What Drives the Business Cycle in a Small Open Economy?
Evidence from an estimated DSGE Model of the Danish Economy*

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Abstract

Estimated DSGE models have become the standard workhorse model for empirically based macroeconomic analysis in recent years. In this paper, we present an estimated DSGE model for Denmark. The model has been estimated using Bayesian methods and a dataset consisting of 22 variables spanning the sample period 1st quarter 1995 to 4th quarter 2012. We use the model to identify the most important determinants of business cycle fluctuations in Denmark. Our results indicate that shocks to foreign economies explain around 50 pct. of the variation in Danish real GDP. The build up to the financial crisis and the bust was also driven by foriegn shocks as well as high domestic demand leading to price pressures, which fiscal policy did not combat sufficiently.

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1 Introduction

Over the last decade, a large number of policy institutions have adopted a class of structural models for forecasting and policy analysis. These so-called Dynamic, Stochastic General Equilibrium (DSGE) models mostly derive from the New-Keynesian tradition (see Woodford [2003] or Galí [2009] for textbook treatments), and are thus in accordance with state-of-the-art academic research in macroeconomics; see e.g. Blanchard [2009]. Following Smets and Wouters [2003] and Smets and Wouters [2007], the models are typically estimated with Bayesian techniques. In recent years, the DSGE models of many institutions have been improved through the addition of a number of empirically relevant features, most notably frictions in the labor market (e.g., Galí et al. [2011]) and in financial markets (e.g., Christiano et al. [2010]). As a result, a number of central banks now use estimated DSGE models as their primary tool for forecasting and policy analysis.¹

In contrast, in Denmark there is a longstanding tradition of using large-scale macroeconometric models for these purposes. This includes the MONA model of Danmarks Nationalbank, the ADAM model of the Ministry of Finance and Statistics Denmark, and the SMEC model of the Danish Economic Councils.² While the use of similar models in different policy institutions has a number of advantages in terms of transparency, communication, and model development, traditional macroeconometric models come with a set of problems on their own. One is the lack of forward-looking behaviour by private agents, another is the exposure of these models to the Lucas Jr [1976] critique. In addition, as already mentioned, the vast majority of academic research in the field of monetary macroeconomics uses DSGE models as the theoretical workhorse, whereas macroeconometric models have largely disappeared from the research agenda over the last couple of decades.

In this paper, we present an estimated DSGE model of the Danish economy. The model builds on recent academic research as well as on models developed by other central banks, implying that many features of the model are quite standard in the literature. Notably, we model Denmark as a small open economy as in Gali and Monacelli [2005]. While the exchange rate is fixed towards the Euro area, reflecting Denmark’s currency peg, we allow for fluctuations in the exchange rate towards the rest of the world. Moreover, the model features a fairly detailed description of fiscal policy, which is the central stabilization tool in the Danish economy. Finally, we model labor market frictions following Galí et al. [2011], introducing a role for involuntary unemployment.

The model is estimated using Bayesian techniques. The estimated parameter values are generally in line with estimates from similar studies. More interestingly, we compare the estimated impulse responses from the model to impulse responses estimated from structural VAR models of the Danish economy. We find that the impulse responses from the DSGE model to shocks to domestic

²One notable exception is the DREAM model, which is, however, mainly a model for the long run.
government spending and to foreign output are roughly in line with the VAR-based evidence. For a shock to the policy rate of the European Central Bank, the DSGE model delivers a quantitative impact on Danish output in line with the data, although the shape of the impulse responses are somewhat different.

We use the estimated model to shed light on the driving forces behind business cycle fluctuations in Denmark. The main finding of this analysis is that foreign shocks are the most important source of movements in Danish GDP. Our variance decomposition shows that for the period 1995-2012, foreign shocks account for around 50 percent of output fluctuations in Denmark at all frequencies. This result is in contrast to other recent studies of the effects of foreign shocks in small open economies, e.g. Justiniano and Preston [2010] and Adolfson [2007], who - somewhat surprisingly - find that foreign shocks explain less than 5 percent of output fluctuations in Canada and Sweden, respectively. The main reason for this difference is Denmark’s fixed exchange rate towards the euro, which opens up an important channel through which foreign shocks are transmitted directly to the Danish economy. For example, the interest rate decisions of the European Central Bank have a direct effect on consumption and investment decisions of Danish households. Indeed, as discussed by Aastveit et al. [2013], part of the explanation for the surprisingly small results obtained by Justiniano and Preston [2010] is the lack of a direct effect of foreign shocks on domestic variables, and the lack of other transmission mechanisms than the international trade channel. Furthermore, our finding of an important role for foreign shocks is in line with empirical evidence based on VAR-studies for Denmark (see Ravn and Spange [2013]) as well as for other small open economies (see, e.g., Cushman and Zha [1997], or Aastveit et al. [2013]).

We also perform a historical decomposition of deviations in Danish output from its trend during the period 1995-2012. This decomposition confirms the importance of foreign shocks. In particular, the large boom in the Danish economy in the years leading up to the recent crisis, as well as the crisis itself, were to a large extent driven by shocks originating abroad. However, domestic demand also increased rapidly in the years preceding the crisis, with domestic consumption and investment shocks contributing significantly to the overheating of the Danish economy by the time the financial crisis hit. In turn, this is likely to have aggravated the subsequent drop in economic activity.

Due to Denmark’s fixed exchange rate, it is the job of fiscal policymakers to stabilize the domestic economy. In the years before the crisis, the high levels of domestic and foreign demand called for a tight fiscal policy. Our analysis shows, however, that fiscal policy was not tightened sufficiently during these years so as to counteract the boom. Instead, fiscal policy shocks exerted a neutral or even stimulating effect on the Danish economy in these years.

The remainder of this paper is structured as follows: In section 2, we give a general introduction to fluctuations in the Danish economy over the period we consider. We then describe the model in section 3, and the estimation of the model in section 4. In section 5, we analyze some of the properties of the estimated model, including its ability to match impulse responses from VAR studies. We then perform a historical decomposition of the quarter-to-
quarter movements in the output gap in Denmark in section 6. Finally, section 7 concludes. In the appendix, we provide additional details on the model as well as a number of tables and graphical illustrations.

2 Some stylized facts about the Danish economy

We start with a brief tour of the development of the main Danish macroeconomic variables to set the stage for the modeling task which lies ahead. The Danish economy experienced repeated devaluations of the krone during the 1970’s and the beginning of the 1980s. This was the case until 1982 when a newly elected government introduced a currency peg vis-a-vis the German mark, and from 1999, against the euro. The fixed exchange rate regime has succesfully been defended ever since. As can be seen in figure (1), top-left, in our data sample there has only been a noteworthy positive spread between the Danish monetary policy rate and the equivalent in the euroarea on two occasions; during the EMS crisis at the beginning of the 1990s and during the recent financial crisis. This confirms the high credibility of the Danish exchange rate regime.

The top-right figure in figure (1) shows one of the challenges in estimating a DSGE model for Denmark: The quarterly growth rate of Danish real GDP is quite volatile relative to real GDP in the euro-zone as an example. This is not surprising: Denmark is a very small and very open economy. We have included data for the financial crisis although it imposes another challenge when having a model with positive trend growth: This raises some end-point problems, which we address by including forecasted data, as explained in section (4.2).

The bottom-left figure in figure (1) shows year-on-year inflation rates. A necessary condition for the success of the fixed-exchange rate regime is that the Danish inflation rate on average equals the inflation rate in the euro-zone. Although closely related, there is some evidence that Danish prices have increased slightly more than prices in the euro-zone in our sample. Moreover, inflation in Denmark is more volatile than inflation in the euro-zone.

Turning to the components on the national accounts balance, private consumption makes up an increasing share of GDP in the Danish economy starting from the early 00’s until the outbreak of the financial crisis, after which it plummets and has since not recovered. Investment as a share of GDP drops significantly during the brief international recession in the late 1990’s/early 2000’s, but recovered strongly until the financial crisis, see figure (1), bottom-right. Like most other countries, Denmark is affected by globalisation, which is reflected in increasing export and import shares, as showed in figure (2), top-left. The collapse in world trade during financial crisis is clear in Danish data. It is noteworthy that during the entire sample, Denmark has had positive net exports. The trend in exports and imports exceeds the trend in output. In the estimation, this additional growth is removed following, for example, Adolfson et al. [2013].

Even though the nominal Danish exchange rate is fixed towards the euro,
that does not imply that the effective Danish exchange rate is constant, which can be seen in figure (2), top-right. These movements clearly indicate that a two-country setup for Denmark and the euro-area is not sufficient. As an example, the depreciation of the effective Danish kron from the onset of the financial crisis is likely to have helped Danish exporters survive the meltdown of global trade. We include these effects in the model through a three-country setup: Denmark, the euroarea, and the rest of the world. The rest of the world consists of the weighted sum of Denmark’s trading partners excluding of course the trading partners which reside within the euroarea.

The Danish labour market has since the early 90’s experienced profound structural reforms. This is reflected in a decrease in the natural rate of unemployment and a corresponding decrease in actual unemployment, as can be observed in figure (2), bottom-right. In the same period the real wage has increased more or less with output growth, while increased above trend during the boom before the financial crisis. We have not attempted to incorporate this downward trend in structural unemployment, which instead is left for future research.

Finally, turning to public debt and expenditures, Denmark’s public finances displayed a fairly large primary surplus (relative to GDP) during the years before the crisis. This turned into a large deficit after the crisis, as seen in figure (2), bottom-left. One reason behind the worsening of public finances is the expansionary fiscal policy conducted in order to mitigate the effects of the financial crisis. This can easily be seen from the large increase in the public consumption to GDP ratio starting from 2009, although much of the increase in this ratio is due to the drop in output. We will come back to the role of fiscal policy during the boom-bust cycle around the financial crisis. First we need to set up a model which can explain the movements in Danish data as presented in this section.

3 The Model

This section sets up the model. The model is a modified version of a number of existing medium sized DSGE models like the Christoffel et al. [2008], Adolfson [2007], Burriel et al. [2010]. The main building blocks of the previous mentioned models are however modified to reflect key aspects of the danish economy. Denmark is a small open economy with a fixed exchange rate against the euro. In the model it is assumed that this regime is 100 percent credible and given leaving room for assuming that Denmark is a part of a currency union with the euro-zone. However, that does not imply that exchange rate effects have not played a role in the determination of the Danish business-cycle, as an example the second and third biggest trading partners are outside the euro-zone being Sweden and the U.K. Consequently, Denmark has seen some fluctuations in the effective exchange rate and the model needs to reflect that.
The model therefore puts Denmark inside a currency union with the euro-zone but also trades with what will be denoted Rest-of-the-World, RoW, which consists of the Danish trading partners excluding countries within the euro-zone. It is assumed that the exchange rate vis-a-vis the RoW is floating. The two foreign countries, the euro-zone and RoW, are assumed to be exogenously given and independent of each other and especially of Denmark reflecting the small open economy assumption.

The problem of the household sector is setup in section (3.2), production is presented in section (3.3) and (3.4) while the consumers choice between home produced goods and foreign produced goods is presented in section (3.5). Fiscal and monetary policy are presented next in section (3.6), the labour market in section (3.7), and lastly section (3.9) presents the market for exports, and section (3.10) imports.

3.1 Trends

Fundamentally, there are two ways of dealing with the presence of non-stationary data in an estimated DSGE model. One is to write a stationary model, and detrend all non-stationary variables before matching them to their model counterparts. The other option is to introduce growth in the relevant variables in the model, so as to be able to estimate the model using the non-filtered, non-stationary data series. In recent years, the latter approach has become best practice in the literature, not least because the process of de-trending variables that may have different trend growth rates is complicated and involves a loss of information. For this reason, we introduce growth in our model.

The first step is to identify the relevant trends in the data. Many recent studies based on US data include two trends in the model; a total factor productivity (TFP) trend to account for the growth rate in output, and an investment-specific trend to account for the continuous decline in the relative price of investment goods, such as computers, in terms of consumption goods.\footnote{See among others the studies by Justiniano et al. [2011], Christiano et al. [2013] and Liu et al. [2013].} We follow this practice after confirming that the same two trends are present in Danish data for our sample period, i.e. 1990-2015. We can write the overall growth rate of the economy as:

\[ d\Gamma_t = (dA_t dZ_t^\alpha)^{\frac{1}{1-\alpha}}, \]

where \( dA_t \) and \( dZ_t \) denote the growth rate of TFP and the (inverse) relative price of investment, respectively. Finally, while the share of imports and exports to GDP has shown an upward trend over our sample period we have decided to detrend these variables, so that the data for imports and exports used in the estimation follow the same trend as domestic GDP, see also section (2). This greatly simplifies the modeling task as concerns the import and export sectors.
3.2 Household Sector

The problem of the representative household is to choose consumption, \(C_t\), holdings of domestic, \(B_t^{DK}\), and international, \(B_t^I\), real bonds, capital, \(K_t\), capital utilization, \(u_t\), and the level of investment, \(I_t\), so as to maximize its stream of discounted future utility, which is given by:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{\text{con}_t} \log (C_t - hC_{t-1}) \],

where \(0 < \beta < 1\) is the discount factor, \(h > 0\) measures the degree of (external) habit formation in private consumption, and \(\text{con}_t\) is a shock to the household’s preference for consumption today versus tomorrow, which is given by:

\[ \frac{\text{con}_t}{\text{con}} = (\frac{\text{con}_{t-1}}{\text{con}})^{\rho_{\text{con}}} \exp \varepsilon_{t}^{\text{con}}, \]

where \(\text{con} > 0\), \(0 < \rho_{\text{con}} < 1\), and where \(\varepsilon_{t}^{\text{con}}\) is an i.i.d. stochastic process with mean zero and variance \(\sigma^{\text{con}}\). Utility maximization is subject to the following budget constraint:

\[
(1 + \tau^V_{t}^{AT}) \frac{P^C}{P_t} C_t + \frac{P^I}{P_t} I_t + B_t^{DK} + B_t^I + T_t \\
= \Pi_t + \left( (1 - \tau^K_t) r^K_t u_t + \tau^K_t \delta^K t - \gamma(u_t) \right) K_{t-1} + \frac{R_{t-1} B_{t-1}^{DK}}{\pi_{t}^{DK}} + \\
+ \frac{R^{ECB}}{\pi_{t}^{DK}} \exp(-\psi_d \left( \frac{B_{t-1}^{DK}}{\pi_{t}^{DK}} - \frac{\pi_{t}^{DK}}{\pi_{t}^{DK}} \right) \left( \frac{B^{DK}}{\pi_{t}^{DK}} \right) B_{t-1}^I - \tau^B_t B_{t-1}^{DK} (R_{t-1} - 1) + \\
+ (1 - \tau^U_t) w_t N_t + \kappa^B w_t U_t^N, \]

where \(P_t\) is the overall price level to be defined below, \(T_t\) denotes real lump-sum taxes, and \(\Pi_t\) is the profits obtained from firms in the intermediate goods sector. Moreover, \(r^K_t\) is the real rental rate on capital, and \(R_t\) and \(R^{ECB}\) denote the Danish and the foreign risk-free rate of interest. \(Y_t\) is output, while \(N_t\) denotes hours worked, with \(w_t\) representing the corresponding real wage rate, \(w_t \equiv \frac{W_t}{P_t}\). \(U_t^N\) is the unemployment rate, to be defined later. We let \(\tau^V_{t}^{AT}, \tau^K_t, \tau^B_t\) and \(\tau^U_t\) be the tax rates on consumption (i.e., a VAT), capital, bond returns and labor, while \(0 < \kappa^B < 1\) is the compensation rate in unemployment benefits. \(\delta^K > 0\) is the capital depreciation rate. Finally, we assume that if the ratio of foreign debt to GDP exceeds its steady state level, Danish households will have to pay a risk premium on top of the interest rate set by the ECB. This reflects that foreign investors will be less willing to hold Danish debt. In turn, the higher interest rate will make it less attractive for Danish households to borrow abroad, so that eventually the debt-to-output ratio will return to its steady state level. In this respect, \(\psi_d > 0\) measures the sensitivity of the risk premium with respect to the net level of holdings of foreign bonds, or equivalently, Denmark’s net foreign asset position. The assumption of a risk premium on foreign bonds is only
made to ensure a stationary model as in . Without such an assumption it would be possible for the consumers to borrow indefinitely in the international bond market and consume the proceeds. We assume that each household does not internalize the effects on Denmark’s net foreign asset position, and thus on the risk premium, of changes in its individual international borrowing or lending. We let \( RPD_t \) denote a shock to the risk premium. This shock evolves as:

\[
\frac{R_{PD_t}^t}{R_{PD_t}^{t-1}} = \left( \frac{R_{PD_t}^{t-1}}{R_{PD_t}^{t-2}} \right)^{\rho_{R_{PD}}} \exp \varepsilon_{t}^{R_{PD}},
\]

where \( R_{PD} = 1, 0 < \rho_{R_{PD}} < 1 \), and where \( \varepsilon_{t}^{R_{PD}} \) is an i.i.d. stochastic process with mean zero and variance \( \sigma_{R_{PD}} \).

