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A suggested solution to the problem set at the exam in **Advanced Macroeconomics 2** January 8, 2010

(3-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates (a) accurate and thorough understanding of the concepts, methods, and models in the course, (b) knowledge of the major empirical regularities for aggregate economic variables, and (c) ability to use these theoretical tools and this empirical knowledge to answer macroeconomic questions.

1. Solution to Problem 1

We consider a Blanchard OLG model for a closed economy with dynamics described by the differential equations

$$\dot{C}_t = (F_K(K_t, L) - \delta - \rho)C_t - m(\rho + m)(K_t + B_t), \tag{1.1}$$

$$\dot{K}_t = F(K_t, L) - \delta K_t - C_t - G, \tag{1.2}$$

$$\dot{B}_t = [F_K(K_t, L) - \delta]\, B_t + G - T_t, \tag{1.3}$$

the condition

$$\lim_{t\to\infty} B_t e^{-\int_0^t [F_K(K_s,L)-\delta]ds} = 0, \tag{1.4}$$

and a requirement that households satisfy their transversality conditions.

a) The parameters:

δ is the capital depreciation rate,

ρ is the pure rate of time preference, i.e., households' utility discount rate (a measure of impatience),

[1] The solution below contains more details and more precision than can be expected at a three hours exam.

m is the mortality rate, i.e., the expected number of deaths per time unit, say per year, relative to the size of population.

Individuals have finite, but uncertain remaining lifetime. The model relies on the simplifying assumption that for a given individual, the probability of having a remaining lifetime, X, longer than some arbitrary number τ is $P[X > \tau] = e^{-m\tau}$, the same for all independently of age. It follows that for any person the probability of dying within one year from now is approximately equal to m. Since a constant population is assumed, also the birth rate equals m.

Individuals can buy life annuity contracts from life insurance companies. The depositor's rate of return on these contracts is $r + m$ until death, where r is the risk-free (real) rate of interest (the life annuity contracts are actuarially fair).

The differential equation (1.1) describes how the increase in aggregate consumption per time unit is determined. In this version of the Blanchard model retirement from the labor market and technical progress are ignored.

The instantaneous utility is logarithmic so that the Keynes-Ramsey rule for an individual born at time v is

$$\dot{c}_{v,t} \equiv \frac{\partial c_{v,t}}{\partial t} = [r_t + m - (\rho + m)]\, c_{v,t} = (r_t - \rho)c_{v,t}. \tag{1.5}$$

Combining this with the individual transversality condition implies the consumption function

$$c_{v,t} = (\rho + m)(a_{v,t} + h_t), \tag{1.6}$$

where $a_{v,t}$ is financial wealth and h_t is human wealth (PV of future labor earnings),

$$h_t \equiv \int_t^\infty (w_s - T_s/L)e^{-\int_t^s (r_\tau + m)d\tau} ds. \tag{1.7}$$

Here, w_s is the real wage at time s and T_s/L is the per capita tax at time s.

The first term in (1.1) comes from aggregation of the Keynes-Ramsey rule (1.5) and the fact that $r_t = F_K(K_t, L) - \delta$ in competitive equilibrium. The subtraction of the term $m(\rho + m)(K_t + B_t)$ in (1.1) is due to the *generation replacement effect* and the fact that aggregate financial wealth, A_t, must equal $K_t + B_t$. In every short instant some people die and some people are born. The arrival of newborns is Lm per time unit, and since they have no financial wealth, the inflow of these people lowers aggregate consumption by $m(\rho + m)A_t$ per time unit. Indeed, the average financial wealth in the population is A_t/L, and the consumption effect of this is $(\rho + m)A_t/L$, cf. (1.6). This implies, *ceteris paribus*, that aggregate consumption is reduced by

$$Lm(\rho + m)\frac{A_t}{L} = m(\rho + m)A_t$$

per time unit, where $A_t = K_t + B_t$.

The second differential equation, (1.2), is easier. Essentially, it is just a way of writing the national income identity for a closed economy: gross investment, $\dot{K}_t + \delta K_t$, equals gross national income, $F(K_t, L)$, minus the sum of private and public consumption, $C_t + G$.

The third differential equation, (1.3), says that the increase per time unit in real public debt equals the real budget deficit, that is, total government expenditure (interest payments plus spending on goods and services) minus net tax revenue. This tells us that the budget deficit is entirely debt-financed (i.e., no money financing).

Finally, equation (1.4) reflects a No-Ponzi-Game condition saying that the government debt should in the long run at most grow at a rate less than the long-run interest rate. So the government cannot run a fiscal policy involving permanent debt-roll-over. The reason government solvency requires this constraint lies the fact that the long-run interest rate, r^*, in the economy is higher than the long-run output growth rate, which is 0 (see below).

A No-Ponzi-Game condition in itself would only require $\leq$ instead of $=$ in (1.4). Since there is $=$ in (1.4), the model assumes that the government does not procure more tax revenue than needed to just satisfy its No-Ponzi-Game condition.

b) Given B_0 and a balanced budget for all $t \geq 0$, we have, from (1.3),

$$T_t = (F_K(K_t, L) - \delta)B_0 + G. \tag{1.8}$$

We introduce two baseline values of K, namely the golden rule value, K_{GR}, and the "critical" value, $\bar{K}$, defined by,

$$F_K(K_{GR}, L) - \delta = 0, \qquad \text{and} \qquad F_K(\bar{K}, L) - \delta = \rho,$$

respectively. In view of the Inada conditions and $\delta > 0$, both values exist and are unique (since $F_{KK} < 0$). We have $\bar{K} < K_{GR}$, since $\rho > 0$ and $F_{KK} < 0$.

Given $B_t = B_0,$ equation (1.2) shows that $\dot{K} = 0$ for

$$C = F(K, L) - \delta K - G,$$

cf. the strictly concave $\dot{K} = 0$ locus in Fig. 1.1.

