, from a LOLR perspective, is the main focus of our analysis. Yet without prejudice to information problems between banks and the official sector, those problems are assumed away in our analysis where we consider subsidised (actually free) LOLR insurance as a vehicle to influence the investment decisions of banks via first-best prudential liquidity regulation.

Sleet and Smith (2000) provide a rigorous analysis of the design of a safety net for the banking system in the presence of both full deposit insurance and a LOLR. Using a general equilibrium framework they argue that the pricing of deposit insurance is irrelevant from a welfare perspective, although the same does not hold as regards the operation of the discount window (LOLR). Sleet and Smith (2000) illustrate that, in the presence of high discount-window rates, the ex-post costs, associated with banks’ perverse incentives to shift risk to the government, offset the ex-ante benefits of inducing banks to invest prudently. But in the presence of multiple equilibria, the LOLR can be used as an \textit{equilibrium selection method}. However, Sleet and Smith (2000) consider regulation only implicitly via a lump-sum tax that is levied to depositors, allowing the official sector to run a balanced budget. That way, they illustrate that taxing deposits does not affect aggregate welfare, which can either be due to the general equilibrium framework that they employ, or due to the assumption that taxes are levied on deposits rather than, for example, on banks’ profits.

In this paper we discuss optimal liquidity regulation in conjunction with a LOLR policy by the central bank, when the banking sector faces both funding constraints and the
possibility of information-induced bank runs. Debt constraints in the model arise from depositors’ rational anticipation of bankers’ moral hazard problems which, in the spirit of Hellmann, Murdock and Stiglitz (2000), could relate to the extent of financial liberalisation of the banking sector. Also, the possibility of information-induced bank runs aims at capturing banks’ inherent fragility due to high leverage, short-term funding and asymmetric information which, in the presence of adverse economic conditions, may lead to loss of confidence to banking institutions and unanticipated foreclosures of wholesale interbank lines. In the presence of such frictions, the analysis provides a necessary and sufficient condition for prudential liquidity regulation to be socially desirable (proposition 4.1), showing that the more debt-constrained the banking sector is and the higher its profit opportunities, the more prudential liquidity regulation becomes socially desirable by augmenting the insurance value of LOLR safety net.

The remainder of the paper is organized as follows: Section 1 discusses the economic environment that we analyse. Section 2 presents the benchmark case of no liquidity regulation and section 3 solves for the optimal regulatory contract. Section 4 considers welfare implications of liquidity regulation and section 5 concludes. Proofs are included in the appendix.

1. BASIC ENVIRONMENT

We consider a model along the basic setup of Holmström and Tirole (1998, 2000) with three dates \( t = 0, 1, 2 \) and three active sectors of risk neutral agents: i) commercial bankers that receive uninsured wholesale deposits and extend loans, ii) fund-managers who manage depositors’ funds that are kept with the bank and iii) a central bank that represents the official sector acting both as a banking regulator and a LOLR.

1.1. Bankers

At \( t = 0 \) the bank can invest in a perfectly diversified portfolio of risky loans with constant returns to scale, where the total amount of investment \( I \) is a continuous variable that can be chosen freely. Bank’s investment portfolio is financed through capital \( A \) and a volume of deposits \( D \). The bank is assumed to invest on behalf of everyone in this economy by choosing an amount of investment that maximises its expected surplus, while it keeps any remaining funds in liquid assets \( l \) that pay no interest. Thus, as of \( t = 0 \) the bank faces the following budget constraint:

\[
I + l = A + D
\]

We assume that the payoff per unit of investment depends on the realisation of a productivity shock \( \phi \) that represents the proportion of loans in bank’s portfolio that succeed at \( t = 2 \), or alternatively, the probability of success of bank’s investment. In particular, \( \hat{\phi} \) is assumed uniformly distributed \( \phi \sim U(1 - \bar{\phi}, \bar{\phi}) \), where \( \frac{1}{2} \leq \bar{\phi} \leq 1 \). Also, investment payoffs are binary, where one unit of investment at \( t = 0 \) pays either \( R > 1 \) units at \( t = 2 \) with probability \( \phi \), or 0 with probability \( 1 - \phi \).

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9That is when banks face a broader range of investment opportunities and possibly less restrictions on the classes of asset they can invest in.
10See, for example, Calomiris and Gorton (1991) and Gorton (1988).
11See, for example, Rochet and Vives (2002).
As in Diamond (1984), the bank is assumed to act as delegated monitor (manager) of its loan investment whose quality is subject to moral hazard from the bank’s side between periods $t = 1$ and $t = 2$. By this is meant that loan quality is affected by an unobservable decision of bank’s managers either to manage their loans prudently or to engage in excess risk taking, in which case the probability of success of bank’s investment $\phi$ is reduced to $\beta \phi$, with $\beta > 1$. Yet excess risk taking is assumed to yield a private benefit $B$ to bank’s managers per unit of investment, but that benefit is paid out only if the bank’s investment succeeds rather than whenever bankers misbehave.

That is consistent with Kane (1989) and Cole, McKenzie and Lawrence (1995) who document that banks opt for a risky investment strategy that pays out a private benefit (bonus) if the gamble succeeds, but leaves depositors with the losses if the gamble fails. As discussed later, such moral hazard problems impose a debt constraint to the bank whose ability to raise deposits is limited by the extent of bankers’ perverse incentives to shift risks to depositors\footnote{We model this here by allowing depositors to foresee bankers’ moral hazard problems and ration their credit extension to the bank to an incentive compatible amount. Other studies, which discuss the role of official sector intervention in terms of crisis management and prevention, also allow for moral hazard problems to impose financial constraints on banks, yet from a different perspective. In Hellmann et al. (2000), for example, depositors have full deposit insurance and moral hazard problems are dealt by regulators after the deposit raising stage. Also, in Repullo (2004), uninsured depositors account for moral hazard both at the deposit-raising stage by requiring an interest rate above that of the safe asset, while at the monitoring stage, they decide whether to keep their funds with the bank on the basis of a private signal about bank’s investment return.}. We then search for conditions under which prudential liquidity regulation, in conjunction with central bank’s role as LOLR, could improve welfare by enhancing the benefits of financial intermediation.

1.2. Depositors (fund managers)

Deposits in this model are typically repaid at $t = 2$, but can also be withdrawn at $t = 1$. By this is meant that depositors are late and they do not face liquidity needs in the interim period. Thus, the emphasis here is shifted from depositors’ liquidity insurance to a situation where bank runs may occur because well, though not perfectly, informed creditors of the bank may refuse to renew their credit lines\footnote{We recognise that in a model with only late depositors, any problems arising from premature liquidation of deposits could be avoided by prohibiting such an action explicitly in the deposit contract. We could consider early depositors in the model by introducing preference shocks a la Diamond and Dybvig (1983), in which case the bank should hold an additional amount of asset as liquid reserves. For simplicity, we do not distinguish between early and late consumers.}. Moreover, to avoid undue complications in dealing with depositors’ state-contingent payoffs and incentives to undertake certain actions, we follow Rochet and Vives (2003) by assuming that the management of deposits is delegated to a continuum of intermediaries (fund managers) of total measure $D$. Those fund managers are assumed to be able to costlessly monitor the performance of bank loans and decide whether to rollover their funds with the bank, or to withdraw, on the basis of noisy private signals about the productivity shock $\phi$. In particular, at $t = 1$ fund managers receive private signals $s_i$ about the realisation of $\phi$, of the form:

$$s_i = \phi + \epsilon_i$$
where, \( \{ \tilde{e}_i \} \) are i.i.d. with \( \tilde{e}_i \sim U (-\varepsilon, +\varepsilon) \), where \( \varepsilon \) is a positive constant\(^{14}\).

By introducing information asymmetries among bank’s creditors, the analysis allows for endogenous liquidity shocks to interrelate with the general economic environment, such as the prudential liquidity and LOLR policy that the official sector has in place, as well as the extent of creditors’ heterogeneity with respect to creditors’ beliefs and incentives to undertake certain actions\(^{15}\). In particular, if a high proportion of creditors observe bad signals, then early deposit withdrawals may occur in a large scale and the bank may face liquidity problems if its liquid assets are insufficient to meet the demand for depositors’ funds. Yet what makes a signal bad in this context is not only the realisation of shock \( \phi \), but also what policies are in place by the official sector, as well as what are creditors’ beliefs about other creditors’ actions\(^{16}\). We consider the following definition of bank liquidity crisis:

**Definition 1.1.** A bank liquidity crisis occurs if bank’s stock of liquid assets at \( t = 1 \) is insufficient to meet fund managers’ demand to withdraw their deposits.

