Competition for Order Flow
and
Smart Order Routing Systems

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Abstract

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We analyze a model of competition for order flow between an incumbent and an entrant exchange. Both exchanges are organized as pure limit order markets. With the assistance of a smart order routing system, traders can costlessly implement routing strategies taking advantage of the liquidity available in both markets. Otherwise, checking offers in each market and routing orders accordingly is costly. Hence, by force of habit, traders who are not equipped with smart order routing systems only trade in the incumbent market to economize on search costs. We show that there is a critical mass of routing systems’ users below which the entrant cannot attract order flow. Above this critical mass, both exchanges in general co-exist (except if one exchange charges a much smaller fee than its competitor). In this case, intermarket competition reduces the cumulative depth in the incumbent’s limit order book but increases the cumulative depth in the consolidated limit order book. Hence, market fragmentation improves overall market liquidity and traders using automated routing systems bear smaller trading costs. Yet, they may fail to invest in order routing systems, even if the cost of developing these systems is small. Thus, we identify the risk of a coordination failure in the adoption of order routing systems as an impediment to competition between trading mechanisms. We conclude this analysis with preliminary empirical evidences regarding the effect of competition between two pure limit order markets on consolidated depth.
"When I was CEO of Tradepoint (now virt-X), my team and I spent a considerable amount of effort ‘selling’ the exchange to traders. However, although they all signed up as member, they did not use the market. One major reason was that access to the market was not connected to their trading systems. Even when better bids and offers appeared on our order book, the (momentarily) inferior prices available on the LSE were hit and lifted. Potential users simply could not see, nor easily access the market. If the Tradepoint terminal was at the end of the desk, it was not accessible. The solution […] was to get Tradepoint integrated into the main order management systems […] This proved to be easier negotiated than implemented […] The traders had many other priorities and we could not demonstrate the required liquidity. Think chicken and egg again!" (in "Is exchange liquidity contestable?", by Nic Stuchfield, The handbook of World Stock, Derivative and Commodity Exchange.)

1 Introduction

Recent years witnessed dramatic changes in trading organization. Automation has decreased the cost of introducing new trading systems, fostering the emergence of new electronic marketplaces and competition for the order flow. Examples abound. In the U.S, Island and other ECNs’ have captured a substantial part of the order flow in Nasdaq stocks. In Europe, new entrants like Tradepoint or Easdaq attempted to challenge incumbent exchanges such as Euronext or the London Stock Exchange (LSE). Recently (in May 2004), the LSE launched a new trading system (EuroSETS) competing directly with Euronext for Dutch stocks.

Automation has changed the nature of trading for intermediaries, as well. When several trading venues co-exist for a single security, brokers can reduce execution costs by splitting their orders across trading venues.¹ Order splitting, however, requires brokers to incur search costs as they must inquire about the depth of each liquidity pool. The advent of smart order routing systems (e.g. LavaTrading, CyBerTrader, etc…) has considerably decreased the cost of searching for best execution. These systems act as “robots” attending brokers in the placement of their orders. They consolidate the offers standing in different liquidity pools and may even automatically split an order so as to minimize total price impact.² Hence, these systems alleviate market fragmentation and, maybe, mitigate the need for some form of mandated consolidation.³

Arguably, this evolution (the rise of electronic trading systems coupled with the automation of order submissions) changes the nature of competition between trading venues. In particular, it should facilitate order flow contestability. It has long been recognized that competition for order flow is hampered by positive network externalities, as liquidity is positively related to market size. These externalities create a tendency for order clustering and lead to situations where a

¹For evidences on order splitting across multiple trading venues, see Menkveld (2004).
²In a way, these systems emulate, for financial markets, the internet search engines which offer price comparisons in good markets (e.g. www.shopper.com).
single market dominates (Admati and Pfleiderer (1988), Pagano (1989), Chowdhry and Nanda (1991)). Thus, they acts as a barrier to entry as incumbent markets often possess a large base of captive investors.\footnote{Positive size externalities also imply that fragmentation of trading across trading venues might be harmful for liquidity (Mendelson (1987)) and traders’ welfare (Economides and Siow (1988) and Pagano (1989)).}

As smart order routing systems reduce the cost of searching for best execution, they decrease the proportion of captive investors and, for this reason, they facilitate the co-existence of multiple trading venues. In these conditions, the success of a new marketplace is likely to hinge upon incentives for brokerage firms to automate their routing decision. The rivalry between the London Stock Exchanges and Euronext provides a vivid illustration of this point. The business press has echoed concerns that the LSE’s ability to make a substantial inroad in the Dutch market would depend on the adoption of routing systems. For instance, on the day after the LSE started its operations in the Netherlands, the FT was writing:

"The London Stock Exchange which yesterday started an assault on Amsterdam stock is drawing attention to traders’ increasing need for smart order routing to take advantage of increased competition" ("LSE tries the smart order route", Financial Times, May 25 2004).

Our purpose in this paper is then twofold. First, we analyze competition between markets in presence of smart order routing systems. Second, we study the decision to adopt automated routing systems by brokerage firms. Specifically, we consider the case of an incumbent exchange and an entrant exchange organized as pure limit order markets. We address a variety of questions: (i) What are the effects of intermarket competition and smart order routing systems on liquidity? (ii) What is the role of fees on limit orders? (i) How are the market shares of competing exchanges affected by smart order routing systems usage? (iv) What are the determinants of the adoption of smart order routing systems by brokers?

Our approach builds upon the model of competition between exchanges developed by Parlour and Seppi (2003). We extend this model in several directions. First we focus on the case in which competing exchanges are pure limit order markets. This is of interest because limit order markets become prevalent and increasingly engage in head-to-head competition for order flow (e.g. Euronext and the LSE for Dutch stocks). In contrast, Parlour and Seppi (2003) consider competition between a pure limit order market and an hybrid trading system (like the NYSE) with multiple types of liquidity providers. Second, we allow competing exchanges to charge different fees on limit order traders. This is relevant because the pricing structure of competing exchanges often differ. Finally, we analyze the effect of smart order routing systems on competition for order flow. Our main results are as follows:

1. **Critical Mass and Routing Systems.** Co-existence of the incumbent and the entrant markets is possible when the proportion of traders using smart order routing systems exceeds a critical mass. Below this critical mass, the entrant attracts no limit orders (even if it
charges lower fees on limit orders) and, for this reason, it attracts no trading.

2. **Fragmentation and liquidity.** When the two markets co-exist, fragmentation of the order flow between the entrant and the incumbent market reduces the depth of the incumbent limit order book but **always** increases the consolidated depth posted in the market. Thus, competition between two pure limit order markets reduces execution costs for traders using smart order routing systems.

3. **Fees on limit orders.** Trading can remain fragmented even when (a) one exchange charges smaller fees on limit orders and (b) all traders use smart order routing systems.

4. **Routing Systems Adoption.** A coordination failure among brokerage firms is possible in equilibrium. That is, adoption of smart order routing systems is not guaranteed even if (i) all traders would benefit from adopting these systems and (ii) the cost of designing such systems is negligible.

Interestingly, trading activity in the entrant market cannot take off if the proportion of brokerage firms adopting automated routing systems is below a critical level. The intuition for this result is as follows. The presence of limit orders in the entrant limit order book is a pre-requisite for trades taking place in the entrant market. Execution probabilities of limit orders placed in the entrant market increase with the proportion of brokerage firms using routing systems. Now, as limit order submission is costly, the expected profits on the limit orders placed in the entrant market are negative if their execution probabilities are too small. Thus, the entrant market does not attract limit orders and for this reason is not viable when the proportion of brokerage firms using routing systems falls below a critical threshold.

When the two markets co-exist, competition for order flow (and the ensuing fragmentation) always enhances consolidated depth in our model. Actually, co-existence of multiple trading venues opens new profit opportunities for limit order traders by enabling them to follow “queue-jumping” strategies. To see this point, suppose that all traders use smart order routing systems and consider a trader who is about to place a limit order at the best ask price. At this price, a large quantity is offered in the incumbent market whereas no limit order is, yet, posted at this price in the entrant market. In this case, posting the limit order in the entrant market yields a larger profit because its likelihood of execution is higher. Actually this limit order does not yield time priority to pre-existing limit orders as priority rules are not enforced across markets. In other words, the placement of a limit order in the entrant market is a way to jump in front of the queue of limit orders in the incumbent market.

This effect fosters competition among limit order traders and, in this way, enhances market liquidity. Of course, "queue-jumping" reduces execution probabilities for the limit orders placed in the incumbent market. For this reason, the depth posted in the incumbent market is smaller when the two markets co-exist. However, in equilibrium, the first effect dominates and opening
the possibility of "queue-jumping" increases consolidated depth. A related point, but in a different model, is made by Glosten (1998).

Another intriguing result is the possibility that traders might not adopt smart order routing systems although they would be better off if they all adopt this technology. Smart order routing systems allow traders to reduce their trading costs. We assume that traders adopt an automated routing system when the reduction in trading costs outweights the cost of acquiring and using the system. Key to our result is the fact that the benefit of using a smart order routing system is strictly positive iff the entrant market attracts limit orders. But, the entrant market attracts limit orders iff the proportion of traders using smart order routing systems is large enough. Thus, the traders might be trapped in a “bad” equilibrium in which (a) the entrant market does not attract limit orders because too few traders route their orders to the entrant market and (b) traders do not find optimal to design smart order routing systems because the entrant market is not deep enough. Anecdotal evidences (see, for instance, the opening quotation of this paper) suggest that this "egg and chicken" problem is a serious impediment to entry in the market for trading platforms.\footnote{See also "LSE tries the smart order route", Financial Times, May 25 2004.}

Our paper is related to the literature on competition for the order flow. Glosten (1994) analyzes competition between two limit order markets when order splitting is costless and limit order traders earn zero expected profits.\footnote{Glosten (1994) considers the case in which there is an infinite number of limit order traders. Biais, Martimort and Rochet (2000) relax this assumption and show that liquidity suppliers earn rents when their number is finite.} Instead, we assume that manual order splitting is costly and that automated order splitting is costless but requires a technological investment. Moreover, in our model, limit order traders do not earn zero expected profits. Instead, as in Parlour and Seppi (2003), the quantities offered in the book adjust in such a way that marginal limit orders at each price break-even while infra-marginal orders earn a positive expected profit. In contrast with Parlour and Seppi (2003), we find that intermarket competition always enhances the depth of the consolidated book. This difference stems from the fact that we consider competition between pure limit order markets while Parlour and Seppi (2003) analyze competition between an hybrid market and a pure limit order market. Other papers have analyzed competition for order flow across trading venues with different market designs. For instance, Hendershott and Mendelson (2000) study competition between a dealer market and a crossing network. Viswanathan and Wang (2002) or Sabourin (2004) analyze competition between a pure limit order market and a dealer market. None of these papers have analyzed the impact of order routing systems and differences in limit order fees on competition between pure limit order markets, as we do in the present article.

There also exist many empirical studies on the effects of competition and market fragmentation.\footnote{For instance Biais, Bisière and Spatt (2004), Barclay, Hendershott and McCormick (2003)) or Weston (2001). Lee (2002) provides a survey of the main empirical results.} Some of these studies (e.g. Battalio (1997), Mayhew (2003), DeFontnouvelle et al. (2003),
Boehmer and Boehmer (2004)) have specifically focused on comparing measures of market liquidity before and after entry of new competitors in the provision of trading services. These studies usually find that spreads are tighter after entry, which suggest that intermarket competition is a useful complement to intramarket competition. Biais, Bisière and Spatt (2004) also obtain results pointing in this direction.

