05.01.2007. Christian Groth

A suggested solution to the problem set at the exam in **Advanced Macroeconomics** January 24, 2007

(4-hours closed book exam)[1]

1. Solution to Problem 1

For convenience we repeat the basics. We consider a Blanchard OLG model for a closed economy. The dynamics of the model can be reduced to two differential equations:

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \tilde{c}_t - (\delta + g + n)\tilde{k}_t, \tag{1.1}$$

$$\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho - g\right]\tilde{c}_t - (n+p)(\rho+p)\tilde{k}_t, \tag{1.2}$$

where $\tilde{k}_t \equiv K_t/(T_t N_t)$ and $\tilde{c}_t \equiv C_t/(T_t N_t) \equiv c_t/T_t$. Here, K_t and C_t are aggregate capital and aggregate consumption, respectively, at time t, N_t is population (= labour force), T_t is the technology level, $T_t = T_0 e^{gt}$, and f is an intensive production function, satisfying $f(0) = 0$, $f' > 0$, $f'' < 0$ as well as the Inada conditions. The remaining symbols represent parameters which are all positive. Households satisfy their transversality condition.

a) In the Blanchard OLG model individuals have finite, but uncertain remaining lifetime. The parameter p is the death rate, i.e., p is the expected number of deaths per time unit, say per year, relatively to the size of population. The model relies on the simplifying assumption that for a given individual the probability of having a remaining lifetime, X, longer than some arbitrary number x is $P[X > x] = e^{-px}$, the same for all (i.e., independent of age). It follows that for any person the probability of dying within one year from now is approximately equal to p. The birth rate, b, is also assumed constant so that $n = b - p$ is the population growth rate. The model appeals to the law of large numbers and considers the actual number of deaths (and births) per year to be indistinguishable from the expected number.

[1] The solution below contains more details than can be expected at a four hours exam.

Individuals can buy life annuity contracts from life insurance companies. These companies have negligible administrative costs so that in equilibrium with free entry (zero profits) the rate of return on these contracts is $r_t + p$ until death, where r_t is the risk-free real rate of interest. The actuarial premium p is financed through the wealth transfer to the insurance sector (the cancelling of the annuity liabilities of the insurance companies) that occurs when people die.

The equation (1.1) is essentially just national income accounting in per unit of effective labour terms. Isolating $f(\tilde{k}_t)$ on one side we have aggregate gross income per unit of effective labour on this side and consumption plus gross investment per unit of effective labour on the other side.

As to the first term on the right-hand side of (1.2), notice that instantaneous utility in the Blanchard OLG model is logarithmic, so that the individual Keynes-Ramsey rule is simply $\dot{c}_t = [r_t + p - (\rho + p)]\, c_t = (r_t - \rho)c_t$, where ρ is the pure rate of time preference (impatience), and $\rho + p$ is the effective rate of discount of future utility (the addition of p to this discount rate reflects the probability of not being alive at the date in question). In general equilibrium with perfect competition $r_t = f'(\tilde{k}_t) - \delta$, where δ is the capital depreciation rate. The corresponding growth-corrected Keynes-Ramsey rule is then $\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho - g\right] \tilde{c}_t$, where g is the rate of (Harrod-neutral) technical progress. This explains the first term on the right-hand side of (1.2).

The second term on the right-hand side of (1.2) represents the generation replacement effect. The arrival of newborns is Nb per time unit and since they have no financial wealth, the inflow of these people lowers aggregate consumption by $b(\rho+p)A_t$ per time unit, where A_t is aggregate financial wealth. Indeed, the average financial wealth in the population is A_t/N_t and the consumption effect of this is $(\rho + p)A_t/N_t$, since the consumption function of the "average individual" is $c_t = (\rho + p)(A_t/N_t + h_t)$, where ρ is the pure rate of time preference and h_t is average human wealth (equal to everyones human wealth since there is no retirement and all have the same expected remaining lifetime). Thus, *ceteris paribus*, aggregate consumption is reduced by $N_t b(\rho + p)A_t/N_t = b(\rho + p)A_t = (n + p)(\rho + p)A_t$ per time unit, since $b = n + p$. In general equilibrium in the closed economy (without government debt), $A_t = K_t$. Transforming into the per unit of effective labour variables, we end up with a lowering of $\dot{\tilde{c}}_t$ equal to $(n + p)(\rho + p)\tilde{k}_t$. This explains the second term in (1.2).

b) There are two baseline values of $\tilde{k}$, namely the golden rule value, $\tilde{k}_{GR}$, and the "critical" value, $\bar{\tilde{k}}$, defined by,

$$f'\left(\tilde{k}_{GR}\right) - \delta = n + g, \qquad \text{and} \qquad f'(\bar{\tilde{k}}) - \delta = \rho + g, \tag{1.3}$$

respectively. (Why $\bar{\tilde{k}}$ is a "critical" value becomes clear below.) Since f satisfies the Inada conditions, both values exist and are unique (since $f'' < 0$). We have $\bar{\tilde{k}} \lesseqgtr \tilde{k}_{GR}$ for $\rho \gtreqless n$, respectively.

Equation (1.1) shows that

$$\dot{\tilde{k}} \ = 0 \text{ for } \tilde{c} = f(\tilde{k}) - (\delta + n + g)\tilde{k}. \tag{1.4}$$

The locus $\dot{\tilde{k}} = 0$ is shown in Fig. 1.1; it starts at the origin O, reaches its maximum at the golden rule capital intensity and crosses the horizontal axis at point B where $f(\tilde{k})$ $= (\delta + n + g)\tilde{k}$.

