

- h) Will the bequest motive be operative? Why or why not?
- i) Suppose π is increased (a little) to a higher level without σ_t being immediately adjusted correspondingly. Is resource allocation affected? Why or why not?
- j) Given π , suppose a tax cut occurs so that for some periods a budget deficit is run. Is resource allocation affected? Why or why not?
- k) If $r > R$, a model like this runs into trouble as a model for a small open economy. Why?
- ℓ) In a few words give your opinion of the Barro model of infinitely-lived families linked through bequests.

II.12 *Can a fall in labor supply increase aggregate labor income?* Consider a closed economy with two factors of production, K and L , that is, capital and labor, respectively. Assume a neoclassical aggregate production function with CRS: $Y = F(K, L) = LF(k, 1) \equiv Lf(k)$, where $k \equiv K/L$. Assume perfect competition.

- a) Find the equilibrium real wage, w , as a function of k and show that the gross capital income share is $f'(k)k/f(k)$.
- b) Define $\alpha(k) \equiv f'(k)k/f(k)$. If the production function is Cobb-Douglas, $\alpha(k)$ equals a key parameter of the function. What parameter?
- c) Show that the elasticity of w wrt. k equals $\alpha(k)/\sigma(k)$, where $\sigma(k)$ is the elasticity of substitution between K and L . *Hint:* Let MRS denote the marginal rate of substitution of K for L , i.e., $MRS = F_L(K, L)/F_K(K, L)$ and the elasticity of substitution between K and L generally be called $\eta_{k, MRS}$ (i.e., the elasticity of k wrt. MRS). Then, for a general production function $F(K, L)$,

$$\eta_{k, MRS} \equiv \frac{MRS}{k} \frac{dk}{dMRS}|_{Y=\bar{Y}} = -\frac{F_K F_L (K F_K + L F_L)}{K L [(F_L)^2 F_{KK} - 2 F_K F_L F_{KL} + (F_K)^2 F_{LL}]}.$$

(For derivation of this formula, see for example Sydsaeter and Hammond (2002).)

When $F(K, L)$ has CRS, the formula simplifies to

$$\eta_{k, MRS} = \frac{F_K(K, L) F_L(K, L)}{F(K, L) F_{KL}(K, L)} = -\frac{f'(k)(f(k) - f'(k)k)}{k f(k) f''(k)} \equiv \sigma(k).$$

- d) Apply your result and your general empirical knowledge to assess whether a lower L (i.e., a rise in the scarcity of labor) is likely to decrease or increase aggregate labor income, $w(K/L)L$.