One prediction of our model is that competition between pure limit order markets enhances market liquidity. In the last part of our paper, we propose a test of this prediction using limit order book snapshots for stocks traded in both EuroSETS and NSC (the trading system of Euronext). Our findings indicate that consolidated depth has substantially increased after the introduction of EuroSETS, in line with our prediction. These findings are very preliminary and deserve a more thorough investigation. Subsequent results will be reported in future versions of this paper.8

The plan of the paper is as follows. In the next section, we describe the model. Section 3 defines and characterizes the equilibrium. Section 4 shows that the entrant will be able to make inroads into its competitor’s market share iff a sufficiently large proportion of traders use smart order routing systems. It also shows that competition enlarges aggregate market depth. Section 5 studies the adoption of smart order routing systems by traders. Section 6 provides a case study and Section 7 concludes.

2 A Model of Competition for Order Flow with Smart Order Routing Systems.

In this section, we describe a model of competition between limit order markets in presence of smart order routing systems. Our approach builds upon the model developed by Parlour and Seppi (2003).

There are two exchanges : (a) the incumbent exchange, denoted I and (b) the entrant exchange, denoted E. The two exchanges trade the same risky security with expected value $v_0$. Both are organized as limit order markets. In each exchange, traders can post limit orders at prices $\{..., p_{-n}, p_{-(n-1)}, ..., p_0, ..., p_n..\}$ with $p_i < p_{i+1}$. The difference between two successive prices on this grid is called the tick size and is denoted by $\Delta$. For simplicity, we assume that the security’s expected value is on the grid : $p_0 = v_0$. It will become clear below that it is never optimal to submit a sell (resp. buy) limit order below (resp. above) $p_0$. From now on, we exclusively focus on the determination of the number of shares offered on the "ask side" of the book (that is at prices $\{p_1, ..., p_n\}$). It is straightforward to derive symmetric results for the "bid side" of the

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8To the best of our knowledge, the impact of competition for order flow on market liquidity has not been analyzed empirically in the case of pure limit order markets. Board and Wells (2000) study competition between SETS and Tradepoint (2 UK based limit order markets). They do not study the effect of this competition on market liquidity, however.
book.

We denote by \( S_{jk} \) the number of shares offered for sale at price \( p_k \) in market \( j \). The cumulative depth available at price \( p_k \) in market \( j \) is denoted \( Q_{jk} \). By definition, observe that

\[
Q_{jk} = \sum_{l=1}^{k} S_{jl}.
\]

Finally, we denote by \( Q_k \) the consolidated depth at price \( p_k \), that is:

\[
Q_k = Q_{Ik} + Q_{Ek}.
\]

The consolidated depth is simply the total number of shares offered up to price \( p_k \) when limit order books in each market are consolidated.

There are 2 periods in the model. In period 1, traders fill the book in each market. In period 2, a broker seeks to execute a market order for \( \tilde{X} \) shares.

**Limit Order Traders.**

As in Seppi (1997), limit orders in each market are submitted by value traders who arrive with stochastic arrival times during period 1. The value trader who arrives first in market \( j \in \{I, E\} \) sees an empty book and fills the book. Then a new value trader arrives, observes the book, and decides whether or not to add depth to the book. The process stops when the last trader arriving in the market decides that there is no price in the book at which submitting a limit order is profitable.

Then, the trading game proceeds to period 2. Price priority and time priority are enforced within each market. Thus, when a buy (resp. sell) market order arrives in market \( j \), it is executed against the limit orders standing in this market at progressively higher (resp. lower) prices (price priority) until completion of the order. Moreover limit orders standing at a given price fill in the order in which they arrived to the market (time priority). As in reality, price priority and time priority are not enforced across markets.

Limit order traders bear two distinct costs. First, they bear an execution cost \( f_j > 0 \), per share executed, when their order submitted in market \( j \) executes. This cost includes the execution fee charged by the exchange but also the order-handling cost incurred by limit order traders.\(^9\)

Second, limit order traders bear an order entry cost \( c_j > 0 \) per share, whether their order is executed or not. This cost includes the entry fee (if any) charged by the exchange but also the costs associated with the time required to submit and then monitor the order for instance. Importantly, this cost is paid whether execution takes place or not, in contrast to the execution fee \( f_j \). Order-handling costs and monitoring costs are likely to be identical across markets. This does not imply that \( c_I = c_E \) or \( f_I = f_E \) as exchange fees may differ, however.\(^{10}\) Hence, we

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\(^9\)Some exchanges offer rebates to limit orders in case of execution. This may result in a negative execution costs if the rebate is large enough. For simplicity, we restrict our attention to the case in which \( f_j > 0 \).

\(^{10}\)For instance, EuroSETS charges no order entry fees in contrast to Euronext.
will interpret differences in order entry or execution costs across exchanges as resulting from differences in the pricing structure of the exchanges. We will assume throughout that the fee structure of the entrant is at least as competitive as the fee structure of the incumbent so that \( c_E \leq c_I \) and \( f_I \leq f_E \).

**Market Orders and Smart Order Routing Systems**

The broker arriving in period 2 must fill a buy (resp. sell) market order with probability \( \alpha \) (resp. \( 1-\alpha \)). The aggregate size of this market order, \( \tilde{X} \), is random with cumulative probability distribution \( F(.) \) on \([0, \mathcal{Q}]\) and \( F(x) \overset{df}{=} 1 - F(x) \). Thus, \( F(x) \) is the probability that \( \{ \tilde{X} \geq x \} \). The broker does not buy (resp. sell) at a price larger (resp. smaller) than \( p_m \) (resp. \( p_{-m} \)) with \( 1 < m < \infty \). Hence, \( p_m \) (resp. \( p_{-m} \)) is her reservation price on buy orders.\(^{11}\) As limit orders at prices larger than \( p_m \) have a zero execution probability, the book is empty at any price strictly larger than \( p_m \). For this reason, from now on, we exclusively focus on the limit orders placed at price \( p_k \leq p_m \).

The broker has one of two types: (i) either she is equipped with a routing system (probability \( 1 - \lambda \)) or (ii) she handles orders manually (probability \( \lambda \)). Hence parameter \( \lambda \) is the proportion of brokerage firms using smart order routing systems. For the moment we take this parameter as given. We endogenize it in Section 5.

An order routing system automatically and optimally splits the broker’s order between markets \( L \) and \( I \) so as to minimize her total trading costs given the offers standing in each limit order book. If the broker’s order size is \( x > Q_m \) then her routing system executes all limit orders placed at prices \( p_1, p_2, \ldots, p_m \) in each market. The residual portion is left unexecuted. If the broker’s order size is \( x \in [Q_{k-1}, Q_k] \) with \( k \leq m \) then her routing system executes all limit orders placed at prices \( p_1, p_2, \ldots, p_k \) in each market. Following Parlour and Seppi (1997) we refer to the price at which the last share of a market order executes, as being the *stop-out price*.

As time priority is not enforced across markets, the residual portion, \( (x - Q_{k-1}) \), of the market order which executes at the stop-out price can be split between the two competing markets in many different ways. We assume that routing systems are programmed to operate in one of two modes for the allocation of the residual portion: either (a) it trades first in market I and then executes the residual (if any) in market \( E \) or (b) it trades first in \( E \) and then chooses I. We call "FI" ("First I"), the first mode and "FE" ("First E"), the second mode. We assume that the routing systems choose one of these two sequences randomly with equal probabilities. Hence if the remaining portion of the order is first routed to market \( j \) then market \( j \) receives an order with size \( \text{Min}\{x - Q_{k-1}, S_{jk}\} \) and the competing market receives an order with size \( \text{Max}\{x - Q_{k-1} - S_{jk}, 0\} \).\(^{12}\)

\(^{11}\)This assumption avoids the unrealistic case in which part of an order would execute at an infinite price because the book lacks liquidity to fill the order.

\(^{12}\)We assume implicitly that trading fees are ignored in the trading costs optimization performed by smart order routing systems. This seems in accordance with the way these systems operate in reality.
When the broker is not equipped with a smart order routing system, we assume that the cost of checking the offers in each market, consolidating these offers and optimally splitting the order is prohibitively high. Consequently, the broker only considers market $I$ as a possible trading venue.\footnote{In fact this might be the case even if search costs are relatively small as suggested by this excerpt of the opening quotation in the paper: "Potential users simply could not see, nor easily access the market. If the Tradepoint terminal was at the end of the desk, it was not accessible [...]."}

This assumption captures the idea that the incumbent has a natural advantage: by force of habit, brokers who do not use smart order routing systems primarily consider this market as a possible choice. This implies that the market order routed to market $I$ might execute at worst prices than if it had been optimally routed between the two markets. This is possible because price priority is not enforced across markets.\footnote{There are evidences that some trades may not always optimally execute in multi-market environment. For instance, Battalio et al. (2002) analyze options which trade on several markets in the U.S. They find that a large number of trades take place at prices inferior to the best quotes. See also the opening quotation in the introduction of the paper.}

For the rest of the analysis, it is useful to define $M_{jk}(\tilde{X})$ as the number of shares executed at price $p_k$ in market $j$ given the broker’s aggregate order size, $\tilde{X}$. For $k \leq m$, we have:

$$M_{Ik}(\tilde{X}) = \begin{cases} 
\min \{\max\{\tilde{X} - Q_{Ik-1}, 0\}, S_{Ik}\} & \text{if the broker does not use a routing system.} \\
\min \{\max\{\tilde{X} - Q_{k-1}, 0\}, S_{Ik}\} & \text{if the broker’s routing system operates in mode "FI"} \\
\min \{\max\{\tilde{X} - Q_{k-1}, S_{Ek}\}, 0\}, S_{Ik}\} & \text{if the broker’s routing system operates in mode "FE"} 
\end{cases}$$

In the same way, we get:

$$M_{Ek}(\tilde{X}) = \begin{cases} 
0 & \text{if the broker does not use a routing system.} \\
\min \{\max\{\tilde{X} - Q_{k-1} - S_{Ik}, 0\}, S_{Ek}\} & \text{if the broker’s routing system operates in mode "FI"} \\
\min \{\max\{\tilde{X} - Q_{k-1}, 0\}, S_{Ek}\} & \text{if the broker’s routing system operates in mode "FE"}
\end{cases}$$

Notice that $M_{jk}(\tilde{X})$ depends on all the orders standing in each book up to price $p_k$. Let $P_I(Q_{Ik-1}, Q_{k-1}, S_{Ik}, S_{Ek})$ be the probability that all the shares offered at price $p_k$ in market $I$ get executed, i.e.:

$$P_I(Q_{Ik-1}, Q_{k-1}, S_{Ik}, S_{Ek}) \overset{\text{def}}{=} \Pr(\text{Prob}(M_{Ik}(\tilde{X}) = S_{Ik}))$$