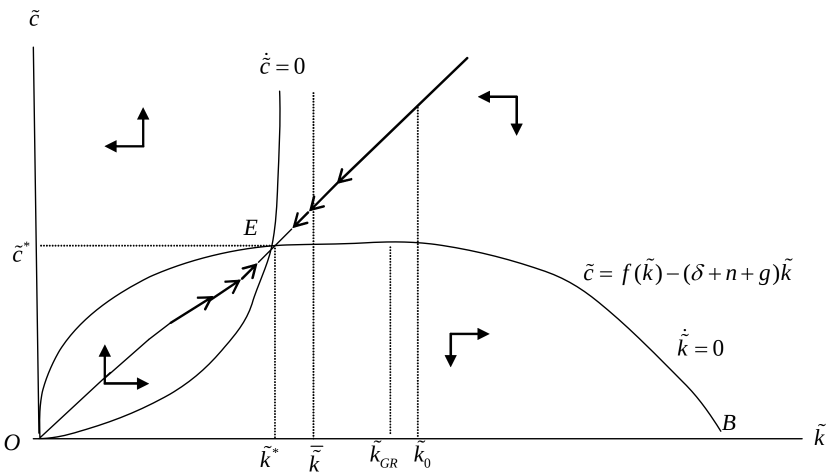

Figure 1.1:

Equation (1.2) shows that

$$\dot{\tilde{c}} \ = 0 \text{ for } \tilde{c} = \frac{(n+p)\,(\rho+p)\,\tilde{k}}{f'(\tilde{k}) - \delta - \rho - g}. \tag{1.5}$$

Hence,

$$\text{along the } \dot{\tilde{c}} \ = 0 \text{ locus, } \tilde{k} \nearrow \bar{\tilde{k}} \Rightarrow \tilde{c} \to \infty,$$

so that the $\dot{\tilde{c}} = 0$ locus is asymptotic to the vertical line $\tilde{k} = \bar{\tilde{k}}$. Moving along the $\dot{\tilde{c}} \ = 0$ locus in the other direction, we see from (1.5) that $\tilde{k} \searrow 0 \Rightarrow \tilde{c} \to 0$, as illustrated in Fig. 1.1. The diagram also shows the steady-state point E where the $\dot{\tilde{c}} = 0$ locus crosses the $\dot{\tilde{k}} = 0$ locus. The corresponding capital intensity is $\tilde{k}^*$, to which corresponds the (growth-corrected) consumption level $\tilde{c}^*$. Given our assumptions (including the Inada conditions), there exists one and only one steady state with positive capital intensity. Whether $\bar{\tilde{k}} \lesseqqgtr \tilde{k}_{GR}$ depends on whether $\rho \gtreqqless n$. Fig. 1.1 illustrates the case $\bar{\tilde{k}} < \tilde{k}_{GR}$. If $\rho < n$, so that $\bar{\tilde{k}} > \tilde{k}_{GR}$, dynamic inefficiency ($\tilde{k}^* > \tilde{k}_{GR}$) is possible.

The direction of movement in the different regions of the phase diagram is determined by the differential equations (1.1) and (1.2) and are shown by arrows. The arrows taken together show that the steady state E is a saddle point. Since we have one predetermined variable, $\tilde{k}$, and one jump variable, $\tilde{c}$, the steady state is saddle-point stable. The saddle path is the only path that satisfies *all* the conditions of general equilibrium (individual utility maximization for given expectations, profit maximization by firms, continuous market clearing and perfect foresight). The other paths in the diagram either violate the transversality condition of the households (paths that in the long run point South-East) or the NPG condition[2] of the households (paths that in the long run point North-West). Hence, equilibrium initial consumption, $\tilde{c}(0)$, is determined as the ordinate, $\tilde{c}_0$, to the point where the vertical line $\tilde{k} = \tilde{k}_0$ crosses the saddle path. Over time the economy moves from this point, along the saddle path, towards the steady state point E with coordinates $(\tilde{k}^*, \tilde{c}^*)$. Since, by assumption, $\tilde{k}_0 > \tilde{k}^*$, the economy approaches the steady state from above as Fig. 1.1 illustrates.

c) In view of $\tilde{k}^* < \bar{\tilde{k}}$, we have $r^* = f'(\tilde{k}^*) - \delta > f'(\bar{\tilde{k}}) - \delta = \rho + g$. From the hint follows that there exists a value of $\tilde{k}$, $\underline{\tilde{k}}$, satisfying $\underline{\tilde{k}} < \tilde{k}^*$ and $f'(\underline{\tilde{k}}) - \delta = \rho + g + n + p$. Hence, $r^* = f'(\tilde{k}^*) - \delta < \rho + g + n + p$. We conclude

$$\rho + g < r^* < \rho + g + n + p. \tag{1.6}$$

d) A shift to a higher g moves the $\dot{\tilde{k}} = 0$ locus downward (although it still starts at the origin). At the same time the $\dot{\tilde{c}} = 0$ locus is turned counter-clockwise. Fig. 1.2 illustrates the new steady state E' with coordinates $(\tilde{k}^{*\prime}, \tilde{c}^{*\prime})$. Since $\tilde{k}^{*\prime} < \tilde{k}^*$, the new interest rate in steady state, $r^{*\prime}$, is higher than the old.

Interpretation can be given along the following complementary lines:

1. Notice that per capita consumption in steady state, $c_t \equiv \tilde{c}_t T_t$, in steady state always must grow at the rate of technical progress, g. This is because in steady state c_t

[2] And therefore also the transversality condition.