The function \( z^{u}(u_t) \) measures the cost of changing the degree of capital utilization, which we assume takes on the following functional form:

\[
z^{u}(u_t) = c_1 (u_t - \overline{u}) + c_2 \left( u_t - \overline{u} \right)^2,
\]

where \( c_1, c_2 > 0 \) are parameters, and \( \overline{u} \) is the steady state level of capital utilization, which we set to 1. Finally, the household also takes the law of motion for capital into account:

\[
K_t = \left( 1 - \delta^K \right) K_{t-1} + (1 - S_t) Z^{T}_t I_t,
\]

where \( S_t = \frac{\kappa_t}{2} \left( \frac{I_t}{I_{t-1}} - \gamma^t \right)^2 \) is the investment adjustment cost function, with the parameter \( \kappa_t > 0 \) measuring the cost of changing the investment level, and where \( \gamma^t > 0 \) denotes the steady state growth rate of investment. \( Z^{T}_t \) is a transitory investment-specific technology shock, which evolves according to:

\[
\frac{Z^{T}_t}{Z^{T}_{t-1}} = \left( \frac{Z^{T}_{t-1}}{Z^{T}_{t-2}} \right)^{\rho_{Z}} \exp \varepsilon_{t}^{Z},
\]

with \( Z^{T} > 0, 0 < \rho_{Z} < 1 \), and where \( \varepsilon_{t}^{Z} \) is an i.i.d. stochastic process with mean zero and variance \( \sigma_{Z}^{2} \). Moreover, the model features a permanent investment-specific technology shock \( Z^{P}_t \), so that \( Z_t = Z^{T}_t Z^{P}_t \). The permanent component follows the process:

\[
\frac{Z^{P}_t}{Z^{P}_{t-1}} = \lambda_{zt},
\]

where, in turn,

\[
\frac{\lambda_{zt}}{\overline{\lambda}_z} = \left( \frac{\lambda_{zt-1}}{\overline{\lambda}_z} \right)^{\rho_{\lambda}} \exp \varepsilon_{t}^{\lambda},
\]

Thus, \( \lambda_{zt} \) denotes the time \( t \) growth rate of investment-specific technology, while \( \overline{\lambda}_z > 0 \) is the steady state growth rate. \( \varepsilon_{t}^{\lambda} \) is an i.i.d. stochastic process with mean zero and variance \( \sigma_{\lambda}^{2} \), while \( 0 < \rho_{\lambda} < 1 \).
The first-order conditions related to the utility maximization problem of the household are as follows:

\[
\frac{P_C^t}{P_t^t} \lambda_t = \frac{1}{(C_t - hC_{t-1})(1 + \tau_t^{r,TF})},
\]

where \( \lambda_t = \beta \frac{E_t \lambda_{t+1}}{E_t \pi_{t+1}^{PB} \lambda_t} \exp(-\psi_d \left( \frac{B_t^T}{Y_t} - \frac{B_t^F}{Y_t} \right)) \left( \frac{RDP_t}{RPD_t} \right) \),

\[ Q_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \tau_{t+1}^R (1 - \tau_{t+1}^K) u_{t+1} + \delta^K \tau_{t+1}^K - z^u(u_{t+1}) + \left( 1 - \delta^K \right) Q_{t+1} \right) \right], \]

\[ (1 - \tau_t^R) \tau_t^K = z^u(u_t), \]

\[ \frac{P_t^I}{P_t^I} = Q_t Z_t^T [1 - S_t - S_t^I I_t] + \beta E_t \left[ Q_{t+1} Z_{t+1}^T \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^I I_t \left( \frac{I_{t+1}}{I_t} \right)^2 \right]. \] (14)

Here, we let \( Q_t \equiv \frac{\mu_t}{A_t} \) denote the price of installed capital, which differs from the price of new capital (i.e., the price of investment) due to the presence of investment adjustment costs. \( \lambda_t \) and \( \mu_t \) denote the Lagrange multipliers associated with the budget constraint and the law of motion for capital, respectively, in the optimization problem. Moreover, given the functional form for \( z^u(u_t) \), it follows that \( z^u(u_t) = c_1 + c_2 (u_t - 1) \), while for \( S_t \), we obtain that \( S_t^I = \frac{\mu_t}{I_t} \left( \frac{I_{t-1}}{I_{t-1}} - \gamma^t \right) \).

### 3.3 Intermediate Goods Producers

There is a continuum (of unit length) of firms in the intermediate goods sector, each of which operates under monopolistic competition. These firms are owned by the household. Each firm \( j \) uses private and public capital as well as labor to produce a firm-specific output according to the following production function:

\[ Y_t(j) D_t = A_t \left( \bar{K}_{t-1}(j)^{1-\eta} (K_{t-1}^{G})^{\eta} \right)^{\alpha} (N_t(j))^{1-\alpha}, \] (15)

where \( \alpha, \eta > 0 \) are parameters, \( \bar{K}_{t}(j) = u_t K_t(j) \) is the effective capital stock being utilised in a given period, \( D_t \) is a measure of price dispersion as described below, and \( A_t \) measures aggregate total factor productivity (TFP). It is assumed that \( A_t \) consists of two terms; a transitory component \( A_t^T \), and a permanent component \( A_t^P \), so that \( A_t = A_t^T A_t^P \). The transitory component evolves according to:

\[ \frac{A_t^T}{A_t^T} = \left( \frac{A_{t-1}^T}{A_t^T} \right)^{\rho_A} \exp \varepsilon_t^A, \] (16)

with \( A_t^T > 0, \ 0 < \rho_A < 1 \), and where \( \varepsilon_t^A \) is an i.i.d. stochastic process with mean zero and variance \( \sigma^A \). The permanent component follows the process:

\[ \frac{A_t^P}{A_{t-1}^P} = \lambda_{At}, \] (17)
where, in turn,
\[
\lambda_{At} = \left( \frac{\lambda_{At-1}}{\lambda_A} \right)^{\rho_{\lambda_A}} \exp \varepsilon_t^{\lambda_A},
\]
with \( \lambda_{At} \) measuring the growth rate in aggregate technology or TFP, while \( \lambda_A \) is the steady state growth rate, \( 0 < \rho_{\lambda_A} < 1 \), and where \( \varepsilon_t^{\lambda_A} \) is an i.i.d. stochastic process with mean zero and variance \( \sigma^{\lambda_A} \).

The problem of each firm is to maximize its profits subject to the production function. This problem gives rise to the following first-order conditions, where we have dropped the \( j \)'s for simplicity:
\[
\tau_t^R = \frac{\alpha Y_t mc_t}{w_t K_{t-1}},
\]
\[
(1 + \tau_t^n) w_t = \frac{(1 - \alpha) Y_t mc_t}{N_t},
\]
where \( mc_t \) is the marginal cost of production, which is identical to the Lagrange multiplier associated with the production function in the optimization problem.

We introduce sticky prices into the model by assuming that intermediate goods firms are subject to staggered price setting. In particular, following Calvo [1983] each firm is only allowed to change its price in any given period with probability \( (1 - \theta_p) < 1 \). Since all firms are identical ex ante, this implies that only a fraction \( (1 - \theta_p) \) of firms will reset their price each period. Of the remaining \( \theta_p \) firms, we allow a fraction \( \Gamma^I \) to index their price to the steady state rate of inflation, while the remaining fraction of firms keep their price unchanged. When a given firm is allowed to re-optimize its price, it solves a dynamic optimization problem, taking into account that the price it sets is likely to prevail for \( \frac{1}{1-\theta_p} \) periods. We can write the resulting first-order condition as:
\[
\bar{P}_t(j) = \frac{\varepsilon_t^P}{\varepsilon_t^n - 1} E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \frac{\lambda_{t+s}}{\lambda_t} Y_{t+s}(j) mc_{t+s} P_{t+k},
\]
where \( \varepsilon_t^P(j) \) is the price set by intermediate firm \( j \) if it is allowed to change its price in period \( t \). As all firms are identical, this price will be the same for all firms. Note also that we use the stochastic discount factor of households, as these are the owners of the firms. Finally, \( \varepsilon_t^P \) is the elasticity with which final goods producers substitute between different varieties of the intermediate good, and is given by:
\[
\left( \frac{\varepsilon_t^P}{\varepsilon_t^n} \right) = \left( \frac{\varepsilon_t^P}{\varepsilon_t^n} \right)^{\rho_{\varepsilon^n}} \exp \varepsilon_t^{\varepsilon^n},
\]
where \( \varepsilon_t^{\varepsilon^n} \) is an i.i.d. stochastic process with mean zero and variance \( \sigma^{\varepsilon^n} \), and where \( 0 < \rho_{\varepsilon^n} < 1 \). We can then write the evolution of the aggregate price index as (tjek dette udtryk)
\[
P_t = \left[ \theta_p P_{t-1}^{1-\varepsilon_t^P} + (1 - \theta_p) \left( \bar{P}_t \right)^{1-\varepsilon_t^P} P_{t-1} \right]^{\frac{1}{1-\varepsilon_t^P}},
\]
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highlighting that the share \((1 - \theta_p)\) of prices are reset in each period, and where \(\pi_t\) is the domestic inflation rate of the Danish producer price index. Finally, \(D_t\) measures the loss associated with price dispersion, and is given by

\[
D_t = (1 - \theta_p) \left( \frac{P_t - \theta_p \pi_t}{\theta_p \pi_t} \right)^{\delta} + \theta_p (\pi_t)^{\delta} D_{t-1}.
\]

(24)

### 3.4 Final Goods Producers

Firms in the final goods sector operate under perfect competition. They collect a variety of intermediate goods and repackage these into a final good to be used for consumption or investment. In doing so, they solve a cost minimization problem by choosing intermediate input goods so as to produce the final output, \(Y_t\), at the lowest possible price. Final goods producers aggregate intermediate goods according to:

\[
Y_t = \left( \int_0^1 Y_t(j) \frac{C_t}{P_t} dj \right)^{\frac{1}{\gamma}}.
\]

(25)

We can write the price index of domestically produced final goods as:

\[
P_t = \left( \int_0^1 P_t(j)^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}},
\]

(26)

where, \(P_t(j)\), is the price set by intermediate goods firm \(j\).

### 3.5 Final Consumption and Investment Goods

We assume that households combine domestically, \(C_t^{DK}\), and foreign, \(C_t^F\), produced goods into the final composite consumption good, \(C_t\), according to a constant elasticity of substitution (CES) technology:

\[
C_t = \left( \vartheta_c^{\frac{1}{\nu_c}} (C_t^{DK})^{1-\frac{1}{\nu_c}} + (1 - \vartheta_c) \frac{1}{\nu_c} ((1 - \chi_t^C) C_t^F)^{1-\frac{1}{\nu_c}} \right)^{\frac{1}{1-\frac{1}{\nu_c}}},
\]

(27)

where \(\nu_c > 1\) measures the elasticity of substitution between foreign and domestic goods, and \(\vartheta_c > 0\) measures the steady state share of foreign and domestic goods in the consumption basket, and thus also the degree of home bias in consumption. Moreover, we follow Erceg et al. [2000] and Christoffel et al. [2008] and assume that there is a cost to adjusting the share of imported consumption goods, represented by the function \(\chi_t^C\), which is given by:

\[
\chi_t^C = \frac{\bar{\chi}_C}{2} \left( \frac{C_t^F}{C_t^{DK}} \omega_t - 1 \right)^2,
\]

(28)
with \( \chi_C > 0 \) measuring the adjustment cost, and where \( \omega^f_t \) is an import shock, which follows the process:

\[
\left( \frac{\omega^f_t}{\omega^f_{t-1}} \right) = \left( \frac{\omega^f_{t-1}}{\omega^f_t} \right)^{\rho_{\text{im}}} \exp \left( \epsilon^\text{im}_t \right),
\]

(29)

with \( \omega^f > 0, 0 < \rho_{\text{im}} < 1 \), and where \( \epsilon^\text{im}_t \) is an i.i.d. stochastic process with mean zero and variance \( \sigma^\text{im} \).

As in Erceg et al. [2000], the optimal composition of final consumption is found by choosing the values of \( C_t^{DK} \) and \( C_t^F \) that solve a cost-minimization problem subject to (27). The two resulting first-order conditions are:

\[
\frac{P_t^{DK}}{P_t^C} = \left( \frac{\partial_c}{C_t^{DK}} \right)^{\frac{1}{\nu_c}} \left( \frac{1}{\nu_c} \left( C_t^{DK} \right)^{1-\frac{1}{\nu_c}} + \left( 1 - \partial_c \right)^{\frac{1}{\nu_c}} \left( \left( 1 - \chi_C C_t^F \right)^{1-\frac{1}{\nu_c}} \right) \right)^{\frac{1}{1-\nu_c}},
\]

\[
\frac{P_t^F}{P_t^C} = \left( \frac{\partial_{1-f}}{C_t^F} \right)^{\frac{1}{\nu_{1-f}}} \left( \frac{1}{\nu_{1-f}} \left( C_t^F \right)^{1-\frac{1}{\nu_{1-f}}} + \left( 1 - \partial_{1-f} \right)^{\frac{1}{\nu_{1-f}}} \left( \left( 1 - \chi_C C_t^F \right)^{1-\frac{1}{\nu_{1-f}}} \right) \right)^{\frac{1}{1-\nu_{1-f}}},
\]

(30)

which can be combined to yield:

\[
C_t^{DK} = \frac{\partial_c}{1 - \partial_c} \left( \frac{P_t^F}{P_t^{DK}} \right)^{\nu_c} \left( 1 - \chi_C C_t^F \right) \left( 1 - \chi_C - \left( \chi_C' C_t^F \right) \right)^{-\nu_c},
\]

(31)

where \( P_t^{DK} \) and \( P_t^F \) denote the price of domestic and foreign goods, respectively. Note that in the absence of adjustment costs, the optimal composition would depend only on the relative price, the elasticity of substitution and the steady state consumption shares.