Similarly, let $P_E(Q_{k-1}, S_{Ik}, S_{Ek})$ be the probability that all the shares offered at price $p_k$ in market $E$ get executed:

$$P_E(Q_{k-1}, S_{Ik}, S_{Ek}) \overset{\text{def}}{=} \Pr(\text{Prob}(M_{Ek}(\tilde{X}) = S_{Ek}))$$

We refer to these probabilities as being the execution probability of the marginal order at price $p_k$ in market $j$. Actually, as time priority is enforced within each market, probability $P_j$ is the execution probability for a trader adding an infinitesimal number of shares at price $p_k$ in market $j$ given the state of the book in each market.
Lemma 1: Given the priority rules and the specification of brokers’ routing systems, the execution probabilities of the marginal order placed at price $p_k$ in the incumbent and the entrant markets are, respectively:

$$P_I(Q_{I,k-1}, Q_{k-1}, S_{Ik}, S_{Ek}) = \alpha \lambda F(Q_{I,k-1} + S_{Ik}) + \frac{(1 - \lambda)}{2} (F(Q_{k-1} + S_{Ik}) + F(Q_{k-1} + S_{Ik} + S_{Ek}))$$

and

$$P_E(Q_{k-1}, S_{Ik}, S_{Ek}) = \frac{\alpha(1 - \lambda)}{2} (F(Q_{k-1} + S_{Ek}) + F(Q_{k-1} + S_{Ik} + S_{Ek}))$$

Notice that the execution probability of the marginal order at price $p_k$ in market $j$ depends on the orders submitted at better prices in both markets. For instance, an increase in the cumulative depth offered at price $p_{k-1}$ in market $E$ (i.e. $Q_{Ek-1}$) reduces the execution probability of the marginal order standing at price $p_k$ in market $I$. This follows from the fact that brokers using smart order routing systems optimally split their orders between each book. This creates an interdependence between the two limit order books. For this reason, as we will shortly see, the equilibrium depth in market $j$ at price $p_k$ depends on the cumulative depth at smaller price levels in market $j$ but also in the competing market.

3 The Competitive Equilibrium

3.1 Definition of the competitive equilibrium

Consider a trader who submits a limit order for an infinitesimal number of shares at price $p_k \leq p_m$ in market $j$. Moreover, suppose that $S_{jk}$ shares are already offered at this price. This order becomes the marginal limit order at price $p_k$. The trader’s expected profit is simply his revenue in case of execution times his execution probability minus the order entry cost (that he incurs whether he is executed or not). Thus, if the trader operates in market $I$, his expected profit (per share) is:

$$\Pi^m_{Ik}(Q_{I,k-1}, Q_{k-1}, S_{Ik}, S_{Ek}) = P_I(Q_{I,k-1}, Q_{k-1}, S_{Ik}, S_{Ek})(p_k - v_0 - f_I) - c_I$$

Similarly, if the trader operates in market $E$, his expected profit is:

$$\Pi^m_{Ek}(Q_{k-1}, S_{Ik}, S_{Ek}) = P_E(Q_{k-1}, S_{Ik}, S_{Ek})(p_k - v_0 - f_E) - c_E.$$

These are the expected profits on the marginal limit order posted at price $p_k$ in each market. The execution probability of the marginal order at a given price is decreasing with the depth quoted at this price (see Equations (3) and (4)). Hence, as the depth quoted at price $p_k$ in market $j$ increases, the expected profit on the marginal order at this price decreases and is eventually zero. At this point, no trader finds optimal to add depth at price $p_k$ in market $j$. If
this zero profit condition on the marginal limit order holds at each price in the book, there are no profit opportunities left in the book and, in this sense, a competitive equilibrium is reached (see Seppi (1997), Sandas (2001) or Parlour and Seppi (2003)).  

Let \( Q_k^i, Q_{jk}^i, S_{jk}^i \) denote the equilibrium values of the consolidated, cumulative and quoted depths at price \( p_k \). Formally, a competitive equilibrium is defined as follows.

**Definition 1**: A competitive equilibrium is a set of depths \( \{S_{I1}, S_{I2}, ..., S_{Ik}, ..., S_{E1}, S_{E2}, ...\} \) such that the expected profit of the marginal limit order at each price \( p_k \) in each limit order book is non-positive if \( S_{jk}^* = 0 \) and nil if \( S_{jk}^* > 0 \). Formally, for \( k \in [1, m] \) :

\[
\Pi_{I_k}^*(Q_{I_k-1}^i, Q_{I_k-1}^j, S_{I_k}^i, S_{E_k}^i) = 0 \text{ if } S_{I_k}^i > 0 \quad \text{and} \quad \Pi_{I_k}^*(Q_{I_k-1}^i, Q_{I_k-1}^j, S_{I_k}^i, S_{E_k}^i) \leq 0 \text{ if } S_{I_k}^i = 0 \\
\Pi_{E_k}^*(Q_{E_k-1}^j, S_{I_k}^j, S_{E_k}^j) = 0 \text{ if } S_{E_k}^j > 0 \quad \text{and} \quad \Pi_{E_k}^*(Q_{E_k-1}^j, S_{I_k}^j, S_{E_k}^j) \leq 0 \text{ if } S_{E_k}^j = 0 
\]

Observe that providing liquidity at a given price is not profitable if the likelihood of execution at this price is too small. Actually, in this case, the expected revenue on the limit order does not cover the order entry cost. Accordingly, there might be prices at which no shares are offered (\( S_{jk}^* = 0 \)). There might even be cases in which posting a limit order in one market is not profitable at any price. In this case, the market is not viable as it features an empty book and attracts no trading. This leads us to distinguish two possible equilibrium outcomes: (i) either each market attracts some limit orders or (ii) one market does not attract any limit order. In the first case, the execution probability of the limit orders standing in each book must be strictly positive, which means that trades will occur in both markets. Hence, we say that the two markets *co-exist* when the books in each market are non-empty in equilibrium. In the second case, obviously, trading concentrates in the market which attracts limit orders. We say that this market is the *dominant market*.

**Definition 2**: Market \( j \) is dominant if the book of the competing market is empty for all possible brokers’ reservation prices, that is: \( S_{-jk}^*(\lambda) = 0, \forall k, \forall m \). Otherwise, i.e. if there exists \( k \geq 1, l \geq 1 \) and \( m < \infty \) such that \( S_{Ik}^* (\lambda) > 0 \) and \( S_{Ek}^* (\lambda) > 0 \), we say that Market E and I co-exist.

There may be cases in which a market dominates when the broker’s reservation price is small but not when it is large enough. Thus, in order to avoid obtaining conclusions driven by the value of the broker’s reservation price, we say that a market dominates when its competitor’s book is empty for all possible values of the broker’s reservation price.

---

\(^1\)See Definition 2 in Parlour and Seppi (2003).
Using Equations (3), (4), (5), (6) and the equilibrium definition, it is immediate that the various measures of depths must satisfy the following conditions in equilibrium:

\[
2\lambda F(Q_{I_{k-1}} + S_{I_k}^*) + (1 - \lambda)(F(Q_{k-1}^* + S_{I_k}^*) + F(Q_{k-1}^* + S_{E_k}^*)) = \frac{2c_I}{(p_k - v_0 - f_I)\alpha} \quad \text{if } S_{I_k}^* > 0
\]

\[
2\lambda F(Q_{I_{k-1}} + S_{I_k}^*) + (1 - \lambda)(F(Q_{k-1}^* + S_{I_k}^*) + F(Q_{k-1}^* + S_{E_k}^*)) \leq \frac{2c_I}{(p_k - v_0 - f_I)\alpha} \quad \text{if } S_{I_k}^* = 0
\]

and

\[
(1 - \lambda)(F(Q_{k-1}^* + S_{E_k}^*) + F(Q_{k-1}^* + S_{I_k}^* + S_{E_k}^*)) = \frac{2c_E}{(p_k - v_0 - f_E)\alpha} \quad \text{if } S_{E_k}^* > 0
\]

\[
(1 - \lambda)(F(Q_{k-1}^* + S_{E_k}^*) + F(Q_{k-1}^* + S_{I_k}^* + S_{E_k}^*)) \leq \frac{2c_E}{(p_k - v_0 - f_E)\alpha} \quad \text{if } S_{E_k}^* = 0
\]

The depth quoted at each price in each book (the \( S_{I_k}^* \) and \( S_{E_k}^* \)) can be found by solving this system of equations recursively (from \( k = 1 \) to \( k = m \)). These values are determined by the values of the ratios:

\[
R_{jk} = \frac{c_j}{(p_k - v_0 - f_j)\alpha}, \quad k \in \{1, ..., m\}, j \in \{E, I\}.
\]

Notice that an increase in the order entry cost \( (c_j) \) or in the order execution cost \( (f_j) \) triggers an increase in \( R_{jk} \). Thus, qualitatively, an increase in the order execution cost has the same effect on the equilibrium as an increase in the order execution cost of this exchange. For instance, the impact of a decrease in \( f_E \) is similar to the impact of a decrease in \( c_E \). Thus, in order to simplify the exposition, we can restrict our attention to the case in which \( f_I = 0 \) and \( f_E = 0 \).

We have checked that all our results are robust when \( 0 < f_I \leq f_E \). In particular, qualitatively, our statements regarding the effect of order entry costs also apply to the effect of order execution costs. In order to simplify notations, we define:

\[
\tilde{c}_j \overset{\text{def}}{=} \frac{c_j}{\alpha}.
\]

Henceforth, we assume that:

\[
\tilde{c}_I < p_1 - v_0.
\]

This assumption is not crucial for the results. The level of \( \tilde{c}_I \) simply determines the ask price at which providing liquidity starts being profitable. Under Assumption A.1 this price is \( p_1 \).

### 3.2 A benchmark: a single market

We are interested in analyzing the effect of entry on the consolidated depth offered at a given price in the book. Hence we will compare the consolidated depth offered at price, say, \( p_k \) when the two markets co-exist with the consolidated depth offered at price \( p_k \) when the incumbent
market operates alone. Obviously, the equilibrium in this case obtains as a particular case of our model by considering the case in which $\lambda = 1$.

When market I operates alone, the expected profit on the marginal limit order at price $p_k$ in market I is:

$$\Pi_{Ik}^m = \alpha [\overline{F}(Q_{Ik})(p_k - v_0) - \widehat{c}_I] \quad \text{for } k \leq m.$$  

In a competitive equilibrium, the cumulative depth offered at price $p_k$ is such that the expected profit on the marginal limit order is equal to zero, i.e.:

$$\overline{F}(Q_{Ik}^*(1))(p_k - v_0) - \widehat{c}_I = 0 \quad \text{for } k \leq m. \quad (10)$$

The equilibrium cumulative depth offered at price $p_k$ is obtained by solving this equation. There is always a solution to this equation because $\overline{c}_I < p_k - v_0$, $\forall k \geq 1$ and $\overline{F}(\cdot)$ decreases. Notice that in all cases: $Q_{Ik}^*(1) < \overline{Q}$ because $\widehat{c}_I > 0$. Moreover $Q_{Ik}^*(1)$ strictly increases with $k$ because $\overline{F}(\cdot)$ is decreasing.