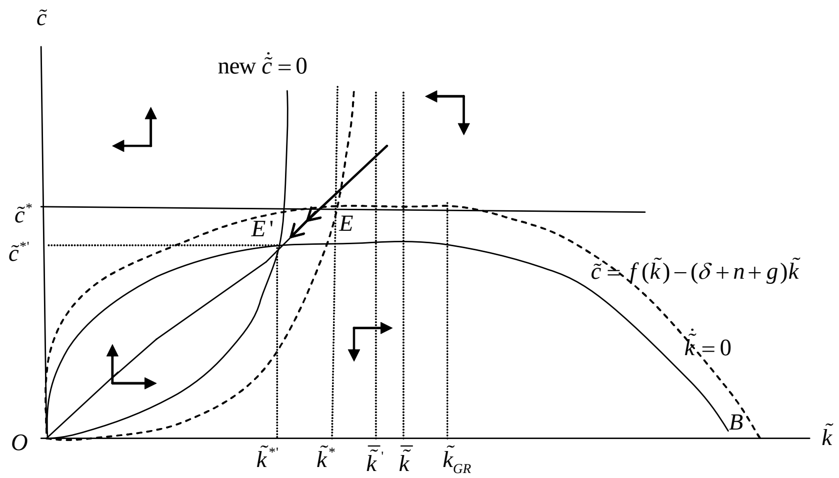

Figure 1.2:

$= \tilde{c}^* T_t = \tilde{c}^* T_0 e^{gt}$. Thus, a higher rate of technical progress implies in the long run a higher rate of growth in c, which requires that individuals chooses a higher growth rate for their consumption. To induce the required willingness to save, a larger gap between r and ρ is needed (as we know from the individual Keynes-Ramsey rule).

2. Since the technology level is in the denominator of the "effective capital intensity", $\tilde{k} \equiv K/(TN)$, a higher rate of technical progress implies more "dilution" of the capital intensity, implying a higher marginal product of capital.

3. When T grows at a higher rate, "effective labour" grows at a higher rate. Thus, everything else equal, capital becomes more scarce (relative to effective labour) and demands a higher rate of return.

e) In steady state the left-hand sides of (1.4) and (1.5) are equal to each other. And so are the right-hand sides. After ordering this implies

$$\left[\frac{f(\tilde{k})}{\tilde{k}} - (\delta + n + g)\right]\left[f'(\tilde{k}) - \delta - \rho - g\right] = (n + p)(\rho + p). \tag{1.7}$$

The steady state value, $\tilde{k}^*$, satisfies this equation, which can thus formally be written $\varphi(\tilde{k}^*, g) = (n + p)(\rho + p)$. We are asked to find the elasticity

$$\eta_{r^*,\, g} \equiv \frac{g}{r^*}\frac{\partial r^*}{\partial g}.$$

Since $r^* = f'(\tilde{k}^*) - \delta$, let us first find $\partial \tilde{k}^*/\partial g$. Taking the total differential on both sides of (1.7) w.r.t. $\tilde{k}^*$ and g gives

$$\varphi_{\tilde{k}} d\tilde{k}^* + \varphi_g dg = 0, \qquad \text{where}$$

$$\begin{aligned}
\varphi_{\tilde{k}} &= \left[\frac{f(\tilde{k}^*)}{\tilde{k}^*} - (\delta + n + g)\right] f''(\tilde{k}^*) + (f'(\tilde{k}^*) - \delta - \rho - g)\frac{f'(\tilde{k}^*) - f(\tilde{k}^*)/\tilde{k}^*}{\tilde{k}^*}, \quad (1.8) \\
\varphi_g &= -\left[\frac{f(\tilde{k}^*)}{\tilde{k}^*} - (\delta + n + g)\right] - (f'(\tilde{k}^*) - \delta - \rho - g). \quad (1.9)
\end{aligned}$$

In view of $\tilde{c}^* > 0$, we see from (1.4) that the term in square brackets is positive. And since $\tilde{k}^* < \bar{\tilde{k}}$, $f'(\tilde{k}^*) - \delta - \rho - g > 0$. Finally, due to concavity of $f(\tilde{k})$ we have $f'(\tilde{k}^*) < f(\tilde{k}^*)/\tilde{k}^*$ (look at a graph of $f(\tilde{k})$). It follows that $\varphi_{\tilde{k}}$ has the sign pattern $(+)(-) + (+)(-)$ and φ_g the sign pattern $-(+) - (+)$. Thus, $\varphi_{\tilde{k}} < 0$ and $\varphi_g < 0$ so that

$$\frac{\partial \tilde{k}^*}{\partial g} = -\frac{\varphi_g}{\varphi_{\tilde{k}}} < 0,$$

and we get

$$\frac{\partial r^*}{\partial g} = \frac{\partial r^*}{\partial \tilde{k}^*}\frac{\partial \tilde{k}^*}{\partial g} = f''(\tilde{k}^*)\frac{\partial \tilde{k}^*}{\partial g} > 0.$$

Hence,

$$\eta_{r^*,g} \equiv \frac{g}{r^*}\frac{\partial r^*}{\partial g} = \frac{g}{f'(\tilde{k}^*) - \delta} f''(\tilde{k}^*)\frac{\partial \tilde{k}^*}{\partial g} = -\frac{g}{f'(\tilde{k}^*) - \delta} f''(\tilde{k}^*)\frac{\varphi_g}{\varphi_{\tilde{k}}} > 0,$$

where (1.8) and (1.9) can be inserted.

