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A suggested solution to the problem set at the exam in **Advanced Macroeconomics 2** January 17, 2011

(3-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates (a) accurate and thorough understanding of the concepts, methods, and models in the course, (b) knowledge of the major empirical regularities for aggregate economic variables, and (c) ability to use these theoretical tools and this empirical knowledge to answer macroeconomic questions.

1. Solution to Problem 1

The model is:

$$\dot{Y}_t = \lambda(D(Y_t, R_t, \alpha) + G - Y_t), \quad \lambda > 0, \quad 0 < D_Y < 1, \quad D_R < 0, \quad D_\alpha > 0, \tag{1.1}$$

$$\frac{M_t}{P_t} = L(Y_t, i_t), \qquad L_Y > 0, \ L_i < 0, \tag{1.2}$$

$$\pi_t = \bar{\pi}, \tag{1.3}$$

$$R_t = \frac{1}{Q_t}, \tag{1.4}$$

$$\frac{1 + \dot{Q}_t^e}{Q_t} = r_t, \tag{1.5}$$

$$r_t = i_t - \pi_t^e, \tag{1.6}$$

The model is essentially Blanchard's dynamic IS/LM model, which in continuous time describes the adjustment in the "very short" run towards a "short-run equilibrium" with respect to output and interest rates. The adjustment of output to demand takes time and during the adjustment process also demand changes (since the output level and asset prices are among its determinants). There are three financial assets: money, a long-term bond, and a short-term bond. Whereas the standard version of the model assumes $\bar{\pi} = 0$, here the more realistic case of ongoing positive inflation is considered.

[1] The solution below contains more details and more precision than can be expected at a three hours exam.

Equation (1.1) tells how output adjusts to demand; the parameter λ is the speed of adjustment. Naturally, output demand depends positively on income, Y, and negatively on the long-term interest rate, since investment is likely to depend negatively on this rate. And because of the substitution and wealth effects, so is also consumption.

Equation (1.2) expresses equilibrium in the money market. Naturally, real money demand depends positively on Y (a proxy for the number of transactions per time unit) and negatively on the short-term nominal interest rate, the opportunity cost of holding money. Although in the short-run perspective of the model, production may deviate from demand, the asset markets are assumed to clear instantaneously.

Equation (1.3) says that within the short-run perspective of the model, the rate of inflation is constant at an exogenous (pre-determined) level. The text says that Q_t is the real price of a consol paying one unit of output per time unit forever. Since R_t appears as the reciprocal of Q_t in equation (1.4), R_t must be the internal rate of return on the consol, i.e., the solution for x in the equation

$$Q_t = \int_t^{\infty} 1 \cdot e^{-x(s-t)} ds = \frac{1}{x}.$$

In brief, R_t is the real long-term interest rate.

Since equation (1.6) says that r_t equals the nominal short-term interest rate minus the expected rate of inflation, r_t must be the expected real short-term interest rate. Finally, equation (1.5) is a no-arbitrage condition saying that the expected rate of return on holding the consol one time unit equals the expected rate of return, r_t, on the short-term bond.

We now assume that expectations are rational and that speculative bubbles never arise.

b) In view of rational expectations and absence of stochastic elements in the model, there is perfect foresight as long as a once-for-all shock does not occur. Hence, $\pi_t^e = E_t\pi_t = \pi_t = \bar{\pi} > 0$ for all t, implying that equation (1.6) reduces to $r_t = i_t - \bar{\pi}$. Similarly, $\dot{Q}_t^e = E_t\dot{Q}_t = \dot{Q}_t$. Combining the equations (1.5) and (1.6) we thus get

$$\frac{1}{Q_t} + \frac{\dot{Q}_t}{Q_t} = R_t - \frac{\dot{R}_t}{R_t} = r_t = i_t - \bar{\pi}. \tag{1.7}$$

