# Credit and Business Cycles

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- Financial Hierarchy
- Bank Lending Channel
- Balance Sheet Channel
- Credit Cycles

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- Modigliani-Miller theorem (MM, 1958): financial structure is irrelevant to real outcomes
- Perfect substitutability between different forms of funding
- Cost of capital is the same regardless of the way funds are raised
- This hypothesis is implicit in the IS-LM framework

- MM theorem is based on the assumption of perfect and complete information
- What's at the root of imperfect substitutability?
  - Gap between internal finance and total investment
  - Asymmetric information between lenders and borrowers
- Asymmetric information or imperfect enforceability of financial contracts:
  - Wedge between the cost of funds raised externally and the opportunity cost of internal funds: *external finance premium (EFP)*
  - EFP: cost implicit in the principal-agent problem characterizing the relationship between financial intermediaries and borrowers

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### **Financial Hierarchy**

- The cost of different sources of external funding increases in the degree of asymmetry between borrowers and lenders
- Pecking order theory (or financial hierarchy): Myers and Majluf (Journal of Finance, 1984)
- Firms prefer internal funds
- If external finance is required, firms will resort to:
  - debt finance
  - 2 bonds
  - equity finance

Financial gap: firms need to resort to the credit market

- New equity issues are too costly due to adverse selection phenomena
- At a given share price, only overvalued firms are willing to sell their shares
- Potential shareholders anticipate that these companies are adversely selected→ no trade on the equity market
- Under these conditions the announcement of an equity issue is generally interpreted as bad news by the investors
- The stock market becomes a typical *market for lemons* (Akerlof, 1970)

# A sketch of the Financial Hierarchy theory

- Internal finance (A): retained profits (cash flow tout-court) which in turn constitute the net worth
- The cost of internal finance can be quantified based on the concept of opportunity cost (OC)
- The OC of internal finance equals the interest rate on a financial investment the firm could undertake and that it must renounce if the real investment is entirely covered with internal finance (r)

- Demand for capital K = f(c) f' < 0
- This can be derived from a profit maximization problem
- In this case investment takes place up to the point where

MCK = MPK

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# Funding Opportunities



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- In a MM world, external and internal finance would have the same cost (r).
- Thus *c* = *r*
- The  $i^{th}$  firm would invest up to  $K^M$
- The gap  $K^M A_0$  is covered with external finance and leverage equals:

$$rac{K^M-A_0}{K^M}=1-a_0$$

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- In the anti-MM world, information is asymmetric
- External finance is more expensive than internal finance
- Cost of external finance:  $\rho$
- The wedge between r and  $\rho$  depends upon the amount of external finance:

$$ho = r + arepsilon \left( {{{oldsymbol{{\cal K}}}^M} - {{oldsymbol{{\sf A}}_0}} 
ight)$$
 ,  $arepsilon > 0$ 

Thus, from  $K^M = A_0$  the cost for external finance schedule is upward sloping

# Conclusions

- Inefficient exploitation of resources
- Relevance of financial structure, compared to the MM paradigm:
  - Assume net worth increases from  $A_0$  to  $A_1$

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- Bank Lending channel: emphasis on banks' balance sheet and substitutability between different forms of finance to extend credit
- Balance Sheet channel (Financial Accelerator): emphasis on borrowers' balance sheet and the premium for external finance

#### Some Literature

- Bank Lending view: Bernanke and Blinder (AER, 1988)
- Balance Sheet channel: Greenwald and Stiglitz (QJE, 1993), Bernanke and Gertler (AER, 1989) and Kiyotaki and Moore (JPE, 1997)
- These frameworks emphasize the role financial frictions and the interaction of heterogeneous agents
- Renewed interest for the Wicksellian view that credit has both relevance in the propagation of business cycles and MPTM
- Distinct role played by financial assets and liabilities and the need to distinguish between different types of non-monetary assets

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- The bank lending channel highlights the nature of credit and the special role played by commercial banks in the credit markets
- Essence of the mechanism: monetary policy can affect the external financial premium by influencing the supply of intermediated credit
- Model economy: commercial banks represent the only source of external funding
- Banks' liabilities: deposits
- Balance sheet:

ASSETSLIABILITIESReserves (r)Deposits (D)Loans (D - r)

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- Banks' specialization in overcoming informational asymmetries in the provision of both transaction services and credit to business
- Commercial banks: in most of the countries the main source of external funding for small and medium-sized firms
- Monetary policy decisions affecting the reserve position of the banks can in turn generate adjustments in the interest rates and in the banking sector's balance sheet
- Policy induced reductions in banks' reserves, for instance, are likely to lead to a reduction in the level of deposits, which should be matched by a fall in the supply of loans, at least to the extent that banks cannot adjust their position in reserves by issuing new non reservable liabilities
- Furthermore, changes in the interest rates can in turn reflect in effects on the demand for money, consumption and investment decisions of firms and households