Equation (1.1) shows that $\dot{C} = 0$ for

$$C = \frac{m(\rho + m)(K + B_0)}{F_K(K, L) - \delta - \rho}. \tag{1.9}$$

Thus, along the $\dot{C} = 0$ locus,

$$K \nearrow \bar{K} \Rightarrow C \to \infty$$

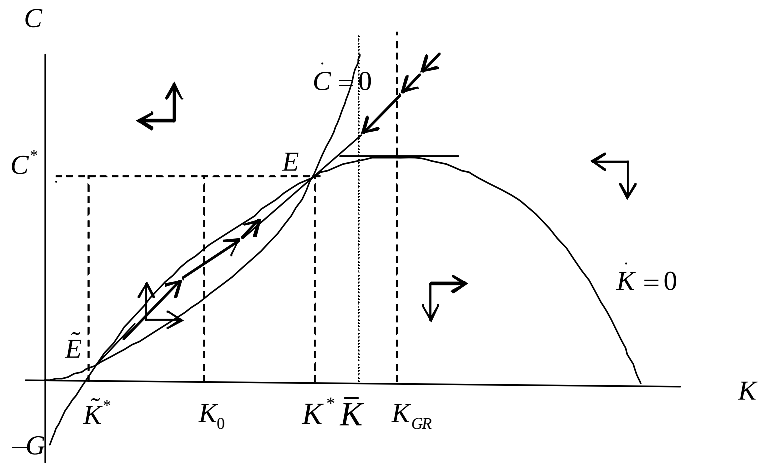

Figure 1.1:

and

$$K \searrow 0 \Rightarrow C \to 0,$$

the latter result following from the lower Inada condition. The $\dot{C} = 0$ locus is shown as the strictly convex curve in Fig. 1.1.

It is assumed that, given K_0, G and B_0 are "modest" relative to the production possibilities of the economy. Then the $\dot{C} = 0$ curve crosses the $\dot{K} = 0$ curve for *two* positive values of K. Fig. 1.1 shows these steady states as the points E and Ẽ with coordinates (K^*, C^*) and $(\tilde{K}^*, \tilde{C}^*)$, respectively. Obviously, $\tilde{K}^* < K^* < \bar{K}$.

The direction of movement in the different regions of Fig. 1.1, as determined by the differential equations, (1.1) and (1.2), are shown by arrows. It is seen that E is a saddle point, whereas Ẽ is totally unstable. Since G and B_0 are "modest", we have that the lower steady-state value, $\tilde{K}^*$, is smaller than K_0, as shown in the figure.

The capital stock is predetermined, whereas consumption is a jump variable. And since the slope of the saddle path is not parallel with the C axis, it follows that the system is saddle-point stable. The only trajectory satisfying *all* the conditions of general equilibrium (individual utility maximization for given expectations, continuous market clearing, perfect foresight) is the saddle path. The other trajectories in the diagram either violate the TVCs of the individual households (paths that in the long run point South-East in Fig. 1.1) or the NPG condition[2] of the households (paths that in the long run point North-West in the diagram). Hence, initial consumption, C_0, is determined as the ordinate to the point where the vertical line $K = K_0$ crosses the saddle path, and

[2] And therefore also the transversality condition.

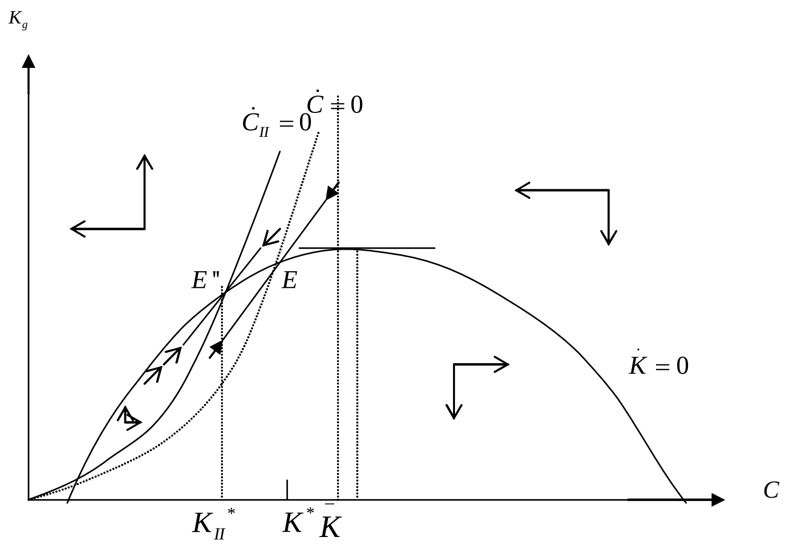

Figure 1.2:

over time the economy moves along the saddle path, approaching the steady state point E with coordinates (K^*, C^*).

c) Let $B_0 = B_0^I < B_0^{II}$ and $K_0 = K_0^I = K_0^{II}$. Based on (1.9), Fig. 1.2 illustrates. In the long run country II has less capital and a lower consumption level, due to the crowding-out effect of government debt in a full-employment economy.

d) Until time $t_0 > 0$ country I has been in the steady state E with a balanced government budget. From now, the variables refer exclusively to country I although the index I is suppressed. The level of public debt in the steady state is $B_0 > 0$ and tax revenue is, by (1.8),

$$T = (F_K(K^*, L) - \delta)B_0 + G \equiv T^*,$$

a positive constant, in view of $F_K(K^*, L) - \delta > \rho > 0$.

At time t_0 the government cuts taxes to a lower level $\bar{T}$, holding public consumption unchanged. That is, at least for a while after time t_0 we have

$$T_t = \bar{T} < T^*. \tag{1.10}$$

As a result $\dot{B}_t > 0$. The tax cut make current generations feel more wealthy, hence they increase their consumption. The rise in C combined with unchanged G implies negative net investment so that K begins to fall, implying a rising interest rate, r. For a while all the three differential equations that determine changes in C, K, and B are active. These dynamics are complicated and cannot, of course, be illustrated in a two-dimensional phase diagram.