Likewise, firms combine foreign and domestic investment goods into a final investment good using a similar CES technology:

\[
I_t = \left( \frac{1}{\nu_t} \left( I_t^{DK} \right)^{1-\frac{1}{\nu_t}} + \left( 1 - \partial_t \right)^{\frac{1}{\nu_t}} \left( \left( 1 - \chi^F I_t^F \right)^{1-\frac{1}{\nu_t}} \right) \right)^{\frac{1}{1-\nu_t}},
\]

(32)

where the parameters are defined as above. The adjustment cost function \( \chi^f_t \) is defined similarly to that for consumption goods, while the import shock is the same. Cost minimization by firms therefore implies that:

\[
\frac{I_t^{DK}}{I_t^F} = \frac{\partial_t}{1 - \partial_t} \left( \frac{P_t^F}{P_t^{DK}} \right)^{\nu_I} \left( 1 - \chi^I_t \right) \left( 1 - \chi^I_t - \left( \chi^I_t' I_t^B \right) \right)^{-\nu_I}.
\]

(33)

Finally, we can write the relative prices of consumption and investment goods as follows:

\[
\frac{P_t^C}{P_t} = \left( \frac{\partial_c + (1 - \partial_c)}{1 - \chi_C C_t^F} \right)^{\frac{1}{\nu_c}} \left( \frac{P_t^F}{P_t^{DK}} \right)^{\nu_c} \left( 1 - \chi_C - \left( \chi_C' C_t^B \right) \right)^{-\nu_c},
\]

(34)
\[
\frac{P_t}{\bar{P}_t} = \left( \vartheta_t + (1 - \vartheta_t) \left( \frac{\frac{\bar{P}_t^\phi}{\frac{\bar{P}_{t-11}}{1 - \chi_t^\phi}}}{1 - \chi_t^\phi} \right)^{\frac{1}{1 - \sigma_t^\phi}} \right),
\]

while the related relative inflation rates are defined as:

\[
\pi_t^C = \frac{\frac{\bar{P}_t^\phi}{\frac{\bar{P}_{t-11}}{1 - \chi_t^\phi}}}{\frac{\bar{P}_{t-11}}{1 - \chi_{t-1}^\phi}},
\]

\[
\pi_t^I = \frac{\frac{P_t}{\bar{P}_{t-11}}}{\frac{\bar{P}_{t-11}}{1 - \chi_{t-1}^\phi}}.
\]

### 3.6 Fiscal and monetary policy

The role of the public sector in the model is to raise taxes to be used for public consumption, public investment, and transfers. Public consumption, \( C^G_t \), evolves according to:

\[
C^G_t = \left( \frac{C^G_{t-1}}{\bar{C}^G} \right)^{\rho_G} \exp(\varepsilon^G_t),
\]

where \( \varepsilon^G_t \) is an i.i.d. stochastic process with mean zero and variance \( \sigma^G \), \( 0 < \rho_G < 1 \), and where \( \bar{C}^G \) is given by:

\[
\bar{C}^G = G^Y Y
\]

where \( Y \) denotes total steady state output, and \( G^Y \) is the steady state share of government spending of goods and services produced by the intermediate goods producers and public production. As for government investments, we assume that these are implemented with a lag. Specifically, we assume that an investment that is decided on in period \( t \) can only be initiated in period \( t + M \) and is finalized in period \( t + N \). In other words, we allow for time to build as well as time to plan as in Leeper et al. [2010]. To this end, we need to distinguish between planned public investment denoted by \( I^G_{t,B} \) and implemented public investment denoted by \( I^G_t \). Planned public investment evolves according to:

\[
\frac{I^G_{t,B}}{\bar{T}^G} = \left( \frac{I^G_{t-1,B}}{\bar{T}^G} \right)^{\rho_{IG}} \exp(\varepsilon^IG_t),
\]

where \( \varepsilon^IG_t \) is an i.i.d. stochastic process with mean zero and variance \( \sigma^IG \), \( \bar{T}^G \) is the steady state level of government investment, and \( 0 < \rho_{IG} < 1 \). Due to our assumption of time to build, implemented investment only adds to the stock of public capital with a lag:

\[
K^G_t = \left( 1 - \delta^G \right) K^G_{t-1} + I^G_{t-N},
\]
where $\delta^G > 0$ is the depreciation rate of public capital, and $N$ is the number of periods it takes from an investment project is decided upon and until the investment is finalized. Note that investment-specific technology shocks also affect the accumulation of the public capital stock. This ensures a stable long-run relationship between the size of the public and the private capital stock along the balanced growth path.\(^5\) Moreover, to take into account that planned investments affect the actual investment level (and hence, economic activity) with a lag, we let actual public investment be given by:

\[ I_t^G = \sum_{i=M}^{N-1} \phi^I_i I_{t-i}^{G,B}, \tag{42} \]

with $\phi^I_i > 0$, and where $M$ is the number of periods that pass from a project is decided on until it is initiated. $I_t^G$ is thus a measure of all ongoing government investment projects at time $t$, as given by the projects that were initiated at least $M$ periods ago and have not yet been finished.

On the revenue side, the government raises six different types of taxes: A labor income tax, $\tau_t^N$, a capital income tax, $\tau_t^K$, a value added tax, $\tau_t^{VAT}$, a tax on domestic bond returns, $\tau_t^B$, and a lump-sum tax $T_t$. By adjusting the tax rates, the government ensures that its intertemporal budget constraint, to be presented below, is always satisfied. This is done via the following type of tax rule:

\[ X_t = \left( \frac{X_{t-1}}{X} \right)^{\rho_X} \varepsilon_t^X \left( \frac{B_{t-1}/Y_{t-1}}{\omega^D} \right)^{(1-\rho_X)\epsilon_X^D \zeta_X}, \]

for $X = \{ \tau_t^N, \tau_t^K, \tau_t^{VAT}, \tau_t^B, T_t \}$. Here, $\bar{X}$ is the steady state value of $X$, while $0 < \rho_X < 1$. $\varepsilon_t^X$ is a white noise shock. Moreover, $\zeta_X > 0$ measures how strongly each fiscal instrument reacts to deviations of the debt-to-GDP ratio from its long-run target value, $\omega^D$, reflecting that if the debt-to-GDP ratio overshoots its long-run target, one or more of the tax rates will eventually have to be raised. Finally, the dummy variable $\epsilon_X^D \zeta_X$ essentially switches the adjustment term on or off. We can set this to zero in order to undertake simulation experiments in which the government only starts raising taxes after a certain number of periods.

We are now ready to present the government’s intertemporal budget constraint, which takes the following form:

\[ B_t^{DK} + TR_t = \frac{R_{t-1}}{\pi_t^{DK}} B_{t-1}^{DK} + G_t + w_t U_t^N \kappa_t^B, \tag{43} \]

where we have defined tax revenues $TR_t$ and government expenditure $G_t$ as:

\[ TR_t = T_t + \tau_t^{VAT} \frac{PC}{P_t} + \tau_t^K K_t - \delta^K K_{t-1} + \tau_t^N N_t + \tau_t^B \frac{R_{t-1}-1}{\pi_t^{DK}} B_{t-1}^{DK}, \tag{44} \]

As we shall see, the growth in investment-specific technology is related to the negative trend in the relative price of investment goods such as high-tech products, IT, software etc. Since many public investments also comprise such products, it seems reasonable to assume that public investments are also affected by the negative trend in the relative price of these.\(^5\)
Moreover, recall that $B_t$ denotes unemployment benefits, while $U_t$ is the unemployment rate in the model, to which we return in the following subsection. Finally, we assume that only 20 percent of the public debt needs to be refinanced in each period, so that changes in the interest rate set by the ECB only has a 20 percent impact on the interest rate on public debt. Moreover, we assume that the interest rate at which the government borrows will increase if the ratio of government debt to output exceeds its steady state level. This reflects that the household sector, which buys the bonds issued by the Danish government, will demand a risk premium if they are to hold the bonds. In technical terms, this gives rise to the following condition:

\[ R_t = \left( \frac{R_{t-1}}{R} \right)^{\rho_{RDK}} \exp\left( \psi_G \left( \frac{B_{t-1}}{Y_{t-1}} - \omega^D \right) \right)^{(1-\rho_{RDK})}, \]  

(46)

where $\rho_{RDK} = 0.8$ and $\psi_G > 0$. 

3.7 The Labor Market

We model the labor market following Galí et al. [2011] and we refer to that paper for the details. The model of Galí et al. [2011] may be seen as a somewhat simpler alternative to the well-known search-and-match approach in the tradition of Diamond, Mortensen and Pissarides. As discussed by Galí [2010], what really matters (at least quantitatively) for the dynamics of unemployment fluctuations is nominal wage rigidities and not search frictions. For our purposes, it therefore seems natural to stick to the formulation of Galí et al. [2011], as it involves adding fewer equations (as compared to the search-and-match approach) to an already large model, and brings in fewer additional parameters to be estimated using Bayesian methods.

The main building block of our labor market are wage setting household and sticky wages. As for the pricing behaviour on behalf of the firms in our model, sticky wages are achieved by assumption using the theory of Calvo [1983]. We assume the existence of a representative household with a continuum of members indexed by

\[(i, j) \in [0, 1] \times [0, 1]\]

Here index $i$ refers to differentiated labour services. Hence, we assume the existence of heterogenous types of labour specialized in various fields. This implies that each labour has some market power to set its wage. We assume the existence of a continuum of labour unions each representing the different labour types. Index $j$ refers to the household members disutility from work. Hence, the household consists of many labour types who each have a certain degree of disutility from work.

We assume full consumption risk sharing across the household and we give that household a utility function. The full risk sharing implies that we do not
need to take care of various consumption levels and hence marginal utilities and that the individual members of the household have the household in mind when maximizing utility. When these members work, they work a full day. Hence, movements on the labour markets are movements on the intra-margin and not on the inter-margin. The assumption about household setting wages and working a full day implies that the employment level is determined on the firm side - the household simply supply the given number of workers at the going real wage.

When the household chooses its labor supply it equalizes the marginal rate of substitution between supplying more labour and consumption to the real wage

$$MRS_t = \frac{\chi_t O_t (N_t)^\phi}{\lambda_t} = w_t$$

where $MRS_t \equiv -\frac{U_{N_t}}{U_{C_t}}$ is the household’s marginal rate of substitution between consumption and leisure, and $\phi > 0$ is the inverse of the Frisch elasticity. That is, $\frac{1}{\phi}$ measures by how much the households’ labor supply changes in percent when the real wage increases by one percent holding consumption constant. Intuitively, in optimum the disutility of working more must be compensated by what the real wage can buy in utility terms. If not, then the household would be able to reshuffle between consumption and labour inputs and achieve a higher utility.

The variable $O_t$ is defined as:

$$O_t = \frac{z_t}{(C_t - hC_{t-1}) \left( 1 + \tau_t V^{AT} \right) \left( \frac{P_C}{P_t} \right)} = z_t \lambda_t,$$

with $z_t$ evolving according to:

$$z_t = z_{t-1}^{1-\nu} \left[ (C_t - hC_{t-1}) \left( 1 + \tau_t V^{AT} \right) \left( \frac{P_C}{P_t} \right) \frac{1}{\text{cont}} \right]^\nu,$$

where $\nu \in [0, 1]$. Following Galí et al. [2011], we may interpret $z_t$ as a smooth trend for (habit-adjusted) aggregate consumption. In other words, $O_t$ is smaller than one when consumption grows faster than this smooth trend. As seen from (47), this implies a drop in the marginal disutility of labor, so that each individual will be willing to work at a lower wage rate, ceteris paribus. The parameter $\nu$ determines the strength of the wealth effect on labor supply. That is, by how much labor supply is affected by changes in wealth: If $\nu$ is close to 1, the wealth effect is quite strong, while the wealth effect disappears when $\nu$ tends to 0.\footnote{As discussed by Galí et al. [2011], a low value of $\nu$ is necessary to ensure that not only employment, but also the labor force moves in a procyclical fashion in response to shocks originating from the demand side.}

Finally, in (47), the term $\chi_t$ represents an exogenous shock to labor supply, which evolves according to:

$$\frac{\chi_t}{\chi} = \left( \frac{\chi_{t-1}}{\chi} \right)^{\theta_x} \exp(\xi_t^x),$$

\[ \text{(50)} \]
where $\varepsilon_t^\chi$ is an i.i.d. stochastic process with mean zero and variance $\sigma^\chi$, while $0 < \rho^\chi < 1$.

In equilibrium, a given individual will participate in the labor market if and only if the net benefits from doing so exceed that individual’s total disutility of labor. We can write this condition as:

$$\lambda_t (1 - \tau_t^\eta) w_t \geq Y_t (j),$$  \hspace{1cm} (51)

where the left-hand side measures the after-tax real wage rate as measured in utility units, and where $Y_t (j) \equiv \chi_t O_t^0 j^\phi$ represents total disutility from working. Here it is important that the individuals for each type of labor $i$ are ordered by their disutility of labor and that the condition is related to the marginal disutility of work to the household. Total disutility from working thus consists of the exogenous shock to labor supply $\chi_t$, the endogenous process $O_t$ as described above, and individual-specific labor disutility.

This implies that the labor force will consist of all individuals for which the above condition is satisfied. We can write the labor force $L_t$ in a symmetric equilibrium as:

$$L_t = \left( \frac{(1 - \tau_t^\eta) w_t}{\chi_t z_t} \right)^{\frac{1}{\phi}}.$$ 

That is, the labour force is the $j$ for which condition (51) is satisfied with equality. Summing over these participation rates across labour types gives the model aggregate labour force. Notice that the labour participation is time varying. It can as example increase due to labour supply shocks which decreases the marginal disutility of work.

Next we define our notion of unemployment as $U_t \equiv \frac{L_t - N_t}{L_t}$, i.e. the ratio between the labor force and total employment. Notice that this definition differs slightly from official unemployment rate which is given by $\frac{L_t - N_t}{L_t}$. However, around a log-linear approximations, these definitions of unemployment are equal for small levels of unemployment. Define the (log) average wage markup in the economy as the difference between the real wage and the marginal rate of substitution between consumption and work as

$$\mu_t^w \equiv \log(w_t) - \phi (z_t + n_t + \log(\chi_t))$$

The wage markup varies as long as wages are not fully flexible and is non-zero as long as the labour market is not fully competitive. We can use this expression together with the participation condition, (52), to write

$$\mu_t^w = \phi u_t$$

, where $u_t \equiv \log(U_t)$. Notice that the natural rate of unemployment rate is given by $\mu^w = \phi u_t^w$. Hence, unemployment in this model is solely due to a non-competitive labour market in which heterogenous types of labour can set a wage above the market clearing wage, and unemployment varies due to changes in the average wage markup in the economy. That is, due to wage rigidities. The
natural rate of unemployment is higher the higher is the degree of monopolistic competition and the higher is the of frisch labour elasticity; that is by how much does the household increase their labour supply when the realwage increases by 1 pct.: The higher is this elasticity the more willing are the members of the household to substitute in and out of employment.