We are told that conditions are such that the central bank maintains the real money supply, $m_t \equiv M_t/P_t$, at a constant level. We denote this level m_0. Equation (1.2) then reads $L(Y_t, i_t) = m_0$. This equation defines i_t as an implicit function of Y_t and m_0, i.e.,

$$i_t = i(Y_t, m_0), \quad \text{with } i_Y = -L_Y/L_i > 0, \quad i_m = 1/L_i < 0. \tag{1.8}$$

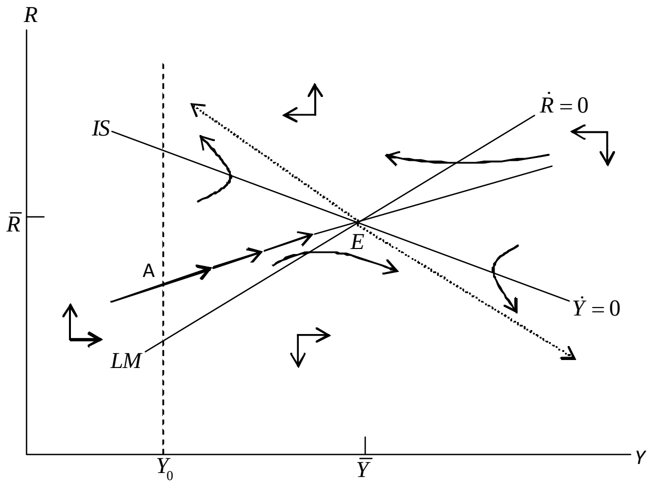

Figure 1.1: Phase diagram.

Inserting this into (1.7), we have

$$\dot{R}_t = \left[R_t - (i(Y_t, m_0) - \bar{\pi})\right] R_t, \tag{1.9}$$

which together with (1.1) constitutes our dynamic system in the two endogenous variables, Y_t and R_t.

Given $R > 0$, (1.9) implies

$$\dot{R} \gtreqless 0 \qquad \text{for} \qquad R \gtreqless i(Y, m_0) - \bar{\pi}, \qquad \text{respectively.} \tag{1.10}$$

We have $\frac{\partial R}{\partial Y}|_{\dot{R}=0} = i_Y = -L_Y/L_i > 0$. The $\dot{R} = 0$ locus is thus an upward-sloping curve, named the LM curve in Fig. 1.1.

From (1.1) we have

$$\dot{Y} \gtreqless 0 \qquad \text{for} \qquad D(Y, R, \alpha) + G \gtreqless Y, \qquad \text{respectively.} \tag{1.11}$$

Hence, $\frac{\partial R}{\partial Y}|_{\dot{Y}=0} = (1 - D_Y)/D_R < 0$. The $\dot{Y} = 0$ locus is thus a downward-sloping curve, named the IS curve in Fig. 1.1.

In addition, the figure shows the direction of movement in the different regions, as described by (1.10) and (1.11). We see that the steady state point, E, with coordinates $(\bar{Y}, \bar{R})$, is a saddle point. This implies that two and only two solution paths − one from each side − converges towards E. These two saddle paths, which together make up the stable arm, are shown in the figure (their slope must be positive, according to the arrows). Also the unstable arm is displayed in the figure (the negatively sloped stippled line).

We are told that the nominal short-term interest rate, $\bar{i} = \bar{R} + \bar{\pi}$, is positive (or equivalently, $\bar{R} > -\bar{\pi}$).