2. Solution to Problem 2

For convenience, we repeat the basic information here:

$$\begin{aligned}
Y_t &= \text{GDP at time } t, \\
T_t &= \text{net tax revenue } (= \text{ gross tax revenue} - \text{transfer payments) at time } t, \\
G_t &= \text{government spending on goods and services at time } t, \\
B_t &= \text{government debt at time } t.
\end{aligned}$$

All government debt is short-term, taxes and transfers lump-sum, no uncertainty, the budget deficit is exclusively financed by debt issue.

$$\begin{aligned} Y_t &= Y_0 e^{(g+n)t}, \\ T_t &= \tau Y_t, \\ G_t &= \gamma Y_t, \qquad 0 < \gamma < 1, \\ B_0 &> 0. \end{aligned}$$

To begin with, it is assumed that $\tau = \gamma$.

a) In view of the budget deficit being bond financed, we have

$$\dot{B}_t = rB_t + \gamma Y_t - \tau Y_t = rB_t - (\tau - \gamma)Y_t,$$

so that with $b_t \equiv B_t/Y_t$ (the debt-income ratio) we get

$$\frac{\dot{b}_t}{b_t} = \frac{\dot{B}_t}{B_t} - \frac{\dot{Y}_t}{Y_t} = r - \frac{(\tau - \gamma)Y_t}{B_t} - (g + n),$$

or

$$\dot{b}_t = (r - g - n)b_t - (\tau - \gamma). \tag{2.1}$$

Let the primary surplus ratio be denoted s. Thus

$$s \equiv \frac{T_t - G_t}{Y_t} = \tau - \gamma.$$

The current fiscal policy has $\tau = \gamma$ so that (2.1) gives

$$\dot{b}_t = (r - g - n)b_t. \tag{2.2}$$

But with $r = g + n$, the debt-income ratio remains constant. Thus, the current fiscal policy is sustainable.

From now, for $t \geq 0$ the interest rate is $r' > r$. Thus, $r' > g + n$.

b) In the new situation, $\dot{b}_t = (r' - g - n)b_t$. Thus, $b_t \to \infty$ for $t \to \infty$, hence the current fiscal policy is not sustainable. With positive debt, but zero primary surplus, the debt-income ratio explodes when the interest rate is higher than the growth rate.

c) One approach to finding $\bar{s}$ is based on (2.1), which in the present situation can be written

$$\dot{b}_t = (r' - g - n)b_t - s. \tag{2.3}$$

Fiscal sustainability requires that b_t is non-exploding, that is, $\dot{b}_t \leq 0$. In particular, for $t = 0$ we need $\dot{b}_0 \leq 0$. By (2.3), this requires $s \geq (r' - g - n)b_0$. The minimum required s is therefore

$$\bar{s} = (r' - g - n)b_0. \tag{2.4}$$

Another approach is based on the intertemporal government budget constraint. Since the real interest rate is above the growth rate, fiscal sustainability requires that the government satisfies the intertemporal budget constraint

$$\int_0^\infty G_t e^{-r't} dt \leq \int_0^\infty T_t e^{-r't} dt - B_0.$$

In the present case (now allowing τ to differ from γ) this amounts to

$$\int_0^\infty (\tau - \gamma) Y_0 e^{(g+n)t} e^{-r't} dt = sY_0 \int_0^\infty e^{-(r'-g-n)t} dt = sY_0 \frac{1}{r' - g - n} \geq B_0,$$

that is,

$$s \geq (r' - g - n)\frac{B_0}{Y_0},$$

implying again (2.4).

d) From (2.4) we get

$$\begin{aligned} \frac{\partial \bar{s}}{\partial g} &= -b_0 < 0, \\ \frac{\partial \bar{s}}{\partial n} &= -b_0 < 0, \\ \frac{\partial \bar{s}}{\partial r'} &= b_0 > 0. \end{aligned}$$

Comment:

We see that higher growth rate (higher g and higher n) makes the required primary surplus smaller. Thus, it becomes easier to obtain fiscal sustainability, given an initial positive debt. With higher $g + n$, not only do G and T grow at a higher rate, but $T - G$, which is positive, grows at a higher rate. This makes it easier to confine the growth in B such that b does not grow.

On the other hand, with higher r', the snowball effect of compound interest is stronger. Hence, it becomes harder to confine the growth in B such that b does not grow.

3. Solution to Problem 3

For convenience, the equations of the model are repeated here:

$$
\begin{aligned}
\dot{Y}_t &= \lambda(D(Y_t, r_t, x_t) - Y_t), \qquad \lambda > 0, 0 < D_Y < 1, D_r < 0, D_x > 0, && (3.1)\\
\frac{M}{P} &= L(Y_t, i_t), \qquad\qquad L_Y > 0,\ L_i < 0, && (3.2)\\
i_t &= i^* + \frac{\dot{X}_t^{\,e}}{X_t}, && (3.3)\\
r_t &\equiv i_t - \pi_t^e, && (3.4)\\
x_t &\equiv \frac{X_t P^*}{P}. && (3.5)
\end{aligned}
$$

The variables M, P, P^* and i^* are exogenous and constant. The initial level, Y_0, of output is given. Expectations are rational, there is no uncertainty and speculative bubbles never occur.

a) Evidently, the model is the Blanchard-Fischer version of Dornbusch's "overshooting" model, i.e., a dynamic IS-LM model for a SOE with a floating exchange rate. It is a model of short-run mechanisms, since the price level P is an exogenous constant. The function $D(Y_t, r_t, x_t)$ in equation (3.1) represents aggregate output demand The signs of the partial derivatives of D have the following interpretation: $D_Y > 0$, because private consumption and perhaps also investment depends positively on aggregate production and income (which affect aggregate wealth); a further possible source to the positive dependence of demand on current income is that liquidity constraints may be operative; $D_Y < 1$ because the marginal propensity to spend can, realistically, be assumed less than one (that net exports tend to depend negatively on Y helps in this direction); $D_r < 0$, because consumption (primarily through the wealth effect) and investment depend negatively on the real rate of interest; $D_x > 0$, because net exports depend positively on the real exchange rate $x \equiv XP^*/P$, cf. (3.5); indeed, x is an indicator of competitiveness.

