Advanced Macroeconomics. Exercises

10.11.2008. Christian Groth

Problem set VIII

VIII.1 *Short questions.*

a) What is an inflation bubble?

b) Is an inflation bubble possible if all agents have rational expectations?

c) Two economists − one from MIT and one from Chicago − are walking down the street. The MIT economist sees a 100 dollar note lying on the sidewalk and says: "Oh, look, what a fluke!". "Obviously it is false", says the Chicago economist, "if it wasn't, someone would have picked it up!" Discuss.

VIII.2 *Term structure of interest rates.* Consider the following dynamic model in continuous time for a closed economy:

$$\begin{aligned} \dot{Y}_t &= \lambda(D(Y_t, R_t, \tau) + G - Y_t), \quad \lambda > 0, 0 < D_Y < 1, D_R < 0, -1 < D_\tau < 0, \\ \frac{M_t}{P} &= L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0. \\ R_t &= 1/Q_t, \\ \frac{1 + \dot{Q}_t^e}{Q_t} &= r_t = i_t, \end{aligned}$$

where a dot over a variable denotes the derivative wrt. time t, and the superscript e denotes subjective expectation. Further, Y_t = output, R_t = real interest rate on a consol, τ = tax parameter, G = government spending on goods and services, M_t = money supply, P = output price, i_t = nominal short-term interest rate, π_t = rate of inflation, Q_t = real price of a consol. The variables G, P, and τ are exogenous positive constants. The initial value of Y, Y_0, is historically given. To begin with we assume that $M_t = M$, an exogenous positive constant.

a) Briefly, interpret the model, including the parameters.

Suppose expectations are rational and that speculative bubbles never arise.

b) To characterize the movement over time of the economy, derive from the model a dynamic system in Y and R. Draw the corresponding phase diagram and illustrate the path that the economy follows. Comment.

c) How does the steady-state value of Y depend on G and τ?

Now we will consider effects of shifts in policy. Suppose that the economy has been in its steady state until time t_0.

d) Then an unanticipated downward shift in G occurs. But after this shift everybody expects G to remain unchanged forever. Illustrate by a phase diagram and by graphical time profiles what happens to Y_t, R_t, r_t, and Q_t. Comment. *Hint:* the following formula may be helpful:

$$R_t = \frac{1}{Q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds}.$$

e) Suggest a broader interpretation of G and the downward shift in G.

f) Sign the slope of the yield curve after the policy shock. Comment.

Change the model so that i_t becomes the instrument and equal to a positive constant, i, as long as the monetary authority does not decide to change it. Now M_t is endogenous.

g) Answer question b) in this new situation.

Suppose that the economy has been in its steady state until time t_0.

h) Then an unanticipated downward shift in i occurs, but after this shift everybody expects i to remain unchanged forever. Illustrate by a phase diagram and by graphical time profiles what happens to Y_t, M_t, R_t, r_t, and Q_t. Comment.

i) Assume instead that at time t_0, the monetary authority credibly announces a downward shift in the instrument variable to take place at time $t_1 > t_0$. Illustrate by a phase diagram and by graphical time profiles what happens to Y_t, R_t, r_t, Q_t, and M_t for $t \geq t_0$. Comment.

j) Briefly discuss the model.

VIII.3 Consider the model from Problem VIII.2 and let the short-term nominal interest rate, i, be the policy instrument.

a) Determine the short-term and long-term real interest rates in steady state and find an implicit solution for Y in steady state. How does Y in steady state depend on i?

Now, suppose that the economy has been in its steady state until time t_0.

b) Then an unanticipated upward shift in the short-term rate occurs, but apart from this shift, everybody expects the short-term rate to remain unchanged forever. Illustrate by a phase diagram and by graphical time profiles what happens to Y_t, R_t, r_t, Q_t, and M_t for $t \geq t_0$. Comment.

c) Assume instead that at time t_0, the monetary authority credibly announces an upward shift in the instrument variable to take place at time $t_1 > t_0$. Illustrate by a phase diagram and by graphical time profiles what happens to Y_t, R_t, r_t, Q_t, and M_t for $t \geq t_0$. Comment.

VIII.4 *Uncovered interest parity in discrete time.* Starting with discrete time (period analysis), let Japan be the domestic country and define:

$$\begin{aligned} X_t &= \text{nominal exchange rate (¥ per \$)}, \\ P_t &= \text{domestic price level}, \\ P^* &= \text{foreign price level}, \\ i_t &= \text{nominal (short-term) interest rate}, \\ i^* &= \text{foreign (short-term) nominal interest rate}. \end{aligned}$$

The uncovered interest parity (UIP) condition is

$$1 + i_t = \frac{1}{X_t}(1 + i_t^*)E_t X_{t+1}.$$

a) Interpret.

b) Show that for i_t^* and $\frac{E_t X_{t+1} - X_t}{X_t}$ "small",

$$i_t \approx i_t^* + \frac{E_t X_{t+1} - X_t}{X_t}.$$

Hint: You may use that $\frac{E_t X_{t+1}}{X_t} = \frac{E_t X_{t+1} - X_t}{X_t} + 1$ and that the product of two small numbers is "*very* small".

—