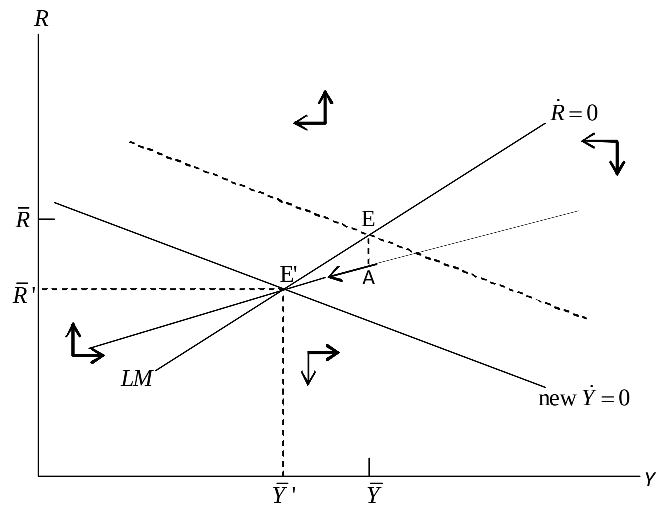

Figure 1.2:

c) Since Y_0 is pre-determined, at time $t = 0$ the economy must be somewhere on the vertical line $Y = Y_0$. Since speculative bubbles are by assumption ruled out, neither the explosive nor the implosive paths in Fig. 1.1 can materialize. We are then left with the saddle path, the path AE in the figure, as the unique solution to the model. So the economy will at $t = 0$ be at point A in Fig. 1.1 and then gradually approach the steady state, E, over time, moving along the saddle path.

d) The effect of the downward shift in the level of confidence, α, is shown in Fig. 1.2. When α shifts down, the LM curve is not affected, but the IS curve is shifted downward. Hence, the long-term interest rate immediately jumps down to R_A, reflecting that the market value of the consol jumps up. This is where the given formula,

$$R_t = \frac{1}{Q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds}, \tag{1.12}$$

is useful. This formula indicates that the long-term interest rate is a kind of average of the expected future short-term rates.

The mechanism behind the jump is as follows. The lower α implies lower output demand. This triggers an expectation of decreasing Y and therefore also an expectation of decreasing i and r in view of the lower transaction demand for money and the unchanged inflation rate (cf. (1.8) and (1.6) with $\pi_t^e = \bar{\pi}$). The implication is, by (1.12), a higher Q, hence a lower R, already immediately after time t_0, as illustrated in Fig. 1.2. As time proceeds and the economy gets closer to the expected low future values of r, these lower values gradually become dominating in the determination of R. Hence, R gradually decreases toward its new steady-state value, the same as that for r.

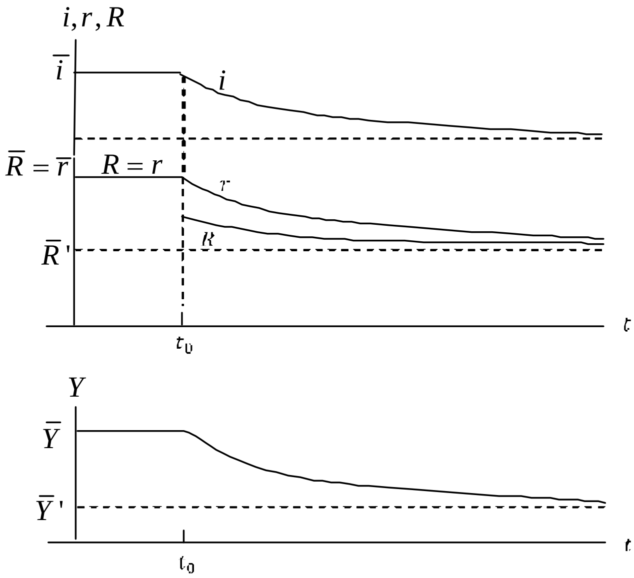

Figure 1.3:

Time profiles showing the evolution of i_t, r_t, R_t, and Y_t for $t \geq 0$ are given in Fig. 1.3. After t_0, output Y and the short-term rates, i and r, gradually decrease toward their new steady state values. These new steady state values of the three interest rates are denoted $\bar{i}'$, $\bar{r}'$, and $\bar{R}'$, respectively. We have

$$\bar{i}' = i(\bar{Y}', m_0) = \bar{r}' + \bar{\pi} = \bar{R}' + \bar{\pi}. \tag{1.13}$$

e) The yield curve is defined as the graph of the internal rate of return on a bond at a given date as a function of time to maturity of the bond. Immediately after the shock, we have $R < r$. Hence, the slope of the yield curve is negative.