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# Kiyotaki and Moore (1997) (KM hereafter)

- Asymmetric information between borrowers and lenders
- Moral hazard: lenders face a maximum limit to the credit they can obtain
- Role of collateralizable assets
- Collateral constraints affect the scale of production: underutilization of resources
- The higher the collateral value, the higher the credit obtained and in turn investment spending and production

- The economy is composed by a large number long-lived agents: 2 categories
- Farmers: financially constrained agents who produce by means of inalienable human capital
- Gatherers: agents endowed with alienable human capital, and hence financially unconstrained
- The distribution of the agents according to their nature is exogenously postulated by KM
- Two goods:
  - fruit (output)
  - land: a durable and collateralizable good, whose supply is exogenously imposed to  $\overline{K}$

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#### Farmers

Output, produced by means of a technology of both labour and land, can be consumed or lent at a constant gross rate of return R = (1 + r). Farmers' technology:

$$y_t^f = (\alpha + c)k_{t-1}^f \tag{1}$$

- $y_t^f$ : output produced by the farmer in t
- $k_{t-1}^{f}$ : land available to the farmer at time t
- *α* and *c*: positive productivity parameters
- c: share of non-tradeable output that can only be employed in the productive process (bruised fruit)

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#### Gatherers

Gatherers access to the following technology:

$$y_t^g = f(k_{t-1}^g) \tag{2}$$

- $y_t^g$ : output of the gatherer in t
- $f(\cdot)$ : decreasing returns to scale technology
- $k_{t-1}^g$ : land available to the gatherer at time t-1
- Production process: time-to-build technology

### Inalienability and Impatience

- Each farmer's technology is individual-specific
- Once the production has started, no one can successfully complete the productive process but the farmer
- In principle, farmers have incentive to threaten their creditors to withdraw their labour and default
- KM assume that lenders cannot force borrowers to repay their debts unless previously secured
- $\beta < \beta'$  : heterogeneity is characterized by different discount rates across households
- Furthermore, if discount rates are equal then the steady state income distribution would be indeterminate: a number of hypothesis are necessary

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### **Collateral Constraint**

Gatherers collateralize farmers' land, by imposing the following constraint:

$$b_t^f = \frac{q_{t+1}k_t^f}{R} \tag{3}$$

 $b_t^f$ : loan  $q_{t+1}$ : price of the land at time t + 1. Flow-of-funds constraint

$$y_t^f + b_t^f = q_t \Delta k_t^f + Rb_{t-1}^f + c_t^f$$
(4)

 $c_t^f$ : farmers' consumption

Substituting the collateral constraint into the budget constraint:

$$c_t^f = (\alpha + c)k_{t-1}^f - \mu_t k_t^f \tag{5}$$

User cost of land:  $\mu_t = q_t - (q_{t+1}/R)$ . Preferences are such that farmers only consume bruised fruit,  $ck_{t-1}^f$ . In order to determine the consumption/saving behavior of the farmer, we recall that one unit of tradable output can be employed in three ways:

- Investment. It consists in investing  $1/\mu_t$  in land, which yields  $c/\mu_t$  non-tradable fruit and  $\alpha/\mu_t$  tradable fruit at date t + 1. The non-tradable fruit is consumed and the tradable fruit is invested which yields  $(\alpha/\mu_t)(c/\mu_{t+1})$  non-tradable fruit and  $(\alpha/\mu_t)(\alpha/\mu_{t+1})$  tradable fruit at date t + 2, and so on
- **Saving**. The second one consists in saving the unit of tradable output and use the return to saving R to begin a strategy of investment like the first one from t + 1 onward
- **Consumption**. The third option consists in consuming right away one unit in *t*

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Alternative paths:

$$\begin{array}{rcl} \text{Inv.} & : & 0, \frac{c}{\mu_t}, \frac{\alpha}{\mu_t} \frac{c}{\mu_{t+1}}, \frac{\alpha}{\mu_t} \frac{\alpha}{\mu_{t+1}} \frac{c}{\mu_{t+2}}, \dots \\ \text{Sav.} & : & 0, 0, R \frac{c}{\mu_{t+1}}, R \frac{\alpha}{\mu_{t+1}} \frac{c}{\mu_{t+2}}, \dots \\ \text{Cons.} & : & 1, 0, 0, \dots \end{array}$$

Steady state:

Inv.: 
$$0, \frac{c}{\mu}, \frac{\alpha}{\mu} \frac{c}{\mu}, \frac{\alpha}{\mu} \frac{\alpha}{\mu} \frac{c}{\mu}, \dots$$
  
Sav. : 
$$0, 0, R \frac{c}{\mu}, R \frac{\alpha}{\mu} \frac{c}{\mu}, \dots$$
  
Cons. : 
$$1, 0, 0, \dots$$

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Discounted steady-state utility associated with the three strategies:

- $U_I = \beta c / (1 \beta) \alpha$  (investment)
- $U_5 = R\beta^2 c / (1 \beta) \alpha$  (saving)
- $U_C = 1$  (consumption)

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Need to impose a condition in order to ensure that farmers will actually invest, rather than saving/consuming:

$$U_I > U_S$$
  
 $U_I > U_C$ 

#### Assumption 1

Using the fact that

 $U_I > U_S$ 

entrepreneurs eat all their non-tradable output iff:

$$\frac{1}{\beta} > R = \frac{1}{\beta'}$$

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#### Assumption 2

Farmers consume no more than the bruised fruit and use all tradable output for making deposits:

 $U_I > U_C$ 

which in turn translates into

$$c > \left(rac{1}{eta} - 1
ight)lpha.$$

Farmer's demand for land is given by

$$k_t^f = \frac{1}{\mu_t} [(\alpha + q_t) k_{t-1}^f - Rb_{t-1}^f] = \frac{\alpha}{\mu_t} k_{t-1}^f$$
(6)

Substituting this expression in the collateral condition:

$$b_t^f = \frac{q_{t+1}\alpha k_{t-1}^f}{R\mu_t} \tag{7}$$

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Alienability of human capital for the gatherers implies the existence of a single constraint:

$$y_t^g + Rb_{t-1}^g = q_t \Delta k_t^g + b_t^g + c_t^g$$
(8)

As  $k_t^f = \overline{K} - k_t^g$ :  $c_t^g = f(k_{t-1}^g) + \mu_t(\overline{K} - k_t^g)$ (9)

The maximization problem for the gatherer implies that his preferences are such that  $R\mu_t = f'(k_t^g)$ . Demand for land:

$$k_t^g = f'^{-1}(R\mu_t)$$
 (10)

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For a given level of farmers' landholding and debt at the previous date an equilibrium from date *t* onward is characterized by the path:

$$\left\{\left.\left(q_{t+s}, K_{t+s}, B_{t+s}\right)\right| s \geq 0\right\}$$

This satisfies flow-of-funds constr., coll. constr. and Euler equation of the gatherer.

Rule out bubbles (transversality condition):

$$\lim_{s\to\infty}\left\{\left(\beta^{'}\right)^{s}q_{t+s}\right\}=0$$

Using these three equations we can recover a unique steady state,  $\{q, k^f, b\}$  with associated user cost,  $\mu$ :

$$q = \frac{\alpha R}{R - 1}$$
$$b = \frac{\alpha}{R - 1} k^{f}$$
(11)

$$\mu = \alpha$$
 (12)

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$$k_t^g = k_{t-1}^g = \overline{K} - k^f = \overline{K} - f^{'-1}(R\alpha)$$
(13)



• We log-linearize the model economy around the steady state:

$$\left[ egin{array}{c} \widehat{k}^{f}_{t} \ \widehat{q}_{t} \end{array} 
ight] = \left[ egin{array}{c} rac{\eta}{1+\eta} & 0 \ -rac{R-1}{\eta} & R \end{array} 
ight] \left[ egin{array}{c} \widehat{k}^{f}_{t-1} \ \widehat{q}_{t-1} \end{array} 
ight]$$

 η : elasticity of the residual supply of farmers' land with respect to the user cost at the steady state:

$$\frac{1}{\eta} = \left. \frac{d \log \mu\left(k_t^f\right)}{d \log k_t^f} \right|_{k_t^f = k^f} = - \left. \frac{d \log f'(k_t^g)}{d \log k_t^g} \right|_{k_t^g = \overline{K} - k^f} \times \frac{k^f}{\overline{K} - k^f}$$

- We consider  $f''(\cdot) < 0$  to ensure that  $\eta$  is positive
- Jacobian Matrix: lower triangular

### **Productivity Shock**

- At time t 1, we assume the model economy is at the steady state
- We introduce an unexpected one-period shock to farmers' productivity, denoted by Δ.
- From time tand t + 1 the production technologies of the two agents return to their equilibrium level
- Perfect foresight: the shock is known to be temporary

Let us consider the BC of the farmer:

$$\mu_t k_t^f = [(\alpha + q_t)k_{t-1}^f - Rb_{t-1}^f]$$

Log-linear approximation around the steady state:

$$RHS \rightarrow [\alpha + \Delta \alpha + q_t - q]k^f \rightarrow [\alpha + \Delta \alpha + q\hat{q}_t]k^f$$
$$LHS \rightarrow \left[1 + \left(1 + \frac{1}{\eta}\right)\right]\hat{k}_t^f$$

- After time t the system returns to its original position
- The financial constraint faced by the farmer holds with equality  $(\beta < \beta^{'})$