### 3.3 An Example

We first consider an example in which we can solve for the equilibrium in closed-form. We use this example to survey the results that we will establish more systematically in the subsequent sections. In this example, we assume that the broker’s order size ($\bar{X}$) has a uniform distribution on $[0, \overline{Q}]$. The equilibrium depth when exchange I operates alone is easily obtained in this case. Solving Equation (10) when $\overline{F}(x) = \frac{(Q-x)}{Q}$ yields:

$$Q_{Ik}^*(1) = \overline{Q}(1 - \frac{\widehat{c}_I}{k\Delta}). \quad (11)$$

When two exchanges compete for the order flow, we focus our attention on the case in which $\Delta > 2\overline{c}_I$. Moreover, we assume that the parameters satisfy the following conditions:

$$\Delta(1 + \lambda)(1 - \lambda) \geq 2(\overline{c}_I + \overline{c}_E - \lambda(\overline{c}_I - \overline{c}_E)). \quad (12)$$

and

$$\lambda \leq \frac{\Delta + \overline{c}_I - 2\overline{c}_E}{\Delta + \overline{c}_I} \quad (13)$$

The set of parameter values which satisfy these conditions is non-empty (take for instance $\lambda \leq 0.3$, $\overline{c}_E \leq \overline{c}_I = 0.02$ and $\Delta = \frac{4}{3}$). In this case, the cumulative depths posted at each price when $\lambda > 0$ are:

$$Q_{Ik}^*(\lambda) = \begin{cases} 
\frac{2\overline{Q}}{3 + \lambda}((1 + \lambda) - \frac{2\overline{c}_E - \overline{c}_I}{\Delta}) & \text{for } k = 1 \\
Q_{Ik-1}^*(\lambda) \text{ for } k \geq 2, \text{ if } \overline{Q}(1 - \frac{\overline{c}_I}{k\Delta}) \leq Q_{Ik-1}^*(\lambda) \\
\overline{Q}(1 - \frac{\overline{c}_I}{k\Delta}) \text{ for } k \geq 2 \text{ if } \overline{Q}(1 - \frac{\overline{c}_I}{k\Delta}) > Q_{Ik-1}^*(\lambda) 
\end{cases}$$

in market I and

$$Q_{Ek}^*(\lambda) = \frac{2\overline{Q}}{3 + \lambda} \left((1 - \frac{2\overline{c}_E - (1 - \lambda)\overline{c}_I}{(1 - \lambda)\Delta}) \right)$$
in market E. In the particular case in which $\lambda = 0$, the cumulative depths at each price are:

\[ Q_{Ik}(0) = \frac{2\bar{Q}}{3} (1 - \frac{2\tilde{c}_I - \tilde{c}_E}{\Delta}), \forall k \]

in market I and

\[ Q_{Ek}(0) = \frac{2\bar{Q}}{3} ((1 - \frac{2\tilde{c}_E - \tilde{c}_I}{\Delta}), \forall k \]

in market E. We do not provide a formal proof of these claims for brevity. They can be checked by substitution of the expressions for the cumulative depth in equations (7) and (8).

The equilibrium has several interesting properties. First, in equilibrium, the two markets co-exist. Actually, under condition (13), $Q_{Ek}(\lambda) > 0$. Moreover, $Q_{Ik}(\lambda) > 0$ as $\Delta > 2\tilde{c}_I$. This means that, in equilibrium, there are cases in which all trading does not concentrate in a single market. Second, fragmentation does not reduce the overall market liquidity. In fact, it is easily checked that Condition (12) implies that the consolidated depth at price $p_1$ exceeds the maximal market order size, i.e. $Q_{1}(\lambda) > \bar{Q}$. This immediately imply that the consolidated depth at any price is larger when the two markets co-exist than when market I operates alone.

Also, observe that the cumulative depth posted in market E decreases in $\lambda$ while the cumulative depth posted in market I increases in $\lambda$. As the proportion of brokers using smart order routing systems declines, limit orders’ execution probabilities in market E decrease while they increase in market I. Furthermore, in line with intuition, the cumulative depth in each market decreases with the order entry fee charged in this market. Interestingly, it also increases with the order entry fee charged in the other market. To understand this result, suppose that market I raises its order entry fee. Cumulative depth in this market decreases as providing liquidity in this market becomes less profitable. But this implies that the likelihood of execution for limit orders placed in the competing market becomes larger. Accordingly, this market attracts more orders at each price and becomes deeper.

We illustrate these results in the particular case in which $\tilde{c}_E = \tilde{c}_I = 0.02$, $\Delta = 0.12$ and $\bar{Q} = 100$. Figures 1(a) and 1(b) illustrate the effect of a decrease in $\lambda$ on cumulative depths (up to 7 ticks behind the best offer) posted in market I and E, respectively. Figure 1(c) depicts the consolidated depth posted at various prices in the book. Observe that, for each value of $\lambda$, the market is deeper than in the benchmark case. For instance at price $p_1$, the consolidated depth is $Q_{1}(1) = 84$ shares when market I operates alone and jumps to $Q_{1}(0.3) = 111.6$ shares when both markets co-exist and $\lambda = 0.3$. The effect of a change in $\lambda$ on the consolidated depth is non-monotonic. At each price, the consolidated depth is larger when $\lambda = 0.2$ or $\lambda = 0.3$ than when...
Figure 2 gives the fraction of the consolidated depth posted at a given price accounted by the entrant market. It shows that this fraction increases in $\lambda$. This suggests that, for given order entry fees, time series variations in this fraction should reflect limit order traders’ beliefs regarding usage of smart order routing systems.

Figure 3 illustrates the effect of a decrease in the fee charged by exchange $E$ when $\lambda = 0$. The parameter values are such that the number of shares offered at prices larger than $p_1$ in each market is zero. This means that the cumulative and consolidated depths at each price are equal to the cumulative and consolidated depths at price $p_1$. For this reason, we just report cumulative and consolidated depths at price $p_1$ when $\hat{c}_E = 0.02$, $\hat{c}_E = 0.01$, $\hat{c}_E = 0.005$. As $\hat{c}_E$ declines, cumulative depth in market $E$ increases while cumulative depth in market $I$ decreases. The drop in market $I$ liquidity is more than compensated by the increase in liquidity in market $E$, however. Thus, overall, consolidated depth improves.

In the subsequent sections, we show that these properties do not specifically derive from the parametric assumptions in this example. Furthermore, we derive the conditions under which the two markets co-exist or under which one market dominate.

4 Is Market Fragmentation Harmful for Market Liquidity?

We first study the conditions under which market $I$ or $E$ co-exist or not (Propositions 1 and 2). In particular, we show that exchange $E$ is viable only if the proportion of brokers using a routing system exceeds a critical mass. This critical mass is determined by the order entry fees charged by both exchanges. Next, we restrict our attention to the case in which market $E$ can attract some order flow and we analyze the effects of inter-market competition on market liquidity.

**Proposition 1 (critical mass):** Let $\lambda^* (\hat{c}_E, \hat{c}_I) \overset{def}{=} \max \{ \Delta + \hat{c}_I - \frac{2\hat{c}_E}{\Delta + \hat{c}_I}, \frac{3\hat{c}_E - \hat{c}_I}{3\hat{c}_I} \}$. If $\lambda \geq \lambda^* (\hat{c}_E, \hat{c}_I)$ then market $I$ is dominant in equilibrium. In contrast, if $0 < \lambda < \lambda^* (\hat{c}_E, \hat{c}_I)$ then both markets co-exist.

Figure 4 illustrates this proposition. For a given order entry cost in market $I$ ($\hat{c}_I$), it depicts the set of values for $\lambda$ and $\hat{c}_E$ such that (a) market $I$ dominates or (b) the two markets co-exist. Observe that $\lambda^* (\hat{c}_E, \hat{c}_I) < 1$ (for $\hat{c}_E > 0$). This implies that, for given fees in each market, there always exists a value of $\lambda$ large enough ($\lambda \geq \lambda^*$) such that entry is unsuccessful and the incumbent market remains dominant. This result outlines the importance of routing systems for the entrant. The latter is able to capture part of the trading activity in the security iff a sufficiently large proportion of brokerage firms, use smart order routing systems. This proportion must be larger than $z^* (\hat{c}_E, \hat{c}_I) \overset{def}{=} 1 - \lambda^* (\hat{c}_E, \hat{c}_I)$. In this sense, $z^* (\hat{c}_E, \hat{c}_I)$ can be seen as the critical mass for the entrant market.
The critical mass depends on the pricing strategies of the entrant and the incumbent. Not surprisingly, the critical mass for the entrant market increases as \( b_cE \) becomes larger or as \( b_cI \) becomes smaller (because \( \lambda^* (\tilde{c}_E, \tilde{c}_I) \) decreases with \( \tilde{c}_E \) and increases with \( \tilde{c}_I \)). Hence, charging relatively small order entry fees can be a way for the entrant market to reduce the required critical mass to attract some trading activity. Conversely, cutting its order entry fee is a way for the incumbent market to prevent the incumbent market from reaching its critical mass.\(^{17}\)

Interestingly, if \( 0 < \lambda < \lambda^* (\tilde{c}_E, \tilde{c}_I) \), then the two markets co-exist, no matter how small is the order entry fee charged by Exchange E (see Figure 4). There are two reasons for this result. First, the brokers which do not use smart order routing systems constitute a captive clientele for Exchange I. They will send their orders to this exchange even if its fee is non-competitive. The second reason is more subtle. Placing a limit order in one market is a way to bypass time priority in the competing market (we formalize this point after the next proposition). This feature implies that limit order traders use both markets even if they charge different fees on limit orders. In turn, when each market attracts limit orders, brokers have an incentive to split their orders between the two markets because splitting orders reduces their total trading costs. We show this point more formally after stating our second result.

**Proposition 2**: If \( \lambda = 0 \) then market E and I co-exist if \( \tilde{c}_E \geq \text{Min}\{\frac{2}{3}\tilde{c}_I, 2\tilde{c}_I - \Delta\} \). Otherwise, i.e. if \( \tilde{c}_E < \text{Min}\{\frac{2}{3}\tilde{c}_I, 2\tilde{c}_I - \Delta\} \) then market E is dominant.

This result shows that even when all brokers use smart order routing systems (no captive clientele) and exchanges charge different fees, all the trading does not necessarily gravitate to the market charging the lowest fee. In fact, the results so far indicate that the entrant exchange will capture the whole market iff all traders use routing systems and the entrant’s order entry fee is smaller than the incumbent’s fee by at least one-third.

This result may seem surprising. Why would some traders decide to submit limit orders in the market charging the largest fee (the incumbent market) in absence of search costs (\( \lambda = 0 \))? To see how this may happen, suppose that \( S_{E1} \) shares are offered at price \( p_1 \) in market E while no limit orders are posted in market I, yet. Then, consider a trader submitting a limit order for an infinitesimal quantity at price \( p_1 \) in market E. His execution probability is then (see Equation (4)):

\[
P_E(0, 0, S_{E1}) = \alpha F(S_{E1})
\]

In contrast, if the trader submits his order in market I, he obtains an execution probability equal to (see Equation (3)):

\[
P_I(0, 0, S_{E1}) = \alpha \left( \frac{F(0) + F(S_{E1})}{2} \right) = \alpha \left( \frac{1 + F(S_{E1})}{2} \right) \geq P_E(0, 0, S_{E1}).
\]

\(^{17}\)Interestingly, the reaction of Euronext to the entry of EuroSETS in the market for Dutch stocks has been to suppress entirely its order entry fee for about 2 months.
Hence, the trader gets a larger execution probability if he places his limit order in the incumbent market. Actually, in this case, his limit order does not yield priority of execution to the limit orders already submitted in market E. For this reason, placing a limit order in the incumbent market can be optimal even if it charges a higher fee on limit orders. This will be the case if:

\[ P_I(0, 0, 0, S_E) (p_1 - v_0) - c_I > P_E(0, 0, S_E) (p_1 - v_0) - c_E, \]

that is

\[ (1 - F(S_E)) \Delta > 2(\bar{c}_I - \bar{c}_E) \]

Note that the L.H.S of this equation is bounded by 1. This explains why market E becomes dominant if the order entry cost in market I is large enough compared to the order entry cost in market E. In this case, the benefit of a relatively high execution probability in the incumbent market is not sufficient to counterbalance the larger order entry cost in this market.