The fiscal policy $(G, \bar{T})$ is called sustainable if the government stays solvent under this policy. We claim that the fiscal policy $(G, \bar{T})$ is not sustainable. There are at least three different approaches to the proof of this.

Approach 1. In view of $K^* < \bar{K} < K_{GR}$, we have $r^* = F_K(K^*, L) - \delta > F_K(\bar{K}, L) - \delta = \rho > 0$. Therefore, r_t $(\geq r^*)$ is strictly positive and thereby larger than the long-run growth rate of output (income). In this situation, a sustainable fiscal policy must, as seen from time t_0, satisfy the NPG condition

$$\lim_{t\to\infty} B_t e^{-\int_{t_0}^t r_s ds} \leq 0. \tag{1.11}$$

This requires that there exists an $\varepsilon > 0$ such that

$$\lim_{t\to\infty} \frac{\dot{B}_t}{B_t} < \lim_{t\to\infty} r_t - \varepsilon, \tag{1.12}$$

i.e., the growth rate of the public debt is not in the long run as high as the long-run interest rate.

The fiscal policy $(G, \bar{T})$ implies increasing public debt B_t. Indeed, we have, for $t > t_0$,

$$\begin{aligned} \dot{B}_t &= r_t B_t + G - \bar{T} \\ &> r^* B_0 + G - \bar{T} > r^* B_0 + G - T^* = 0, \end{aligned} \tag{1.13}$$

where the first inequality comes from $B_t > B_0 > 0$ and $r_t = F_K(K_t, L) - \delta > r^* = F_K(K^*, L) - \delta$, in view of $K_t < K^*$. This implies $B_t \to \infty$ for $t \to \infty$. Hence, dividing by B_t in (1.3) gives

$$\frac{\dot{B}_t}{B_t} = r_t + \frac{G - \bar{T}}{B_t} \to r_t \qquad \text{for} \qquad t \to \infty. \tag{1.14}$$

But this violates the NPG condition (1.12) and the fiscal policy $(G, \bar{T})$ is not sustainable.

Approach 2. An alternative argument is the following. Since $K_t < K^*$, we have $Y_t < Y^* = F(K^*, N)$ at the same time as $B_t \to \infty$ for $t \to \infty$. Hence, the debt-income ratio, B_t/Y_t, tends to infinity for $t \to \infty$, thus confirming that the fiscal policy $(G, \bar{T})$ is not sustainable.

Approach 3. Yet another way of showing absence of fiscal sustainability is to start out from the intertemporal government budget constraint and check whether the primary budget surplus, $\bar{T} - G$, which rules after time t_0, satisfies

$$\int_{t_0}^{\infty} (\bar{T} - G) e^{-\int_{t_0}^t r_s ds} dt \geq B_{t_0},$$

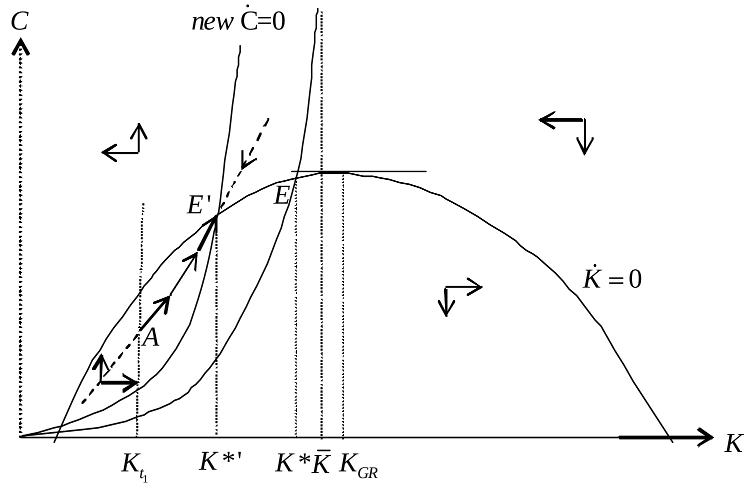

Figure 1.3: The adjustment after fiscal tightening at time t_1, presupposing $t_1 - t_0$ "large".

where $B_{t_0} = B_0 > 0$. Obviously, if $\bar{T} - G \leq 0$, this is not satisfied. Suppose $\bar{T} - G > 0$. Then

$$\int_{t_0}^{\infty} (\bar{T} - G) e^{-\int_{t_0}^{t} r_s ds} dt < \int_{t_0}^{\infty} (\bar{T} - G) e^{-r^*(t-t_0)} dt = \frac{\bar{T} - G}{r^*} < B_0 = B_{t_0},$$

where the first inequality comes from $r_t = F_K(K_t, N) - \delta > r^*$, the first equality from the hint, and, finally, the second inequality from the last equality in (1.13) and the fact that $\bar{T} < T^*$. So the intertemporal government budget constraint is not satisfied. The current fiscal policy is unsustainable.

e) After time t_1 there is again a balanced budget. The most straightforward interpretation is that this is obtained by raising taxation at time t_1 and letting T_t adjust over time ssuch that

$$T_t = (F_K(K_t, N) - \delta) B_{t_1} + G, \tag{1.15}$$

for $t \geq t_1$. Dynamics are then again governed by a two-dimensional system:

$$\dot{C}_t = [F_K(K_t, N_t) - \delta - \rho] C_t - m(\rho + m)(K_t + B_{t_1}), \tag{1.16}$$

$$\dot{K}_t = F(K_t, N) - \delta K_t - C_t - G, \tag{1.17}$$

and phase diagram analysis can again be used. The new constant level of public debt is $B_{t_1} > B_0$. The new initial K is K_{t_1}, which is smaller than the previous steady-state value, K^*, because of the negative net investment in the time interval (t_0, t_1). The phase diagram for $t \geq t_1$ is depicted in Fig. 1.3. Relative to Fig. 1.1 the $\dot{K} = 0$ locus is unchanged (since G is unchanged), but the $\dot{C} = 0$ locus has turned counter-clockwise.