In this model, fund managers’ payoffs are assumed to depend on their actions at \( t = 1 \) (i.e. to withdraw or to rollover). In particular, we consider the following structure of state-contingent payoffs:

<table>
<thead>
<tr>
<th>Fund Managers</th>
<th>NC&amp;ND</th>
<th>C&amp;ND</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Withdraw</td>
<td>( B (1 - k) )</td>
<td>( B (1 - k) )</td>
<td>( B (1 - k) )</td>
</tr>
<tr>
<td>Rollover</td>
<td>( B )</td>
<td>( B (1 - k) )</td>
<td>0</td>
</tr>
</tbody>
</table>

NC denotes the events of no liquidity crisis and ND that of no bank default. Also, C stands for the event of liquidity crisis and D for default. Fund managers are assumed to receive a private benefit (bonus) \( B \) if the bank faces no problems (i.e. no liquidity crisis and no default). Otherwise, their bonus is reduced by a proportion \( k \in (0, 1) \) if they withdraw their deposits prematurely for whatever reason – given that they where supposed to keep them with the bank for two periods – or if they fail to foresee a liquidity crisis and shift their deposits elsewhere. In case of bank default, fund managers are assumed to lose their total bonus. Thus, in the NC&ND scenario, fund managers would have the incentive to rollover their deposits, in the C&ND case they would be indifferent between withdrawing and rolloving their deposits, while in the D case they would prefer to withdraw.

The above structure of fund managers’ payoffs aims at capturing an increased incentive to withdraw/rollover the higher the proportion of other fund managers undertaking the same action. That allows us to consider information-induced bank runs in this model using a simple global game argument, which preventing the analysis from becoming messier under a fully-fledged bank run model á la Diamond and Dybvig\(^{17}\).

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\(^{14}\)In order to preclude either negative signals, or signals greater than one, the following regularity condition is imposed: \( \varepsilon \leq \frac{1}{1 + \phi} \).

\(^{15}\)That is in contrast to the exogenous liquidity shocks that are assumed, for example, in Holmström and Tirole (1998, 2000).

\(^{16}\)That is in contrast to the exogenously specified liquidity shocks in Holmström and Tirole (1998, 2000).

\(^{17}\)Given non-empty upper and lower dominance regions and global strategic complementarities – whereby the incentive of an agent to undertake a certain action increases with the proportion of other agents undertaking the same action – the standard global game argument of Morris and Shin (2002) can be
1.3. Central bank

The central bank is assumed to have a non-profit motive, acting both as a banking regulator and a potential LOLR. In relation to LOLR policy, we conjecture a level $\phi^{**}$ of productivity shock $\phi$ such that, if the bank faces a liquidity crisis, the central bank will bail the bank out if and only if $\phi \geq \phi^{**}$. Consequently, given the interpretation of shock $\phi$, the central bank could be considered as more forbearing in accommodating adverse liquidity shocks to the bank, the lower the threshold $\phi^{**}$ that determines its LOLR policy. In that sense, ambiguity about LOLR intervention is not constructive, but it simply stems from uncertainty about the actual realisation of shock $\phi$.

As discussed later, the central bank is assumed to set its LOLR policy in a way that satisfies a two-fold objective: First, to maximise the expected surplus from financial intermediation. Second, to maintain a zero expected fiscal cost of LOLR intervention, i.e. to break even in loans extended under its LOLR facility without systematically losing money under its LOLR operations, which would inevitably require it to draw on its fiscal backup. Moreover, we assume that any LOLR loan would rank pari passu with depositors' claims and the central bank has no preferential access to collateral\(^{18}\). We also abstract from the possibility that the central bank charges penalty rates on LOLR loans\(^{19}\).

In relation to its role as banking regulator, the central bank is assumed to prescribe certain holdings of liquid assets by the bank as a proportion to its deposits. As discussed in section 3, given central bank's LOLR policy $\phi^{**}$, such a liquidity requirement aims at implementing the loan investment by the bank that is socially optimal. In that sense, one may also consider prudential liquidity regulation as an expedient for the central bank to precommit to its LOLR policy. The figure below summarises the sequence of moves:

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Fig. 1. Time line of the model.
First, the central bank precommits to a LOLR policy $\phi^{**}$ requiring the bank to maintain a certain ratio of liquid assets to deposits, while investing all remaining funds in risky loans. As discussed in section 3, such a prudential liquidity ratio turns out to be higher than in the absence of both regulatory restrictions and a LOLR facility and we find a necessary and sufficient condition for such an official intervention to be socially desirable.

Second, a productivity shock $\phi$ hits the bank’s loan portfolio and, on the basis of private signals about $\phi$, fund managers decide whether to rollover their deposits with the bank, or to withdraw. If as a result of fund managers’ withdrawals the bank faces a liquidity crisis then, if the realised productivity shock $\phi$ is greater than $\phi^{**}$, the central bank intervenes and bails-out the bank, otherwise it lets the bank to fail. If continuation occurs, either because of no liquidity crisis or due to a bail-out by the central bank, bank’s managers face a moral hazard problem due to risk-taking opportunities for a private benefit $B$. Finally, investment payoffs are realised, bank’s creditors are repaid and any residual value is assumed by the bank’s shareholders.

The above time-line of events aims at capturing the basic — yet not necessarily uncontestable — presumption that liquidity crises occur abruptly and precede solvency problems that develop more gradually, for example, due to misconduct of business by bankers (moral hazard) or, bad luck resulting from an unexpected deterioration of general economic conditions. The following section summarises a number of technical restrictions for our model parameters. Those conditions essentially allow us to focus on short/medium term prudential liquidity policy, abstracting from long-term solvency issues that may arise from bankers’ misbehavior. In this model instead, such solvency problems are taken care of by depositors who require their deposit funds to be incentive compatible. As we will see, incentive compatibility of deposits translates into a debt constraint for the bank and imposes some scarcity of funds to finance productive investments.

1.4. Basic assumptions

Given the characterisation of the bank in section 1.1, we impose a number of necessary conditions for the proper functioning of the bank in our model as a financial intermediary. Those conditions are summarised in the following assumptions:

**Assumption 1:** $R > \frac{2(2\phi - 1)}{\phi}$.

**Assumption 2:** $R - \frac{B}{\beta - 1} > 0$.

**Assumption 3:** $R - \frac{B}{\beta - 1} < 1$.

Assumption 1 implies that bank’s investment has a positive net present value\(^{20}\), i.e. the risky investment is socially optimal, and assumption 2 that bank’s investment has a higher net present value when bank’s managers behave properly rather than when they shirk\(^{21}\).

\(^{20}\)To see this, for a given $\phi^{**}$ the expected return per unit of risky investment by the bank is $E\left(\bar{R}\right) = [1 - F(\phi^{**})] \frac{1 + \phi^{**} - \phi}{2} R$. If $R \leq \frac{2(2\phi - 1)}{\phi}$, then it is easy to show that $E\left(\bar{R}\right) = 1 - \frac{\phi^{**}}{\phi} < 1$, which implies a negative net present value of bank’s investment.

\(^{21}\)If managers monitor bank’s investments properly, the expected return per unit of investment becomes: $E\left(\bar{R} \mid \text{monitoring}\right) = [1 - F(\phi^{**})] \frac{1 + \phi^{**} - \phi}{2} R$. Otherwise, if they shirk, the expected return becomes: $E\left(\bar{R} \mid \text{shirking}\right) = [1 - F(\phi^{**})] \frac{1 + \phi^{**} - \phi}{2} R + B$. It easily follow that $E\left(\bar{R} \mid \text{monitoring}\right) > E\left(\bar{R} \mid \text{shirking}\right)$ holds if and only if $R - \frac{B}{\phi - \phi^{**}} > 0$. 


Finally, assumption 3 implies that the bank is sufficiently debt constrained that it needs also capital to finance all its investment\(^{22}\).

### 1.5. Equilibrium in fund managers’ strategies

This section describes the liquidity shock that might hit the bank at \(t = 1\). Such a liquidity shock is described here in terms of the equilibrium strategy of fund managers following the realisations of their private signals \((s_i)\). That allows us to characterise the extent of the liquidity shock in terms of the realised value \(\phi\) of bank’s fundamentals, the extent of noise \(\varepsilon\) in fund managers’ information and the proportion of deposits that the bank maintains as liquid assets.