Equation (3.1) reflects that the adjustment of output to demand takes time; the parameter λ is the speed of adjustment. Equation (3.2) expresses equilibrium at the money market. Naturally, real money demand depends positively on Y through the "transaction motive" and negatively on the (short-term) nominal rate of interest, the opportunity cost of holding money. Equation (3.3) is the UIP assumption, saying that domestic and foreign financial assets pay the same expected rate of return (measured in the same currency). Indeed, on the left-hand side appears the interest rate on a bond denominated in domestic currency (henceforth a "domestic bond"). On the right-hand side appears the expected rate of return on investing in a bond denominated in foreign currency (henceforth a "foreign bond") plus the expected rate of depreciation of the domestic currency; this sum is in fact the expected rate of return on the foreign asset when measured in the domestic currency.

b) The assumption of rational expectations (here perfect foresight) implies $\pi_t^e = \pi_t = 0$ for all t, since the price level P is constant in the model. Therefore, equation (3.4) reduces to $r_t = i_t$. In view of (3.2) we can write i_t as a function of Y_t and M/P, that is, $i_t = i(Y_t, M/P)$, where $i_Y = -L_Y/L_i > 0$ and $i_{M/P} = 1/L_i < 0$. Inserting into (3.1) gives

$$\dot{Y}_t = \lambda(D(Y_t, i(Y_t, M/P), X_t P^*/P) - Y_t). \tag{3.6}$$

Similarly, $\dot{X}_t^e = \dot{X}_t$, so that (3.4) can be written

$$\dot{X}_t = (i(Y_t, M/P) - i^*)X_t. \tag{3.7}$$

In this way the model has been reduced to two coupled differential equations in Y_t and X_t, where Y_t is predetermined and X_t is a jump variable (forward-looking variable).

c) To draw the corresponding phase diagram, note that (3.6) implies that

$$\dot{Y} = 0 \text{ for } D(Y, i(Y, M/P), XP^*/P) = Y. \tag{3.8}$$

Take the total differential on both sides w.r.t. Y, X and M (for later use):

$$\begin{aligned}(D_Y + D_r i_Y)dY + D_r i_{M/P}\frac{1}{P}dM + D_x\frac{P^*}{P}dX &= dY \Rightarrow \\ (1 - D_Y - D_r i_Y)dY &= D_r i_{M/P}\frac{1}{P}dM + D_x\frac{P^*}{P}dX. \end{aligned} \tag{3.9}$$

Setting $dM = 0$, we find

$$\frac{dX}{dY}|_{\dot{Y}=0} = \frac{1 - D_Y - D_r i_Y}{D_x P^*/P} > 0. \tag{3.10}$$

It follows that the $\dot{Y} = 0$ locus (the "IS curve") is upward-sloping as shown in Fig. 3.1.

Equation (3.7) implies that $\dot{X} = 0$ for $i(Y, M/P) = i^*$. The value of Y satisfying this equation is unique (because $i_Y \neq 0$) and is called $\bar{Y}$. That is, $\dot{X} = 0$ for $Y = \bar{Y}$, which says that the $\dot{X} = 0$ locus (the "LM curve") is vertical, cf. Fig. 3.1. The figure also

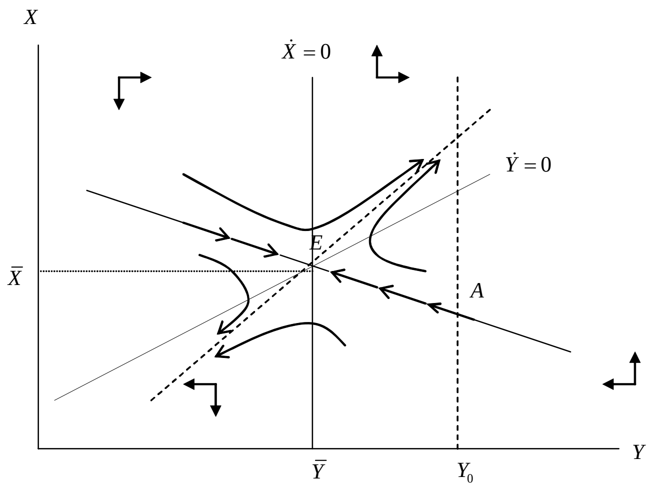

Figure 3.1:

indicates the direction of movement in the different regions, as determined by (3.6) and (3.7). We see that the steady state point, E, is a saddle point. This implies that two and only two solution paths − one from each side − converge towards E.