It is worth stressing that the previous reasoning crucially relies on the fact that time priority is not enforced across markets. To see this, suppose instead that time priority is enforced across markets and consider again the previous situation. If the trader submits his limit order in market I, he has to wait for execution of the limit orders standing at price \( p_1 \) in market E before being executed. Thus, his execution probability is \( F(S_E) \). It is identical to his execution probability in market E. In this case, the trader’s expected profit is larger if he submits his limit order in market E because this market charges the lowest fee. Hence, if time priority was enforced across markets, limit orders would be submitted only in the market charging the lowest order entry fee and markets would not coexist.

**Proposition 3**: When the two markets co-exist or when exchange E is dominant, the cumulative market depth in the incumbent market is smaller than when the incumbent market operates alone. This means that:

\[ Q^*_i(\lambda) \leq Q^*_k(1) \quad \forall k \geq 1 \quad \lambda \in [0, \lambda^*]. \]

The intuition for this result is as follows. When the two markets co-exist, limit order traders in the incumbent market have a smaller execution probability than when the incumbent market operates alone. This follows from the fact that market orders are optimally split between the two competing trading venues. Thus, the expected profit of joining the queue of limit orders at any given price in the incumbent market is reduced compared to the pre-entry period. Actually, the expected benefit is smaller because the execution probability is reduced but the order entry cost is unchanged.18 Accordingly, the aggregate number of shares offered at each price in the incumbent book is smaller.

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18 We assume that the order entry fee charged by the incumbent exchange is identical before and after entry of a new market. This proposition is not necessarily valid if instead the order entry fee decreases in the post-entry period. But this will reinforce our claim that competition between pure limit order markets raise market liquidity (Proposition 4).
It is worth stressing that the last proposition is derived holding the order entry fee charged by the incumbent exchange constant before and after entry. If inter-market competition triggers a reduction of its fee by the incumbent, then the cumulative depth in the incumbent market might be larger after entry of exchange E.

**Proposition 4:** When the two markets co-exist or when exchange E is dominant, the consolidated market depth is larger than when the incumbent market operates alone, i.e. \( Q_k^*(\lambda) \geq Q_k^*(1), \forall k \geq 1 \) when \( \lambda \in [0, \lambda^*] \). More specifically, there exists \( k_0 \) such that:

\[
Q_k^*(\lambda) > Q_k^*(1), \quad \forall k \geq k_0,
\]

and

\[
Q_k^*(\lambda) = Q_k^*(1), \quad \forall k < k_0.
\]

Moreover, if \( \lambda = 0 \) (which includes the case in which E dominates), then \( k_0 = 1 \).

Thus, entry of a new limit order market increases the consolidated depth. In other words, in this model, market fragmentation is not harmful for overall market liquidity. There are two different effects which explain the previous result. First, suppose that the parameters are such that exchange E operates alone. This happens only if \( \tilde{c}_E \) is small enough compared to \( \tilde{c}_I \). In this case, consolidated market depth is larger than when market I operates alone simply because order entry cost is smaller after entry. This effect also operates when the two markets co-exist and \( \tilde{c}_E < \tilde{c}_I \).

Differences in order entry costs are in no way necessary for obtaining this result, however. Actually, it also holds when \( \tilde{c}_I = \tilde{c}_E \) (in which case the two markets necessarily co-exist for \( \lambda \in [0, \lambda^*] \)). This means that there is another mechanism which leads to the improvement in consolidated depth. Intuitively, creation of a new limit order market enables traders to engage in "queue-jumping" strategies. That is, a trader can bypass time priority on orders standing at a given price in a given market by submitting his limit order at the same price in the competing market. This possibility induces traders to collectively quote larger depth in the consolidated market.\(^{19}\)

A formal explanation is as follows. Consider the case in which all traders use smart order routing systems \((\lambda = 0)\) and exchanges feature identical order entry costs \((c_I = c_E = c)\). When only exchange I operates, the depth posted at price \( p_1 \) is determined by the following equilibrium condition:

\[
\alpha(F(Q_1^*(1)))(p_1 - v_0) = c. \tag{14}
\]

\(^{19}\) Anecdotal evidences suggest that limit order traders do use queue-jumping strategies in presence of multiple trading venues. For instance, the August 2004 issue of the EuroSETS newsletter mentioned that: "[... ] some firms are using the relatively lower volumes on the spread offered by the Dutch Trading Service to "queue-jump" rather than waiting elsewhere for execution" (EuroSETS newsletter, Issue August 2004, 4).
Now, suppose that the consolidated depth posted at price $p_1$ when the two exchanges co-exist is just equal to its level when only exchange I operates, i.e. $Q_1(0) = Q_1^*(1)$. In this case, the expected profit of submitting a small limit order at price $p_1$ in market $j$ is

$$\Pi_{j1}^m = \left[ \alpha \left( \frac{F(S_{j1}(0)) + F(Q_1(0))}{2} \right) \right] (p_1 - v_0) - c,$$

and $\alpha \left( \frac{F(S_{j1}(0)) + F(Q_1(0))}{2} \right)$ is the execution probability of the limit order. Observe that this execution probability is such that:

$$F(S_{j1}(0)) + F(Q_1(0)) > F(Q_1(1)),$$

in at least one market. This follows from the fact that $F(.)$ is decreasing and $S_{j1}(0)$ is strictly smaller than $Q_1(0)$ for at least one market. But then, Condition (14) and this observation imply that $\Pi_{j1}^m > 0$. That is, adding depth in at least one of the two markets is profitable if $Q_1(0) = Q_1^*(1)$. Intuitively, by submitting an order in market $j$, a trader bypasses time priority on the limit orders standing in the competing market. For this reason, in similar market conditions ($Q_1(0) = Q_k^*(1)$), his execution probability is therefore strictly larger than his execution probability when a single market operates. It follows that in equilibrium $Q_1^*(0) > Q_1^*(1)$. More generally, a similar logic explains why $Q_k^*(\lambda) > Q_k^*(1)$.

Proposition 4 generates a simple testable implication. Consider a situation in which an incumbent limit order market is challenged by a new entrant. In this case, if the new entrant attracts some order flow, consolidated market depth should be larger after entry. We offer a test of this prediction in Section 6. The next proposition offers another testable prediction.

**Proposition 5**: Consider two levels for the proportion $\lambda$, namely $\lambda_0$ and $\lambda_1$, such that both exchanges co-exist for each level and $0 < \lambda_0 < \lambda_1 < \lambda^*$. In equilibrium, the consolidated depth of market I is larger when $\lambda = \lambda_1$ and the consolidated depth of market E is smaller when $\lambda = \lambda_1$, i.e. $Q_{kI}(\lambda_1) > Q_{kI}(\lambda_0)$ and $Q_{kI}(\lambda_1) < Q_{kI}(\lambda_0)$.

This proposition suggests that the evolution of cumulative depth in the entrant and the incumbent markets should be driven by liquidity suppliers’ beliefs regarding the proportion of brokers who check offers in both markets (i.e. $(1 - \lambda)$). For instance, if liquidity suppliers anticipate an increase in this proportion (i.e a decrease in $\lambda$), we expect an increase in the cumulative depth displayed in market E at the expense of the cumulative depth displayed in market I. This assumes implicitly that there are time-series variations in parameter $\lambda$. This may be the case if brokers do not adopt order routing technology at the same point in time. Alternatively, brokers may decide on a daily basis whether they bear the costs of checking offers systematically in market E or not. This would also create time series variations in $\lambda$.

Designing a test of this prediction is not straightforward as $\lambda$ is not easily observed. The model suggests a proxy, however. Consider the case in which market E is active at the best ask
price in the consolidated market (i.e. \( Q_{1E}^* > 0 \)).\(^{20}\) Conditional on this state for the limit order book, the likelihood that the next buy order will be routed (at least partially) to market E is given by:

\[
P_E(0, Q_{1I}^*(\lambda), 0) = \frac{\alpha(1 - \lambda)}{2} \times (1 + F(Q_{1I}^*(\lambda))).
\]

This probability clearly decreases with \( \lambda \).\(^{21}\) There are two effects which concur to this result: (a) as \( \lambda \) increases, the proportion of brokers checking offers in both markets decreases and (b) even brokers who consider both markets are less likely to consume liquidity in market E as market I is deeper (\( Q_{1I}^*(\lambda) \) increases with \( \lambda \)). Thus, the likelihood of observing a trade in market E conditional on both markets posting the inside quotes could be used as a proxy for \( \lambda \) to conduct a test of Proposition 5.

5 Smart Order Routing Systems Adoption

In this section, we study brokerage firms' incentives to adopt smart order routing systems. Firms with smart order routing systems bear smaller trading costs because they can engage (costlessly) in multi-market trading. Designing a smart order routing system is costly, however. Thus, we assume that a brokerage firm adopts a smart order routing system if the expected reduction in trading costs is larger than the cost of developing the routing system.\(^{22}\)

There is a continuum of brokerage firms. Each firm has two possible actions: "A" (Adoption of a routing system) and "NA" (Non Adoption). We assume that brokerage firms make their adoption decision simultaneously and we focus on Nash equilibria in pure strategies. We denote by \( \lambda^e \), the proportion of brokerage firms adopting a routing system in equilibrium.

Let \( T^er(\lambda) \) be the expected trading cost of a brokerage firm, which uses a smart order routing system (as indicated by superscript "r"), for a given \( \lambda \). This is the expected average markup that the brokerage firm expects to incur on a buy order in \emph{equilibrium} (as indicated by superscript "e"). That is,

\[
T^er(\lambda) = \frac{\lambda}{s(\lambda, \bar{X}, r) - 1} \left( \sum_{k=1}^{s(\lambda, \bar{X}, r) - 1} (p_k - v_0)(S^r_{E_k}(\lambda) + S^e_{E_k}(\lambda)) + (p_{s(\lambda, \bar{X}, r)} - v_0)(N_{s(\lambda, \bar{X}, r)}(\bar{X})) \right),
\]

where \( p_{s(\lambda, \bar{X}, r)} \) is the stop-out price and \( N_{s(\lambda, \bar{X}, r)}(\bar{X}) \) is the number of shares purchased at the

\(^{20}\)These include two different cases which can emerge in equilibrium: (a) market E is the only market active at the best ask price (\( Q_{1I}^*(\lambda) = 0 \)) or (b) market I is also active at the best ask price (\( Q_{1I}^*(\lambda) > 0 \)).