Finally, we describe the wage formation. Observe that households supply differentiated types of labor services, giving rise to monopolistic competition for labor. Furthermore, we assume that households face Calvo-style wage stickiness. The nature of the problem implies that all households who can reoptimize the wage rate in a given period choose the same wage $w_t^P$ according to the following first-order condition:

$$w_t^P (i) = \frac{\epsilon_t^W}{\epsilon_t^W - 1} \sum_{s=0}^{\infty} (\beta \theta_W)^s \frac{\lambda_{t+s} N_{t+s}^P (j) X_{t+s}}{N_{t+s}^P (j)} Y_{t+s+1}^P (j),$$

(53)

$$\bar{P}_t (j) = \frac{\epsilon_t^P}{\epsilon_t^P - 1} \sum_{s=0}^{\infty} (\beta \theta_P)^s \frac{\lambda_{t+s} Y_{t+s} (j) m c_{t+s} P_{t+k}}{Y_{t+s} (j)} P_{t+k},$$

(54)

where $0 < \theta_W < 1$ is the wage stickiness parameter, and $\epsilon_t^W$ is the elasticity of substitution between labor types, which evolves according to:

$$\left( \frac{\epsilon_t^W}{\epsilon_t^W} \right) = \left( \frac{\epsilon_{t-1}^W}{\epsilon_t^W} \right)^{\rho^W_t} \exp \epsilon_t^W,$$

(55)

where $\epsilon_t^W$ is an i.i.d. stochastic process with mean zero and variance $\sigma^W$, and where $0 < \rho^W < 1$. Finally, we can write the evolution of the wage level in the private sector as:

$$w_t^P = \left[ \theta_w \left( w_{t-1}^P \right)^{1-\epsilon_t^W} + (1 - \theta_w) \left( w_{t-1}^P \right)^{1-\epsilon_t^W} \right]^{1-\epsilon_t^W}. $$

(56)

### 3.8 Trade and the two foreign economies

As discussed in the introduction, Denmark’s fixed exchange rate towards the euro implies that we need to include two foreign economies in the model: One (the euro area, EA for short) towards which Denmark has a fixed exchange rate, and one (which we label the Rest of the World; RoW for short) towards which the exchange rate is fully flexible. The two foreign economies are otherwise completely identical, and are taken as completely exogenous, so that movements in the Danish economy does not affect the foreign economies. Each of them is described by a basic 3-equation New Keynesian model, so that for $j = (EA, RoW)$ we have:

$$\frac{Y_t}{\bar{Y}_t} = \left( \frac{Y_t^{EA}}{\bar{Y}_t} \right)^{\rho_t^{EA}} \left( \frac{Y_t^{RoW}}{\bar{Y}_t} \right)^{1-\rho_t^{RoW}} \left( \frac{R_t}{\bar{R}_t} \right)^{-\rho_t^{RoW}} \left( \frac{\epsilon_{t}^{EA}}{\epsilon_{t}^{RoW}} \right)^{-\rho_t^{RoW}}.$$  

(57)
Here, (57) is a hybrid dynamic IS curve that links output to the real interest rate, (58) is a version of a hybrid New Keynesian Phillips Curve linking the rate of inflation to real activity, and (59) is a Taylor rule that determines monetary policy in each of the two regions as a function of inflation and economic activity. See Galí [2009] for a detailed exposition of the 3-equation New Keynesian model. In turn, the shock processes in each of these equations are given as AR(1)-processes:

\[ \frac{\pi^j_{t+1}}{\pi^j_t} = \left( \frac{\pi^j_{t+1}}{\pi^j_t} \right) \rho^j_{\pi} + \left( \frac{\pi^j_{t-1}}{\pi^j_t} \right)^{1-\rho^j_{\pi}} \left( \frac{Y^j_{t+1}}{Y^j_t} \right) \phi^j_\pi \left( \frac{\epsilon^{\pi, j}_{t+1}}{\epsilon^\pi_{t+1}} \right), \]  

(58)

\[ \frac{R^j_t}{R^j_{t-1}} = \left( \frac{R^j_{t-1}}{R^j_t} \right) \rho^j_R + \left[ \left( \frac{\pi^j_{t}}{\pi^j_t} \right) \left( \frac{Y^j_{t}}{Y^j_t} \right) \right]^{1-\rho^j_R} \left( \frac{\epsilon^{R, j}_{t+1}}{\epsilon^R_{t+1}} \right). \]  

(59)

The six exogenous shocks (the \( \epsilon^{j, k}_{t} \)'s) are included in the estimation to account for the contribution to the Danish business cycle of foreign shocks.

Finally, we can write world output and inflation as:

\[ \frac{Y^W_{t}}{Y^W_t} = \left( \frac{Y^\text{EA}_{t}}{Y^\text{EA}_t} \right)^\Omega \left( \frac{Y^\text{RoW}_{t}}{Y^\text{RoW}_t} \right)^{1-\Omega}, \]  

(61)

\[ \frac{\pi^W_{t}}{\pi^W_t} = \left( \frac{\pi^\text{EA}_{t}}{\pi^\text{EA}_t} \right)^\Omega \left( \frac{FX^t}{FX^t} \right)^{1-\Omega}, \]  

(62)

where the parameter \( \Omega \) measures the relative size of the euro area, and where \( FX \) denotes the change in the effective exchange rate of the Danish krone.

### 3.9 Exports

The role of the export sector is to buy final domestic goods, differentiate them, and sell them to import firms in the euro area or the rest of the world. We can write the world demand for Danish exports as:

\[ Ex_t = x_t^2 Y^W_t \left( \frac{P^X_t}{P^X_{t-1}} \right)^{-\zeta^W_t}, \]  

(63)

\footnote{\textsuperscript{7}It may be difficult to distinguish interest rate smoothing from persistence in the shocks hitting the interest rate rule. We therefore decide to eliminate the latter by fixing the parameter \( \rho^j_{\zeta_R} = 0 \) for \( j = (\text{EA}, \text{RoW}) \).}
where the parameter $\varepsilon^W$ denotes the elasticity with which world consumers substitute between Danish and foreign goods. The demand for Danish exports is thus increasing in world output and decreasing in the ratio between the relative price of Danish exports, $P_t^X$, and the relative world market price, $P_t^W$. We define the latter as:

$$P_t^W = P_{t-1}^W \frac{\pi_t^W}{\pi_t^K}, \quad (64)$$

where $\pi_t^W$ is the world inflation rate, as described above. The relative price of Danish exports, $P_t^X$, is defined as:

$$P_t^X = P_{t-1}^X \frac{\pi_t^X}{\pi_t^K}, \quad (65)$$

where $\pi_t^X$ is the inflation rate in Danish exports price, as described below. Finally, the export demand shock $x_t^Z$ evolves according to:

$$x_t^Z = x_{t-1}^Z \exp \left( \varepsilon_t^Z \right), \quad (66)$$

where $\varepsilon_t^Z$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{\varepsilon^Z}$, and where $0 < \rho_{\varepsilon^Z} < 1$.

Firms in the export sector are faced with price rigidities of the same form as in the domestic sector. We can therefore write the optimal export price $\tilde{P}_t^X$ set by a given firm $j$ in the export sector that is allowed to change its price in period $t$ as:

$$\tilde{P}_t^X (j) = \frac{\varepsilon_t^X}{\varepsilon_t^X - 1} \sum_{s=0}^{\infty} (\beta \theta_X)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{Y_{t+s}^X (j) m_{t+s}^X P_{t+s}^X}{Y_{t+s}^W (j)}; \quad (67)$$

where $\theta_X$ is the Calvo stickiness parameter in the export sector, and $m_{t+s}^X$ is the marginal cost for the export firms, which is simply given by the inverse of the export price; $m_{t+s}^X = \frac{1}{P_{t+s}^X}$. Finally, $\varepsilon_t^X$ is the elasticity of substitution between the goods produced by each individual firm in the export sector, which follows the process:

$$\left( \frac{\varepsilon_t^X}{\varepsilon_t^X} \right) = \left( \frac{\varepsilon_{t-1}^X}{\varepsilon_{t-1}^X} \right)^{\rho^{\varepsilon^X}} \exp \varepsilon_t^X, \quad (68)$$

where $\varepsilon_t^X$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{\varepsilon^X}$, and where $0 < \rho_{\varepsilon^X} < 1$. Finally, the inflation rate in Danish export prices will then satisfy:

$$1 = \theta_X (\pi_t^X)^{\varepsilon_t^X - 1} + (1 - \theta_X) \left( \frac{\tilde{P}_t^X}{P_t^X} \right)^{1-\varepsilon_t^X}. \quad (69)$$
3.10 Imports

The structure of the importing sector can be described as follows: A continuum of import differentiators import a homogenous final good from foreign exporters, differentiate the good (say, by adding brand names), and sell the differentiated products to Danish households and firms, who, as described above, solve a cost minimization problem when they choose between imported and domestically produced goods. The world market price of import goods, which in turn determines the marginal cost of Danish import differentiators, is computed as a weighted average of prices in the euro area and the rest of the world.\(^8\) We can write the marginal cost for an import differentiator as:

\[ mc^M_t = \frac{Px^W_t}{\pi^M_t}, \]

(70)

where, as described in the previous subsection, \(Px^W_t\) is the relative world market price, and \(\pi^M_t\) is the price of imported goods relative to Danish goods:

\[ P^M_t = P^M_t \frac{\pi^M_t}{\pi^D_t}, \]

(71)

We define the inflation rate of import prices in Denmark, \(\pi^M_t\), below. Just like domestic and exporting firms, the firms in the import sector face sticky prices as in Calvo [1983]. We can therefore write the optimal price \(\bar{P}^M_t\) chosen by a given import differentiator \(j\) that is allowed to change its price in period \(t\) as:

\[ \bar{P}^M_t (j) = \frac{\epsilon^M_t}{\epsilon^M_t - 1} \sum_{s=0}^\infty (\beta \theta^M_t)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{\text{Im}_{t+s} (j) mc^M_{t+s} P^M_{t+k}}{\text{Im}_{t+s} (j)}, \]

(72)

where \(\theta^M_t\) is the Calvo stickiness parameter in the import sector. \(\text{Im}_t\) denotes Danish demand for imported goods, which consists of two terms: Imports used for consumption by households, and imports used as investment goods by Danish firms. As shown in the appendix, we can write Danish import demand as:

\[ \text{Im}_t = \frac{P^C_t (C_t - C_{DK}^t) + P^I_t (I_t - I_{DK}^t)}{P^M_t}. \]

(73)

In the expression for the optimal price, \(\epsilon^M_t\) is the elasticity of substitution between the goods of each import differentiator, which follows the process:

\[ \left( \frac{\epsilon^M_t}{\pi^M_t} \right) = \left( \frac{\epsilon^M_{t-1}}{\pi^M_{t-1}} \right)^{\rho^M} \exp \epsilon^M_t, \]

(74)

\(^8\)Our modeling of the import sector involves one important drawback: Consider for example a situation where the US dollar appreciates against the Danish krone. In turn, this drives up the aggregate import price faced by Danish households and firms, who in turn choose to buy fewer imported goods from the rest of the world AND from the Euro area, even though the exchange rate towards the Euro is unaffected.
where $\varepsilon^x_t$ is an i.i.d. stochastic process with mean zero and variance $\sigma^x$, and where $0 < \rho_x < 1$. Finally, analogous to the previous subsection, the inflation rate in Danish import prices satisfies:

$$1 = \theta_M \left( \pi^M_t \right)^{1-M} + (1 - \theta_M) \left( \frac{\pi^M_t}{\bar{P}^M} \right)^{1-M}. \quad (75)$$

### 3.11 Market Clearing

We can write the aggregate resource constraint of the Danish economy as:

$$Y_t = \frac{P C_{DK_t}}{P_t} + \frac{I_t^{DK}}{P_t} + G_t + \frac{P_t}{P_t} I_t^G + z^a (u_t) K_{t-1} + \frac{P_t}{P_t} Ex_t. \quad (76)$$

Moreover, Denmark’s net foreign asset position is given by:

$$B_t^{LB} = \frac{R_{t-1} \exp(-\psi d \left( \frac{B^I_{t-1}}{P_t} - \frac{\pi_t^I}{P_t} \right))}{\pi_t^I} B^I_{t-1} + \frac{P_t}{P_t} E x_t - \frac{P_t}{P_t} Im_t, \quad (77)$$

so that net holdings of foreign assets increase if Danish exports exceed imports in a given period.

### 3.12 Stationary equilibrium and steady state

As already described, the model features two deterministic trends: growth in total factor productivity, $A_i$, and in investment-specific technology, $Z_t$. This implies that aggregate macroeconomic variables, such as output and consumption, fluctuate around a balanced growth path. In order to solve the model, we therefore need to rewrite the equations in terms of detrended stationary variables and find the steady state of the stationary model. Observe that we can write the compounded trend growth of these two variables as $\Gamma_t \equiv (A_i Z_t)^{1-\nu}$, where we have taken into account that both public and private capital are affected by investment-specific technological progress.

To obtain a stationary equilibrium, we then make the following transformations of the endogenous variables: We define $\bar{Y}_t = \frac{Y_t}{\Gamma_t}$ as the stationary counterpart of $Y_t$. Similarly, we define $\bar{C}_t = \frac{C_t}{\Gamma_t}$, $\bar{G}_t = \frac{G_t}{\Gamma_t}$, $\bar{T}_t = \frac{T_t}{\Gamma_t}$, $\bar{B}^{DK}_t = \frac{B^{DK}_t}{\Gamma_t}$, $\bar{w}_t = \frac{w_t}{\Gamma_t}$, and so forth, and we define $\bar{K}_t = \frac{K_t}{\Gamma_t}$, $\bar{I}_t = \frac{I_t}{\Gamma_t}$, and $\bar{K}_G^G = \frac{K_G^G}{\Gamma_t}$, where we have taken into account that capital and investment grow at a faster rate than output in the non-stationary model. We also define $\lambda_t = \lambda_t \Gamma_t$ so as to ensure that the shadow price of consumption remains stable as the level of consumption grows, and we let $\bar{Q}_t = Q_t Z_t$, so that the relative price of investment goods changes over time along with investment-specific technological progress.
3.12.1 Steady state

We normalize GDP and the price level in all three economies to 1 in steady-state. These normalisations give us the rest of the prices in the economy: Import-, export-, investment-, and consumer prices and their relative prices. We also assume that the exchange rate is constant against the euro and 1 against rest-of-world. Given the monetary policy regime in place in Denmark, the domestic nominal interest rate is equal to the ECB nominal interest rate. In steady state risk premia for holding foreign bonds are zero.

We further impose that adjustment costs are zero in steady state: cost for changing the import content of consumption and investments, steady state utilization costs, and investment costs are all zero in steady state.

Given steady state exports imposing the law of one price and normalizations of foreign output, the CES-functions for private consumption and investments, and steady state investments, we can derive steady state imports and the net foreign asset position in the steady state.

Following Galí et al. [2011], we set the elasticity of substitution among labour varieties, $\varepsilon^{\text{labW}}$, such that unemployment in steady states equals the Nationalbanken estimate of the natural rate of unemployment of around 4 pct., see Andersen and Rasmusen [2011].

The average quarterly growth rate of Danish real GDP in our sample is around 0.4 pct. The average change of the relative investment price is -0.2. We use these estimates to determine the steady states of the two processes which together determines the growth in the model, $A^A_t, A^Z_t$.

Finally, regarding the fiscal policy side of the model, we can from data observe steady state ratio of public consumption (public expenditures on goods and services as well as public employment), public investments, all tax rates as well as the debt ratio. At the very end of the model, steady state lump sum taxes are determined such that public debts eventually can be expected to be honored.

4 Estimating the model

Our goal is to estimate all the structural shocks in our model and a majority of the parameters. We outline the econometric approach in section 4.1, the data and description of the shocks used in the estimation in section 4.2, the calibrated parameters in the model in section 4.3, while our parameter estimates and prior distributions are discussed in section 4.4.

4.1 Econometric methodology

We confront the model with data using Bayesian methods. In this section we only outline the methodology. For a more thorough introduction to Bayesian estimation of DSGE-models, see among many Smets and Wouters [2003], Smets and Wouters [2007], An and Schorfheide [2007], or a series of papers by Jesus
Fernandez-Villaverde and coauthors; (Fernández-Villaverde [2010], Fernández-Villaverde et al. [2009] or Fernández-Villaverde and Rubio-Ramírez [2007]). We follow the Bayesian approach for a number of reasons. Firstly, as is well-known the use of priors allows us to introduce presample information and to reduce the dimensionality problem associated with the large parameters to data ratio. Secondly, Bayesian methods have well-known and important computational advantages over maximum likelihood in larger DSGE models. That is, the use of simulation provides us with a much easier method to derive the marginal distribution of the parameters in the model than the traditional frequentist maximum likelihood approach involving maximisation. The use of priors gives curvature to a highly dimensional likelihood-function, which is likely to be flat in many dimensions due to poorly identified parameters. Even the most sophisticated algorithms find it hard to find the global maximum of such a function; it is much easier to simulate the posterior distribution of the parameters than to maximize the likelihood function.