For any given $K \in (0, \bar{K})$, the value of C required for $\dot{C} = 0$ is higher than before, cf. (1.9). The new saddle-point stable steady state is denoted E' and it has capital stock $K^{*\prime} < K^*$ and consumption level $C^{*\prime} < C^*$.

The level of consumption immediately after t_1, where the fiscal tightening sets in, is found where the line $K = K_{t_1}$ crosses the new saddle path, i.e., the point A in Fig. 1.3. As the figure is drawn, K_{t_1} is smaller than $K^{*\prime}$. This reflects that the tax cut has lasted a long time ($t_1 - t_0$ "relatively large"). The movement of the economy after t_1 implies gradual lowering of the capital stock and consumption until the new steady state, E$''$, is "reached". This induces a low consumption level – so low that net investment becomes positive. Then the capital stock and output increase gradually during the adjustment to the steady state E$''$.

In a situation where the tax cut did not last long ($t_1 - t_1$ "relatively small"), the point A in Fig. 1.2 would be to the right of the new steady state. Then the movement of the economy after t_1 would imply gradual lowering of the capital stock and consumption until the new steady state is "reached".

Thus, in both cases the long-run effect of the temporary budget deficit is qualitatively the same, namely that the larger supply of government bonds crowds out physical capital of the private sector. The time profiles of T_t, B_t, C_t, and K_t for $t \geq 0$ are shown in Fig. 1.4.

Is there an alternative way for fiscal policy to change such that a balanced budget for $t \geq t_1$ is obtained? One might contemplate maintaining taxation at the level $\bar{T}$, but lowering public consumption, G. Within the model, G is a constant so only a once-for-all shift in public consumption is consistent with the model. Can such a once-for-all shift be consistent with a permanently balanced budget for $t \geq t_1$, given the constant tax revenue $\bar{T}$? By (1.3), the required new level of public consumption would have to satisfy

$$G' = \bar{T} - (F_K(K_t, L) - \delta)B_{t_1}, \tag{1.18}$$

which in turn would require K_t constant at the level K_{t_1} for all $t \geq t_1$, that is, require the economy to be in a new steady state already from time t_1. This would not generally be possible.

Finally, a balanced budget for $t \geq t_1$ could of course be obtained by a *combination* of a lower level of government consumption, G'', and a time-varying tax revenue satisfying (1.15) for $t \geq t_1$, with G replaced $G'' < G$. Then the $\dot{K} = 0$ locus would be raised somewhat and the economy would in the long run settle down in a "higher" steady state than in Fig. 1.3.

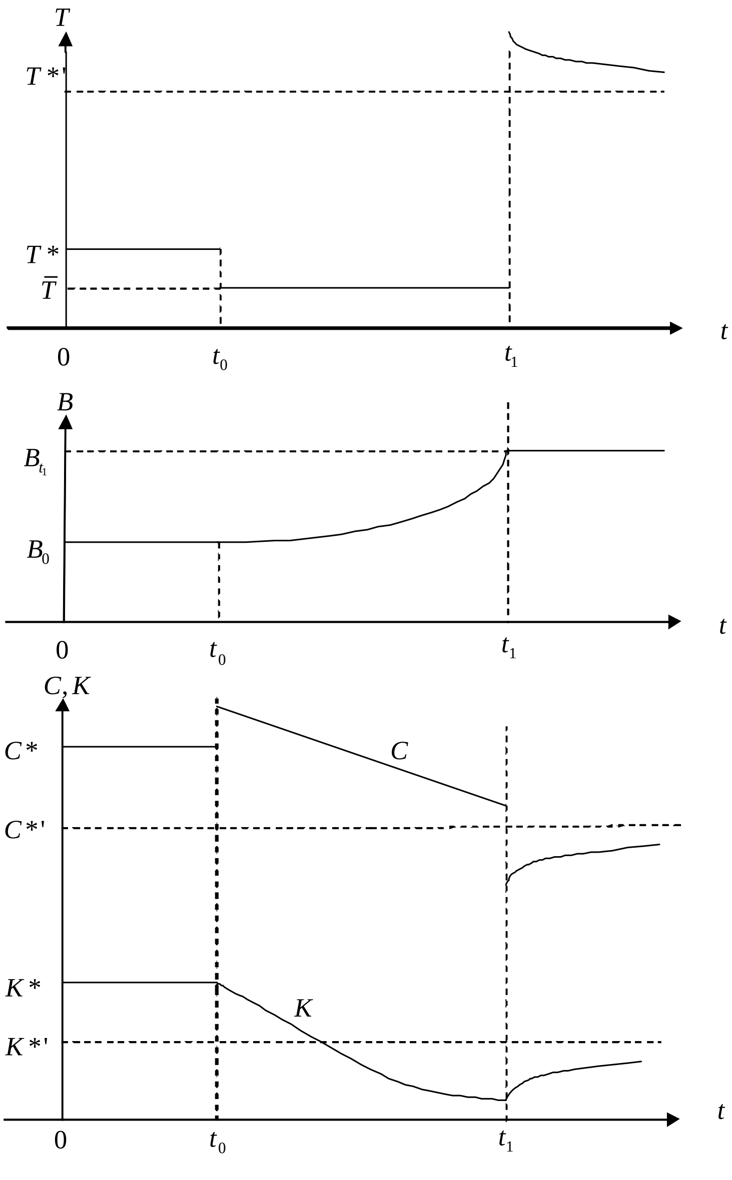

Figure 1.4: Time profiiles of $T_t,\ B_t,\ C_t$,and K_t, $t_1{-}t_0$ “large”.

All the results in this whole analysis of course hinge on the model's assumption of full capacity utilization.