\(^{21}\)Of course if \( Q_{1I}^*(\lambda) = 0 \) then \( \text{Prob}(X > Q_{1I}^*) = 1 \) but this does not invalidate our reasoning.

\(^{22}\)The following comment of an investment banker illustrates the relevance of this trade-off between execution costs and developing costs: "We are driven by what our clients want if they wanted us to trade Dutch stocks in London, we would have a solution in there. But execution costs are not the only costs we look at, we have to look at development costs too!" (Financial Times, May 24, 2004: "LSE tries the smart order route").
stop-out price for an order of size $\tilde{X}$, when a broker uses a smart order routing system.\footnote{Observe that $N_{s(\lambda,\tilde{X},r)}(\tilde{X}) = \min \{X - Q_{s(\lambda,\tilde{X},r)}^{-1}, S_{I_{s(s(\lambda,\tilde{X},r))}^{-1}} + S_{E_{s(s(\lambda,\tilde{X},r))}} \}$.} In a symmetric way, we denote by $T^{\text{enr}}(\lambda)$ the expected trading cost of a brokerage firm, which does not use a smart order routing system. We have:

$$T^{\text{enr}}(\lambda) = E_{\tilde{X}} \left( \sum_{k=1}^{s(\lambda,\tilde{X},r)-1} (p_k - v_0)(S^*_{I_{s(\lambda,\tilde{X},r)}}) + (p_{s(\lambda,\tilde{X},r)} - v_0)(N_{s(s(\lambda,\tilde{X},r))}(\tilde{X})) \right),$$

where $p_{s(s(\lambda,\tilde{X},r))}$ is the stop-out price for an order of size $\tilde{X}$ and $N_{s(\lambda,\tilde{X},r)}(\tilde{X})$ is the number of shares purchased at the stop-out price for an order of size $\tilde{X}$ for a firm that does not use a smart order routing system.

\textbf{Lemma 2} : In equilibrium, we have : (a) $T^{\text{enr}}(\lambda) = T^{\text{er}}(\lambda)$ if $\lambda \geq \lambda^*$ and (b) $T^{\text{enr}}(\lambda) > T^{\text{er}}(\lambda)$ if $\lambda < \lambda^*$.

The intuition is straightforward. When $\lambda < \lambda^*$, the two exchanges co-exist or Exchange E is dominant. In either case, the consolidated depth available at each price $p_k$ is larger than the cumulative depth at price $p_k$ in market I, with a strict inequality at at least one price (see Propositions 3 and 4). Thus, traders who split their orders between the two trading venues achieve smaller trading costs. In contrast, when $\lambda \geq \lambda^*$, the book in market E is empty. In this case the possibility of order splitting is useless.

Let $K$ be the per trade cost of developing and using a smart order routing system. For given choices of the other brokerage firms, a brokerage firm optimally adopts a routing system if:

$$T^{\text{enr}}(\lambda) - T^r(\lambda) > K.$$

To make things interesting, we assume that $K$ is negligible, so that\footnote{Recall that the development cost is per trade. Actually, it is fixed and it can therefore be amortized over many trades. Hence, assuming that is negligible is reasonable.}:

$$A.2 \quad T^{\text{enr}}(0) - T^r(0) > K.$$

Thus, collectively, all brokerage firms would benefit from adopting routing systems.

There are only two possible equilibria in pure strategies. In the first equilibrium, all brokerage firms adopt a routing system and $\lambda^c = 0$. If a brokerage firm anticipates that all other brokerage firms adopt routing systems then it becomes optimal for the firm to develop such a system (under Condition A.2). The polar situation ($\lambda^c = 1$) is also an equilibrium. Given non-adoptions by other firms, each firm rationally anticipates that there is no gain in using a routing system because the book in market E will feature no orders (since $\lambda^c \geq \lambda^*$). This prophecy is self-fulfilling.
Proposition 6: The adoption game has two possible equilibria in pure strategies: (a) all the brokerage firms adopt smart order routing systems or (b) no brokerage firms adopt smart order routing systems.

Thus, brokerage firms may fail to coordinate on the equilibrium in which they all adopt routing systems, even though their execution costs are smaller in this equilibrium. In this case, entry by Exchange E is impossible.\textsuperscript{25} The coordination problem comes from the positive externality associated with the adoption of a routing system by a brokerage firm. Adoption of a smart order routing system enhances the execution probability of limit orders in the entrant market and, in this way, it helps the entrant market to reach the critical mass it needs to co-exist with the incumbent market. In turn, this effect is beneficial to other brokerage firms using a smart order routing systems because these systems are useful only if the entrant reaches the critical mass.

The risk of coordination failure constitutes a barrier to entry in the provision of trading services. Interestingly, the pricing policy followed by the entrant exchange is not in itself sufficient to remove this barrier. Actually, the equilibrium in which no exchange adopts routing systems exists no matter how large is \( \lambda^* \), that is no matter how small is the fee charged by the entrant on limit orders. Intuitively, this is due to the fact limit order traders have no incentive to place limit orders in the entrant market if their execution probabilities are too small because order submission is costly \( (\tilde{c}_E) \). Thus, in order to overcome the risk of coordination failure, the entrant must raise the likelihood of execution for limit orders. One possibility for the entrant is to subsidize the development of smart order routing systems.

To explore this point further, assume that the entrant distributes a routing system for free to a proportion \( \Phi_i \) of brokerage firms. Clearly, all brokerage firms who receive the system for free will adopt it. These firms constitute an “initial base” for market E. Assume that \( \Phi_i \) is chosen such that:

\[
T^{enr}(1 - \Phi_i) - T^r(1 - \Phi_i) > K, \quad (15)
\]

which requires \( \Phi_i > 1 - \lambda^* \) but not necessarily \( \Phi_i = 1 \). Now let \( a \) be the proportion of firms adopting a routing system in the set of firms which are not in the "initial base" of market E. Also, let \( \lambda(a, \Phi_i) \) be the proportion of these firms which do not adopt the routing system:

\[
\lambda(a, \Phi_i) = (1 - \Phi_i)(1 - a).
\]

In this case, it is dominant strategy for all brokerage firms to invest in a routing system. Actually, under Condition (15), we have:

\[
T^{enr}(\lambda(a, \Phi_i)) - T^r(\lambda(a, \Phi_i)) > K, \quad \forall a \geq 0.
\]

Hence, the only Nash equilibrium is such that all the firms adopt a routing system, i.e. \( a^e = 1 \) and \( \lambda^e = 0 \). This result yields the following conclusion.

\textsuperscript{25}This risk of coordination failure is similar to the problem of excess inertia in the adoption of new technologies in presence of networks effects (see Farell and Saloner (1986) for instance).
Proposition 7: Exchange E can suppress the risk of coordination failure by subsidizing the development of routing systems for a subset of brokerage firms.

Of course, this policy is costly for Exchange E. In order to minimize its investment in building an initial base of users, the entrant exchange should choose the smallest possible value of $\Phi_i$ such that Condition (15) is satisfied. When $K$ is small, the smallest value of $\Phi^i$ such that Condition A.3 is satisfied will be close to $(1 - \lambda^*(c_E, c_I))$. As $\lambda^*(c_E, c_I)$ increases with $c_E$, exchange E can reduce its initial investment in building an initial base of users by decreasing its fee. In this case, the pricing policy followed by the entrant exchange complements the policy which consists in sponsoring the development of smart order routing systems.

6 Empirical Findings (preliminary-to be completed)

The recent introduction (on May, 24 2004) by the London Stock Exchange (LSE) of a new trading system-EuroSETS- for Dutch Stocks constitutes an ideal experiment to test this prediction. EuroSETS competes with NSC (operated by EuroNext), the traditional trading system for Dutch stocks in the AEX and AMX indices. The nature of competition between EuroSETS and NSC comes close to our theoretical model for several reasons. First, EuroSETS and NSC are very similar trading platforms. They are both limit order markets and they use the same price grid. Moreover, time and price priorities are enforced within NSC and EuroSETS but not across the two trading systems. Second, most brokerage firms have signed up to be members of both exchanges and they can clear their trades in EuroSETS and NSC through the same clearing system ("Clearnet"). This feature is important. For many attempts in the past, the domestic exchange, effectively, kept the entrant at bay, as clearing and settlement was not allowed to take place through the incumbent’s system. Or, if it was allowed, it was costly. This is not the case here. Hence, the main cost of using both systems is the cost of splitting orders between each system, as assumed in our model.

The fee structure chosen by EuroSETS also fits well our assumptions on the pricing strategies of the entrant and the incumbent. In particular, there are no order entry fees on EuroSETS in contrast to NSC (i.e. $c_E < c_I$). Moreover brokers receive rebates (on their total monthly trading fees) in case of execution of their limit orders on EuroSETS but not on NSC (i.e. $f_E < f_I$).

For 23 stocks, we have collected data on limit order books in EuroNext and EuroSETS before and after the entry of EuroSETS. Specifically, we obtained snapshots of the EuroSETS limit order book from the London Stock Exchange from April, 1 until September, 30 2004. For each stock in our sample, these snapshots give us the number of shares offered at the 10 best bid and ask prices every 5 minutes in each trading day. Furthermore, for each stock in our sample, we obtained snapshots of the EuroNext limit order book, which are observed at the same point in time as those obtained for EuroSETS. They give the number of shares offered at the 5 best bid
and ask prices. Thus, with these data, we can measure the consolidated depth at the 5 best prices on each side of the book. Finally, for each stock, we have collected data on trades occurring in each trading system and the quotes at these times. Thus, we can analyze the evolution of market shares for both trading systems.

Table 1 reports the monthly market share of EuroSETS for the stocks in our sample from June to September 2004. As it can be seen from the table, the two markets co-exist as EuroSETS’s market share is not zero. There is some cross-sectional (and time-series) variations in EuroSETS market share. For instance, in July, it ranges from a high of 10% to a low of 0%. On average, EuroSETS’s market share is small (below 2%). This is surprising since our data reveal that, at least for some stocks, quotes are competitive in EuroSETS. This suggests that a large proportion of traders do not take advantage of the liquidity available in EuroSETS, presumably because they are slow to adopt smart order routing systems. This interpretation is confirmed by information on market developments reported in its monthly newsletter by the LSE. The LSE repeatedly encouraged market participants to use such systems.\footnote{For instance, it advertizes order routing developers on its web site.}

Table 2 reports the average daily quoted spread and the average daily quoted depth for each stock in our sample before and after the entry of EuroSETS (the pre-entry and the post-entry periods cover 30 trading days). After entry, the quoted spread is the difference between the best bid and offer in the consolidated market. In the same way, the quoted depth is the average number of shares offered at the best bid and offer in the consolidated market (i.e. $Q_1 = Q_{1E} + Q_{1I}$ in the model). The quoted spread is slightly smaller in the post-entry period. This reduction is economically insignificant, though, as it is less than 1% for the full sample. The quoted depth in the consolidated market, however, is significantly larger (at the 5%-level) in the post-entry period for 18 stocks. The changes are also economically significant as average depth increases by roughly one-third! Observe that this change cannot be attributed to an increase in the quoted spread in the post entry period as we already noticed that, if anything, the quoted spread is smaller in the post-entry period. Thus the impact of EuroSETS’ entry on the quoted depth supports our Proposition 4. Fragmentation of order flow between two pure limit order markets can enhance the consolidated depth.