As of \(t = 0\), all model parameters, \(\phi^{**}\) and the prior distribution of \(\tilde{\phi}\) are assumed common knowledge. Moreover, the realised sample distribution of fund managers is assumed to be the common distribution of their signals \(\{s_i\}\) and fund managers decide whether to rollover their deposits, or not, on the basis of those signals. We consider the following definition of a fund manager’s strategy:

**Definition 1.2.** A fund manager’s trigger strategy \(s^*\) is a rule of action that maps the realization of her signal \(s_i\) to one of the following actions: to withdraw her deposits, or to rollover.

As of the above definition, a trigger strategy \(s^*\) can be considered as a measure of fund managers’ impatience to keep their deposits with the bank. That is, the higher the \(s^*\) the higher the fund managers’ impatience and vice versa. Let now fund managers follow trigger strategies around a critical signal level \(s^*\), i.e. they unilaterally withdraw their deposits if \(s_i < s^*\), or they rollover if \(s_i \geq s^*\). If such a \(s^*\) exists and is unique, then it would be the only dominant solvable equilibrium strategy\(^{23}\). We prove the following lemma:

**Lemma 1.1.** The critical level of productivity shock \(\phi^*\) below which a liquidity crisis occurs, is given by:

\[
\phi^* = s^* + \varepsilon \left(1 - \frac{2l}{D}\right)
\]

*Proof. See Appendix.*

We may easily observe that in the presence of liquid asset holdings by the bank the minimum level of productivity shock \(\phi^*\) that the bank could sustain without incurring a liquidity crisis is decreasing in the proportion of deposits that the bank maintains as liquid assets and increasing in the strategy that is followed by fund managers. In equilibrium, such a strategy by fund managers is given by the following lemma:

**Lemma 1.2.** Fund managers’ equilibrium in trigger strategies \(s^*\) is given by:

\[
s^* = \phi^{**} + \varepsilon \left(1 - \frac{2k}{1 - kD}\right)
\]

\(^{22}\)This point is discussed in more detail in section 3.2.

\(^{23}\)By this is meant that \(s^*\) becomes the only strategy that can survive the iterated deletion of strictly dominated strategies. See, for example, Morris and Shin (2002).
where, $\phi^{**}$ is the level of productivity shock below which the bank is not bailed out and $\frac{l}{D}$ is the ratio of liquid assets to deposits that is maintained by the bank.

Proof. See Appendix.

Generally speaking, given a fund manager’s private information, the higher the trigger strategy $s^*$ the more reluctant that fund manager becomes in rollovering his funds with the bank. As of lemma 1.2, such a reluctance increases with the extent of asymmetric information $\varepsilon$ among fund managers about fundamentals $\tilde{\phi}$. In addition, the willingness to rollover deposits with the bank increases in the penalty parameter $k$, as well as in bank’s liquidity ratio $\frac{l}{D}$ and the extent of central bank’s willingness to extend the LOLR facility in case of crisis.

In what follows we analyse the basic case without liquidity regulation and LOLR safety net. That will be subsequently used as a benchmark to judge whether or not prudential liquidity regulation, *quid pro quo* for LOLR insurance, is socially desirable. Yet we abstract from information problems that the central bank might face at the time of a potential LOLR intervention, concentrating instead on the interaction between prudential liquidity and LOLR policies and their impact on expected benefits of financial intermediation.

### 2. NO-REGULATION BENCHMARK

In this paper, central bank’s commitment to a LOLR policy is viewed as conditional on the bank maintaining a certain stock of liquid assets relative to deposits. As discussed later in section 3.3, an optimal liquidity requirement should aim at reconciling the intermediation benefit of LOLR insurance with the potential costs of LOLR intervention. Before turning to analyse such an optimal liquidity regime, we consider the basic benchmark where the bank is free to choose the amount of its liquid assets by maximising its expected surplus from loan investment. Considering LOLR insurance and liquidity regulation as a *package*, under the no-regulation benchmark the central bank would simply react to a liquidity crisis, rather than committing ex-ante to a certain LOLR policy. In that case, emergency liquidity assistance, if any, could also be extended by an efficient interbank market, where only a bank with sufficient collateral to pledge would be able to raise the funds necessary to deal with a liquidity crisis and avoid liquidation.

Having assumed that the bank holds a fixed amount of capital $A$, let $I_0$ be bank’s investment under no liquidity regulation, $l_0$ the amount of liquid assets that the bank would hold voluntarily and $D_0$ the amount of deposits that the bank would be able to attract. Let also $\phi_0$ be a threshold for the productivity shock below which the bank would be liquidated absent any official sector intervention. Such a threshold would then satisfy the following break-even condition:

$$\phi_0 RI_0 = D_0 - l_0$$

---

24 In fact, our model specification implies that any shortfall in bank’s liquid assets would also reveal to the central bank the actual realisation of $\tilde{\phi}$, because of *i.i.d.* noise terms in fund managers’ signals. However, full revelation of the productivity shock could be prevented by introducing some friction in the model, such as a random proportion of deposits that are withdrawn for non-fundamental reasons. Yet this is beyond the scope of our present analysis.
or, by substituting \((D_0 - l_0)\) from the budget constraint (1) into (2) and rearranging:

\[
\phi_0 = \frac{I_0 - A}{RI_0}
\]  

(3)

Let also \(m(I_0)\) be the marginal net expected return per unit of investment \(I_0\). Then, the expected surplus of bank’s investment would be such that \(U(I_0) = m(I_0)I_0\).

### 2.1. Voluntary liquidity holdings

In this model, both the creditors of the bank and the central bank are assumed to make no profits. Thus, the surplus of bank’s investment is equal to the bank’s net expected utility from investment \(I_0\). Also the bank’s optimisation problem can be written as follows:

\[
\max_{I_0} U(I_0)
\]

subject to:

\[
\frac{\bar{\phi} + \phi_0}{2} (RI_0 - D_0) - \frac{\bar{\phi} + \phi_0}{2\beta} (RI_0 - D_0 + BI_0) 
\]

(4a)

\[
I_0 + l_0 = A + D_0
\]

(4b)

where, (4a) is bank’s incentive compatibility condition, (4b) is bank’s budget constraint and \(\phi_0\) is given by (3). It is important to realise that the amount of deposits \(D_0\) is set before the bank’s choice of its optimal investment and is such that, taking into account the bank’s equilibrium investment \(I_0\), bankers have no incentive to take excessive risks for the private benefit \(B\). Thus, the incentive compatibility condition (4a) can be written as follows:

\[
D_0 \leq \left( R - \frac{B}{\beta - 1} \right) I_0
\]

(4a)

Moreover, as of assumption 1, bank’s investment has positive net present value and also the cost of borrowing from depositors (normalised to zero) is less than the expected net payoff per unit of investment. Thus, given risk neutrality, the bank has always an incentive to take more deposits in order to increase its investment and, as a result, the incentive compatibility condition (4a) binds:

\[
D_0 = \left( R - \frac{B}{\beta - 1} \right) I_0
\]

(4a)

We may now prove the following proposition:

**Proposition 2.1.** In a laisser-faire environment, i.e. with no regulatory requirements and a LOLR safety net, the liquidity ratio \(\frac{l_0}{D_0}\) that the bank opts to maintain is given by:

\[
\frac{l_0}{D_0} = 1 - \frac{1}{R - \frac{B}{\beta - 1}} + \frac{R\sqrt{\frac{1}{\beta^2} - C^2}}{(R - \frac{B}{\beta - 1})}
\]
where $C_2 \equiv \sqrt{\phi - \frac{2(2\phi - 1)}{R}}$.

Proof. See Appendix.

In relation to proposition 2.1, we also consider the following corollary:

**Corollary 2.1.** In the absence of prudential liquidity regulation, the bank opts to hold a positive amount of liquid assets if and only if its loans-to-deposits ratio is such that:

$$\frac{I_0}{D_0} < \frac{1}{1 - \sqrt{1 - R^2C_2^2}}$$

where $C_2 \equiv \sqrt{\phi - \frac{2(2\phi - 1)}{R}}$.

Proof. It follows immediately from the incentive compatibility condition (4a) and proposition 2.1 by setting $l_0 > 0$.

It is worth emphasising that any voluntary liquidity holdings in this model can be considered as *spare liquidity* in the presence of bank capital $A$ and an optimal amount of loan investment $I_0$ by the bank. Thus, a stock of liquidity in our *laisser-faire* case plays solely a residual role, rather than serving a deeper economic purpose. Nevertheless, deriving explicitly those stocks is important in order to offer a consistent benchmark for comparison with the case where prudential liquidity and LOLR policies interact, which is discussed in the following section.