2. Solution to Problem 2

The model is:

$$
\begin{aligned}
\dot{Y}_t &= \lambda(D(Y_t, R_t, \omega) + G - Y_t), \quad \lambda > 0, 0 < D_Y < 1, D_R < 0, 0 < D_\omega < 1, && (2.1)\\
\frac{M_t}{P} &= L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0, && (2.2)\\
i_t &= \alpha + \beta Y_t, \qquad \beta > 0, && (2.3)\\
R_t &= \frac{1}{Q_t}, && (2.4)\\
\frac{1 + \dot{Q}_t^e}{Q_t} &= r_t = i_t - \pi_t^e. && (2.5)
\end{aligned}
$$

The model is Blanchard's dynamic IS/LM model, which is a short-run model where the adjustment of output to demand takes time and where there is a distinction between a long-term bond (a consol) and a short-term bond. Equation (2.1) tells how output adjusts to demand; the parameter λ is the speed of adjustment. Naturally, output demand depends positively on income $= Y$, and negatively on the long-term interest rate, since investment is likely to depend negatively on this rate. And because of the wealth effect so is in fact also consumption.

Equation (2.2) expresses equilibrium in the money market. Naturally, real money demand depends positively on Y (a proxy for the number of transactions per time unit) and negatively on the short-term nominal rate of interest, the opportunity cost of holding money. Although in the short-run perspective of the model, production may deviate from demand, the asset markets are assumed to clear instantaneously.

Equation (2.3) is the counter-cyclical monetary policy rule. It says that the central bank raises the short-term interest rate when output goes up. In this way fluctuations in Y are dampened (which explains the term "counter-cyclical").

The inverse relation between the consol interest rate, R_t, and the real price of a consol in equation (2.4) comes from the definition of the consol interest rate as the internal rate of return on the consol, i.e., the solution for x in the equation

$$Q_t = \int_t^\infty 1 \cdot e^{-x(s-t)} ds = \frac{1}{x}.$$

Equation (2.5) is a no-arbitrage condition saying that, absent uncertainty, the expected rate of return on holding the consol one time unit is equal to the expected rate of return,

r_t, on the short-term bond. The equation indicates that the consol pays an annuity of one unit of account (worth one output unit) per time unit forever. Finally, the last equality in (2.5) defines r_t as the short-term nominal interest rate minus the expected rate of inflation.

Parameters: λ is the speed of adjustment; α and β are parameters characterizing the counter-cyclical monetary policy; ω is an index of market participants' general degree of confidence.

We now assume expectations are rational and that speculative bubbles never arise.

b) In view of rational expectations and absence of stochastic elements in the model, there is perfect foresight as long as a once-for-all shock does not occur. Hence, $\dot{Q}_t^e = E_t\dot{Q}_t = \dot{Q}_t$, and so the first equation in (2.5) gives

$$\frac{1}{Q_t} + \frac{\dot{Q}_t}{Q_t} = R_t - \frac{\dot{R}_t}{R_t} = r_t. \tag{2.6}$$

Since the price level P is an exogenous constant, we have $\pi_t^e = E_t\pi_t = \pi_t = 0$ for all t so that the second equation in (2.5) reduces to $r_t = i_t$. Substituting this and (2.3) into (2.6) and rearranging, we have

$$\dot{R}_t = (R_t - (\alpha + \beta Y_t))\, R_t. \tag{2.7}$$

We have another differential equation in R_t and Y_t directly given in (2.1). The differential equations (2.7) and (2.1) constitute the dynamic system of the model.

Given $R > 0$, (2.7) implies

$$\dot{R} \gtreqless 0 \qquad \text{for} \qquad R \gtreqless \alpha + \beta Y_t, \qquad \text{respectively.} \tag{2.8}$$

Thus, $\frac{\partial R}{\partial Y}|_{\dot{R}=0} = \beta > 0$. The $\dot{R} = 0$ locus is illustrated as the upward sloping line, LM, in Fig. 2.1.

From (2.1) we have

$$\dot{Y} \gtreqless 0 \qquad \text{for} \qquad D(Y, R, \omega) + G \gtreqless Y, \qquad \text{respectively.} \tag{2.9}$$

Hence, $\frac{\partial R}{\partial Y}|_{\dot{Y}=0} = (1 - D_Y)/D_R < 0$. The $\dot{Y} = 0$ locus is illustrated as the downward sloping curve, IS, in Fig. 2.1. In addition, the figure shows the direction of movement in the different regions, as described by (2.8) and (2.9). We see that the steady state point, E, with coordinates $(\bar{Y}, \bar{R})$, is a saddle point. This implies that two and only two solution paths – one from each side – converges towards E. These two saddle paths, which together make up the stable arm, are shown in the figure (their slope must be

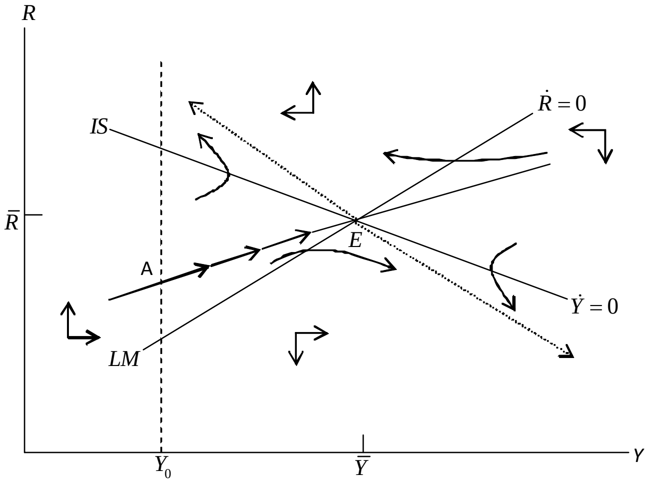

Figure 2.1: Phase diagram.

positive, according to the arrows). Also the unstable arm is displayed in the figure (the negatively sloped stippled line).