Of course, a full analysis of this question requires to study the evolution of the depth offered behind the best quotes after entry of EuroSETS. We plan to do so in future empirical work. In particular, our data give us the possibility to analyze the impact of entry on the trading costs for hypothetical orders of various order sizes, assuming that orders are optimally split between the two markets. If entry of EuroSETS has increased consolidated depth, in accordance with Proposition 4, then we should find smaller trading costs for all order sizes.
7 Conclusion

We have analyzed competition for order flow between two pure limit order markets—an incumbent market and an entrant market. We relate the outcome of this competition to the usage of smart order routing systems and to the fees charged by each market on limit orders. We obtain several intriguing results. We show that there is a critical mass of routing systems’ users below which the entrant cannot attract order flow, even if it charges very low fees on limit orders. Above this critical mass, both exchanges in general co-exist and trading does not necessarily gravitate to the exchange charging the lowest execution fees on limit orders. In this case, intermarket competition reduces the depth in the incumbent’s limit order book but increases the consolidated depth. Thus, traders benefit from using order routing systems because they bear smaller trading costs. In spite of this benefit, traders may fail to invest in order routing systems, even if the corresponding investment outlay is small. This risk of coordination failure in the adoption of order routing systems constitutes a barrier to entry. The entrant can alleviate this barrier by sponsoring the development of automated routing systems.

Competition for order flow between the LSE and EuroNext in Dutch stocks provides an ideal natural experiment to test the model. In the last part of the paper, we provide preliminary evidences on this competition and relate them to our framework. Interestingly, we find that the consolidated quoted depth has increased after the introduction of its EuroSETS trading system by the LSE. We are currently investigating empirically competition between the LSE and EuroNext more thoroughly, in light of the implications of our model.

8 Proofs

Proof of Lemma 1

Let $H_0$ be the following event : \{in period 2, the broker who submits a market order has no smart order routing system\}. Let $H_1$ be the following event : \{in period 2, the broker who submits a market order has a smart order routing system operating in mode FI\}. Finally let $H_2$ denotes the event : \{in period 2, the broker who submits a market order has a smart order routing system operating in mode FE\}.

By definition

$$P_I = \lambda \Pr (M_{Ik}(\tilde{X}) = S_{Ik} \mid H_0) + \left(1 - \frac{\lambda}{2}\right)(\Pr (M_{Ik}(\tilde{X}) = S_{Ik} \mid H_1) + \Pr (M_{Ik}(\tilde{X}) = S_{Ik} \mid H_2)).$$

Then, using the definition of $M_{Ik}(\tilde{X})$, we obtain :

$$P_I = \lambda \Pr (\tilde{X} \geq Q_{Ik-1} + S_{Ik}) + \left(1 - \frac{\lambda}{2}\right)(\Pr (\tilde{X} \geq Q_{k-1} + S_{Ik}) + \Pr (\tilde{X} \geq Q_{Ik-1} + S_{Ik} + S_{Ek})).$$
As \( Pr(X \geq x) = \alpha F(x) \), this rewrites:

\[
P_I = \alpha (\lambda F(Q_{I,k-1} + S_{I,k}) + \frac{(1-\lambda)}{2} (F(Q_{k-1} + S_{I,k}) + F(Q_{k-1} + S_{I,k} + S_{E,k}))).
\]

This proves the first part of the lemma. The second part is obtained following similar steps. We skip the details for brevity. QED

Proof of Proposition 1

**Part 1:** We first show that market I is dominant if and only if \( \lambda \geq \lambda^* \). When I is dominant, \( S_{E,k} = 0 \) (the book in market E is empty) and \( Q_k^* = Q_{I,k}^* \), \( \forall k \) (i.e. the consolidated depth is equal to the cumulative depth in market I at each price level). This means that the following condition is satisfied:

\[
(1-\lambda) (\lambda F(Q_{I,k-1} + 2\hat{c}_E) + \lambda (p_k - v_0)) \leq 2\hat{c}_E (p_k - v_0), \forall k \geq 1.
\]

Furthermore, the cumulative depth at each price level in market I satisfies the following condition:

\[
\hat{F}(Q_{I,k}) = \frac{\hat{c}_I}{(p_k - v_0)}, k \geq 1.
\]

Substituting this expression for \( \hat{F}(Q_{I,k}) \) in Equation (16), we deduce that an equilibrium in which market I dominates is obtained if and only if,

\[
(1-\lambda) (\frac{\hat{c}_I}{(p_k - v_0)} + \frac{\hat{c}_I}{(p_k - v_0)}) \leq 2\hat{c}_E (p_k - v_0), \forall k \geq 2.
\]

and

\[
(1-\lambda) (1 + \frac{\hat{c}_I}{(p_1 - v_0)}) \leq 2\hat{c}_E (p_1 - v_0).
\]

Condition (18) is equivalent to

\[
\lambda \geq \frac{3\hat{c}_I - 2\hat{c}_E}{3\hat{c}_I},
\]

while Condition (19) is equivalent to

\[
\lambda \geq \frac{\Delta + \hat{c}_I - 2\hat{c}_E}{\Delta + \hat{c}_I}.
\]

Hence we deduce that an equilibrium in which exchange I dominates can be sustained if and only if:

\[
\lambda \geq \text{Max} \left\{ \frac{\Delta + \hat{c}_I - 2\hat{c}_E}{\Delta + \hat{c}_I}, \frac{3\hat{c}_I - 2\hat{c}_E}{3\hat{c}_I} \right\}.
\]

**Part 2:** We know that if \( 0 < \lambda < \lambda^* \) then Exchange I cannot dominate. This leaves us with two possibilities: (a) either exchange E dominates or (b) both exchanges co-exist. We show that a situation in which exchange E dominates is impossible when \( 0 < \lambda < \lambda^* \). The proof proceeds by contradiction. Suppose that there exists an equilibrium in which E is dominant. Then, \( S_{I,k}^* = 0 \).
(the book in market I is empty) and \( Q_k^* = Q_{Ek}^* \), \( \forall k \) (i.e. the consolidated depth is equal to the cumulative depth in market E at each price level). This means that the following condition is satisfied:

\[
2 \lambda \mathcal{F}(0) + (1 - \lambda)(\mathcal{F}(Q_{Ek-1}^*) + \mathcal{F}(Q_{Ek}^*)) \leq \frac{2 \hat{c}_I}{(p_k - v_0)} \forall k \in [1, m], \forall m
\]

(20)

Now, observe that \( \mathcal{F}(Q_{Ek-1}^*) + \mathcal{F}(Q_{Ek}^*) \geq 0, \forall k \). This implies that the L.H.S of inequality (20) is larger than or equal to \( 2 \lambda > 0 \). The R.H.S of inequality (20) decreases with \( k \) and goes to zero as \( k \) becomes large. Thus, we conclude that, when \( \lambda > 0 \), there always exists \( k_0(\lambda) \) such that inequality (20) is violated for \( k \geq k_0(\lambda) \). This implies that there is no case in which E dominates when \( \lambda > 0 \). QED

Proof of Proposition 2

An equilibrium in which E is dominant satisfies two conditions. First, \( S_{Ik}^* = 0 \) (the book in market I is empty) and \( Q_k^* = Q_{Ek}^* \), \( \forall k \) (i.e. the consolidated depth is equal to the cumulative depth in market E at each price level). This means that the following conditions are satisfied when \( \lambda = 0 \):

\[
(\mathcal{F}(Q_{Ek-1}^*) + \mathcal{F}(Q_{Ek}^*)) \leq \frac{2 \hat{c}_I}{(p_k - v_0)} \forall k \geq 1,
\]

(21)

and

\[
\mathcal{F}(Q_{Ek}^*) = \frac{\hat{c}_E}{(p_k - v_0)} \forall k \geq 1,
\]

Hence Condition (21) rewrites:

\[
\frac{\hat{c}_E}{(p_{k-1} - v_0)} + \frac{\hat{c}_E}{(p_k - v_0)} \leq \frac{2 \hat{c}_I}{(p_k - v_0)} \text{ for } k > 1,
\]

(22)

and

\[
1 + \frac{\hat{c}_E}{(p_1 - v_0)} \leq \frac{2 \hat{c}_I}{(p_1 - v_0)} \text{ for } k = 1
\]

(23)

Equation (22) is satisfied for all \( k > 1 \) if and only if:

\[
\hat{c}_E \leq \frac{2}{3} \hat{c}_I,
\]

and Equation (23) is satisfied iff:

\[
\hat{c}_E \leq 2 \hat{c}_I - \Delta
\]

This shows that if \( \hat{c}_E < \text{Min}\{\frac{2}{3} \hat{c}_I, 2 \hat{c}_I - \Delta\} \) and \( \lambda = 0 \) then there is an equilibrium in which exchange E dominates. QED

Proof of Proposition 3

Benchmark: When exchange I operates alone, the consolidated depth up to price \( p_k \) satisfies:

\[
\mathcal{F}(Q_k^*(1)) = \frac{\hat{c}_I}{(p_k - v_0)}.
\]

(24)
Notice that in this case the number of shares offered at each price in the book (up to the broker’s reservation price) is strictly positive, that is $S^*_j(1) > 0$, $\forall k \in [1,m]$ since $\hat{c}_l < p_k - v_0$.

Thus, $Q^*_j(1) = Q^*_k(1) > 0$, $\forall k \in [1,m]$.

Case 1: First consider the case in which exchange E enters and dominates. In this case, $Q^*_j(\lambda) = 0$, $\forall k$ and therefore $Q^*_j(\lambda) < Q^*_j(1)$, with strict inequalities for $k \leq m$.

Case 2: The two exchanges co-exist. The proof uses a recursive argument. Consider first the case in which $k = 1$. If $S^*_1(\lambda) = 0$ then $Q^*_1(\lambda) < Q^*_1(1)$ and the result is true for $k = 1$. If $S^*_1 > 0$ then $Q^*_1(\lambda)$ satisfies:

$$2\lambda F(Q^*_1(\lambda)) + (1 - \lambda) F(Q^*_1(\lambda)) + F(Q^*_1(\lambda) + S^*_E) = \frac{2\hat{c}_l}{(p_1 - v_0)}.$$ (25)

Now observe that:

$$Q^*_1(\lambda) \leq Q^*_1(\lambda) + S^*_E.$$ As $F(.)$ is decreasing function, we deduce from this inequality and equation (25), that:

$$F(Q^*_1(\lambda)) \geq \frac{\hat{c}_l}{(p_1 - v_0)}.$$ This inequality implies (using equation (24)) that $Q^*_1(\lambda) \leq Q^*_1(1)$.