Our goal is to report features of the posterior distribution. The Bayesian methodology provides a mapping from the prior distribution to the posterior through data. Let \( \Theta \) denote all the parameters in the model we aim to estimate and let \( Y^T \) denote all the observed data we will use in the estimation. Let \( p(\Theta) \) denote the prior distribution over these parameters. The model implies a likelihood \( p(Y^T|\Theta) \) and we then have a posterior distribution of \( \Theta \):

\[
p(\Theta|Y^T) = \frac{p(Y^T|\Theta)p(\Theta)}{p(Y^T)}
\]

The posterior distribution equals the prior distribution times the likelihood function divided by a scaling factor. It can perhaps be helpful to think of Bayesian analysis as traditional maximum likelihood with a penalty function in form of the prior distributions, \( p(\Theta) \). That is, the prior distribution assign low values to parameter values which the Bayesian econometrician finds implausible.

The posterior distribution, \( p(\Theta|Y^T) \), summarizes the uncertainty regarding the parameter values. The posterior is difficult to characterize and we consequently generate draws from it using a Metropolis-Hastings algorithm, which is our main reason behind using the Bayesian approach. The resulting empirical distribution is used to obtain point estimates, impulse response functions etc. which we will discuss in section (5).

4.2 Data and shocks

4.2.1 Data

We estimate the model on data running from 1990 to 2015. We use the first 5 years as 'training' sample for the Bayesian estimation which we afterwards discard in the analysis. This has the advantage that initial conditions in the historical shock decomposition are likely to have vanished in the sample period, see also section (6). We add the most recent forecast provided by Nationalbanken for the 3 year period after the sample to get better estimates of the long
term trend in data. The latter must be seen as a consequence of the crises period which might bias the long-run growth downwards leaving a worse fit at the end of our sample. Strictly speaking we should allow for a structural break but we leave that for future work. We use data from 1995 although the European currency union was not in place before 1999. Hence, we weight the costs of having less data less than the costs of using data from a group of countries within a currency union, which was not in place at that point in time. We believe, however, that the initial euro area countries to some extent shared business cycles already in 1995, as also studied by Dam [2008].

In estimating the model, we use time series for 22 macroeconomic variables. The following time series for Danish variables are taken from Statistics Denmark and Nationalbanken:

<table>
<thead>
<tr>
<th>Real GDP</th>
<th>Unemployment (net)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private consumption</td>
<td>Industry nominal wages deflated by CPI</td>
</tr>
<tr>
<td>Government consumption including public production</td>
<td>Investment deflator</td>
</tr>
<tr>
<td>Government investment</td>
<td>PPI index</td>
</tr>
<tr>
<td>Exports</td>
<td>Import deflator</td>
</tr>
<tr>
<td>Imports</td>
<td>Export deflator</td>
</tr>
<tr>
<td>Labour income tax</td>
<td>Total investment including inventories and construction</td>
</tr>
<tr>
<td>Private employment</td>
<td></td>
</tr>
</tbody>
</table>

As explained in the text, the rest of the world variables are defined as the weighted sum of the GDP, inflation or policy rate of the Danish trading partners excluding trading partners within the euro area. The two economies are approximately of equal size. Data is taken from the Ecown data base and OECD. For the foreign economies we use the following variables:

- Inflation in the euro area and in the rest of the world
- Real GDP in the euro area and in the rest of the world
- ECB policy rate
- Implied rest of the world policy rate
- Effective exchange rate

Prior to estimation, we transform the time series into quarter-on-quarter growth rates, approximated by the first difference of their logarithm. As explained above, in the model, we include a trend in productivity and in the
relative price of investments goods. The variables in the model therefore have
trends and consequently, we do not demean data. Also, a number of additional
transformations are made to ensure that variable measurement is consistent with
the properties of the model’s growth path. Firstly, we remove the sample growth
erate differentials between the export and import variables and Danish GDP, as
these variables in the sample have grown faster than GDP reflecting globalisa-
tion, see figure (2). Secondly, for the effective exchange rate we band-pass filter
and demean the data. Lastly, we HP-filter data for the foreign economies as we
do not model trends in the foreign economies.

Data for Denmark including the effective exchange rate against the rest-of-
world is shown in figures (3), (4) and (5). The time-series used in the estimation
for the foreign economies are shown in figure (6).

4.2.2 Shocks

The model includes all in all 26 structural shocks but we only use 21 including
the shocks from the exogenous AR-models for public consumption, public
investments and labour income tax in the estimation of the model. Finally, we
use the 6 shocks from the DSGE-model for the two foreign economies:

- Public consumption shock, $\epsilon^G$
- Perm tech shock, $\epsilon^{perm}$
- Wage markup shock, $\epsilon^W$
- Temp. tech shock, $\epsilon^A$
- Consumption shock, $\epsilon^{prf}$
- Price markup shock, $\epsilon^P$
- Investment shock, $\epsilon^{Inv}$
- Export shock, $\epsilon^{XEx}$
- Import shock, $\epsilon^{Im}$
- Import price markup shock, $\epsilon^M$
- Export price markup shock, $\epsilon^X$
- Riskpremia shock, $\epsilon^{Rp}$
- Public investment shock, $\epsilon^{ig}$
- Tax on labour income shock, $\epsilon^{ta}$
- EA price shock, $\epsilon^{piEA}$
- EA output shock, $\epsilon^{gEA}$
- ECB policy rate shock, $\epsilon^{rEA}$
- RoW output shock, $\epsilon^{yRW}$
- RoW price shock, $\epsilon^{piRW}$
- RoW interest rate shock, $\epsilon^{rRW}$
- UIP shock, $\epsilon^{UIP}$

All shocks are assumed to follow first-order autoregressive processes, except
for the shock to the ECB policy rate, the shock to the policy rate for the rest of
the world, which both are white noise, and the shocks to public spending, public
investment and the labour income tax, which follows higher-order autoregressive
processes.

The shocks which are not used in the estimation are shocks to three taxes,
capital income tax, $\epsilon^K$, VAT, $\epsilon^VAT$, and tax on bond income, $\epsilon^B$. These tax
rates have been kept almost fixed throughout the sample but we keep the shocks
in the model for policy purposes. We also do not include shocks to the trend in the relative price of investment goods, $\xi_{zt}$. Lastly, we do not use the labour supply shock, $\xi_{lt}$, as the model was not able to distinguish between this shock and the wage markup shock in the estimation. In the estimation we assume that all data are measured with measurement error except for the ECB policy rate.\(^9\)

### 4.3 Calibration

It is well known that some parameters in DSGE-models are hard to identify and we do not assume that our model is any different. Also, some parameters such as the depreciation rate on private capital, $\delta$, are better estimated using micro data. The parameters we have calibrated in the estimation are shown in table (2). Also, we do not estimate the standard deviations of non-estimated shocks as described above, $\varepsilon^{t_o}$, $\varepsilon^{VAR}$, $\varepsilon^{tk}$, and $\varepsilon^{L}$, nor their autoregressive parameters. These are only in use for policy analysis.

### 4.4 Parameter estimates and Prior Distributions

Tables (3), (4), and (5), show our assumptions regarding the priors and the results of the estimation. That is, the posterior mode estimates of the structural parameters and the shocks in the model.

#### 4.4.1 Priors

The prior distribution for the estimated parameters are shown in the left column of the tables. Regarding the priors, we in general follow the literature and make broadly similar assumptions about our priors, see Christoffel et al. [2008], Adolfson et al. [2013], Burriel et al. [2010]. The prior distributions for the parameters are chosen in conformity with the constraints on the parameter space implied by theory. For those parameters that are bounded between 0 and 1, we choose a standardised beta distribution. For parameters that are bounded from below at zero, we have chosen either a gamma or an inverse gamma distribution to model the prior distributions. An easier way to evaluate the choices we have made regarding our priors is to plot them. First, we discuss the posterior distributions.

#### 4.4.2 Posterior estimates

The right hand side of tables (3), (4), and (5) show the posterior distribution of the parameters for our preferred specification of the model. The entries in the posterior-mode column refer to the values of the estimated parameters that are obtained by maximizing the model’s posterior distribution. The distributions

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\(^9\)We do assume measurement errors for the interest rate in the rest-of-world, as this variable is, so to speak, an artificial variable constructed mainly by aggregating the monetary policy rates in Sweden, U.K. and the U.S.
are computed based on a Markov chain with 1.000.000 draws. We use Dynare build-in figures for convergence (not shown for brevity). We found that the 1.000.000 draws was enough to obtain convergence.

We highlight the following in our estimation results. On the labour market, we find a elasticity of labour supply of 2.9. The inverse of this parameter determines the size of the elasticity of employment with respect to the real wage holding consumption constant. We note that the elasticity is with respect to employment and not hours. Hence, the elasticity determines how many workers substitute in and out of employment in response to changes in wages, and the parameter is not related to the hours worked by a particular person. We find a value for \( v \) of around a 2/3. This implies some degree of wealth effect on the labour supply. This has implications for as an example the size of the fiscal multiplier, see Monacelli and Perotti [2008].

On the nominal side of the economy, we find some differences across sectors of the economy. The estimate of the Calvo parameter is quite low for domestic goods, \( \theta_p \), with a value of around 1/3. This a quite low and much lower than found in other studies. It is also in contrast much larger for import prices while almost the same for export prices. Wages are according to the model estimates more sticky than the prices. This might reflect that a large share of Danish wages are set according to 2-3 year wage agreements. The relative flexibility of export prices might reflect that Danish exporters can not to a great extend rely on the nominal exchange rate to do the adjustment to changes in market conditions. Also, the indexation of export prices is almost zero, while quite large for wages with a value of around a half. Regarding the the shocks, we do not find that some shocks dominates in terms of having a big standard deviation.

Another way to check the quality of the estimation is by comparing the prior and posterior distributions of each parameter. As discussed above, this is also a method to evaluate the choices we have made regarding the priors. This is done in figures (7) to (13). In general the figures show that the data is informative about the posterior distribution. That is, the posterior distribution is not equal to the prior. However, some parameters do seem to be 'defined' by their prior; as an example the capital utilization cost parameter. This may reflect that our data sample is not informative about these parameters. Our parameter estimates can from this perspective be regarded as calibrated or can be viewed as being estimated with a high degree of outside information.

As revealed by the figures, some priors are set quite tight. That reflects to a large degree a necessity; without these tight priors the model would not work well in some important dimensions like impulse response functions. Finally, the point estimates for the autoregressive parameters of shock processes show that some shocks are very persistent, especially those related to temporary technol-
ogy, \( \rho_A \), consumption shocks, \( \rho_c \), and export shocks, \( \rho_{XE} \). This can suggest that the model has some difficulties in generating the level of endogenous persistence present in the data and therefore the model opts for these exogenous shocks to be highly persistent.

Figures (14) to (17) display the smoothed estimates of the structural shocks. We have 21 shocks so we have chosen to highlight the following observations.
Regarding the foreign shocks, the financial crisis is clearly visible in the ECB policy rate and euroarea inflation, while less so for rest of world prices. The crisis period can also be recognized in the wage markup shock which increases up until 2007, whereafter it falls sharply. The same pattern can be observed for the domestic price markup shock. The build-up to the crisis is also recognizable in the import shocks and especially for the consumption shock. As expected, the consumption shock is much less volatile than investment shocks as theory tells us it should be. The tax shock is quite small but tax reforms do show up in the shock. The shock to inflation and output in rest-of-world has some evidence of non-stationarity. As in common in the literature, we also find quite large price markup shocks, while wage markup shocks on the contrary are quite small.

5 Model properties

Having looked at the prior-posterior plots, point estimates and smoothed shocks, we now move on to study the models’ empirical properties. We do this by reporting the models’ impulse-response functions in section (5.1). Here we also compare for selected impulse-response functions with the impulse-response functions from a structural VAR. We move on to study forecast-error-variance decompositions for various horizons in section (5.2).

We first highlight some results from the model which may seem in contrast to comparable results from other medium-sized DSGE-models like the ones in Christoffel et al. [2008], Adolfson et al. [2013], or Burriel et al. [2010]. The key to the differences is straightforward: Denmark’s fixed exchange rate regime.

Firstly, the variance decomposition shows that foreign shocks are the most important drivers of the Danish business cycle. This stands in contrast to existing results, see, e.g., Justiniano and Preston [2010]. The fixed exchange rate regime implies that shocks originating in the Euro area have a direct effect on the Danish economy. Hence, euro-area shocks are transmitted directly to the Danish economy through the interest rate, and therefore have a large impact on the Danish business cycle on top of the effects via international trade. Secondly, all shocks which affect inflation cause an over- or undershooting. This is because in the model all goods are tradable and because the Danish economy can effect neither foreign prices nor exchange rates due to the fixed exchange rate regime and the small open economy assumption. This implies that the steady state level of Denmark’s terms of trade needs to be achieved eventually. This provides the economy with a nominal anchor and ensures determinacy of the model. The nominal anchor imposes a lot of restrictions on the economy and give rise to differences between the effect of shocks in the model in this paper and the previously mentioned papers.

\footnote{For example, while the effects on the Danish economy of a rise in the interest rate in each of the two foreign regions are qualitatively very similar, the impact is roughly twice as big after a rise in the euro-area interest rate as compared to a rise in the interest rate in the rest of the world.}
5.1 Impulse-response functions

We focus on a few of the most important shocks in our analysis of the impulse-response functions, namely a transitory technology shock, a domestic price markup shock, a shock to foreign output, a shock to the ECB policy rate, and the most important policy instrument for Denmark, government spending. The interest rate shock provides a view on the monetary policy transmission mechanism from the ECB to the Danish economy, while the government spending shock both provides a fiscal multiplier and an example of a demand shock. The first three shocks constitute examples of supply, cost-push, and foreign demand shocks respectively. We further compare the monetary policy shock, the public consumption shock and the foreign consumption shock with impulse-response functions from a structural VAR. We focus on the effect on a subset of the endogenous variables in the model.

5.1.1 Consequences of a temporary technology shock

The impulse-response to a positive shock to the temporary technology shock of size one standard deviation is shown in figure (18). A temporary technology shock decreases real marginal costs and hence domestic prices initially. However, the forward looking firms correctly anticipates higher supply and demand in the economy. As it turns out, the higher demand effect in the estimated economy rapidly mitigates the increase in inflation.

The increase in aggregate demand both stems from the initial lower prices but also from wealth effects on consumption: The forward looking households realize that they have become more productive and hence richer which induces them to consume some of the wealth today. This is amplified by a subsequent drop in the real interest rate due to higher inflation, which is not combatted through nominal interest rate increases. The latter reflects the fixed-exchange rate regime in Denmark. The more productive capital leads to an investment boom further increasing demand. Exports fall due to the subsequent rise in inflation. Together with increasing imports, this counteracts the rise in domestic demand from investments and consumption.

Turning to the labour market, higher productivity increases supply of goods in the economy leading to an increase in the demand for labour. That is, the workers have become more productive which causes a outward shift in the labour demand curve leading to higher real wage and employment. This is the so-called classic effect, but not the only effect. Workers have become more productive and richer which makes them decrease their labour supply - a wealth effect on employment. This puts a further upward pressure on real wages but downward pressure on employment. If the model was model fully "Keynesian", then employment would fall if aggregate demand did not rise, as more goods in response to a technology shock can be produced with fewer workers. However, as explained above, aggregate demand does rise and initially pushes employment up.
5.1.2 Consequences of a domestic price markup shock

The impulse-response to a one standard deviation positive shock to the domestic price markup is shown in figure (19). A positive shock can be interpreted as a situation in which domestic firms set a higher markup over marginal costs. That is, the firms set a higher price all else being equal. This causes inflation to rise, but only in the first few quarters, despite the fact that the shock is quite persistent with a half-life of around $2\frac{1}{2}$ quarters. This is because the forward-looking firms anticipate that the price increase leads to a decline in economic activity and thereby lower marginal costs. Ultimately, firms seek to reestablish the optimal relationship between prices and marginal costs and therefore decrease their prices over time.