### 3. REGULATORY CONTRACT

For the purposes of our analysis, the optimal regulatory contract provides for two things: First, an optimal LOLR policy by the central bank. Second, an optimal liquidity ratio that the bank is required to maintain. An LOLR policy stipulates an intervention threshold $\phi^{**}$ such that the expected surplus of bank’s investment is maximised under the following constraints: i) The amount of deposits $D$ that the bank is able to attract is incentive compatible, i.e. bankers are given appropriate incentives not to engage in excess risk-taking in the expense of depositors. ii) The central bank expects to break even under the LOLR facility, i.e. not to make systematic losses under the LOLR facility. iii) The budget constraint (1) of the bank is satisfied.

As in the no-regulation benchmark, bank’s creditors are assumed to make no profits. Thus, the surplus of bank’s investment is equal to bank’s net expected surplus from loan investment $I$. Let $m(\phi^{**})$ be the marginal net expected return per unit of investment. Then the expected surplus $U$ of bank’s investment is $U(\phi^{**}) = m(\phi^{**}) I(\phi^{**})$ and the central bank’s optimisation problem can be written as follows:

$$\max_{\phi^{**}} U(\phi^{**})$$

(5)
subject to:

\[
\frac{\theta + \phi^{**}}{2}(RI - D) \geq \frac{\theta + \phi^{**}}{2\beta}(RI - D + BI) \tag{5a}
\]

\[
\frac{\phi^{*} + \phi^{**}}{2}RI \geq D - l \tag{5b}
\]

\[
I + l = A + D \tag{5c}
\]

where (5a) is bank’s incentive compatibility condition, (5b) is central bank’s break-even condition and (5c) is bank’s budget constraint. Also, \(l\) denotes the amount of bank’s liquid asset holdings, while \(A\) the amount of bank’s capital.

However, in order to determine central bank’s optimal LOLR policy, we firstly need to establish what is the optimal amount of bank investment \(I(\phi^{**})\) for a given LOLR policy \(\phi^{**}\).

### 3.1. Optimal investment

As with the no-regulation benchmark, the amount of deposits \(D\) is set before the bank’s choice of its optimal investment, and is such that bank’s managers have no incentive to gamble for the private benefit \(B\). Thus, taking into account bank’s equilibrium investment \(I\), the incentive compatibility condition (5a) is binding and can be written as follows:

\[
D = \left(R - \frac{B}{\beta - 1}\right)I \tag{6}
\]

where we note that the ratio of loans to deposits is fixed and equal to \(\left(R - \frac{B}{\beta - 1}\right)^{-1}\) regardless of the choice of investment amount \(I\).

Given also positive returns to investment and that the central bank makes no profits, the break even condition (5b) of the central bank also binds and can be written as follows:

\[
\frac{\phi^{*} + \phi^{**}}{2}RI = (D - l) \tag{5b}
\]

Central bank’s break even condition (5b) implies that, conditional on LOLR intervention, the shareholders of the bank are expected to receive just nothing. Whether they will eventually receive something, i.e. \(RI - D - l\), will depend on the actual realisation of productivity shock \(\phi\). Nevertheless, a LOLR policy \(\phi^{**}\) that is consistent with a zero expected cost of the LOLR facility must be such that, conditional on LOLR intervention, shareholders are not expected to receive anything. We are now ready to prove the following proposition:

**Proposition 3.1.** For a given LOLR policy \(\phi^{**}\), the amount of investment \(I(\phi^{**})\) that satisfies simultaneously bank’s incentive compatibility condition (6), central bank’s break even condition (5b) and bank’s budget constraint (5c), is given by:

\[
I(\phi^{**}) = A \left(\frac{\frac{1}{\pi} - \frac{\alpha}{\pi - \frac{\phi^{**}}{\phi^{**}}}}{C_1 - \phi^{**}}\right) \tag{7}
\]
where \( C_1 \equiv \frac{1}{R} - \varepsilon \frac{a(1-a)(R-\frac{R}{\pi\tau})}{(R-\frac{R}{\pi\tau})} \) and \( a \equiv \frac{1}{1-k} \).

Proof. See Appendix.

Having calculated bank’s optimal loan investment for given LOLR policy \( \phi^{**} \), we can now turn to evaluate the optimal LOLR policy \( \phi^{**} \) that maximises the expected surplus from bank’s loan investment.

### 3.2. Optimal LOLR policy

Proposition 3.1 provides the optimal investment \( I(\phi^{**}) \) by the bank for a given LOLR policy \( \phi^{**} \). With \( I(\phi^{**}) \) in hand, the optimal LOLR policy maximises the expected surplus from bank’s investment. We prove the following proposition:

PROPOSITION 3.2. Central bank’s optimal LOLR policy is to bail out the bank if and only if the level of productivity shock \( \phi \) is such that \( \phi \geq \phi^{**} \), where:

\[
\phi^{**} = C_1 - \sqrt{C_1^2 - C_2^2}
\]

where \( C_1 \equiv \frac{1}{R} - \varepsilon \frac{a(1-a)(R-\frac{R}{\pi\tau})}{(R-\frac{R}{\pi\tau})} \), \( C_2 \equiv \sqrt{\phi^2 - \frac{2(2\varepsilon-1)}{R}} \) and \( a \equiv \frac{1}{1-k} \).

Proof. See Appendix.

It is worth noting that central bank’s optimal LOLR policy \( \phi^{**} \) may be considered in terms of the induced probability of liquidity crisis at the optimum. In fact, lemmas 1.1 and 1.2 imply a one-to-one mapping from central bank’s optimal intervention threshold \( \phi^{**} \) to the critical level of productivity shock \( \phi^* \) below which a liquidity crisis occurs. Consequently, by choosing an LOLR policy \( \phi^{**} \), the central bank implicitly induces a certain probability of liquidity crisis in equilibrium. That is consistent with Allen and Gale (1998) and the idea of optimal financial crisis, which could induce banks to hold efficient portfolios of risky assets. Moreover, the provision of optimal partial LOLR insurance, conditional on banks conforming with costly liquidity regulation, is consistent with the principle of proportionality which is widely used in political debate about the extent and intensity of action by the official sector\(^{25}\).

We may now show that there exists a threshold \( \phi_U \) of the productivity shock such that if \( \phi \in [\phi_U, \bar{\phi}] \), all fund managers decide to rollover their deposits with the bank. The range \( [\phi_U, \bar{\phi}] \) is referred as the upper dominance region and corresponds to realisations of fundamentals high enough to prevent any information-induced outflow of funds from the bank. There is also a level of productivity shock \( \phi_L \) such that if \( \phi \in [1 - \bar{\phi}, \phi_L] \), all fund managers decide to withdraw their funds. The range \( [1 - \bar{\phi}, \phi_L] \) is referred as the lower dominance region, corresponding to very weak realisations of fundamentals that induce a

\(^{25}\) Under the proportionality principle, the content and form of actions by the official sector shall not exceed what is necessary to achieve its policy objective. In our case, such an official sector objective is the maximization of the surplus from financial intermediation, conditional on avoiding systematic losses under the LOLR facility.
massive outflow of funds from the bank. In relation to \( \phi_U \) and \( \phi_L \), we derive the following result.

**Corollary 3.1.** The upper and lower dominance region for productivity shock \( \phi \) are defined by the following thresholds \( \phi_U \) and \( \phi_L \), respectively:

\[
\begin{align*}
\phi_U &= \phi^{**} + 2\varepsilon \left( 1 - \frac{k}{1 - k \frac{l}{D}} \right) \\
\phi_L &= \phi^{**} - 2\varepsilon \frac{k}{1 - k \frac{l}{D}}
\end{align*}
\]

where \( \phi^{**} \) is given by proposition 3.2 and defines central bank’s LOLR policy, while \( \frac{l}{D} \) is bank’s ratio of liquid assets to deposits that we solve for in the following section.

**Proof.** It follows from lemmas 1.1 and 1.2 and the fact that \( \text{Pr}(s_i \leq s^* | \phi_U) = 0 \), while \( \text{Pr}(s_i \leq s^* | \phi_L) = 1 \). \( \blacksquare \)

Consequently, depending on the realisation of productivity shock \( \phi \), we may summarise agents actions in the following event line.