Now suppose that the property, $Q^*_1(\lambda) \leq Q^*_1(1)$, is true up to $l = k - 1 \leq m$. Then at price $p_k$, there are two possibilities. If $S^*_k(\lambda) = 0$ then

$$Q^*_k(\lambda) = Q^*_j(1).$$

But then $Q^*_k(\lambda) < Q^*_k(1)$ since $Q^*_j(1) < Q^*_k(1)$ and $Q^*_j(\lambda) \leq Q^*_k(1)$. The second possibility is that $S^*_k(\lambda) > 0$. But then the cumulative depth up to price $p_k$ in market I satisfies:

$$2\lambda F(Q^*_k(\lambda)) + (1 - \lambda) F(Q^*_k(\lambda) + S^*_k) + F(Q^*_k(\lambda)) = \frac{2\hat{c}_l}{(p_k - v_0)}.$$ (26)

Now observe that:

$$Q^*_k(\lambda) \leq Q^*_k(\lambda) + S^*_k(\lambda) \leq Q^*_k(\lambda) + S^*_k(\lambda) + S^*_E.$$ We deduce from equation (26) that:

$$F(Q^*_k(\lambda)) \geq \frac{\hat{c}_l}{(p_k - v_0)},$$ which implies (using equation (24)) that $Q^*_k(\lambda) \leq Q^*_k(1)$. This shows that if $Q^*_{i-1}(\lambda) \leq Q^*_k(1)$ then $Q^*_k(\lambda) \leq Q^*_k(1)$. As the property is true for $k = 1$, the result holds for all $k$. QED

Proof of Proposition 4

Part 1.

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Case 1: First consider the case in which exchange E enters and dominates. In this case, the consolidated depth up to price \( p_k \) after entry solves:

\[
\overline{F}(Q^*_k(\lambda)) = \frac{\widehat{c}_E}{(p_k - v_0)}.
\]  

(27)

Combining equations (24) and (27), we get:

\[
\overline{F}(Q^*_k(1)) - \overline{F}(Q^*_k(\lambda)) = \frac{\widehat{c}_I}{(p_k - v_0)} - \frac{\widehat{c}_E}{(p_k - v_0)} > 0,
\]

where the last inequality follows from the fact that \( \widehat{c}_E \) is strictly smaller than \( \widehat{c}_I \) when Exchange E dominates. As \( \overline{F}(\cdot) \) decreases, we conclude that: \( Q^*_k(\lambda) > Q^*_k(1), \forall k \in [1, m] \).

Case 2. Now, consider the case in which the two markets co-exist. In this case, we know that there exists \( k \leq m \) such that \( S^*_{E_k} > 0 \). Let \( k_0 \) be the smallest integer such that this is the case. For \( k < k_0 \), the equilibrium cumulative depth in the incumbent market must satisfy:

\[
\overline{F}(Q^*_{I_k}(\lambda)) = \frac{\widehat{c}_I}{(p_k - v_0)}.
\]

Moreover as \( S^*_{E_k} = 0 \) for \( k < k_0 \), we have \( Q^*_k(\lambda) = Q^*_I(\lambda) \) for \( k < k_0 \). It immediately follows (see Equation (24)) that \( Q^*_k(\lambda) = Q^*_k(1) \) for \( k < k_0 \). Thus the property stated in the proposition is true up to price \( p_{k_0-1} \).

Now we show that \( Q^*_k(\lambda) > Q^*_k(1) \) for \( k \geq k_0 \). The proof is recursive. First consider the case in which \( k = k_0 \). The following condition must be satisfied in the incumbent market:

\[
2\lambda \overline{F}(Q^*_I k_0) + (1 - \lambda)(\overline{F}(Q^*_{k_0-1} + S^*_{I k_0}) + \overline{F}(Q^*_I k_0)) \leq \frac{2\widehat{c}_I}{(p_k - v_0)}.
\]

(28)

Now since \( S^*_{E k_0} > 0 \) and \( Q^*_{k_0-1} = Q^*_{I k_0-1} \), we have

\[
Q^*_{I k_0} = Q^*_{k_0-1} + S^*_{I k_0} < Q^*_k, \quad k_0 < k.
\]

But then, as \( \overline{F}(\cdot) \) decreases, Condition (28) imposes:

\[
\overline{F}(Q^*_{I k_0}(\lambda)) < \frac{\widehat{c}_I}{(p_k - v_0)},
\]

which rewrites (using Equation (10) in the proof of Proposition 3)

\[
\overline{F}(Q^*_{I k_0}(\lambda)) < \overline{F}(Q^*_{I k_0}(1)).
\]

This implies \( Q^*_{I k_0}(\lambda) > Q^*_{I k_0}(1) \). Now suppose that the property is true up to price \( p_k \) with \( k_0 \leq k \leq m \). At price \( p_k \), the following condition must be satisfied in the incumbent market:

\[
2\lambda \overline{F}(Q^*_{I k}(\lambda)) + (1 - \lambda)(\overline{F}(Q^*_{k-1} + S^*_{I k}) + \overline{F}(Q^*_k(\lambda))) \leq \frac{2\widehat{c}_I}{(p_k - v_0)}
\]

(29)

Now observe that:

\[
Q^*_{I k-1} \leq Q^*_{I k-1}(1) < Q^*_{I k-1}(\lambda),
\]

which concludes the proof.
where the first inequality follows from Proposition 3. This implies:

\[ Q^*_I k^{-1}(\lambda) + S^*_I k(\lambda) < Q^*_k(\lambda) + S^*_I k(\lambda) \leq Q^*_k(\lambda). \]

But then Condition (29) implies that:

\[ F(Q^*_k(\lambda)) < \frac{\tilde{c}_I}{(p_k - v_0)} \]

which rewrites (using Equation (24) in the proof of Proposition 3):

\[ F(Q^*_k(\lambda)) < F(Q^*_k(1)). \]

This implies \( Q^*_k(\lambda) > Q^*_k(1) \). This shows that if \( Q^*_k(\lambda) > Q^*_k(1) \) then \( Q^*_k(\lambda) > Q^*_k(1) \). As the property is true for \( k = k_0 \), the result holds for all \( k > k_0 \).

**Part 2.** Now we show that when \( \lambda = 0 \), it must be the case that \( k_0 = 1 \). For this, we just need to show that in equilibrium it must be the case that \( S^*_E 1 > 0 \). We proceed by contradiction. If \( S^*_E 1 = 0 \) then the following condition must be satisfied:

\[ 1 + F(S^*_I 1) \leq \frac{2\tilde{c}_E}{(p_1 - v_0)}. \]

If \( S^*_I 1 = 0 \) then \( F(S^*_I 1) = 1 \). The previous inequality cannot be satisfied since \( \frac{\tilde{c}_E}{(p_1 - v_0)} < 1 \). If \( S^*_I 1 > 0 \) then \( F(S^*_I 1) = \frac{\tilde{c}_I}{(p_1 - v_0)} \). Again, the previous inequality cannot be satisfied since \( \tilde{c}_E \leq \tilde{c}_I \) and \( \frac{\tilde{c}_I}{(p_1 - v_0)} < 1 \). QED

**Proof of Lemma 2**

When \( \lambda \geq \lambda^* \), the book in market E is empty. Hence, obviously, routing systems are useless and \( T^{entr}(\lambda) = T^r(\lambda) \). Suppose \( \lambda < \lambda^* \). From Proposition 4 and Proposition 3, we know that, at each price, the consolidated depth is larger than the cumulative depth in market I:

\[ Q^*_k(\lambda) \geq Q^*_I (\lambda). \] (30)

This implies that:

\[ s(\lambda, x, r) \leq s(\lambda, x, nr), \forall x. \]

Obviously, these two inequalities implies that, for a given order size, the trading cost borne by a broker who does not use a routing system cannot be smaller than the trading cost borne by a trader who uses a routing system. Now we show that there are trade sizes for which the trading cost of the latter is strictly smaller than the trading cost of the former. This implies

\[ T^{entr}(\lambda) > T^r(\lambda) \]
We know from Propositions 3 and 4, that there is at least one value of \( k \) such that Inequality (30) is strict. Let \( k_0 \) be the smallest value of \( k \) such that \( Q^*_k(\lambda) > Q^*_{Ik}(\lambda) \). This means that book E is empty up to price \( p_{k_0-1} \) but not at price \( p_{k_0} \). Consider an order size \( x \) such that

\[
Q^*_{Ik_0}(\lambda) < x < Q^*_k(\lambda),
\]

and

\[
x > Q^*_{k_0-1}(\lambda).
\]

Such a trade size must exist because \( Q^*_{Ik_0}(\lambda) < Q^*_{Ik_0}(1) < \overline{Q} \). This trade size can be broken into two components: (i) \( Q^*_{k_0-1}(\lambda) \) and (ii) \( x - Q^*_{k_0-1}(\lambda) \). The execution cost on the first component is the same for a broker who does not use a routing system and a broker who does because the book in market E is empty up to price \( p_{k_0-1} \) (as \( Q^*_{k}(\lambda) = Q^*_{k_0}(\lambda), \forall k < k_0 \)). However, the stop-out price is equal to \( p_{k_0} \) for a broker using a routing system and strictly larger than \( p_{k_0} \) for a broker who does not use a system. Hence the execution cost on this order size is strictly smaller for a broker using a routing system.

**Proof of Proposition 5**

To be written.

**Proof of Proposition 6**

The result follows from the argument in the text. \textbf{QED}

**References**


The Economist, 3/6/04 “Dutch Auction; Can the LSE snatch Dutch equities trading from Euronext?” p.65

U.S. Securities and Exchange Commission, 2000, Release N˚34-42450


Figure 1(a): Effect of a change in $\lambda$ on cumulative depth in market I
Figure 1(b): Effect of $\lambda$ on the cumulative depth in market E
Figure 1(c): Effect of $\lambda$ on the consolidated depth

- Benchmark
- $\lambda=0.3$
- $\lambda=0.2$
- $\lambda=0$
Figure 2: Contribution of entrant to consolidated depth
Figure 3: Order Submission Fee and Consolidated Depth

Consolidated Market Depth (at any price)

order submission fee in market E

- aggregate market depth
- aggregate market depth
- aggregate market depth
Figure 4

Market I dominates
The two markets co-exist
Table 1: Evolution of LSE Market Share Post-Entry

Market share is defined in percentage points and based on the number of shares traded in LSE system (EuroSETS) divided by the total number of shares traded.

<table>
<thead>
<tr>
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This table reports the evolution of LSE market share in the months after the introduction.
Table 2: Quoted Spread, Quoted Depth, and LSE Market Share: Before and After Entry

This table reports the average daily quoted spread and quoted depth in the pre- and post-entry period. Each period covers 30 trading days and the event date—the introduction of the LSE trading platform EuroSETS—is May 24, 2004. The * indicates that the change is significant at the 5% level.

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<td>-1.3*</td>
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</table>

\[ QS_t = \frac{1}{T} \sum_{k=1}^{H} QS_{t,k}, \]

\[ QS = \frac{1}{T} \sum_{t=1}^{T} QS_t. \]

\[ a \] Quoted spread is the relative quoted spread in basis points. We define the daily average quoted spread as \( QS_t = \frac{1}{H} \sum_{k=1}^{H} QS_{t,k} \), with \( QS_{t,k} \) equal to the relative quoted spread at intraday time point \( k \) and day \( t \). For a window of \( T \) days before and after entry we calculate the average daily quoted spread, \( QS = \frac{1}{T} \sum_{t=1}^{T} QS_t. \)

\[ b \] Quoted depth is the average depth at the best quotes. Quoted depth is in number of shares and calculated in the same way as quoted spread.

\[ c \] Market share is in percentage points and based on the number of shares traded through the LSE system versus the total number of shares traded.

\[ d \] The pre-entry period runs from 01/04/2004 until 14/05/2004, a period of 30 days.

\[ e \] The post-entry period runs from 02/06/2004 until 13/07/2004, a period of 30 days.