Advanced Macroeconomics. Exercises

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Problem set I: Technology. OLG models in discrete time

I.1 *Short questions* (answering requires only a few well chosen sentences).

a) Assume all firms' technology is described by the same neoclassical production function, $Q = F(K, L)$, with decreasing returns to scale everywhere. Suppose there is "free entry and exit" and perfect competition at all markets. Then a paradoxical situation arises in that no equilibrium with a finite number of firms (plants) would exist. Explain.

b) In the Diamond OLG model as in many other macro models, the technology is assumed to have constant returns to scale (CRS) with respect to capital and labour taken together. Often the so-called *replication argument* is put forward as a reason to expect CRS should hold in the real world. What is the replication argument? Do you think it is valid in the present case? Explain.

c) What is likely to happen in an economy where, for a certain historical period, there has been something close to constant returns to scale and perfect competition, but then, after a shift to new technologies in the different industries, increasing returns to scale arise?

I.2 *Short questions.*

a) Make a list of motives for individual saving. Are some of these motives better represented by OLG models than by the Ramsey model?

b) Briefly give some hints about how you think the Diamond OLG model should be extended to give a more adequate picture of the standard life-cycle pattern of individual saving?

c) In the Diamond OLG model, assume that the number of young people, L_t, grows at the constant rate n. Derive a formula showing the growth of population $P_t \equiv L_t + L_{t-1}$. Comment.

d) In standard long-run models with perfect competition (like Diamond's OLG model or the Ramsey model), the real rate of interest, r_t (i.e., a price on the market for loans), and the real rental rate, $\tilde{r}_t$, for physical capital (i.e., a price on the market for capital services) may or may not coincide for all t. Give a necessary and sufficient condition that they coincide.

I.3 *Short questions.*

a) A steady-state capital intensity can be in the "dynamically efficient" region or in the "dynamically inefficient" region. What is the *golden rule* capital intensity? How are the two mentioned regions defined? Give a simple characterization of the two regions.

b) Compare some long-run properties of the Ramsey model with the corresponding long-run properties of the Diamond OLG model. *Hint:* For example, think of the long-run interest rate and/or the possibility of dynamic inefficiency.

c) The First Welfare Theorem states that, given certain conditions, any competitive equilibrium ($\equiv$ Walrasian equilibrium) is Pareto optimal. Give a list of single circumstances that each tend to make the conclusion untrue.

I.4 *The aggregate saving rate.*

a) Derive a formula for the aggregate (net) saving rate (S^n/Y in our standard notation) in the long run in a "well-behaved" Diamond OLG model (in terms of the rate of population growth and the steady state capital intensity); ignore technical progress. *Hint:* Use that for a closed economy $S_t^n = K_{t+1} - K_t$. Comment!

b) Assume now that the rate of population growth is zero. What does the formula tell you about the level of (net) aggregate savings in this case? Give the intuition behind the result.

c) Suppose there is Harrod-neutral technical progress at the constant rate $g > 0$. Derive a formula for the aggregate (net) saving rate (S^n/Y in standard notation) in the long run in the model in this case.

d) Is your result from b) still true in case of zero population growth? Comment.

I.5 Consider an economy described by a Diamond OLG model, extended with endogenous labor supply. Assume the young has the lifetime utility function (standard notation):

$$U(c_1, 1-\ell, c_2) = \ln c_1 + \gamma \ln(1-\ell) + (1+\rho)^{-1} \ln c_2.$$

a) Write down the two period budget constraints and find the solution for labor supply and saving of the young.

b) What is the effect of a rise in the real wage on labor supply? Comment. *Hint:* relate to the Slutsky decomposition into a substitution, income, and wealth effect.

c) What is the effect of a rise in the real interest rate on saving? Comment.

d) The fundamental difference equation for the model can be shown to be

$$\bar{k}_{t+1} = \frac{w(\bar{k}_t/\ell)}{[2+\rho+(1+\rho)\gamma](1+n)}. \tag{*}$$

The notation is: $\bar{k}_t \equiv K_t/N_t$, where K_t is aggregate capital and $N_t = N_0(1+n)^t$ is the number of young, $k_t \equiv K_t/L_t \equiv K_t/(\ell N_t)$, $w(k) = f(k) - kf'(k)$ and $f(\cdot)$ is the production function on intensive form. Briefly explain (*).

e) Suppose from now that $f(\cdot)$ is Cobb-Douglas. Write down the fundamental difference equation for this case.

f) Illustrate by a phase diagram the dynamic development of the economy.

g) *A brush-up question.* Can the time path of the economy be dynamically inefficient? Why or why not? (A good argument should be based both on intuition and a hint to a proof).

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