At time $t = 0$, the economy must be somewhere on the vertical line $Y = Y_0$. Since speculative bubbles are by assumption ruled out, neither the explosive nor the implosive paths of R in Fig. 2.1 can materialize. We are then left with the saddle path, the path AE in the figure, as the unique solution to the model.

c) In steady state $R = \bar{R}$ and $Y = \bar{Y}$, where

$$\begin{aligned}\bar{R} &= \alpha + \beta\bar{Y}, \\ \bar{Y} &= D(\bar{Y}, \alpha + \beta\bar{Y}, \omega) + G.\end{aligned}$$

Taking the total differential on both sides of the latter equation gives $d\bar{Y} = D_Y d\bar{Y} + D_R\beta d\bar{Y} + D_\omega d\omega + dG$, from which follows

$$\begin{aligned}\frac{\partial\bar{Y}}{\partial G} &= \frac{1}{1 - D_Y - D_R\beta} > 0, \quad \text{and} \\ \frac{\partial\bar{Y}}{\partial\omega} &= \frac{D_\omega}{1 - D_Y - D_R\beta} > 0.\end{aligned}$$

d) The effect of the downward shift in the general degree of confidence, ω, is shown in Fig. 2.2. When ω shifts, the long-term interest rate jumps down to R_A, reflecting that the market value of the consol jumps up. This is where the given formula,

$$R_t = \frac{1}{Q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds}, \tag{2.10}$$

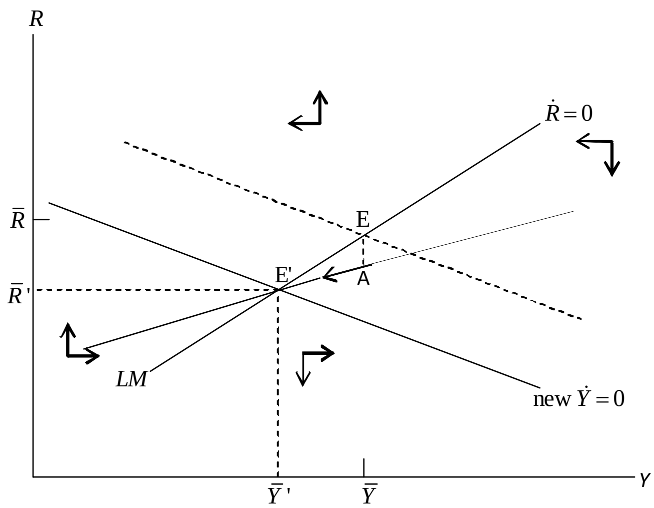

Figure 2.2:

is useful. This formula indicates that the long-term interest rate is a kind of average of the expected future short-term rates.

The mechanism behind the jump is as follows. The lower ω implies lower output demand. This triggers an expectation of decreasing Y and therefore also an expectation of decreasing i and r, in view of (2.3) and (2.5) with $\pi_t^e = 0$. The implication is, by (2.10), a higher Q and a lower R already immediately after time t_0, as illustrated in Fig. 2.2. As time proceeds and the economy gets closer to the expected low future values of r, these lower values gradually become dominating in the determination of R. Hence, after t_0 also R gradually decreases toward its new steady-state value, the same as that for r.

Time profiles showing the evolution of R_t, r_t, Q_t, Y_t, and M_t for $t \geq 0$ are given in Fig. 2.3. After t_0, output Y and the short-term rate r gradually decrease toward their new steady state values, $\bar{Y}'$ and $\bar{R}'$, respectively, as shown by Fig. 2.3. Whether the new long-run M, $\bar{M}'$, is higher or lower than the old, depends on the size of β.

To clarify this, note that even with laissez-faire monetary policy, the response of the nominal interest rate to changes in output is so as to stabilize output compared to what it would be at a constant nominal interest rate. Indeed, by (2.2), $M = PL(Y, i)$ and considering M fixed, we get $0 = P(L_Y dY + L_i di)$ so that

$$\frac{\partial i}{\partial Y}_{|(2.2), M \text{ fixed}} = -\frac{L_Y}{L_i} > 0.$$

On the other hand, with M determined via the policy rule (2.3) we have $M = PL(Y, \alpha +$

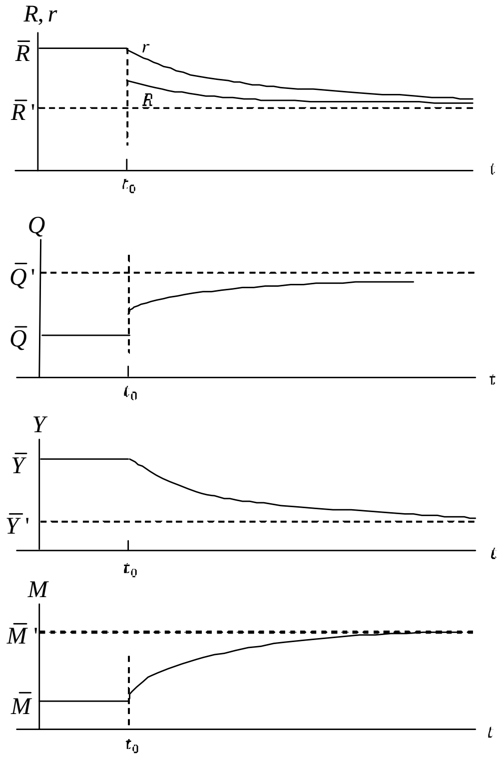

Figure 2.3:

βY). Then $dM = P(L_Y dY + L_i \beta dY)$ so that

$$\frac{\partial M}{\partial Y} = P(L_Y + L_i\beta) \gtreqless 0 \text{ for } \beta \lesseqgtr -\frac{L_Y}{L_i},$$

respectively.

It follows that only if $\beta > -L_Y/L_i$, is the monetary policy (2.3) *more* stabilizing (more counter-cyclical) than the laissez-faire policy of keeping M fixed. We have in Fig. 2.3 assumed this case, implying that money supply is increased when Y decreases (but as the text of the assignment is formulated, it is also OK to explicitly assume the opposite case).

e) The effect of the upward shift in G at time t_1 is shown in Fig. 2.4 and is qualitatively symmetric to that of the fall in ω. On impact, the long-term interest rate jumps up to $R_{A'}$, reflecting that the market value of the consol jumps down. The explanation is symmetric with the story under d). The higher G implies higher output demand. This triggers an expectation of increasing Y and therefore also an expectation of increasing i and r. The implication is, by (2.10), that R immediately jumps up to $R_{A'}$ and then, for rising t, gradually approaches its new higher long-run level, $\bar{R}''$.

