Underemployment, On-the-Job Search, and the Beveridge Curve

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Abstract

This paper derives the implications of on-the-job search for unemployment dynamics and shows how the initial jump in market tightness is influenced by the search behaviour of employed workers. The model predicts that the vacancy: unemployment ratio can either overshoot or undershoot its steady state value in response to a change in the productivity of jobs.

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1 Introduction

The standard 'textbook' model of job vacancies and on-the-job search is not suited to the study of equilibrium dynamics (ref: Pissarides 1994, 2000). However, as is discussed in Pissarides (2000), the model of the present paper offers a tractable framework in which to explicitly study the equilibrium adjustments of an economy to a steady state. This model has workers in bad jobs - the underemployed - who search on-the-job only for good jobs. The model gives two new predictions that are not present in a model without

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1See the endnotes to chapter 4 in Pissarides (2000).
on-the-job search. The first prediction is that the job finding rate of unemployed workers depends positively on the vacancy: unemployment ratio and negatively on the underemployment: unemployment ratio. The second prediction is that the vacancy: unemployment ration can either overshoot or undershoot its steady state in response to a shock in the productivity of jobs. Overshooting/undershooting occurs if the underemployment: unemployment ratio falls/increases in equilibrium. These predictions are highly relevant to the properties of the business cycle, because vacancies appear less sensitive to business fluctuations than can be explained by the standard model without on-the-job search (ref Andolfatto 1996, Shimer 2004).

2 The model

Consider a continuous time model of an infinite horizon economy with infinitely lived workers. Each worker has linear preference over lifetime consumption, a common discount factor $r$ and wishes to maximize the expected present discounted value of his or her income stream. The population is distributed on the unit interval and it consists of $e$ employed workers and $u$ unemployed workers. Thus

$$u = 1 - e.$$  

(1)

The total stock of employed workers consists of $e_b$ workers in bad jobs and $e_g$ workers in good jobs. Hence

$$e = e_b + e_g.$$  

(2)

A worker at a bad job is called underemployed while a worker at a good job is called fully employed. A good job produces an exogenous flow of $y_g$ units of output while a bad job produces an exogenous flow of $y_b < y_g$ units of output. All unemployed workers receive a flow of $a$ units of income.

Unemployed and underemployed workers search for jobs. Both types of workers write resumes and answer job advertisements. However, underemployed workers do not answer the advertisements for bad jobs, because such jobs cannot change their productivity. Therefore, the stock of $s_b$ job searcher in the submarket for bad jobs is

$$s_b = u.$$  

(3)

The supply of job searchers in the market for good jobs includes unemployed and underemployed workers. The total supply of these job searchers is

$$s_g = u + \alpha e_b.$$  

(4)

$^2$Similar predictions are discussed speculatively in Pissarides (1994).
where $\alpha$ is the exogenous effective is on-the- search by underemployed workers relative to unemployed workers.

The total stock of $v$ job vacancies include $v_b$ vacant bad jobs that each have an exogenous flow cost $k_b$, and $v_g$ vacant good jobs that each have an exogenous flow cost $k_g > k_b$. Either type of job can be opened instantaneously.

The total flow of $m$ new job matches is given by an aggregate matching technology, which is a function of the number of job searchers and job vacancies in each labor submarket. We have

$$m = m_b(s_b, v_b) + m_g(s_g, v_g)$$

(5)

Both components of this matching function are assumed to be homogenous of degree one, concave and increasing in the respective arguments. The first component gives the flow of bad job matches in the bad submarket while the second component gives the flow of good job matches.

Matching is assumed to be random. Therefore, the transition probabilities into employment for job searchers, $p_i$, and job vacancies, $q_i$, are given by

$$p_i = m_i(1, \phi_i) = \phi_i q_i \quad \text{for all } i = \{g, b\}$$

(6)

where $\phi_i = v_i/s_i$ denotes the tightness in the submarket. The transition probability, $p$, that an unemployed worker leaves unemployment is given by

$$p = p_b + p_g$$

(7)

while the transition probability of an employed worker moving into a good job is given by $\alpha p_g$. Finally, all jobs are subject to a common exogenous rate of separation, $b$.

The rate at which unemployment changes is given by

$$\dot{u} = (1 - u)b - up$$

(8)

where dot notation is used to denote the time derivative. The number of workers in bad jobs changes according to

$$\dot{e}_b = up_b - e_b(b + \alpha p_g)$$

(9)

and the number of workers in good jobs changes according to

$$\dot{e}_g = up_g + e_b \alpha p_g - e_g b.$$  

(10)

The next section describes how the equilibrium division of output between workers and firms is determined. This division of output determines the equilibrium supply of jobs by a free entry condition.
3 Equilibrium

The equilibrium of the model is characterized by the general approach put forward by Pissarides (1990). In particular, a set of asset equations characterize the expected present discounted values of income of each type of worker and firm as a function of (i) the exogenous discount factor, (ii) the flows of income in different states of the world and (iii) the transition probabilities. In this section, we give the total surpluses of good and bad matches, $\Sigma_g$ and $\Sigma_b$, respectively. These surpluses follow from standard asset equations. The model is closed by the standard Nash Bargaining rule and by the standard free entry condition on vacancies.

At all times, free entry of good and bad job vacancies implies

$$ k_i = q(\phi_i)(1 - \beta)\Sigma_i \quad \text{for all } i = \{g, b\} $$

The left hand side of equation (11) is the cost of posting a particular type of job vacancy and the right hand side is the flow probability this vacancy is matched times the amount of total surplus that goes to the firm - I assume that this sharing rule, $\beta \in [0, 1]$, is the same for all matches.