The shock leads to lower output because households face higher prices. This effect is mitigated by lower real interest rates, as the policy rate does not react to the higher inflation. After a few quarters, the decline in prices leads to an improvement in competitiveness and higher exports, which helps in driving output back towards the steady state. The decline in aggregate demand causes employment to fall and exerts downward pressure on real wages.

5.1.3 Consequences of a shock to foreign demand

While less obvious for the technology and domestic markup shock, we have an observable variable for foreign demand, which can be included into a VAR and compared to the estimated impulse-response functions produced by the DSGE model. We do the same comparison for shocks to the ECB policy rate and public consumption in the subsequent sections. The VAR is identified using a Choleski decomposition. We use the same data as in the estimation of the DSGE models and all series are in logs except for inflation and interest rates, which are included in levels. We order the ‘shock’ variables last in the VAR. This implies that Danish variables in the VAR cannot react in the first quarter to shocks from the relevant foreign variables. By including the relevant variables last in the causal structure we try to ‘control’ for exogenous movements in the other variables in the VAR. This can be considered the most cautious approach, see Abildgren [2010].

The results for a shock to GDP in the rest-of-world region are shown in figure (20). Notice that we do not provide SVAR results for all the endogenous variables in the figure. We look at a one percent increase in rest-of-world output. In the following quarters GDP in Denmark rises both in the SVAR and in the DSGE-model though the effect is much larger in the DSGE-model. Both models predict a negative response from around 4 quarters. The recession is, however, more severe in the DSGE-model than in the SVAR. The initial larger response in the DSGE-model can be attributed primarily to the identification strategy in the SVAR.

Private consumption hardly moves in the DSGE-model while export booms. This boom pushes employment and the real wage upwards with higher marginal costs as a consequence, and thus higher inflation. This higher inflation is, as
usual in our DSGE-model, followed by deflation such that the initial loss of competitiveness can be regained. This deflation is the main cause behind the recession observed for Danish GDP and can be attributed to a drop in exports after around 5 quarters and a subsequent rise in imports. Finally, notice that the uncertainty bounds in the DSGE-model are much smaller than in the SVAR. This can be explained by the estimation procedure in which the foreign block is estimated separately of the Danish model. The uncertainty regarding parameter values in the foreign block consequently does not show up in the confidence bands.

5.1.4 Consequences of a shock to the ECB monetary policy rate

Next we consider the effects of a shock to the ECB policy rate. This is the de facto monetary policy rate for the Danish economy in normal times. In the VAR we order the ECB policy rate after the real variables in the causal structure, following Christiano et al. [1999]. We normalize the shock so that it is equivalent to a 25 basis points increase in the ECB policy rate. The impulse responses from the VAR and the DSGE-model are pictured in figure (21). Notice that again we do not provide SVAR results for all the endogenous variables in the figure.

GDP in the VAR stays unchanged in the first quarter, which is due to the identification of the VAR. GDP in the DSGE model falls around 0.15 percent on impact. The shock is less persistent in the DSGE-model than in the VAR-model and only statistically significant during the first year or so.

Notice that in the DSGE-model a shock to the interest rate in the euroarea does not imply that the interest rate is the only variable that moves. The euroarea block, GDP, inflation and the policy rate, is estimated as a small scale DSGE-model along the line of Galí [2009] and Woodford [2003]. Hence, a positive shock to the policy rate in the euroarea also causes a fall in GDP and inflation in the euroarea. The fall in euroarea GDP puts downward pressure on Danish exports, all else equal. However, the fall in inflation in the euroarea is smaller than the fall in Danish inflation causing an improvement in Danish competitiveness. This depresses imports and stimulates exports and therefore GDP in Denmark. This explains the strong rebound of Danish GDP.

We notice that the initial fall in GDP for Denmark as a consequence of contractionary monetary policy is of about the same size as the estimated effect in DSGE-models for the euroarea, see Christoffel et al. [2008]. We also notice that the effects of a monetary policy shock from the rest-of-world economy are qualitatively similar, but that quantitatively speaking, the effects are only little more than half as big as the effect from the monetary policy shock from the euroarea. This is due to the fact that the monetary policy rate enters directly in the Danish economy, which partly explains why the foreign shocks contribute so much to the variation in Danish GDP, as discussed in section (5.2) below.

5.1.5 Consequences of a government spending shock

Ravn and Spange [2012] did a VAR-based analysis of the effects of a temporary
increase in public consumption. We refer to that paper for a detailed description of the VAR-model. Here we use and compare the results in that paper with the estimated response to expansionary fiscal policy in the DSGE-model. That is, we compare the implied fiscal multipliers. In figure (22) we compare the impulse response functions from Ravn and Spange [2012] to the impulse response functions from the estimated DSGE-model. For a deeper analysis of the effects of fiscal shocks under fixed exchange rates in a DSGE-model we refer to Pedersen [2012].

The following observations from this analysis merit some comments. The confidence bands produced by the VAR are a lot wider than those from the DSGE-model, and the persistence of the shocks are approximately equal. The fiscal multiplier in the DSGE-model resides within the confidence bands produced by the VAR-model although the multipliers are somewhat different. As can be seen from the figure, the DSGE-model predicts a fiscal multiplier of around 0.7, which is substantially smaller than the fiscal multiplier predicted by the VAR of around 1.3. The results are comparable to the findings of Pedersen [2012], who discusses fiscal multipliers in a DSGE-model calibrated to the Danish economy.

The economic effects driving the multiplier are quite standard: Consumption falls due to a wealth effect, imports increase, while exports decrease due to loss of competitiveness as inflation rises. The increase in inflation is, however, shortlived: Danmark needs to regain competitiveness and therefore the economy experiences deflation.

In the labour market, the wealth effect gives rise to a rightwards shift in the labour supply curve as the households feel poorer and thus work more. This puts downward pressure on wages and drives up employment. At the same time, however, the firms wish to meet the extra demand as their prices are sticky and they do so by hiring more labour, pushing real wages up. As seen from figure (22), real wages increase and therefore the demand effect on the labour market wins. That is, the existence of nominal rigidities makes the wedges in the economy move countercyclically, driving up real wages. After some quarters real wages start to fall.

There is one common theme from the analysis of the impulse response functions in this section: Increasing prices, due to either cost-push, productivity shocks or demand shocks, leads to a subsequent deflationary period as the terms of trade need to revert to the initial value. If not, Denmark would be loosing competitiveness indefinitely, and hence a share of world trade. This is because Denmark is a small open economy which cannot influence the macroeconomic state of its trading partners and due to the fixed exchange rate: Deflation is the only means to regain the initial terms of trade.

5.2 Forecast-error-variance decompositions

In tables (6), (7), and (8) we show the contribution of the structural shocks in the model to the forecast error variances of a selected set of observed variables. This exercise tries to provide a quantitative insight as to which structural shocks on
average in the estimated model during the sample period give rise to variability in the endogenous variables in the model. For simplicity we only look at the variance decomposition for the 1st, 4th, and 40th quarter, and in what follows we concentrate on the variance decomposition for real GDP.

The tables illustrate that foreign shocks are very important drivers of the Danish business cycle. At all horizons, the group of structural shocks originating in the two foreign economies (inflation, output, interest rates, and the effective exchange rate) account for between 30 percent of the variations in real GDP at the very short term, 1 quarter, to around 60 percent at longer horizons, 40 quarters. In particular, the shock to output in the euro-area is by far the largest contributor to movements in Danish GDP at the 4- and 40-quarter horizon. This should not be surprising, as Denmark is a very small and open economy with a fixed exchange rate towards the Euro.

Another important source of GDP variations is the temporary technology shock, which contribute to between almost 40 percent at the one quarter horizon falling to around 15 percent of the variations in real GDP at longer horizons. Permanent technology shocks have a contribution of virtually zero at short horizons, but gain some importance at the 40-quarter horizon. Domestic price markup shocks account for around 5 percent at very short horizons, and roughly half that number after 40 quarters. The investment shock accounts for only around 10 percent of GDP fluctuations (but 50 percent of fluctuations in investments themselves) at the shortest horizons.

While our finding that variations in Danish GDP are to a large extent driven by shocks from abroad may not seem very surprising, it does stand in contrast - and remarkably so - to the results of Justiniano and Preston [2010]. After estimating a small open economy model in the tradition of Gali and Monacelli [2005] for Canada, they study the contribution of shocks to the US economy for fluctuations in Canadian GDP. They find that US shocks account for less than 3 percent of the movements in a number of Canadian macroeconomic variables, including GDP, at all horizons. In line with this result, the estimated DSGE model of the Swedish Riksbank, see Adolfsson [2007] also ascribes less than 10 percent of GDP fluctuations in Sweden to foreign shocks. Obviously, the main difference between Denmark and these other small open economies is that Denmark has pegged its currency to the Euro, whereas Canada and Sweden both have a flexible exchange rate. Aastveit et al. [2013] suggest two main reasons for the small explanatory power typically assigned to foreign shocks in estimated small open economy models: The absence of other cross-country linkages than the trade channel (for example via financial markets or consumer and investor sentiments), and the lack of direct effects of foreign-economy shocks on domestic variables (for example through common shocks). A fixed exchange rate fills both of these gaps, as it makes way for another channel, the interest rate channel, through which shocks originating in the Euro area have a direct effect on the Danish economy. As the variance decomposition shows, the contribution of shocks in the Euro area is much larger than that of shocks in the rest of the world for output fluctuations at medium and long horizons, as well as for movements in inflation and private consumption at all horizons. This confirms
that Euro area shocks are transmitted directly to the Danish economy through the interest rate, and therefore have a large impact on the Danish business cycle on top of the effects via international trade.

Finally, even though the transmission of shocks works through the interest rate, the variance decomposition shows that interest rate shocks in the Euro area are much less important than output shocks. The explanation is that movements in the Euro area interest rate, which is set according to a Taylor rule, are primarily driven by shocks to Euro area output (and inflation), whereas monetary policy shocks are less important. These insights, which are confirmed by the variance decomposition of the Euro area interest rate, merely suggest that the rule-based component of monetary policy is much more important than monetary policy shocks, in line with a number of empirical studies, see e.g. Christiano et al. [1999].

6 Applications - Historical Shock Decomposition

In this paper we focus on a historical shock decomposition of Danish real GDP throughout our sample and leave forecasting exercises and counterfactual analysis to future work. We decompose real GDP into the contributions of the models' 21 structural shocks. To facilitate the presentation, we group the structural shocks into five categories: markup, demand, productivity, the foreign economies, and fiscal policy.

- Markups: Domestic, export and import price markup and wage markup shocks
- Demand: Consumption, investment, import, export shocks
- Productivity: Temporary and permanent technology shocks
- Foreign shocks: All shocks to production, inflation, interest rates and UIP-shocks from euro area and rest-of-world
- Fiscal policy: All shocks to taxes and public consumption and investment

We show in figure (23) the combination of structural shocks which according to the model have given rise to the historical development in real GDP for Denmark in the sample period. The thick line in figure (23) shows the actual development in GDP around an estimated growth rate and we will in what follows denote it as the output gap.\(^\text{11}\) The sum of all the structural shocks add up to the quarter-to-quarter growth rate for GDP. A group of variables taken together has contributed positively to growth in GDP in a given quarter if their sum is positive.

\(^{11}\)This is not equivalent to the output gap in DSGE terminology. We leave the 'DSGE output gap' - the difference between actual output and the level of production which would have prevailed in the absence of any nominal rigidities - to future work.
Danish business cycles are easily detectable in figure (23). In the beginning of our sample GDP is below trend but on an upward path after the low-growth period during the beginning of the 90's. This cycle ends at the beginning of the 00's. Around this period, the Danish economy experienced a shortlived and mild economic downturn. This downturn was succeeded by a large upturn culminating at the outbreak of the financial crisis. Denmark was, as many developed economies, hit hard by the financial crisis causing a large output gap, which stays negative for the remainder of the sample.

The next subsections dig deeper into the effects of these 5 subgroups of structural shocks on the historical development of real GDP in Denmark. We will look at the full sample but emphasize the build up to the financial crisis and the following downturn.

6.1 Foreign shocks

The first observation we make is that by comparing the 5 subgroups the group of foreign shocks provides the greatest impact on the historical development of Danish real GDP. This confirms the results in the variance decomposition in section (5.2).

In figure (24) we have decomposed the sum of the foreign shocks in the respective shocks within the group: Production, inflation, interest rates in the two economies as well as shock to the exchange rate. At the beginning of the period the model identifies a negative contribution from shocks which originate in the foreign economies. Likewise, we observe a negative contribution from the foreign shocks from around 1998, which from around 2000 turns into positive contributions due to the economic upturn in both USA and Germany. In 2001 the US economy is hit by a short recession coinciding with lower growth in Germany, one of Denmarks biggest trading partners. This shows up in figure (24) as a negative contribution from the foreign shocks. Finally, the Danish economy booms along with the global economy from 2005 until the outbreak of the financial crisis.

As can be seen from figure (24), at the onset of the financial crisis the contraction in output is counteracted by low interest rates in both the euroarea and rest-of-world. Later on, the zero-lower bound starts to bind and therefore the interest rates contribute negatively to GDP growth; the economic situation would prescribe even lower interest rates but the zero-lower bound restricts the central banks from doing so.

At the end of our sample, the contribution of the foreign sectors is roughly zero. This is the result of two offsetting effects: While the rest-of-world block exerts a positive effect, likely reflecting the economic recovery and maintained expansionary monetary policy in the US, the euro-crisis still contributes negatively to the growth in Danish GDP.
6.2 Demand shocks

Shocks to domestic demand contributed to the strong growth in Danish GDP before the outbreak of the financial crisis, as seen from figure (23). We show in figure (25) the contribution of each shock in this group to movements in the output gap. The subgroup consists, as an example, of investment shocks. A positive shock to investments means that a given investment in capital in the current period leads to a bigger increase in the stock of installed capital in the next period than what would normally be the case. In the model a shock to investment also consists of residential investments as we do not have a separate residential sector in the model, and because the observed data series for investments consists of all investments as we wish to cover the entire national account. We therefore assign an important part of the large, positive investment shocks during the period 2005-2008 to higher residential investments.

From figure (25) we also observe that shocks to Danish households’ preferences for imported goods contributed positively to the output gap during the period 2005-2008. This may reflect that Danish households were more eager to buy Danish goods and services than usually. A possible explanation to this may be that Danish households consumed a lot of ‘residential’ goods and services.

6.3 Markup shocks

We now turn to the group which consists of shocks to the economy’s markup shocks. This subgroup consists of shocks to wage and price (domestic prices, import prices and export prices) markups. We decompose the contribution of the markup shocks to the growth rate in real GDP in figure (26).

In the build-up to the financial crisis, the markup shocks taken together affected the output gap negatively. This is primarily due to the domestic markup shock which was relatively high before the crisis. The economic intuition behind this observation is that domestic producers in those years utilized the extraordinary high domestic and foreign demand to increase their margins more than usually. That is a sign of an overheated economy.

A bit more surprising is that figure (26) shows that the wage markup shock affected GDP positively during the build up to the financial crisis, 2005-2008. This implies that the wage markup was relatively low in these years. Looking back at figure (1) in section (2), a relatively smooth increase can be observed in the industrial wage deflated by the consumer price index, along with a fall in unemployment and a strong increase in employment (not shown). Only in the latter phase of the build-up to the financial crisis an equally strong increase in real wages can be observed. The limited wage increases may be attributed to inflows of foreign labour and increases in the labour force. To explain these movements the model identifies a decrease in the wage markup, which contributes positively to GDP growth.

The subsequent bust leads to a downward pressure on prices due to a drop in demand. As they anticipate falling domestic and foreign demand, domestic producers lower their margins, which affects GDP positively. On the contrary,
the wage markup contributes negatively to the growth rate in GDP. This can be explained by a rapid increase in unemployment, while wages did not fall accordingly, see also figure (1).