Time profiles showing the evolution of R_t, r_t, Q_t, Y_t, and M_t for $t \geq 0$ are given in Fig. 2.5.

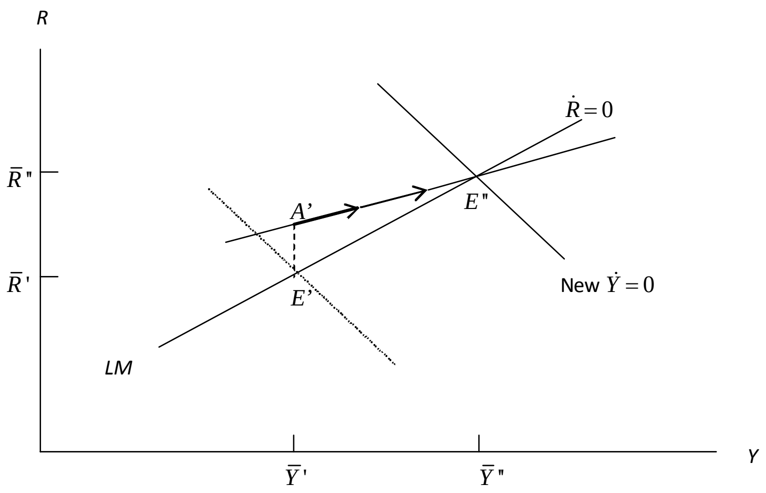

Figure 2.4:

By dampening output demand the higher R implies a financial crowding-out effect on production.[3] After t_1, during the transition to the new steady state, we have $R > r$ because R "anticipates" all the future increases in r and incorporates them, cf. (2.10). We may also note that (2.6) implies

$$R = r + \dot{R}/R \gtreqless r \qquad \text{for} \qquad \dot{R} \gtreqless 0, \qquad \text{respectively.}$$

For example, $\dot{R} > 0$ reflects that $\dot{Q} < 0$, that is, a capital loss is expected. To compensate for this, the *level* of R, which is always $1/Q$, must be higher than r such that the no-arbitrage condition (2.5) is still satisfied.

f) We assume that the private sector at time t_1 becomes aware that G will shift to a higher level at time t_2. The implied expectation that the short-term interest rate will in the future rise towards a higher level, $\bar{R}''$, immediately triggers an upward jump in the long-term rate, R, cf. Fig. 2.6. To what level? In order to find out, note that the market participants understand that from time t_2, the economy will move along the new saddle path corresponding to the new steady state, E', in Fig. 2.6. The market price, Q, of the consol cannot have an *expected* discontinuity at time t_2, since such a jump would imply an infinite expected capital loss (or capital gain) per time unit immediately before $t = t_2$ by holding long-term bonds. Anticipating for example a capital loss, the market participants would want to sell long-term bonds in advance. The implied excess supply would generate an adjustment of Q downwards until no longer a jump is expected to occur at time t_2. If instead a capital gain is anticipated, an excess demand would arise. This would generate

[3] The crowding out is only partial, because Y still increases.

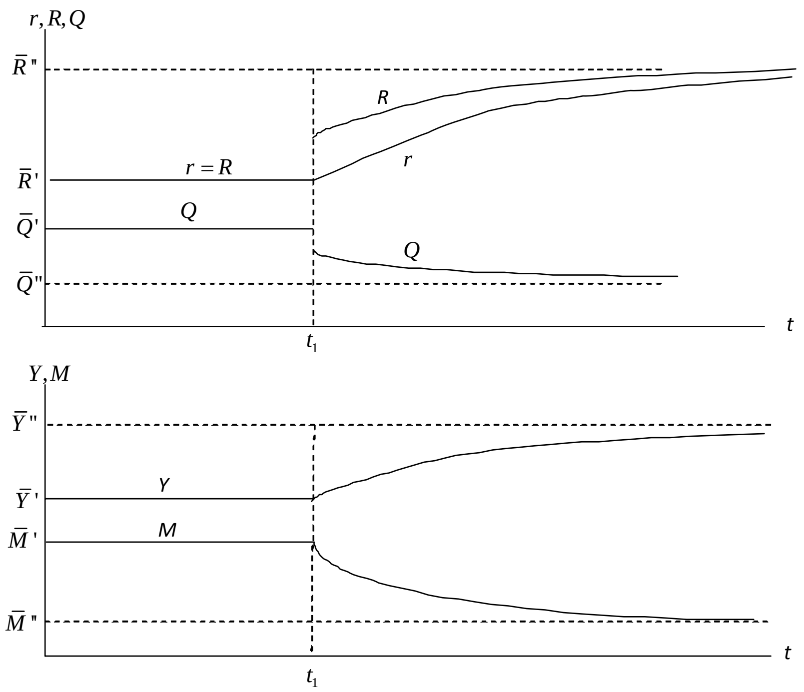

Figure 2.5:

in advance an upward adjustment of Q, thus defeating the expected capital gain. This is the general principle that arbitrage prevents an expected jump in an asset price.

In the time interval (t_1, t_2) the dynamics are determined by the "old" phase diagram, based on the no-arbitrage condition which rules up to time t_2. In this time interval the economy must follow that path (A'B in Fig. 2.6), which, starting from a point on the vertical line $Y = \bar{Y}'$, takes precisely $t_2 - t_1$ units of time to reach the new saddle path. At time t_1, therefore, R jumps to exactly the level $R_{A'}$ in Fig. 2.6. This upward jump has a contractionary effect on output demand. So output starts falling as shown by Fig. 2.6. This is because the potentially counteracting force, the increase in G, has not yet taken place. Not until time t_2, when G shifts to G', does output begin to rise. In the long run both Y, R, and r are higher than in the old steady state.