![Event line depending on realisation of \( \phi \)](image)

**3.3. Optimal liquidity regulation**

Having established central bank’s optimal response to a liquidity crisis and the commensurate optimal investment \( I(\phi^{**}) \) by the bank, that level of investment can be implemented by regulating the level of bank’s liquid assets. We prove the following proposition:

**Proposition 3.3.** Under the optimal regulatory contract, the bank is required to maintain a liquid-assets-to-deposits ratio such that:

\[
\frac{l}{D} = 1 - \frac{1}{R - \frac{B}{\beta - 1}} + \frac{R\sqrt{C_1^2 - C_2^2}}{R - \frac{B}{\beta - 1} - aR\varepsilon}
\]

where \( C_1 \equiv \frac{l}{R - \varepsilon(1 - a)(R - \frac{B}{\beta - 1})} \), \( C_2 \equiv \sqrt{\frac{2}{R^2} - \frac{2(2\varepsilon - 1)}{R}} \) and \( a \equiv \frac{1}{1 - R} \).

**Proof.** See Appendix. \( \blacksquare \)
From proposition 3.3, we can easily show that if, in general, fund managers’ incentive
to keep their deposits with the bank is sufficiently high — i.e. the penalty parameter
is sufficiently high — then the prudential liquidity ratio increases with 

\[
\frac{1}{D}
\]

namely with the extent of asymmetric information about bank’s fundamentals. That implies that the
higher the extent of transparency about bank’s fundamentals — i.e. the lower the 

\[
\epsilon
\]

— the lower the prudential liquidity ratio that the bank would be required to maintain. Also,
from propositions 2.1 and 3.3 it is easy to observe that, as the noise in fund managers’
signals decreases towards zero, the prudential liquidity ratio converges towards the level
that the bank would choose to maintain voluntarily absent any official sector intervention.
Yet for \( \epsilon > 0 \) we prove the following result:

**Proposition 3.4.** *In the absence of official sector intervention, both in terms of pru-
*  

dential liquidity requirements and LOLR insurance, the ratio of liquid assets to deposits
that the bank would opt to maintain would be lower than under the optimal regulatory
* contract.*

**Proof.** See Appendix. ■

We now turn to analyse under what circumstance regulating a bank’s holdings liquid
assets is, if at all, welfare improving of the no-regulation benchmark.

**4. WELFARE IMPLICATIONS**

From section 3, the expected surplus \( U \) of bank’s investment under the optimal regula-
tory contract \( (\phi^{**}, \frac{1}{D}) \) is given by

\[
U = \frac{A}{2(2\phi^{**} - 1)} \left( C_{2} - \phi^{**} \right) \left( 1 - \frac{aR}{R - \frac{B}{\beta - 1}} \right)
\]

where by substituting the optimal LOLR policy \( \phi^{**} \) from proposition 3.2

\[
U = \frac{A}{2\phi^{**} - 1} \left( C_{1} - \sqrt{C_{1}^{2} - C_{2}^{2}} \right) \left( 1 - \frac{aR}{R - \frac{B}{\beta - 1}} \right)
\]

(8)

where

\[
C_{1} = \frac{1}{R} - \varepsilon \frac{a + (1 - a)(R - \frac{B}{\beta - 1})}{H - \frac{B}{\beta - 1}}, \quad C_{2} = \sqrt{\frac{a}{\phi^{**}} - \frac{2(a^{2} - 1)}{H}} \quad \text{and} \quad a \equiv \frac{1}{1 - \xi}.
\]

Similarly, from section 2, the expected surplus \( U_{0} \) of bank’s investment under no liquidity
regulation is given by

\[
U_{0} = \frac{A}{2\phi^{**} - 1} \left[ \frac{1}{R} - \sqrt{\frac{1}{R^{2} - C_{2}^{2}}} \right]
\]

(9)

Clearly, from (8) and (9) follows that asymmetric information plays a pivotal role in the
analysis of welfare implications of liquidity regulation. We observe that as the noise in fund
managers’ signals tends to zero, i.e. \( \epsilon \to 0 \), the expected surplus of bank’s investment under
the optimal regulatory contract becomes equal to the no-regulation benchmark. That
is, when asymmetric information among fund managers dissipates, regulating a bank’s
holdings of liquid assets leads to no welfare improvement/deterioration of the no regulation benchmark.

However, in the presence of asymmetric information, i.e. for $\varepsilon > 0$, liquidity regulation may or may not lead to a welfare improvement of the no-regulation case. As we show next, whether or not liquidity regulation is warranted depends on the bank’s loans-to-deposits ratio. Figure 3, in particular, plots the expected surplus of bank’s investment under the optimal regulatory contract for two different levels of loans-to-deposits ratio ($\frac{L}{D}$), i.e. for 1.4 and 1.95, for different levels of $\varepsilon$. The horizontal line between the two curves corresponds to the expected surplus from bank’s investment under the no-regulation benchmark.

Fig. 3. Welfare implications of liquidity regulation

As of figure 3, one would expect prudential liquidity regulation, combined with an appropriate LOLR policy, to be more socially desirable the more debt-constraint the banking sector is, i.e. the higher the ratio of loans to deposits. But if the debt constraint problems of the banking sector are not substantial then prudential liquidity may turn out to be too costly even after taking into account the insurance value of the LOLR. In the following proposition we derive a threshold for the loans-to-deposits ratio above which liquidity regulation leads to welfare improvement of the no-regulation case:

**Proposition 4.1.** For small values of $\varepsilon$, a necessary and sufficient condition for liquidity regulation to be welfare improving of the no-regulation benchmark is bank’s loans-to-deposits ratio to be such that:

$$\frac{I}{D} > \frac{a - 1}{a \left(1 - \sqrt{1 - R^2 C_2^2}\right)}$$

(10)

where $C_2 \equiv \sqrt{\frac{2}{1 - \phi} - \frac{2(2\phi - 1)}{\bar{R}}}$ and $a \equiv \frac{1}{1 - \varepsilon}$.

**Proof.** See Appendix. \[\Box\]
The main implication of this result is that, since moral hazard problems limit the ability of the bank to raise deposit funds above a certain proportion of its loan investment\textsuperscript{26}, the more pronounced those problems are the more the LOLR insurance could be essential to realise the benefits of financial intermediation. This is because, by offering LOLR insurance against states of the world that the bank faces a liquidity shortage, the central bank increases the bank’s marginal expected return from investment and, thus, bank’s willingness to employ more funds in productive assets\textsuperscript{27}. That also becomes evident from the fact that the RHS of (10) is decreasing in $R$, which implies that inequality (10) is more likely to hold – i.e. liquidity regulation is more likely to be welfare improving of the no-regulation benchmark – the higher the potential return $R$ per unit of investment.

However, an unconditional extension of a safety net by the central bank would lead to zero reserve-holdings by the bank, which would then choose to take on excessive risks by investing all available funds in risky loans up to the maximum amount that is implied by the budget constraint\textsuperscript{28}, i.e. $I = \frac{A - (R - B)}{\beta}$. Thus, in exchange of LOLR insurance, the bank has to meet a regulatory requirement regarding its liquid-assets-to-deposits ratio. Such a reserve requirement turned out to be higher than what the bank would maintain voluntarily in the absence of LOLR\textsuperscript{29} and is important to emphasise that it is not to guarantee the existence of a stock of high-quality assets against which central banks can lend in a crisis\textsuperscript{30}. It is rather to enable the bank to meet liquidity shocks from its own resources and to a certain level of confidence, allowing the central bank to satisfy its ex ante clear-exit objective, while counteracting any excessive risk-taking due to moral hazard that is created by the existence of LOLR. Having said that, one then might ask what are the actual policy implications of proposition 4.1. In other word, what does this result imply about the economic conditions that could possibly justify the regulation of bank liquidity.

Considering an economic environment with similar moral hard problems from the bank’s side as the one we consider here, Hellmann et al. (2000) argue that financial liberalisation grants more freedom to banks in determining their lending portfolios and may result in banks facing more pronounced moral hazard problems due to more gambling opportunities. Then, Hellmann et al. turn to suggest that the introduction of financial liberalisation could lead to more conservative regulatory standards, in particular higher regulatory capital requirements.

Although gambling opportunities in the Hellmann et al. paper are treated ex-post by regulators, through capital requirements and deposit-rate controls, rather than ex-ante by depositors, in either case moral hazard increases the scarcity/cost of funding to the

\textsuperscript{26}See incentive compatibility condition (6).