At time $t = 0$, the economy must be somewhere on the vertical line $Y = Y_0$. In view of the absence of speculative bubbles, the explosive or implosive paths of X in Fig. 3.1 cannot arise. Hence, we are left with the saddle path, the path AE in Fig. 3.1, as the unique solution to the model. Gradually over time the economy moves along the saddle path towards the steady state E. If $Y_0 > \bar{Y}$ (as in Fig. 3.1), output is decreasing and the exchange rate increasing (depreciation) during the adjustment process. If instead $Y_0 < \bar{Y}$, the opposite movements occur.

d) In steady state

$$\frac{M}{P} = L(\bar{Y}, i^*). \tag{3.11}$$

To see how $\bar{Y}$ is affected by the increase in M, we take the total differential on both sides:

$$\frac{1}{P}dM = L_Y d\bar{Y} + L_i di^*.$$

With $di^* = 0$, this gives $\partial\bar{Y}/\partial M = 1/(PL_Y) > 0$, implying

$$d\bar{Y} = (\partial\bar{Y}/\partial M)dM = dM/(PL_Y) > 0, \tag{3.12}$$

since $dM > 0$. Hence, the $\dot{X} = 0$ line is shifted to the right, cf. Fig. 3.2.

As to the shift of the $\dot{Y} = 0$ locus, (3.9) gives, for given X, i.e., $dX = 0$,

$$\frac{\partial Y}{\partial M}_{|\dot{Y}=0, X=X_0} = \frac{D_r i_{M/P} 1/P}{1 - D_Y - D_r i_Y} = \frac{D_r/(PL_i)}{1 - D_Y + D_r L_Y/L_i} > 0.$$

Hence, the increase in M shifts the $\dot{Y} = 0$ locus rightward, cf. Fig. 3.2. (Another way of understanding the shift of the $\dot{Y} = 0$ locus is to consider Y as fixed, i.e., $dY = 0$. Then (3.9) gives

$$\frac{\partial X}{\partial M}_{|\dot{Y}=0, Y=Y_0} = -\frac{D_r i_{M/P} 1/P}{D_x P^*/P} = -\frac{D_r/L_i}{D_x P^*} < 0.$$

Hence, we can also say that the increase in M shifts the $\dot{Y} = 0$ locus downward.)

Since both the $\dot{X} = 0$ locus and the $\dot{Y} = 0$ locus shift right-ward, it might seem ambiguous in what direction $\bar{X}$ moves. However, we know that $\bar{Y}' > \bar{Y}$. Given $r = i^*$, this requires higher competitiveness in the new steady state, i.e., $\bar{X}' > \bar{X}$.

[More formally, the two equations describing steady state are (3.11) and

$$D(\bar{Y}, i^*, \bar{X}P^*/P) = \bar{Y}. \tag{3.13}$$

First, $d\bar{Y}$ is given by (3.12), i.e., determined from (3.11), independently of (3.13). Then, by (3.13), we see that $D_Y d\bar{Y} + D_x \frac{P^*}{P} d\bar{X} = d\bar{Y}$, from which follows

$$\begin{aligned} d\bar{X} &= \frac{1 - D_Y}{D_x P^*/P} d\bar{Y} \\ &= \frac{1 - D_Y}{D_x P^* L_Y} dM > 0, \qquad \text{(by (3.12))} \end{aligned}$$

where the inequality is due to $dM > 0$.]

After time t_0, the economy, (Y_t, X_t), moves along the new saddle path as indicated by the arrows in Fig. 3.2.

The long-run effect of the rise in M is a rise in X, that is, *depreciation* (fall in the value of the domestic currency vis-a-vis the foreign currency). The impact effect is, however, a *larger* depreciation ("overshooting"). The reason is that the rise in money supply immediately leads to a fall in i, since output is given in the short run. This leads to capital outflow, hence depreciation. The capital outflow and depreciation occur instantly. If we for a moment imagine they occurred gradually over time, there would be expected depreciation of the domestic currency, implying that domestic bonds became even less attractive relative to foreign bonds, thereby reinforcing the capital outflow and the rise in X. Hence, the capital outflow and depreciation occur in a jump, that is, X jumps from

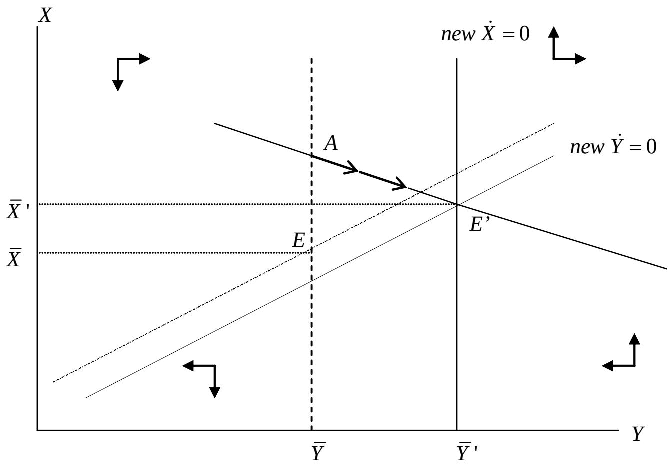

Figure 3.2:

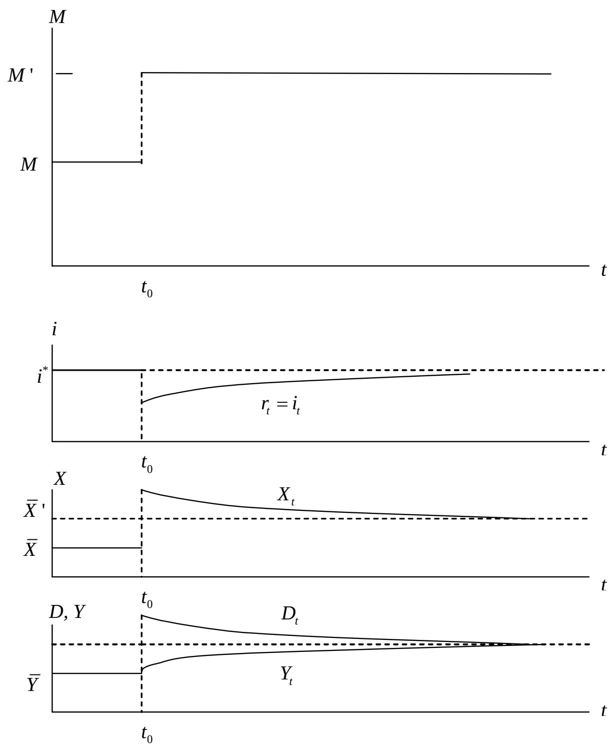

Figure 3.3:

its previous steady state level to a level high enough so that the concomitant *expected gradual appreciation* is at the level needed to make domestic bonds not less attractive than foreign bonds and so that convergence to the new steady state, E', is obtained. This happens where the vertical line $Y = \bar{Y}$ crosses the new saddle path, i.e., at the point A in Fig. 3.2.

For $t > t_0$ the economy moves along the new saddle path: Y gradually rises in response to the high output demand generated by the low interest rate $r_t = i_t$ and the high exchange rate X_t (high competitiveness); X gradually falls as expected (appreciation). The time profiles of the variables are shown in Fig. 3.3.

e) Fig. 3.4 illustrates what happens from time t_0 when people become aware that an increase in the money supply is going to take place at time t_1. As soon as the future expansionary monetary policy becomes anticipated, there is an immediate effect on X in the same direction as the long-run effect, i.e., the exchange rate *jumps* to some specific level, X_B, as shown in Fig. 3.4. This jump is due to the agents' anticipation that from time t_1 the expansionary monetary policy will cause the rate of interest to be lower than i^*. Foreseeing this, agents want more foreign bonds in their portfolios already now. This causes the exchange rate to depreciate in a jump already now. The jump is smaller than in the previous case where the monetary expansion was immediate and unanticipated. After this upward jump at time t_0, the exchange rate, X, adjusts continuously. This is because the exchange rate is an asset price and *anticipated* discrete jumps in an asset price are ruled out by arbitrage. In particular, at time t_1 there can be no jump, because no new information has arrived.

In the time interval (t_0, t_1) the movement of (Y, X) is governed by the "old" dynamics and the corresponding phase diagram. That is, for $t_0 < t < t_1$ the economy must follow some trajectory consistent with the "old" dynamics. The economy will "select" that trajectory (BC in Fig. 3.4), which takes exactly $t_1 - t_0$ time units to reach the new saddle path. This is because agents know that from time t_1 the economy must be on the new saddle path. In Fig. 3.4 the level X_B denotes the required level of X immediately after time t_0.[3] The implied instantaneous depreciation implies higher competitiveness, hence higher output demand, so that output begins its gradual upward adjustment already be-

[3]It can be shown that X_B is unique and this is also what intuition tells us. Indeed, imagine that the jump, $X_B - \bar{X}$, was smaller than in Figure 3.4. Then, not only would there be a longer way along the road to the new saddle path, but the system would also start from a position closer to the steady state point E, which implies an initially lower adjustment speed.

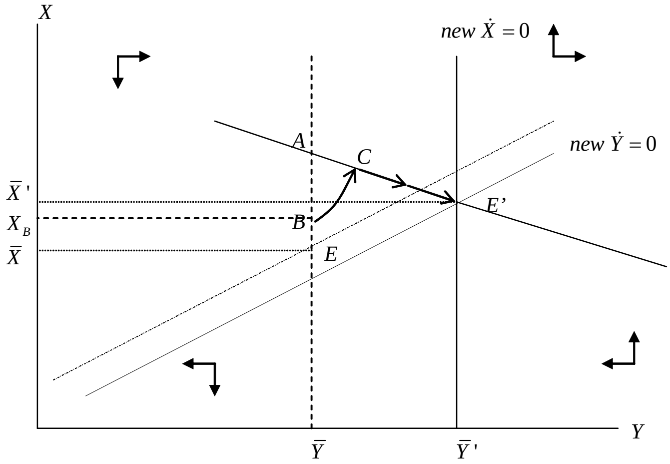

Figure 3.4:

fore monetary policy has been loosened. Immediately after t_0, the exchange rate will begin to *gradually* rise until t_1. This amounts to an expected and actual gradual depreciation ($\dot{X} > 0$) in the time interval (t_0, t_1), and this happens along with a gradual rise in the interest rate i, caused by the higher transaction demand for money along with the rising Y (remember, M has not changed yet). The process continues until the new monetary policy is *actually* implemented at time t_1. Exactly at this time the economy's trajectory, governed by the old dynamic regime, crosses the new saddle path (cf. the point C in Fig. 3.4). The actual rise in M at time t_1 triggers a downward jump in the interest rate, so that for $t > t_1$ we have $i < i^*$ and the new dynamics with gradual appreciation ($\dot{X} < 0$) set in. Thus, for $t > t_1$ the economy moves along the new saddle path, approaching the new steady state E' for $t \to \infty$.

The resulting time profiles of Y_t, X_t and r_t $(= i_t)$ are shown in Fig. 3.5. If the length of the time interval (t_0, t_1) is small enough, X may already immediately after time t_0 be above its new long-run level. However, Fig. 3.4 and Fig. 3.5 depict the opposite case, where the time interval (t_0, t_1) is somewhat larger.

