Written exam for the M. Sc. in Economics, Winter 2012/2013

Advanced Macroeconomics

Master's Course

January 21, 2013

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is:

Problem 1: 40 %, Problem 2: 45 %, Problem 3: 15 %.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 The Blanchard OLG model for a closed economy is described by the two differential equations

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \frac{\lambda + b}{b}\tilde{c}_t - (\delta + g + b - m)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given}, \tag{1}$$

$$\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho + \lambda - g\right]\tilde{c}_t - b(\rho + m)\tilde{k}_t, \tag{2}$$

and the condition that for any fixed pair (v, t_0), where $t_0 \geq 0$ and $v \leq t_0$,

$$\lim_{t\to\infty} a_{v,t} e^{-\int_{t_0}^{t}(f'(\tilde{k}(s))-\delta+m)ds} = 0. \tag{3}$$

Notation: $\tilde{k}_t \equiv K_t/(T_t L_t)$ and $\tilde{c}_t \equiv C_t/(T_t N_t) \equiv c_t/T_t$, where K_t and C_t are aggregate capital and aggregate consumption, respectively; N_t is population, L_t is labor supply, and T_t is the technology level, all at time t; f is a production function on intensive form, satisfying $f(0) = 0$, $f' > 0$, $f'' < 0$, and the Inada conditions. Finally, $a_{v,t}$ is financial wealth at time t of an individual born at time v. The remaining symbols stand for parameters and we assume all these are strictly positive. Furthermore, $\rho \geq b - m \geq 0$ and $\lambda < \delta + \rho + g$.

a) Briefly interpret (1), (2), and (3), including the parameters.

b) Draw a phase diagram and illustrate the path the economy will follow, given some arbitrary positive initial value of $\tilde{k}$. Can the divergent paths be excluded? Why or why not?

The model entails a simple theory of the rate of return, r^*, in the long run. For example, the model implies that r^* must belong to an open interval, defined by the parameters ρ, g, λ, and b.

c) By appealing to the phase diagram you have already drawn, indicate what the lower end point of this interval must be.

d) Indicate what the upper end point of the interval must be. You need not provide a full proof, but may argue on the basis of the fact that it can be proved that the steady-state value of $\tilde{k}$ must be above the value $\underline{\tilde{k}} > 0$ satisfying the equation $f'(\underline{\tilde{k}}) - \delta = \rho + g + b$.

e) Compare the long-run rate of return in this economy with what it would be according to a Ramsey model with logarithmic instantaneous utility function and the same ρ and g as in the present model. Comment.

f) Compare the growth rate of individual consumption through lifetime in steady state with the growth rate of per capita consumption in steady state. Does the first growth rate deviate from the latter in a steady state? If so, why?

In society there is a debate and a concern that demographic change will entail lower b and g in the future with negative effects on the long-run rate of return, thus making financing old-age security harder.

g) How will a lower b affect the long-run rate of return in the present model? *Hint:* you are only supposed to make a comparative analysis, considering b as a shift parameter; although standard curve shifting in the phase diagram does not work here, another graphical argument is possible based on the fact that in steady state the equations for the $\dot{\tilde{k}}_t = 0$ and $\dot{\tilde{c}}_t = 0$ loci are simultaneously satisfied; elimination of $\tilde{c}$ gives an equation in $\tilde{k}$ which can be ordered in a convenient way.

h) How will a lower g affect the long-run rate of return in the present model? *Hint:* consider g as a shift parameter; otherwise the hint is the same as that at g).

Problem 2 Consider a firm supplying a differentiated good in the amount y_t per time unit at time t. The production function is

$$y_t = K_t^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{*}$$

where K_t and L_t are capital and labor input at time t, respectively.

The nominal wage and the nominal *general* price level in the economy faced by the firm are constant over time and exogenous to the firm. So the real wage is an exogenous positive constant, w. The demand, y^d, for the firm's output is perceived by the firm as given by

$$y^d = p^{-\varepsilon} \frac{Y}{n}, \qquad \varepsilon > 1, \tag{**}$$

where p is the price set in advance by the firm (as a markup on expected marginal cost), relative to the general price level in the economy, n is the given large number of monopolistically competitive firms in the economy, Y is the overall level of demand, and ε is the (absolute) price elasticity of demand. The interpretation is that the firm faces a downward sloping demand curve the position of which is given by the general level of demand, which is exogenous to the firm. We assume that within the time horizon relevant for the analysis, Y is constant and the firm keeps p fixed, possibly due to menu costs. Moreover, the analysis will ignore uncertainty.