\textsuperscript{27}From (21) we may observe that, in the absence of a LOLR, the expected marginal return from investment is decreasing in the total amount of investment and, as a result, the willingness of the bank to invest more funds in risky loans also decreases.

\textsuperscript{28}That is because the marginal expected return from loan investment would not depend anymore on the total loan amount and, thus, bank’s expected surplus would be strictly increasing in the total amount lent.

\textsuperscript{29}See proposition 3.4.

\textsuperscript{30}In practice, the bank may have expended all its liquid assets raising funds in the market, in the hope of meeting a liquidity shock by itself, before approaching the central bank for LOLR support. That could be on the assumption that the latter is the only counterparty that will be prepared to lend against lower quality collateral on non-punitive terms and without alerting the market to the bank’s difficulties.
bank. As a result, we believe that the economic implications of moral hazard under both formulations are similar and the argument by Hellmann et al., regarding the impact of financial liberalisation on regulatory requirements, also relevant to our analysis. In fact, we have already seen that the more pronounced the moral hazard problems are, i.e. the higher the ratio of bank loans-to-deposits, the more likely it is for the inequality (10) to hold. That would imply that an optimal liquidity regulation, coupled with an LOLR policy, would possibly have more positive welfare implications in a liberalised financial system than otherwise.

Moreover, our analysis can accommodate a more positive view toward financial liberalisation, whereby it allows banks to diversify and add to their revenues\textsuperscript{31}. In that case, the potential return $R$ per unit of investment would be expected to increase, making inequality (10) more likely to hold and, again, liquidity regulation more likely to be welfare improving of the no-regulation benchmark. Thus, on the basis of our model, one would expect liquidity regulation, coupled by an appropriate bail-out policy, to be more appropriate for banking systems characterised by a broader range of investment opportunities and less restrictions in the asset-class they are able to hold. In the following section we provide a more formal exposition of the basic trade-off between LOLR insurance benefit and the cost of liquidity regulation that gives rise to proposition 4.1.

Finally, it is worth noting that the RHS of 10 is increasing in parameter $k$, which determines the penalty that deposit managers face for premature withdrawal of their deposits with the bank. That implies that the more stable the deposit base is – i.e. the higher the penalty parameter $k$ – the more difficult it is for inequality 10 to be satisfied and, as a result, for prudential liquidity regulation to be welfare improving.

\subsection*{4.1. Regulatory cost vs. insurance benefit}

From lemmas 1.1 and 1.2, the distance between the critical level of productivity shock $\phi^*$, below which a liquidity crisis arises, and the level of shock $\phi^{**}$, below which the central bank does not bailout the bank, can easily be expressed as follows:

\begin{equation}
(\phi^* - \phi^{**}) = 2\varepsilon \left( 1 - \frac{l}{1 - k D} \right)
\end{equation}

Equation (11) implies that, conditional on LOLR intervention, the support of productivity shock $\hat{\phi}$ is increasing in $\varepsilon$, i.e. the extent of noise in fund managers’ signals, and decreasing in the bank’s liquidity ratio $\frac{l}{D}$. Moreover, for a given realisation of $\hat{\phi}$ in the interval $[\phi^*, \phi^{**}]$ and investment $I$ by the bank, the expected recovery rate of a LOLR loan is

$$
\text{Expected recovery rate} = \min \left[ \frac{\phi I R}{D - I}, 1 \right]
$$

or,

$$
\text{Expected recovery rate} = \min \left[ \frac{\frac{I R \phi}{D - \sqrt{\phi}}}{1 - \frac{\phi}{\sqrt{\phi}}}, 1 \right]
$$

\textsuperscript{31}See, for example, Ibanez and Molyneux (2004) who examine structural and performance features of European banking.
Consequently, the minimum level $\phi_e$ of productivity shock below which the central bank expects to recover less than the full amount of a LOLR loan is

$$\phi_e = \frac{1 - lD}{lDR}$$

(13)

We note that $\phi_e$ in (13) is decreasing in both the bank’s liquidity ratio and the ratio of loans to deposits. Not surprisingly, given that the break-even condition (5b) of the central bank is binding, $\phi_e$ must be equal to the expected productivity shock conditional on LOLR intervention, i.e. $\phi_e = \frac{\phi^* + \phi^{**}}{2}$. That is shown in figure 4 and can easily be verified by comparing (5b) with (13).

![Fig. 4. Expected recovery rate of LOLR loan.](image_url)

Let us now suppose that for a given balance-sheet structure of the bank and LOLR policy $\phi^{**}$, $\phi_e$ was higher than $\frac{\phi^* + \phi^{**}}{2}$. Then central bank’s break-even condition (5b) would be violated and, as of $t = 0$, the central bank would be unable to insure bank’s liquidity shocks to a level as low as $\phi^{**}$. Yet, given that $\phi_e$ is decreasing in $lD$, the central bank could provide such an insurance by increasing the reserve requirement to the bank up to the point where $\phi_e$ would become equal to the expected productivity shock conditional on LOLR intervention, i.e. $\phi_e = \frac{\phi^* + \phi^{**}}{2}$.

Thus, by increasing the liquidity requirement to the bank, the central bank would be able to provide LOLR insurance against a wider range of productivity shocks that might hit the bank’s loan portfolio. That, in turn, would result in an increase in bank’s expected profits, but on the downside, would also increase the regulatory burden to the bank by requiring it to hold costly liquid assets in excess of what it would opt to maintain in a laissez-faire regime, i.e. in the absence of official sector intervention. In equilibrium, such a trade-off could result in adverse welfare implications of liquidity regulation, with the regulatory costs from holding liquid assets outweighing the insurance benefits from the LOLR.

However, $\phi_e$ is not only decreasing in $lD$, but also in $lD$. Thus, ceteris paribus, a bank with a higher loans-to-deposits ratio would need to hold a lower proportion of its deposits as liquid assets – in order for the central bank’s break even condition (5b) to be satisfied – compared to a bank with a lower ratio of loans-to-deposits. Consequently, there must also be a threshold for the loans-to-deposits ratio above which the insurance benefits brought by
the LOLR outweigh the regulatory costs from holding costly liquid assets. Such a threshold was explicitly derived in proposition 4.1. Finally, for $\varepsilon \to 0$ and from (11), the analysis collapses to the simple case where $\phi^* = \phi^{**}$ and, in terms of welfare implications, there is no distinction in our model between the cases with and without liquidity regulation.

5. CONCLUDING REMARKS

In the spirit of Posner (1971), we regarded prudential liquidity regulation for banks as quid pro quo for emergency liquidity assistance by the central bank, where the regulatory cost of holding liquid assets was considered as an implicit insurance premium for (partial) LOLR insurance. In the presence of bank funding constraints, asymmetric information among bank’s creditors about the quality of bank’s loan portfolio turned out to be key in describing prudential liquidity policy. It was shown that the more diverse creditors’ beliefs are about bank’s fundamentals the lower the prudential liquidity ratio that the bank would be required to maintain. However, as asymmetric information about bank’s fundamentals dissipates, the prudential liquidity ratio converges towards bank’s voluntarily holdings of liquidity in the absence of any official sector intervention.

It was demonstrated that optimal liquidity regulation implies a ratio of liquid assets to deposits that is higher than what the bank would voluntarily maintain in the absence of LOLR (proposition 3.4). As emphasised, such a higher ratio is not to guarantee the existence of a stock of high-quality assets against which central banks can lend in a crisis, but it is rather to enable the bank to meet liquidity shocks from its own resources and to a certain level of confidence. In other words, we viewed prudential liquidity regulation serving as a first line of defence against bank liquidity problems that allows the central bank to maintain a zero expected cost of LOLR intervention, while counteracting any excessive risk-taking due to LOLR safety net.

We also derived a necessary and sufficient condition for liquidity regulation to be welfare improving of the no-regulation case, showing that this is the case if and only if the bank’s total loans-to-deposits ratio is above a certain threshold that was explicitly evaluated (proposition 4.1). Otherwise, liquidity regulation is too costly from a welfare perspective, even after taking into account the social value of LOLR insurance. Along the lines of Hellmann et al. (2000), we argued that in a liberalised banking system, namely in an environment where banks face a broad range of investment opportunities and limited restrictions on the classes of assets they can invest in, it is more likely for prudential liquidity to have positive welfare implications. That is because, prudential liquidity requirements augment the social value of LOLR insurance over and above the costs that such requirements would imply for the banking sector. Finally, it was shown that the more stable a bank’s deposit base is – i.e. the lower the prima facie incentive of bank’s creditors to foreclose their exposures to the bank – the more unlikely it is for prudential liquidity regulation to be welfare improving.