4. Solution to Problem 4

a) True. According to the extended Slutsky equation the total effect of a wage increase on labour supply can be decomposed into three effects:

$$\text{total effect} \ = \ \text{substitution effect} + \text{income effect} + \text{wealth effect}.$$

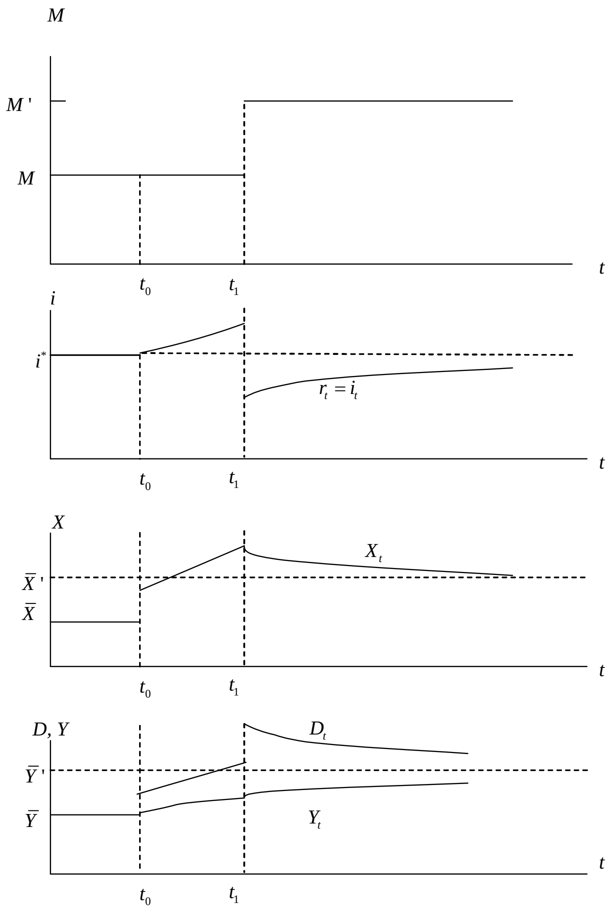

Figure 3.5:

Evidently, the wage increase raises the "price" on leisure and has therefore a *positive substitution effect* on labour supply. At the same time, by making leisure more expensive, the wage increase implies that a given budget can buy less consumption of goods *and leisure*. This amounts to a *positive income effect* of the wage increase on labour supply (assuming leisure is a normal good). Finally, the wage increase raises the present discounted value of expected future labour income and this makes it possible to buy *more* consumption of goods *and leisure*. This amounts to a *negative wealth effect* of the wage increase on labour supply.

b) False. Only the "only if" part of the statement is true. The general principle is that the economy has positive bequests in steady state if and only if $R < r_D^*$, where r_D^* is the real interest rate in steady state of the associated Diamond economy (resulting from replacing the true utility function by the "truncated" utility function where the bequest motive is eliminated). Suppose $n < r_D^* < R$. Then the bequest motive will not be operative. Thus the economy behaves like a Diamond economy and a steady state with $r^* = r_D^*$ is realized. But since $r_D^* > n$, this economy is dynamically efficient.

c) False. The expectations theory of the term structure ignores variations in the term premium (sometimes even by putting the term premium equal to zero) and predicts that the long-term interest rate will be a kind of average of the expected future short-term interest rates. Consider a consol paying a constant stream of one unit of account per time unit in the indefinite future. Then the long-term interest rate is defined as the internal rate of return on the consol, i.e., that number, R_t, which satisfies the equation

$$Q_t = \int_t^\infty 1 \cdot e^{-R_t(s-t)} ds,$$

where Q_t denotes the market value of the consol at time t. By performing the integration we get

$$Q_t = \frac{1}{R_t}. \tag{4.1}$$

Assuming no speculative bubbles, by arbitrage the market value of the consol will be equal to its *fundamental value*, that is, the present value of the future payments on the consol, where present value is calculated with the expected future short-term interest rates as the rates of discount (uncertainty is ignored). Thus,

$$Q_t = \int_t^\infty e^{-\int_t^s r_\tau d\tau} ds, \tag{4.2}$$

where r_τ, $\tau \geq t$, represent the expected future short-term interest rates. Combining this with (4.1) yields

$$R_t = \frac{1}{Q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds}.$$

If r_τ is expected to be a constant, r, we get

$$R_t = \frac{1}{\int_t^\infty e^{-r(s-t)} ds} = \frac{1}{1/r} = r.$$

If r_τ is expected to be decreasing, we get

$$R_t = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds} < \frac{1}{1/r_t} = r_t.$$

Thus, in these two cases the prediction from the expectations theory contradicts the statement. However, if r_τ is expected to be increasing, we get $R_t > r_t$, which is in line with the statement.

d) The *minimum transaction rule* is a key element in new Keynesian macroeconomics, where prices are considered as preset by price-setting firms that face a downward-sloping demand curve (usually in a monopolistic competition framework). Thus, the preset price $\bar{P}_i$ of firm i will typically be above (expected) marginal cost. Output of the firm becomes

$$y_i = \min \left[y^d(\bar{P}_i/P, Y), y^c(\bar{P}_i, W) \right],$$

where P is the general price level (of competing firms supplying more or less differentiated goods), Y is a measure of the aggregate level of demand and W is the wage rate. That is, production is determined as the minimum of demand, $y^d(\bar{P}_i/P, Y)$, and the "classical supply", $y^c(\bar{P}_i, W)$, at the preset price $\bar{P}_i$ (by "classical supply" is meant the output level, where marginal cost equals $\bar{P}_i$). This principle is called *the minimum transaction rule.* It follows that as long as demand $y^d(\bar{P}_i/P, Y)$ defines the most narrow limit (i.e., as long as $y^d(\bar{P}_i/P, Y) < y^c(\bar{P}_i, W)$), production will be determined by demand.

—