The increase per time unit in the firm's capital stock is given by

$$\dot{K}_t = I_t - \delta K_t, \qquad \delta > 0, \qquad K_0 > 0 \text{ given},$$

where I_t is gross investment per time unit at time t and δ is the capital depreciation rate. We assume that p is high enough to always be above actual marginal cost so that it always pays the firm to satisfy demand. Then cash flow at time t is

$$R_t = py^d - wL_t - I_t - G(I_t),$$

where $G(I_t)$ is a capital installation cost function satisfying

$$G(0) = G'(0) = 0, \; G''(I) > 0.$$

a) To obtain $y_t = y^d$, a certain employment level is needed. Find this employment level as a function of K_t and y^d. Let your result be denoted $L(K_t, y^d)$.

The real interest rate faced by the firm is denoted r and is, until further notice, a positive constant. As seen from time 0, the firm solves the following decision problem:

$$\max_{(I_t)_{t=0}^{\infty}} V_0 = \int_0^{\infty} \left[py^d - wL(K_t, y^d) - I_t - G(I_t)\right] e^{-rt} dt \quad \text{s.t.}$$

$$I_t \text{ free (i.e., no restriction on } I_t\text{)},$$

$$\dot{K}_t = I_t - \delta K_t, \qquad K_0 > 0 \text{ given},$$

$$K_t \geq 0 \text{ for all } t.$$

b) Briefly interpret this decision problem.

c) Denoting the adjoint variable q, derive the first-order conditions and state the necessary transversality condition (TVC) for a solution. *Hint:* the TVC has the standard form for an infinite horizon optimal control problem with discounting.

d) The optimal investment level, I_t, can be written as an implicit function of q_t. Show this.

e) Construct a phase diagram for the (K, q) dynamics, assuming that a steady state with $K > 0$ exists. Let the steady state value of K be denoted K^*. For an arbitrary $K_0 > 0$, indicate in the diagram the movement of the pair (K_t, q_t) along the optimal path.

Assume that until time t_1, the economy has been in steady state. Then, unexpectedly, the aggregate demand level, and thereby y^d, shifts to a new constant level $y^{d\prime} < y^d$ and is rightly expected to remain at that level for a long time.

f) Illustrate by the same or a new phase diagram what happens on impact and gradually over time. Comment on the implied effect on investment on impact and in the long run.

Assume instead that it is the interest rate which at time t_1 shifts to a new constant level $r' > r$ and is rightly expected to remain at that level for a long time.

g) Illustrate by the same or a new phase diagram what happens on impact and gradually over time. Comment on the implied effect on investment on impact and in the long run.

h) As a modified scenario, imagine that the fall in demand at time t_1 considered under f) was in fact due to a rise in the interest rate at time t_1. Compare the implied combined effect on investment on impact and in the long run with the isolated effects under f) and g), respectively.

i) Relate the results in f) and g) to the signs of the partial derivatives of the investment function in a standard IS-LM model. Comment.

Problem 3 *Short questions*

a) Define what is meant by a *Beveridge curve.*

In the wake of the full-blown financial and economic crisis in late 2008 a large fall in employment occurred in many countries, not least in the U.S. Two different stories could in principle explain this sharp fall in employment. One is a "Schumpeterian story" emphasizing technological and structural change. The other is a "Keynesian story".

b) During the recession a believer of the Schumpeterian story would expect "total separations", "quits", and "hiring" to rise, while a believer of the Keynesian story would expect "layoffs and discharges" to rise and "hiring" and "quits" to fall (the terms in quotation marks are the terms used by the Bureau of Labor Statistics in the U.S.). Briefly explain why the two types of believers would expect so.

c) What does the data on labor market flows in the U.S. published by the Bureau of Labor Statistics tell us in relation to these two types of explanations?

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