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REFERENCES


6. APPENDIX

Proof of Lemma 1.1
Conditional on productivity shock \( \phi \), fund managers’ signals \( \{s_i\} \) are i.i.d. uniform random variables with support on \( [\phi - \varepsilon, \phi + \varepsilon] \). Given \( \phi \) and fund managers’ equilibrium trigger strategies \( s^* \), the proportion of them who withdraw their deposits is equal to:

\[
\Pr (s_i \leq s^*| \phi) = \frac{s^* - \phi + \varepsilon}{2\varepsilon}
\]  

(14)

From (14), the critical level of shock \( \phi^* \) solves the following expression:

\[
\frac{s^* - \phi^* + \varepsilon}{2\varepsilon} D = l
\]

or

\[
\phi^* = s^* + \varepsilon \left(1 - 2 \frac{l}{D}\right)
\]

(15)

Q.E.D.

Proof of Lemma 1.2
Conditional on observing signal \( s_i = s^* \), let \( P_{00} \) be the conditional probabilities of no liquidity crisis at \( t = 1 \) and no bank default at \( t = 2 \) conditional on signal \( s^* \), i.e. \( P_{00} \equiv \Pr (NC&ND|s^*) \). Similarly, let \( P_{10} \) be the probabilities of liquidity crisis at \( t = 1 \) and no default at \( t = 2 \) conditional on signal \( s_i = s^* \), i.e. \( P_{10} \equiv \Pr (C&ND|s^*) \). Probabilities \( P_{00} \) and \( P_{10} \) are calculated as follows:

\[
P_{00} = \Pr \left( \hat{\phi} > \phi^* \mid s^* \right)
\]

or

\[
P_{00} = \frac{s^* + \varepsilon - \phi^*}{2\varepsilon}
\]

(16)

By substituting \( \phi^* \) from lemma 1.1 into (16) we get:

\[
P_{00} = \frac{l}{D}
\]

(17)

where \( l \) is the level of liquid assets that are held by the bank and \( D \) is the volume of bank deposits. Similarly, given \( \phi^* \) and \( \phi^{**} \), \( P_{10} \) is given by:

\[
P_{10} = \Pr \left( \phi^{**} < \hat{\phi} < \phi^* \mid s^* \right)
\]

or

\[
P_{10} = \frac{\phi^* - \phi^{**}}{2\varepsilon}
\]

(18)

With \( P_{00} \) and \( P_{10} \) in hand, fund managers’ equilibrium in trigger strategies \( s^* \) is such that, given the payoff structure in Table I, a fund manager by observing a signal \( s_i = s^* \)
is indifferent between withdrawing or rollovering. That is, \( s^* \) ought to solve the following equation:

\[
P_{00}B + P_{10}B (1 - k) = B (1 - k)
\]  

(19)

where \( P_{00} \) and \( P_{10} \) are given by equations (17) and (18).

The LHS of (19) is the expected payoff from rollovering, conditional on \( s^* \), while the RHS is the (certain) payoff from withdrawing. By substituting \( P_{00} \) and \( P_{10} \) from (17), (18) into (19), we get the following expression:

\[
\frac{l}{D}B + \frac{\phi^* - \phi^{**}}{2\varepsilon}B (1 - k) = B (1 - k)
\]  

(20)

By substituting \( \phi^* \) from lemma 1.1 into (20), \( s^* \) is given by:

\[
s^* = \phi^{**} + \varepsilon \left( 1 - 2 \frac{k}{1 - k} \frac{l}{D} \right)
\]

Q.E.D.

**Proof of Proposition 2.1**

Equation (3) implies that the net expected return \( m (I_0) \) per unit of investment under no regulation is given by:

\[
m (I_0) = \frac{R}{2 (2\phi - 1)} \left[ C_2^2 - \left( \frac{I_0 - A}{RI_0} \right)^2 \right]
\]  

(21)

where \( C_2 \equiv \sqrt{\phi - \frac{2(2\phi - 1)}{R}} \). Then, the bank’s expected utility from investment \( I_0 \) is given by:

\[
U_0 = m (I_0) I_0
\]

or, from (21),

\[
U_0 = \frac{R}{2 (2\phi - 1)} \left[ C_2^2 - \left( \frac{I_0 - A}{RI_0} \right)^2 \right] I_0
\]  

(22)

Assuming that \( R < \frac{1}{\sqrt{2}} \), we can easily show that the amount of investment \( I_0 \) that maximises \( U_0 \) in (22) is given by:

\[
I_0 = \frac{A}{R \sqrt{\frac{1}{R} - C_2^2}}
\]  

(23)

From (1), (4a) and (23), the amount of liquid assets \( l_0 \) that the bank would hold under no regulation is given by:

\[
l_0 = \left[ 1 - \frac{1}{(R - \frac{B}{\beta - 1})} + \frac{R \sqrt{\frac{1}{R} - C_2^2}}{(R - \frac{B}{\beta - 1})} \right] \left( R - \frac{B}{\beta - 1} \right) I_0
\]  

(24)
Finally, from (4a) and (24), the liquid—assets-to-deposits ratio under no-regulation is given by:

\[ \frac{l_0}{D_0} = 1 - \frac{1}{R - \frac{\beta}{\beta - 1}} + \frac{R \sqrt{\frac{1}{R^2} - C^2_2}}{\left(R - \frac{\beta}{\beta - 1}\right)} \]  

(25)

Q.E.D.

Proof of Proposition 3.1

By substituting (5c) into (5b) we get:

\[ \frac{\phi^* + \phi^{**}}{2} RI = I - A \]  

(26)

where from lemmas 1.1 and 1.2, the term \( \frac{\phi^* + \phi^{**}}{2} \) on the LHS of (26) can be written as:

\[ \frac{\phi^* + \phi^{**}}{2} = \phi^{**} + \varepsilon \left(1 - \frac{1}{1 - kD}\right) \]  

(27)

Thus, by substituting (27) into (26) and setting \( a \equiv \frac{1}{1 - \varepsilon} \), the central bank's break-even condition becomes:

\[ \left[\phi^{**} + \varepsilon \left(1 - a \frac{l}{D}\right)\right] RI = I - A \]  

(28)

and by substituting \( l \) from (5c) into (28):

\[ \left[\phi^{**} + \varepsilon \frac{aI + (1 - a)D - aA}{D}\right] RI = I - A \]  

(29)

By substituting (6) into (29) we get:

\[ (C_1 - \phi^{**}) I = A \left[\frac{1}{R} - \frac{\alpha \varepsilon}{R - \frac{\beta}{\beta - 1}}\right] \]

where \( C_1 \equiv \frac{1}{R} - \varepsilon \frac{a + (1 - a)(R - \frac{\beta}{\beta - 1})}{(R - \frac{\beta}{\beta - 1})} \) and \( a \equiv \frac{1}{1 - \varepsilon} \). Consequently, for a given \( \phi^{**} \), the amount of investment \( I (\phi^{**}) \) that satisfies simultaneously constraints (5a), (5b) and (5c) is given by:

\[ I (\phi^{**}) = \frac{A \left(\frac{1}{R} - \frac{\alpha \varepsilon}{R - \frac{\beta}{\beta - 1}}\right)}{(C_1 - \phi^{**})} \]  

(30)

Q.E.D.