Percentage deviation of land from its steady state:

$$\widehat{k}_{t}^{f} = \frac{\eta R}{\left(R-1\right)\left(\eta+1\right)}\widehat{q}_{t} + \frac{\eta\Delta}{\eta+1},$$
(14)

From time t + 1 onwards, land dynamics will turn on the following path:

$$\widehat{k}_{t+s}^{f} = \left(\frac{\eta}{1+\eta}\right)^{s} \widehat{k}_{t}^{f}, \qquad s > 0$$
(15)

Next, we need to determine the evolution of the asset price,  $q_t$ . From the Gatherer's Euler equation:

$$q_{t} = \beta' f' \left( \overline{K} - k_{t}^{f} \right) + \beta' q_{t+1}$$
(16)

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After log-linearizing the Euler condition and plug it back into the linearized constraint of the farmer:

$$\widehat{q}_{t} = \frac{\left(R-1\right)\left(\eta+1\right)}{\eta\left(\left(\eta+1\right)R-\eta\right)}\widehat{k}_{t}^{f}$$
(17)

We solve the resulting system to obtain:

$$\widehat{q}_{t} = \frac{\Delta}{\eta}, \qquad (18)$$

$$\widehat{k}_{t}^{f} = \frac{\eta}{1+\eta} \left( 1 + \frac{1}{\eta} \underbrace{\frac{R}{R-1}}_{\text{Amplification Factor}} \right) \Delta. \qquad (19)$$

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### The role of elasticity $\eta$

Persistence:

$$\widehat{k}^f_{t+s} = \left(rac{\eta}{1+\eta}
ight)^s \widehat{k}^f_t, \qquad s>0$$

Market Clearing:

$$\left(1+rac{1}{\eta}
ight)\widehat{k}_t^f=\Delta+rac{R}{R-1}\widehat{q}_t$$

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# Extended Version: Section II KM (1997)

- Let us assume that there is a continuum of farmers, as in the original KM framework
- Land as a reproducible asset
- Depreciation
- Probability of investing
- A fraction  $\pi$  of farmers can invest (entrepreneurs), while  $1-\pi$  cannot (households)

#### Entrepreneurs

Flow-of-funds Constraint

$$y_{i,t}^{f} + b_{i,t}^{f} = q_{t} \left( k_{i,t}^{f} - k_{i,t-1}^{f} \right) + \phi \left( k_{i,t}^{f} - \lambda k_{i,t-1}^{f} \right) + R b_{i,t-1}^{f} + c_{i,t}^{f}$$
(20)

 $\phi\left(k_{i,t}^{f} - \lambda k_{i,t}^{f}\right)$ : input for reproduction of capital Collateral Constraint

$$b_{i,t}^f = \frac{q_{t+1}k_{i,t}^f}{R}$$

Assuming that investing is strictly better than consuming, the i<sup>th</sup> entrepreneur faces the following constraint:

$$\left(q_t + \phi - \frac{q_{t+1}}{R}\right) k_{i,t}^f = \left(\alpha + \lambda \phi + q_t\right) k_{i,t-1}^f - Rb_{i,t-1}^f$$

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#### Households

$$k_{j,t}^f = \lambda k_{j,t-1}^f$$

In this case, land is solely subject to depreciation

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### Aggregation

Land Dynamics

$$\begin{split} \mathcal{K}_{t} &= \int k_{j,t}^{f} dj + \int k_{i,t}^{f} di = \\ &= (1-\pi) \, \lambda \mathcal{K}_{t-1} + \frac{\pi}{\left(q_{t} + \phi - \frac{q_{t+1}}{R}\right)} \left[ \left(\alpha + \lambda \phi + q_{t}\right) \mathcal{K}_{t-1} - \mathcal{R}\mathcal{B}_{t-1} \right] \end{split}$$

**Debt Dynamics** 

$$B_{t} = q_{t} \left( K_{t} - K_{t-1} \right) + \phi \left( K_{t} - \lambda K_{t-1} \right) + RB_{t-1}$$

Asset Price (Gatherer's Euler Equation)

$$R = rac{f'\left(\overline{K} - K_t
ight) + q_{t+1}}{q_t}$$

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### Numerical example



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# Numerical Simulations ( $\pi = 0.5$ )



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Numerical Simulations ( $\pi = 0.8$ )



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• Loan-to-value (LTV) ratio: amount of the loan as a percentage of the total appraised value of real property:

$$b_{i,t}^f = \chi rac{q_{t+1}k_{i,t}^f}{R} \quad 0 < \chi \leq 1$$

- χ : LTV
- As  $\chi \uparrow$ , the qualification guidelines for certain mortgage programs become more strict
- Lenders can require borrowers of high LTV loans to buy mortgage insurance

Numerical Simulations ( $\chi = 0.8$ )



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Numerical Simulations ( $\chi = 0.5$ )



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