The time profiles of R_t, r_t, Q_t, Y_t, and M_t for $t \geq t_1$ are shown in Fig. 2.7.

A very good answer mentions both this temporary financial crowding out effect and the fact that the yield curve (also called the term structure of interest rates) "twists" in the time interval $(t_1,\, t_2)$. The long-term rate R rises, because the time where a higher Y (and thereby a higher r) is expected to show up, is getting nearer. But at the same time the short-term rate r is falling because of the falling transaction need for money implied by the initially falling Y, triggered by the rise in the long-term interest rate.

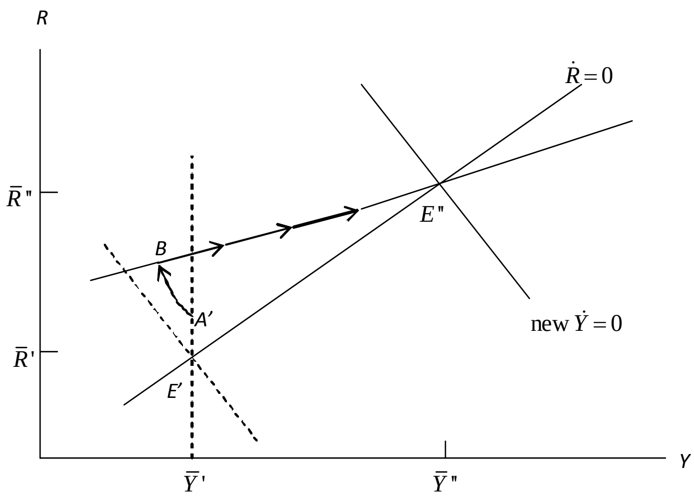

Figure 2.6: Response to an anticipated upward shift in G.

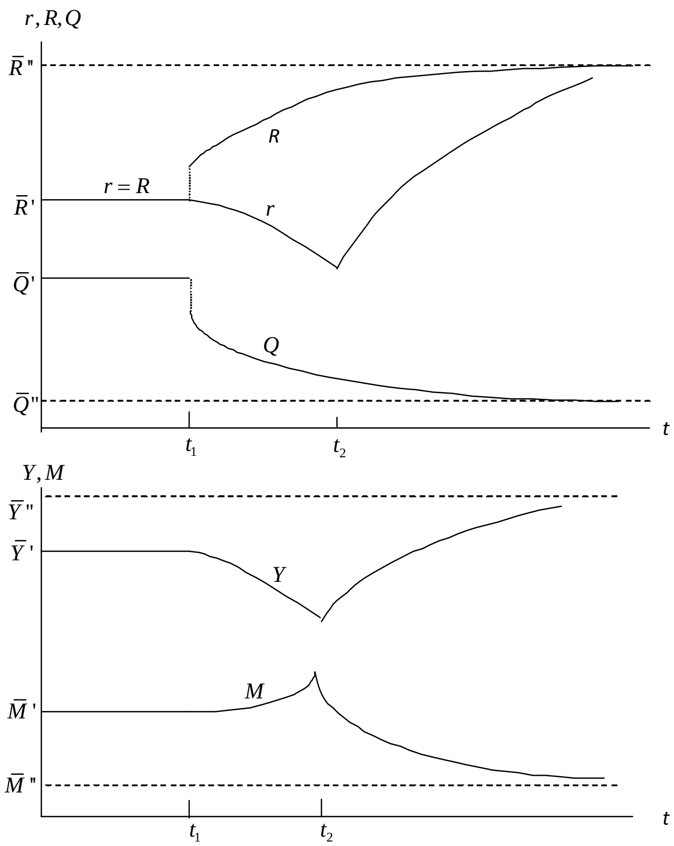

Figure 2.7: Response to an anticipated upward shift in G.

3. Solution to Problem 3

a) As a benchmark case, consider the simple Keynesian money market equilibrium condition.

$$\frac{M}{P} = L(Y, i), \qquad L_Y > 0, L_i < 0. \tag{3.1}$$

Here, since P is sticky in the short run, an increase in M affects the economy via a lower short-term nominal interest rate, i. This is called the *interest rate channel.*

In classical monetary theory money demand is simply proportional to income, $M/P = kY$, and so the income velocity of money V, defined by $V \equiv PY/M$, is a constant, $1/k$, independent of i. The price level is perfectly flexible, whereas output is supply determined. an increase in M affects the economy via a correspondingly higher price level. This may be called the *nominal price channel.*

In an extended Keynesian model where bonds and bank loans are not perfect substitutes, there is, in addition to the interest rate channel, a so-called *credit channel.*

In an open economy with floating exchange rate there is also the *exchange rate channel.*

b) Barro's point of view is that anticipating higher taxes in the future due to current budget deficits households will save more, and leave more as bequests to their descendents, in order that the "dynasty" can be prepared for the higher taxes in the future. The higher saving now more or less neutralizes the otherwise stimulating demand effect of the current US fiscal policy.

Typical counter arguments are:

1) The dynasty-picture of the household sector is misleading. Tax payers today know that future taxes will partly be paid by newcomers in the economy.

2) Because many people are credit constrained, a fiscal easing stimulates aggregate consumption.

3) There is evidence that under economic recession and depression, with a lot of idle resources, fiscal policy multipliers are sizeable.

c) False. The Sidrausky model, a typical neoclassical model, illustrates how the money growth rate may have welfare implications. A higher money growth rate implies higher steady state inflation with the implication that the nominal interest rate will be higher, thereby the opportunity cost of holding money is higher and so real money holding ends up lower. This implies lower liquidity services of money. And since consumption and capital accumulation in the steady state are unaffected by the money supply growth rate (super neutrality), there is lower total lower welfare due to the lower liquidity services (higher "shoe-leather costs").

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