6.4 Fiscal policy

The Danish economy had a large positive output gap before the financial crisis. Our analysis has indicated that this output gap can be explained by a combination of strong global growth and a considerable growth in domestic demand.

In a fixed exchange rate regime such as Denmark’s, the monetary policy rate is determined by the development of the Danish exchange rate vis-a-vis the euro. This implies that monetary policy can not be used to dampen domestic demand and price pressures. This implies that fiscal policy plays the role as the main stabilisation instrument in the Danish economy.

The last subgroup in figure (23) is fiscal policy. This subgroup mainly consists of public consumption and investment as well as the labour income tax. Focusing on the period leading up to the financial crisis, figure (23) shows that fiscal policy throughout most of the period 2005-2007 had a stimulative effect on the economy and hence, added to the domestic demand; or was neutral. Given the circumstances described above, it seems clear that an expansionary (or even a neutral) fiscal policy in these years was not appropriate; instead, fiscal policy should have been strongly contractionary so as to avoid the subsequent overheating of the Danish economy.

As many other developed economies, Denmark conducted massive fiscal stimulus policies after the crisis. These policies show up in figure (23) as contributing positively to the growth in GDP especially in 2009. On the other hand, the subsequent austerity measures in Denmark seem not to have dragged down economic growth significantly.

7 Conclusion

In this paper, we have set up a newly developed DSGE-model for the Danish economy, and shown and explained how to estimate it using Bayesian techniques. We have presented estimation results and examined the empirical properties through impulse-response functions and variance decompositions. Our results indicate that the model has economically plausible properties. Finally, we have used the model to compute a historical shock decomposition of Danish real GDP growth.

The model is, however, not finished and we expect a continuing development in light of economic and academic developments. We can easily point to some improvements: We have not provided a DSGE-based output gap, the model does not have inputs of imported goods in the production function, and finally, and perhaps most importantly, the model does not feature house prices, financial frictions or banking. In the future, we plan to incorporate these elements in the model. Also, we think that the set of possible applications of this model is fairly
large. As an example, we have not talked about forecasting or counterfactual analyses in this paper.
8 Appendix: The euroarea and rest of world models

The main impacts of the foreign countries on the Danish economy work through trade and interest rates. We consequently aim for the most flexible model for these two economies and downplay the microfoundations. Further, we do not aim to estimate a common trend for all the three economies as the data points to different steady state growth rates in output. Denmark’s role as a small open economy implies that we can model the foreign economies as being exogenous to the Danish economy. Also, we do not aim to model the interrelations between the euroarea and rest of the world, and we consequently do not model trade between these two economies. We do, however, include a UIP-relation between the policy rate in the two countries so that we can pin down the effective exchange rate between Denmark and rest of world.

The model equations are shown in section (3.8) while the parameter estimates are shown in table (1). We use the same sample as for the Danish economy. Data are HP-filtered and shown in figure (6). We estimate the international linkages using a two-step procedure: In the first step, we estimate the two separate small-scale DSGE models of each of the foreign blocks. In the second step, where we estimate the main model for Denmark, we include these estimated relations, and then estimate the shocks in the foreign models by including again the data for the foreign economies.

9 Appendix: Market Clearing

In this appendix, we demonstrate that all markets clear in the model. Start from the budget constraint of the household, which reads:

\[ (1 + \tau^V AT) \frac{P^C_t}{P^t} C_t + \frac{P^I_t}{P^t} I_t + B^DK_t + B^I_t + T_t \]

\[ = \Pi_t + \left( (1 - \tau^K_t) r^K_t u_t + \tau^K_t \delta^K_t - z^u(u_t) \right) K_{t-1} + \frac{R_{t-1} B^{DK}_{t-1}}{\pi^DK_t} + \]

\[ + \frac{R^ECB_{t-1} \exp(-\psi_d \left( \frac{B^I_{t-1}}{s_t} - \frac{B^{DK}_{t-1}}{\pi^DK_t} \right)) B^I_{t-1}}{\pi^DK_t} - \frac{\tau^K_t B^{DK}_{t-1} (R_{t-1} - 1)}{\pi^DK_t} + \]

\[ + (1 - \tau^K_t) w_t N_t + \kappa^B w_t U^N_t. \]
Now use the government’s budget constraint to insert for $B_t^{DK}$ on the left-hand side, and rewrite to obtain:

$$(1 + \tau_t^{VAT}) \frac{P_t^C}{P_t} C_t + \frac{P_t^I}{P_t} I_t + \left( \frac{R_t-1}{\pi_t^{DK}} B_{t-1}^{DK} + G_t + w_t U^N t \right) + B_t^I + T_t$$

$$= \Pi_t + \left( (1 - \tau_t^K) r_t^K u_t + \tau_t^K \delta^K - z^u (u_t) \right) K_{t-1} + \frac{R_t-1}{\pi_t^{DK}} B_{t-1}^{DK} +$$

$$\frac{R_{t-1}^{ECB} \exp(-\psi_d \left( \frac{B_t^I}{Y_t} - \frac{\pi_t}{Y_t} \right)) B_t^I}{\pi_t^{DK}} - \tau_t^B B_{t-1}^{DK} \left( R_t-1 \right) - (1 - \tau_t^n) w_t N_t + \kappa^B w_t U^N t.$$

Next, insert the expressions for $TR_t$ and $G_t$ presented in the main text, as well as for profits, which we can write as $\Pi_t = Y_t - w_t N_t - r_t^K u_t K_{t-1}$:

$$(1 + \tau_t^{VAT}) \frac{P_t^C}{P_t} C_t + \frac{P_t^I}{P_t} I_t + B_t^I + T_t - (Y_t - w_t N_t - r_t^K u_t K_{t-1})$$

$$= \left( (1 - \tau_t^K) r_t^K u_t + \tau_t^K \delta^K - z^u (u_t) \right) K_{t-1} - \frac{R_t-1}{\pi_t^{DK}} B_{t-1}^{DK} + \tau_t^B B_{t-1}^{DK} \left( R_t-1 \right) -$$

$$\frac{R_{t-1}^{ECB} \exp(-\psi_d \left( \frac{B_t^I}{Y_t} - \frac{\pi_t}{Y_t} \right)) B_t^I}{\pi_t^{DK}} - (1 - \tau_t^n) w_t N_t - \kappa^B w_t U^N t.$$

We can now begin to cancel out terms:

$$\frac{P_t^C}{P_t} C_t + \frac{P_t^I}{P_t} I_t + B_t^I - Y_t + z^u (u_t) K_{t-1} - \frac{R_{t-1}^{ECB} \exp(-\psi_d \left( \frac{B_t^I}{Y_t} - \frac{\pi_t}{Y_t} \right)) B_t^I}{\pi_t^{DK}}$$

$$= -C_t^G - \frac{P_t^I}{P_t} I_t^G,$$

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which we can rewrite as:

\[ Y_t = \frac{P^C}{P_t} C_t + \frac{P^I}{P_t} I_t + C^G_t + \frac{P^I}{P_t} I^G_t + z^u (u_t) K_{t-1} + \left( B_t^I - \frac{R_{ECB}^{ECB} \exp(-\psi_d \left( \frac{B_{t-1}}{Y_t} - \frac{\pi^I}{\pi^D} \right))}{\pi_t^{DK}} B_{t-1} \right). \]

Now apply the resource constraint as defined in the main text:

\[
\begin{align*}
\frac{P^C}{P_t} C_t + \frac{P^I}{P_t} I_t + C^G_t + \frac{P^I}{P_t} I^G_t + z^u (u_t) K_{t-1} + \left( B_t^I - \frac{R_{ECB}^{ECB} \exp(-\psi_d \left( \frac{B_{t-1}}{Y_t} - \frac{\pi^I}{\pi^D} \right))}{\pi_t^{DK}} B_{t-1} \right) \equiv \frac{P^X}{P_t} E_t - \frac{P^C}{P_t} (C_t - C_t^{DK}) - \frac{P^I}{P_t} (I_t - I_t^{DK}) = B_t^I - \frac{R_{ECB}^{ECB} \exp(-\psi_d \left( \frac{B_{t-1}}{Y_t} - \frac{\pi^I}{\pi^D} \right))}{\pi_t^{DK}} B_{t-1}^I.
\end{align*}
\]

The final step is to apply the equation determining Denmark’s net foreign asset position, which reads as:

\[
B_t^I = \frac{R_{ECB}^{ECB} \exp(-\psi_d \left( \frac{B_{t-1}}{Y_t} - \frac{\pi^I}{\pi^D} \right))}{\pi_t^{DK}} B_{t-1} + \frac{P^X}{P_t} E_t - \frac{P^M}{P_t} \Im_t \Leftrightarrow \]

\[
B_t^I = B_{t-1} - \frac{R_{ECB}^{ECB} \exp(-\psi_d \left( \frac{B_{t-1}}{Y_t} - \frac{\pi^I}{\pi^D} \right))}{\pi_t^{DK}} B_{t-1} = \frac{P^X}{P_t} E_t - \frac{P^M}{P_t} \Im_t.
\]

Insert this to obtain:

\[
\frac{P^X}{P_t} E_t - \frac{P^C}{P_t} (C_t - C_t^{DK}) - \frac{P^I}{P_t} (I_t - I_t^{DK}) = \frac{P^X}{P_t} E_t - \frac{P^M}{P_t} \Im_t,
\]

which we can rewrite as:

\[
\frac{P^C}{P_t} (C_t - C_t^{DK}) - \frac{P^I}{P_t} (I_t - I_t^{DK}) = -\frac{P^M}{P_t} \Im_t \Leftrightarrow \]

\[
\Im_t = \frac{P^C}{P^M} \frac{(C_t - C_t^{DK}) + P^I (I_t - I_t^{DK})}{P^M}, \tag{79}
\]

which is the expression for imports used in the main text.

10 Tables and figures
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<tr>
<th>Parameter</th>
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<td>Policy interest rate, reaction to inflation</td>
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<td>Mean - 0.69 Source - Normal s.d. - 0.68 s.d. - 0.068 Median - 0.69 5 pct. - 0.57 95 pct. - 0.8</td>
</tr>
<tr>
<td>Policy interest rate, reaction to inflation</td>
<td>Calibrated - 0.86 0.1</td>
<td>Mean - 0.9 Source - Beta s.d. - 0.91 s.d. - 0.02 Median - 0.9 5 pct. - 0.9 95 pct. - 0.93</td>
</tr>
<tr>
<td>Policy interest rate, reaction to inflation</td>
<td>Calibrated - 0.86 0.1</td>
<td>Mean - 0.86 Source - Beta s.d. - 0.86 s.d. - 0.067 Median - 0.86 5 pct. - 0.75 95 pct. - 0.97</td>
</tr>
</tbody>
</table>

Table 1: Estimated and calibrated parameters for the foreign countries: This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>0.99</td>
<td>To match annualized SS real rate</td>
</tr>
<tr>
<td>Private capital depreciation</td>
<td>0.0250</td>
<td>Standard value</td>
</tr>
<tr>
<td>Markup of export goods</td>
<td>6</td>
<td>Sub. between good</td>
</tr>
<tr>
<td>Capital share SS labour share in production</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Risk premia for inv. in foreign bonds</td>
<td>0.0100</td>
<td>Small but keep model stationary</td>
</tr>
<tr>
<td>Natural rate of unemployment</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>Danish interest rate risk premia wrt. debt</td>
<td>0.0200</td>
<td>Corsetti et al.</td>
</tr>
<tr>
<td>Average through period</td>
<td>0.5</td>
<td>Relative size of foreign countries</td>
</tr>
<tr>
<td>Microdata, Kamps 2004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depreciation of public capital</td>
<td>0.0250</td>
<td>Standard</td>
</tr>
<tr>
<td>Markup of import goods</td>
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<td>Sub. between import goods</td>
</tr>
<tr>
<td>Immediate implementation of pub. inv.</td>
<td>0</td>
<td>Time-to-build, Leeper et al. [2010]</td>
</tr>
<tr>
<td>Time-to-build, Leeper et al. [2010]</td>
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</tr>
<tr>
<td>Lump-sum taxes wrt. public debt</td>
<td>0.5</td>
<td>Ela. of lump-sum taxes wrt. public debt</td>
</tr>
<tr>
<td>Unemployment compensation LR average for the sample</td>
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<td></td>
</tr>
<tr>
<td>SS capital utilization cost</td>
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<td>Simplification</td>
</tr>
<tr>
<td>To match annualized SS real rate</td>
<td>66.0</td>
<td>Decrement factor</td>
</tr>
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Table 3: **Estimated parameters**: This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model.
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<th>Persistence of shocks</th>
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<th>Mean</th>
<th>s.d.</th>
<th>Mean</th>
<th>Mode</th>
<th>s.d.</th>
<th>Median</th>
<th>5 pct.</th>
<th>95 pct.</th>
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</thead>
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<td>$\rho_A$</td>
<td>Beta</td>
<td>SW07</td>
<td>0.85</td>
<td>0.1</td>
<td>0.93</td>
<td>0.95</td>
<td>0.031</td>
<td>0.94</td>
<td>0.87</td>
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<td>$A_{temp}$</td>
<td>Beta</td>
<td>SW07</td>
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<td>0.1</td>
<td>0.86</td>
<td>0.93</td>
<td>0.058</td>
<td>0.88</td>
<td>0.72</td>
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<tr>
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<td>$\rho_{com}$</td>
<td>Beta</td>
<td>SW07</td>
<td>0.85</td>
<td>0.1</td>
<td>0.87</td>
<td>0.91</td>
<td>0.077</td>
<td>0.88</td>
<td>0.75</td>
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<td>Beta</td>
<td>SW07</td>
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<td>0.1</td>
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<td>0.92</td>
<td>0.087</td>
<td>0.86</td>
<td>0.69</td>
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<tr>
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<td>$\rho_{X_{Es}}$</td>
<td>Beta</td>
<td>SW07</td>
<td>0.85</td>
<td>0.1</td>
<td>0.82</td>
<td>0.94</td>
<td>0.063</td>
<td>0.84</td>
<td>0.63</td>
</tr>
<tr>
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<td>$\rho_{EpsiX}$</td>
<td>Beta</td>
<td>SW07</td>
<td>0.85</td>
<td>0.1</td>
<td>0.87</td>
<td>0.88</td>
<td>0.009</td>
<td>0.88</td>
<td>0.76</td>
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<tr>
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<td>$\rho_{EpsiM}$</td>
<td>Beta</td>
<td>SW07</td>
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<td>0.1</td>
<td>0.84</td>
<td>0.91</td>
<td>0.093</td>
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<td>0.69</td>
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<td>Beta</td>
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<td>0.41</td>
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<tr>
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<td>Beta</td>
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<td>0.1</td>
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<td>0.83</td>
<td>0.11</td>
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<tr>
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<td>Beta</td>
<td>SW07</td>
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<td>0.1</td>
<td>0.29</td>
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<td>0.041</td>
<td>0.93</td>
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<table>
<thead>
<tr>
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<th>Type</th>
<th>Source</th>
<th>Mean</th>
<th>s.d.</th>
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<th>0.00052</th>
<th>0.00031</th>
<th>0.00074</th>
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<td>$e_{L}$</td>
<td>Inv. gamma</td>
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<td>0.004</td>
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<td>0.0057</td>
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<td>0.0018</td>
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<td>Inv. gamma</td>
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<td>0.0079</td>
<td>0.00041</td>
<td>0.0079</td>
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<tr>
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<td>$e_{pr f}$</td>
<td>Inv. gamma</td>
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<td>0.048</td>
<td>0.0019</td>
<td>0.0057</td>
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<td>$e_{P}$</td>
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<td>0.039</td>
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<td>$e_{X_{Es}}$</td>
<td>Inv. gamma</td>
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<td>0.022</td>
<td>0.0033</td>
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<td>Export shock</td>
<td>$e_{X_{Es}}$</td>
<td>Inv. gamma</td>
<td>0.01</td>
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<td>Inv. gamma</td>
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<td>$e_{M}$</td>
<td>Inv. gamma</td>
<td>0.01</td>
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<td>0.0066</td>
<td>0.0045</td>
<td>0.0017</td>
<td>0.0096</td>
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<td>$e_{X}$</td>
<td>Inv. gamma</td>
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<td>0.0032</td>
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<td>$e_{R_{P}}$</td>
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<td>0.003</td>
<td>0.00069</td>
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<td>Public consumption shock</td>
<td>$e_{G}$</td>
<td>Inv. gamma</td>
<td>0.01</td>
<td>2</td>
<td>0.0071</td>
<td>0.007</td>
<td>0.0005</td>
<td>0.0071</td>
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<td>Public investment shock</td>
<td>$e_{igB}$</td>
<td>Inv. gamma</td>
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<td>0.063</td>
<td>0.0044</td>
<td>0.063</td>
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<td>Tax on labour income tax shock</td>
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<td>Inv. gamma</td>
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<td>2</td>
<td>0.11</td>
<td>0.11</td>
<td>0.0077</td>
<td>0.11</td>
</tr>
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<td>EA price shock</td>
<td>$e_{p_{EA}}$</td>
<td>Inv. gamma</td>
<td>0.01</td>
<td>2</td>
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<td>0.0031</td>
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<td>EA output shock</td>
<td>$e_{EA}$</td>
<td>Inv. gamma</td>
<td>0.01</td>
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<td>ECB policy rate shock</td>
<td>$e_{EA}$</td>
<td>Inv. gamma</td>
<td>0.01</td>
<td>2</td>
<td>0.005</td>
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<td>RoW output shock</td>
<td>$e_{RW}$</td>
<td>Inv. gamma</td>
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<td>0.002</td>
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<td>Inv. gamma</td>
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<td>0.0029</td>
<td>0.00051</td>
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</table>