Proof of Proposition 3.2

For a given LOLR policy \( \phi^{**} \), bank’s net expected return per unit of investment is given by:
$m(\phi^{**}) = (1 - F(\phi^{**})) \bar{\phi} + \phi^{**} R - 1$

or

$m(\phi^{**}) = \frac{R}{2(2\bar{\phi} - 1)} (C_2^2 - \phi^{**^2})$  \hspace{1cm} (31)

where $C_2 \equiv \sqrt{\bar{\phi} - \frac{2(2\bar{\phi} - 1)}{R}}$. From (31) we note that the net expected return per unit of investment is positive if and only if $\phi^{**} \leq C_2$. For such a $\phi^{**}$, central bank's optimisation problem (5) can be expressed in terms of the following unconstrained problem:

$max_{\phi^{**}} U(\phi^{**})$

where $U(\phi^{**}) = m(\phi^{**}) I(\phi^{**})$ is given by proposition 3.1 and $m(\phi^{**})$ is the net expected return per unit of investment. Given (31), the central bank's optimisation problem can be restated as:

$max_{\phi^{**}} \frac{A}{2(2\bar{\phi} - 1)} \left( \frac{C_2^2 - \phi^{**^2}}{(C_1 - \phi^{**})} \left( 1 - \frac{aR}{R - \frac{B}{\bar{\phi} - 1}} \right) \right)$

where $C_1 \equiv \frac{1}{R} - \frac{a}{R - \frac{B}{\bar{\phi} - 1}}$, $C_2 \equiv \sqrt{\bar{\phi} - \frac{2(2\bar{\phi} - 1)}{R}}$ and $a \equiv \frac{1}{1-\epsilon}$. That is equivalent to maximise the following expression of $\phi^{**}$:

$f(\phi^{**}) = C_1 + \phi^{**} - \frac{C_2^2 - C_1^2}{\phi^{**^2} - C_1}$  \hspace{1cm} (32)

The first derivative of $f(\cdot)$ with respect to $\phi^{**}$ is:

$\frac{\partial f(\phi^{**})}{\partial \phi^{**}} = 1 + \frac{C_2^2 - C_1^2}{(C_1 - \phi^{**^2})^2}$  \hspace{1cm} (33)

Also the second derivative of $f(\cdot)$ with respect to $\phi^{**^2}$ is:

$\frac{\partial^2 f(\phi^{**})}{\partial \phi^{**^2}} = 2 \frac{C_2^2 - C_1^2}{(C_1 - \phi^{**^2})^3}$  \hspace{1cm} (34)

We consider two cases: i) $C_1 \leq C_2$ and ii) for $C_1 > C_2$. However, we can easily show that inequality $C_1 \leq C_2$ holds if and only if the bank’s loans-to-deposits ratio is such that $\frac{1}{R} \geq \frac{1-(C_2 - \epsilon) R}{2R}$, which for small values of $\epsilon$ implies a high value of $\frac{1}{R}$. Nevertheless, given that the loans-to-deposits ratio under both the regulation and the no-regulation case is equal to $\left( R - \frac{B}{\bar{\phi} - 1} \right)^{-1}$, the case where $C_1 \leq C_2$ can easily be ruled out from corollary 2.1. Thus the only relevant case here to consider is that for $C_1 > C_2$, under which (34) becomes negative and the optimal LOLR policy $\phi^{**}$ is given by:

$\phi^{**} = C_1 - \sqrt{C_1^2 - C_2^2}$  \hspace{1cm} (35)
Q.E.D. 

Proof of Proposition 3.3

From bank’s budget constraint (1), the liquidity ratio \( \frac{l}{D} \) can be written as follows:

\[
\frac{l}{D} = \frac{D + A - I}{D} \tag{36}
\]

By substituting (6) and (30) into (36) we derive the following expression for the liquid-assets-to-deposits ratio:

\[
\frac{l}{D} = 1 - \frac{1}{R - \frac{B}{\beta - 1}} + \frac{R(C_1 - \phi^{**})}{R - \frac{B}{\beta - 1} - aR\varepsilon} \tag{37}
\]

where \( C_1 \equiv \frac{1}{R} - \varepsilon \frac{a + (1 - a)(R - \frac{B}{\beta - 1})}{(R - \frac{B}{\beta - 1})} \) and \( a \equiv \frac{1}{1 - K}, \) \( C_1 \equiv \frac{1}{R} - \varepsilon \left( \frac{2}{R - \frac{B}{\beta - 1}} - 1 \right) \). Then, by substituting \( \phi^{**} \) from proposition 3.2 into (37) and letting \( C_2 \equiv \sqrt{\phi - \frac{2a(2\beta - 1)}{R}} \), the optimal liquid ratio is given by:

\[
\frac{l}{D} = 1 - \frac{1}{R - \frac{B}{\beta - 1}} + \frac{R\sqrt{C^2_1 - C^2_2}}{R - \frac{B}{\beta - 1} - aR\varepsilon} \tag{38}
\]

Q.E.D.

Proof of Proposition 3.4

Propositions 2.1 and 3.3 imply that \( \frac{l}{D_0} < \frac{l}{D} \) if and only if:

\[
\frac{\sqrt{\frac{1}{R^2} - C^2_2}}{R - \frac{B}{\beta - 1}} < \frac{\sqrt{\left[ \frac{1}{R} - \varepsilon \frac{a + (1 - a)(R - \frac{B}{\beta - 1})}{(R - \frac{B}{\beta - 1})} \right]^2}}{R - \frac{B}{\beta - 1} - aR\varepsilon}
\]

or

\[
\left( \varepsilon - \frac{K}{aR} \right)^2 < \frac{1}{(1 - L)} \left( \frac{a + (1 - a)K}{a} \right)^2 \left( \varepsilon - \frac{K}{R[a + (1 - a)K]} \right)^2 - \frac{K^2L}{a^2R^2(1 - L)} \tag{38}
\]

where \( K \equiv R - \frac{B}{\beta - 1}, \) \( L \equiv (RC_2)^2 \) and \( C_2 \equiv \sqrt{\phi - \frac{2a(2\beta - 1)}{R}} \). With respect to \( \varepsilon \), the geometric locuses defined by the LHS and the RHS of (38) are parabolas. The parabola defined by the LHS of (38) has its vertex at the point \( \left[ \frac{K}{aR}, 0 \right] \) and its focal parameter is \( p_{LHS} = \frac{1}{4} \). Similarly, the parabola defined by the RHS of (38) has its vertex at the point \( \left[ \frac{K^2L}{a^2R^2(1 - L)}, \frac{K^2L}{a^2R^2(1 - L)} \right] \) and its focal parameter is \( p_{RHS} = \frac{(1 - L)}{2} \left[ \frac{a + (1 - a)K}{a} \right]^2 \).

From analytical geometry we know that if the focal parameter \( p \) of a parabola is positive then the parabola faces upwards. That is definitely the case for the LHS of (38), i.e. \( \frac{1}{4} \) > 0, while for the RHS of (38) that is true if and only if \( 1 - L > 0 \), or \( C_2 < \frac{1}{\pi} \). But, \( C_1 < \frac{1}{\pi} \).
which implies that $C_2 < \frac{1}{R}$. Consequently, the parabola defined by the RHS of (38) is also facing upwards.

Finally, we observe that both parabolas intersect at $\left[0, \left(\frac{K}{aR}\right)^2\right]$ and their vertices lie on the right of their intersection point. Thus, for small values of $\varepsilon$, a sufficient condition for (38) to hold is $0 > -\frac{K^2L}{a^2R^2(1-L)}$, or $1 - L > 0$. However, given that $C_1 > C_2$, it also follows that $1 - L > 0$, which implies that under the optimal regulatory contract the bank is required to maintain a higher liquidity ratio than it would be the case under no liquidity regulation.

Q.E.D.

Proof of Proposition 4.1

Given that $U|_{\varepsilon=0} = U_0$, a necessary and sufficient condition for $U(\varepsilon) > U_0$ for small values of $\varepsilon$ is that the derivative of $U$ with respect to $\varepsilon$, evaluated at $\varepsilon = 0$, is positive, i.e. $\frac{\partial U(\varepsilon)}{\partial \varepsilon}|_{\varepsilon=0} > 0$. It can be shown that the derivative of $U(\varepsilon)$ at $\varepsilon = 0$ is given by:

$$\frac{\partial U}{\partial \varepsilon}|_{\varepsilon=0} = \frac{A}{(2\phi - 1)} \left(1 - \sqrt{1 - L}\right) \left[a \left(1 - \sqrt{1 - L}\right) + (1 - a) K\right]$$

(39)

where $K \equiv R - \frac{B}{\phi - 1}$, $L \equiv R^2C_2^2$, $C_2 \equiv \sqrt{\phi} - \frac{2(2\phi - 1)}{R}$ and $a \equiv \frac{1}{1 - k}$. We can easily observe that a necessary and sufficient condition for $\frac{\partial U(\varepsilon)}{\partial \varepsilon}|_{\varepsilon=0} > 0$ is:

$$a \left(1 - \sqrt{1 - L}\right) + (1 - a) K > 0$$

or

$$K < \frac{a \left(1 - \sqrt{1 - L}\right)}{a - 1}$$

or, from (6),

$$\frac{I}{D} > \frac{a - 1}{a \left(1 - \sqrt{1 - R^2C_2^2}\right)}$$

Q.E.D.

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32 See regularity condition for $\varepsilon$ in footnote 14.