Standard manipulation of the underlying asset equations yields two expressions for the evolution of the total surpluses of each match, both in and out of steady state (See Appendix). We have

$$ \Sigma_g = (r + b + \beta p_g + (1 - \beta)q_g)\Sigma_g + \beta p_b\Sigma_b - (y_b - a - (-k_g)) $$

and

$$ \Sigma_b = (r + b + \beta p_g + (1 - \beta)q_g)\Sigma_b + \beta p_g\Sigma_g - \alpha p_g(\beta\Sigma_g - \Sigma_b) - (y_b - a - (-k_g)) $$

where $\phi_i(\Sigma_i)$ is given by equation (11).

In a stationary equilibrium, the total surplus of each match is proportional to the net flow of income of a match and is inversely proportional to the discount factor, the job destruction rate and the probability of matching. The total surplus of a bad job is also proportional to productivity improvements derived as the outcome of on-the-job search. The extra benefit of a good match viewed by the current participants of a good match is the worker’s anticipated share of the good match minus the total surplus of a bad match, which is lost if the worker switches employers. The following two dimensional phase diagram illustrates the relationship between the total surpluses of good and bad matches.

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3 A longer description of these standard asset equations are given in Kennes (1996).

4 This description of the two types of match surplus is applicable to Nash Bargaining, because it is independent of the actions of individual buyers and sellers. Hence the ‘pie’ is fixed.
4 The Beveridge Curve

The Beveridge curve is the steady state relationship between unemployment and vacancies (ref: Pissarides 2000). Movements around the Beveridge curve are characterized by changes in the vacancy:unemployment ratio, $\phi$, which is given by

$$\phi = \frac{v_b + v_g}{u}$$
To illustrate the dynamic adjustment behavior of the vacancy: unemployment ratio, it is introduce the underemployment: unemployment ratio, $\psi$, which is given by

$$\psi = \frac{e_b}{u}$$

The vacancy: unemployment ratio is given by

$$\phi = \phi_b + \phi_g (1 + \alpha \psi).$$

The two non-predetermined variables in this expression, $\phi_b$ and $\phi_g$, jump immediately to their steady state if there is a shock to productivity. However, the vacancy: unemployment ratio is a predetermined variable, because the underemployment: unemployment ratio is also a predetermined variable. The steady state value of the underemployment: unemployment ratio is given by

$$\psi^* = \frac{p_b}{b + \alpha p_g}$$

If a positive productivity shock causes an increase/decrease in this steady state underemployment: unemployment ratio, then the vacancy: unemployment ratio must undershoot/overshoot the value to which it converges. These dynamic adjustment paths are illustrated in figure 2.

![Figure 1. Dynamic Adjustment of Vacancies and Unemployment](image)

The overshooting path is given by a to b to d. The undershooting path is given from a to c to d.
5 Conclusions

Pissarides (1994) argues that the sensitivity of labor market tightness to changes in productivity might vary across countries. The present paper offers a potential explanation of such differences using a model of on-the-job search. We found that a dampened response occurs if the underemployment: unemployment ratio rises during an upturn (or falls during a downturn). In this case, the vacancy: unemployment ratio increases very little at the start of an upturn, because the relatively low proportion of underemployed workers in the pool of job searchers attracts a disproportionately small number of additional vacancies. By contrast, there is an amplified response if the underemployment: unemployment ratio falls during an upturn and increases during a downturn. In this case, labor market tightness increases a lot at the start of a upturn, because the relatively high proportion of the underemployed workers in the pool of job searchers at the start of the boom attracts a disproportionately large number of addition vacancies. An amplified response of the vacancy: unemployment ratio appears warranted in the United States. In particular, Andolfatto (1996) calibrates a job matching model without on-the-job search and he finds that the model predicts cyclical movement in vacancies that are substantially smaller than those displayed by the US economy. Related findings are offered by Shimer (2004).

References


Appendix

Let $U, W_b, W_g$ denote the present values of the unemployed, the under-employed (in bad jobs), and the fully employed (in good jobs), respectively. The asset equations for these values are

$$rU = a + p_b(W_b - U) + p_g(W_g - U) + \dot{U}$$

$$rW_b = w_b + \alpha p_g(W_g - W_b) + b(U - W_b) + \dot{W}_b$$

$$rW_g = w_g + b(U - W_g) + \dot{W}_g$$

where $w_b, w_g$ are the wages paid in bad and good jobs, respectively. Let $V_1, V_2$ denote the present values of the bad and good job vacancies respectively. The asset equations are given by

$$rV_b = -k_b + q_b(J_b - V_b) + \dot{V}_b$$

$$rV_g = -k_g + q_g(J_g - V_g) + \dot{V}_g$$

where $J_1, J_2$ are the present values of filled bad and good jobs, respectively. The asset equations for these values are given by

$$rJ_b = y_b - w_b + (b + \alpha p_g)(V_b - J_b) + \dot{V}_b$$

$$rJ_g = y_g - w_g + b(J_g - V_g) + \dot{V}_g$$

The surplus of each type of match is defined by

$$\Sigma_i = W_i - U_i + J_i - V_i$$

where

$$\dot{\Sigma}_i = \dot{W}_i - \dot{U}_i + \dot{J}_i - \dot{V}_i$$

The share of surplus to the worker is always $\beta$. Straightforward, algebraic manipulation yields equations (12) and (13).