Table 4: Estimated parameters: This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model.
<table>
<thead>
<tr>
<th>Type</th>
<th>Source</th>
<th>Mean</th>
<th>s.d.</th>
<th>Mean</th>
<th>Mode</th>
<th>s.d.</th>
<th>Median</th>
<th>5 pct.</th>
<th>95 pct.</th>
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<tbody>
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<td></td>
<td></td>
<td></td>
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<tr>
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<td>0.014</td>
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<td>Output, RoW</td>
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<td>0.017</td>
<td>0.0017</td>
<td>0.016</td>
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</table>

Table 5: **Estimated parameters:** This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model.
Forecast horizon: 1 quarter

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<th></th>
<th>Real GDP</th>
<th>Consumption</th>
<th>Investment</th>
<th>Export</th>
<th>Import</th>
<th>Inflation</th>
<th>Real wages</th>
<th>Employment</th>
<th>Unemployment</th>
</tr>
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<tr>
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<td>0.0204</td>
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<td>0.9543</td>
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<td>1.5228</td>
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<tr>
<td>Wage markup shock</td>
<td>0.7876</td>
<td>0.8891</td>
<td>0.3648</td>
<td>0.3357</td>
<td>0.4509</td>
<td>0.0399</td>
<td>0.0132</td>
<td>1.2901</td>
<td>2.0646</td>
</tr>
<tr>
<td>Temp. tech shock</td>
<td>38.0300</td>
<td>8.5044</td>
<td>8.9213</td>
<td>0.2479</td>
<td>2.9218</td>
<td>0.7222</td>
<td>7.9666</td>
<td>0.3029</td>
<td>1.7826</td>
</tr>
<tr>
<td>Consumption shock</td>
<td>0.2500</td>
<td>1.2813</td>
<td>0.0262</td>
<td>0.1182</td>
<td>0.1760</td>
<td>0.0361</td>
<td>0.0289</td>
<td>0.4064</td>
<td>0.1140</td>
</tr>
<tr>
<td>Price markup shock</td>
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<td>0.7110</td>
<td>1.1579</td>
<td>0.0122</td>
<td>0.4253</td>
<td>0.0338</td>
<td>6.1941</td>
<td>5.5931</td>
<td>3.0954</td>
</tr>
<tr>
<td>Investment shock</td>
<td>10.9933</td>
<td>0.0013</td>
<td>53.0515</td>
<td>13.7798</td>
<td>15.0672</td>
<td>3.6473</td>
<td>4.8318</td>
<td>17.7825</td>
<td>8.1573</td>
</tr>
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<td>0.0854</td>
<td>0.0253</td>
<td>0.0759</td>
<td>0.1315</td>
<td>0.1769</td>
<td>0.2052</td>
<td>0.5124</td>
<td>0.3815</td>
</tr>
<tr>
<td>Import shock</td>
<td>0.0049</td>
<td>0.0001</td>
<td>0.0023</td>
<td>0.2674</td>
<td>0.4777</td>
<td>0.0344</td>
<td>0.0049</td>
<td>0.0081</td>
<td>0.0202</td>
</tr>
<tr>
<td>Import price markup shock</td>
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<td>0.0080</td>
<td>0.0419</td>
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<td>0.2227</td>
<td>0.0167</td>
<td>0.0009</td>
<td>0.0097</td>
<td>0.0179</td>
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<td>0.9266</td>
<td>0.3327</td>
<td>0.4523</td>
<td>1.2856</td>
<td>1.4576</td>
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</tr>
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<td>0.0010</td>
<td>0.0026</td>
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<td>0.0024</td>
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<td>0.2089</td>
<td>0.0027</td>
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<td>1.1979</td>
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<tr>
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<td>0.0890</td>
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<td>1.3159</td>
<td>3.8272</td>
<td>2.8282</td>
<td>3.9259</td>
<td>2.7983</td>
</tr>
<tr>
<td>EA output shock</td>
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<td>20.2023</td>
<td>2.5355</td>
<td>2.7790</td>
<td>17.2934</td>
<td>47.5835</td>
<td>27.1999</td>
<td>14.1143</td>
<td>17.0194</td>
</tr>
<tr>
<td>ECB policy rate shock</td>
<td>8.6906</td>
<td>43.5441</td>
<td>19.3120</td>
<td>46.1225</td>
<td>36.9004</td>
<td>22.2980</td>
<td>25.6345</td>
<td>13.8240</td>
<td>27.5505</td>
</tr>
<tr>
<td>RoW output shock</td>
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<td>0.0509</td>
<td>0.8631</td>
<td>16.0378</td>
<td>0.0255</td>
<td>2.0491</td>
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</tr>
<tr>
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<td>UIP shock</td>
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<td>4.6721</td>
<td>5.3548</td>
<td>12.0805</td>
<td>9.6095</td>
</tr>
</tbody>
</table>

Table 6: Variance decompositions: This table reports posterior mean estimates for the forecast-error-variance decomposition of selected variables at the 1 quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped.
### Forecast horizon: 4 quarter

<table>
<thead>
<tr>
<th></th>
<th>Real GDP</th>
<th>Consumption</th>
<th>Investment</th>
<th>Export</th>
<th>Import</th>
<th>Inflation</th>
<th>Real wages</th>
<th>Employment</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public consumption shock</td>
<td>0.1710</td>
<td>0.0572</td>
<td>0.0514</td>
<td>0.1535</td>
<td>0.0055</td>
<td>0.0986</td>
<td>0.0210</td>
<td>0.5681</td>
<td>0.2996</td>
</tr>
<tr>
<td>Perm tech shock</td>
<td>1.6828</td>
<td>5.9777</td>
<td>0.2615</td>
<td>8.7476</td>
<td>1.8775</td>
<td>0.5590</td>
<td>0.3822</td>
<td>5.1232</td>
<td>1.2760</td>
</tr>
<tr>
<td>Wage markup shock</td>
<td>1.2075</td>
<td>1.1341</td>
<td>1.0615</td>
<td>0.6441</td>
<td>0.6766</td>
<td>0.0312</td>
<td>0.0038</td>
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<td>8.1694</td>
</tr>
<tr>
<td>Consumption shock</td>
<td>0.0888</td>
<td>0.6474</td>
<td>0.0722</td>
<td>0.0435</td>
<td>0.0487</td>
<td>0.0385</td>
<td>0.0131</td>
<td>0.2753</td>
<td>0.0966</td>
</tr>
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<td>Price markup shock</td>
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<td>1.9700</td>
<td>0.5961</td>
<td>0.6647</td>
<td>0.2052</td>
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<td>Investment shock</td>
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<td>0.1758</td>
<td>27.6090</td>
<td>6.4935</td>
<td>6.4771</td>
<td>3.6295</td>
<td>2.3418</td>
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<td>6.7932</td>
</tr>
<tr>
<td>Export shock</td>
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<td>0.1152</td>
<td>0.0064</td>
<td>0.1136</td>
<td>0.1647</td>
<td>0.1492</td>
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<td>0.3681</td>
<td>0.3158</td>
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<tr>
<td>Import shock</td>
<td>0.0394</td>
<td>0.0209</td>
<td>0.2152</td>
<td>0.5304</td>
<td>2.1881</td>
<td>0.0269</td>
<td>0.1724</td>
<td>0.1579</td>
<td>0.0214</td>
</tr>
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<td>0.0085</td>
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<td>0.0136</td>
<td>0.0528</td>
<td>0.0526</td>
<td>0.0163</td>
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<td>1.1369</td>
<td>0.9325</td>
<td>1.5314</td>
<td>1.6031</td>
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<td>Riskpremia shock</td>
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<td>1.5256</td>
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<td>3.1071</td>
<td>2.1334</td>
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<td>0.0214</td>
<td>0.0032</td>
<td>0.0061</td>
<td>0.0009</td>
<td>0.0581</td>
<td>0.0365</td>
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<tr>
<td>Tax on labour income shock</td>
<td>0.4033</td>
<td>0.0504</td>
<td>0.0423</td>
<td>0.0505</td>
<td>0.0125</td>
<td>0.5459</td>
<td>0.4662</td>
<td>2.2456</td>
<td>1.7439</td>
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<td>0.7802</td>
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<td>3.1389</td>
<td>2.3757</td>
<td>2.7218</td>
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<td>1.2827</td>
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<td>55.1692</td>
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<td>0.6461</td>
<td>0.3658</td>
<td>1.2475</td>
<td>1.0741</td>
</tr>
</tbody>
</table>

**Table 7: Variance decompositions:** This table reports posterior mean estimates for the forecast-error-variance decomposition of selected variables at the 1 quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped.
### Forecast horizon: 40 quarter

<table>
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<th>Real GDP</th>
<th>Consumption</th>
<th>Investment</th>
<th>Export</th>
<th>Import</th>
<th>Inflation</th>
<th>Real wages</th>
<th>Employment</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public consumption shock</td>
<td>0.0900</td>
<td>0.0477</td>
<td>0.0429</td>
<td>0.0672</td>
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<td>0.0886</td>
<td>0.0283</td>
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<tr>
<td>Perm tech shock</td>
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<td>11.6933</td>
<td>2.0403</td>
<td>0.5099</td>
<td>0.6792</td>
<td>4.6660</td>
<td>1.1141</td>
</tr>
<tr>
<td>Wage markup shock</td>
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<td>1.2363</td>
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<td>0.5379</td>
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<tr>
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<td>0.0327</td>
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<td>3.2982</td>
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<td>5.9171</td>
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<td>0.2588</td>
<td>0.1347</td>
<td>0.1175</td>
<td>0.1974</td>
<td>0.2750</td>
</tr>
<tr>
<td>Import shock</td>
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<td>1.9113</td>
<td>2.2308</td>
<td>2.1894</td>
</tr>
<tr>
<td>ECB policy rate shock</td>
<td>10.9694</td>
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<td>29.5207</td>
<td>43.6434</td>
<td>27.9471</td>
<td>15.6558</td>
<td>25.0493</td>
<td>31.4537</td>
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<td>RoW output shock</td>
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<td>0.0549</td>
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<td>4.7899</td>
<td>0.1664</td>
<td>1.8774</td>
<td>0.5022</td>
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<td>3.4951</td>
<td>2.3911</td>
<td>5.7241</td>
<td>6.9217</td>
</tr>
</tbody>
</table>

**Table 8: Variance decompositions:** This table reports posterior mean estimates for the forecast-error-variance decomposition of selected variables at the 1 quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped.
Figure 1: Stylized facts about the Danish economy.
Figure 2: Stylized facts about the Danish economy
Figure 3: **Transformed observed time series**
This figure shows the time series of the observed variables used in the estimation of the model.
Figure 4: Transformed observed time series
This figure shows the time series of the observed variables used in the estimation of the model.
Figure 5: Transformed observed time series
This figure shows the time series of the observed variables used in the estimation of the model.
Figure 6: Transformed observed time series
This figure shows the time series of the observed variables used in the estimation of the model.
Figure 7: Prior and posterior
Prior and posterior distributions of the estimated parameters.
Figure 8: Prior and posterior
Prior and posterior distributions of the estimated parameters.
Figure 9: Prior and posterior
Prior and posterior distributions of the estimated parameters.
Figure 10: Prior and posterior distributions of the estimated parameters.
Figure 11: Prior and posterior
Prior and posterior distributions of the estimated parameters.
Figure 12: **Prior and posterior**
Prior and posterior distributions of the estimated parameters.
Figure 13: Prior and posterior
Prior and posterior distributions of the estimated parameters
Figure 14: Smoothed shocks
This figure shows the smoothed estimates of the model's structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.
Figure 15: Smoothed shocks
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model’s structural parameters.
Figure 16: Smoothed shocks
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model’s structural parameters.
Figure 17: Smoothed shocks
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.
Figure 18: Effects of a Temporary Technology shock
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a temporary technology shock equal to one standard deviation. The responses are reported as percentage deviations from steady state.
Figure 19: Effects of a Domestic price markup shock
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a domestic price markup shock equal to one standard deviation. The responses are reported as percentage deviations from steady state.
Figure 20: Effects of a shock to GDP in rest-of-world
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a temporary technology shock equal to one standard deviation. The responses are reported as percentage deviations from steady state. The red and blue lines are from the DSGE-model. The cyan and thick dotted black lines are from a SVAR.
Figure 21: Effects of a shock to the ECB policy rate
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a temporary shock to the ECB policy rate equal to 0.25 basis point increase in the policy rate. The red and blue lines are from the DSGE-model. The cyan and thick dotted black lines are from a SVAR.
Figure 22: Effects of a fiscal policy shock
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a temporary fiscal policy shock equal to one percent of GDP. The responses can therefore be interpreted as multipliers. The red and blue lines are from the DSGE-model. The cyan and thick dotted black lines are from a SVAR.
Figure 23: Historical Shock Decomposition for real GDP
This figure shows the historical quarter to quarter growth rates in Danish real GDP (solid lines) decomposed into the structural shocks in the model. The 22 shocks are grouped into 5 categories as explained in the text. Real GDP growth is reported in deviation of the steady-state mean growth rate of approximately 0.4 percent per quarter. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the models’ structural parameters.
Figure 24: **Historical Shock Decomposition, shocks from foreign economies**

This figure shows the combined contribution of the category named foreign sector (solid lines), as explained in the text. The shocks in this category are shocks to GDP, inflation, interest rates and the UIP shocks. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the models’ structural parameters.
Figure 25: **Historical Shock Decomposition, demand shocks**

This figure shows the combined contribution of the category named demand (solid lines), as explained in the text. The shocks in this category are preference shocks affecting consumption, investment shocks, import and export shocks. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the models' structural parameters.
Figure 26: Historical Shock Decomposition, markup shocks
This figure shows the combined contribution of the category named markup shocks (solid lines), as explained in the text. The shocks in this category are markup shocks to domestic-, import-, and export prices as well as the wage markup shock. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the models’ structural parameters.
References