f) The effect of the downward shift in the inflation rate is shown in Fig. 1.4. When the inflation rate shifts down, the IS curve is not affected, but the LM curve is shifted upward and R immediately jumps up to $R_{A'}$. Hereafter the economy gradually moves along the new saddle path down to the new steady state E". The output level is here even lower than at E'. This is because the lower inflation rate implies a *higher real* interest rate for any given nominal short-term interest rate, hence lower aggregate demand. In analogy with (1.13), in the new steady state we have

$$\bar{i}'' = i(\bar{Y}'', m_0) = \bar{r}'' + \bar{\pi}' = \bar{R}'' + \bar{\pi}'.$$

Since the real money supply is unchanged (at the level m_0) and $\bar{Y}'' < \bar{Y}'$, we have $\bar{i}'' < \bar{i}'$. Since the fall in inflation shifted the LM curve upward, we have $\bar{R}'' > \bar{R}'$. The

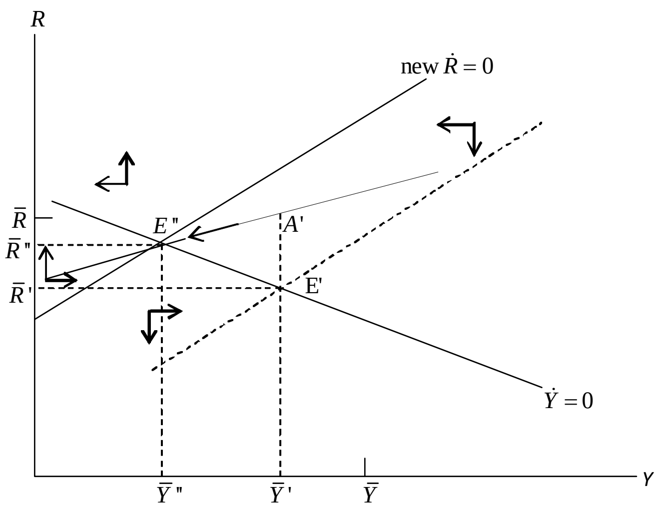

Figure 1.4:

intuition is that for the lower output level to be an equilibrium level, a lower aggregate demand is needed which in turns requires a higher real long-term interest rate in order to satisfy the steady state condition

$$D\left(\bar{Y}'', \bar{R}'', \alpha'\right) + G = \bar{Y}''. \tag{1.14}$$

In view of the marginal propensity to spend being less than one ($0 < D_Y < 1$), there would otherwise be excess demand.

Whether the new steady state level of R exceeds or does not exceed the steady state level before time t_0 is ambiguous. That is, we cannot from the given information know whether $\bar{R}'' > \bar{R}$ or $\bar{R}'' < \bar{R}$ (although, to fix ideas, Fig. 1.5 portrays the latter case). The reason comes to the fore when we consider the condition for steady state before $t = t_0$:

$$D\left(\bar{Y}, \bar{R}, \alpha\right) + G = \bar{Y}.$$

In spite of $\bar{Y}'' < \bar{Y}$, a situation with $\bar{R}'' \leq \bar{R}$ need not imply excess demand in the steady state E", because the level of confidence is lower than before $t = t_0$.

Time profiles showing the evolution of i_t, r_t, R_t, and Y_t for $t \geq 0$ are shown in Fig. 1.5. For $t > t_2$, the vertical distance between the i curve and the r curve is smaller than in Fig. 1.3. This simply reflects that this distance equals the rate of inflation which is now lower.

Note also that during the adjustment, $R_t < r_t$, that is, the long-term rate is lower than the short-term rate (the yield curve is downward-sloping). It must be so in view of

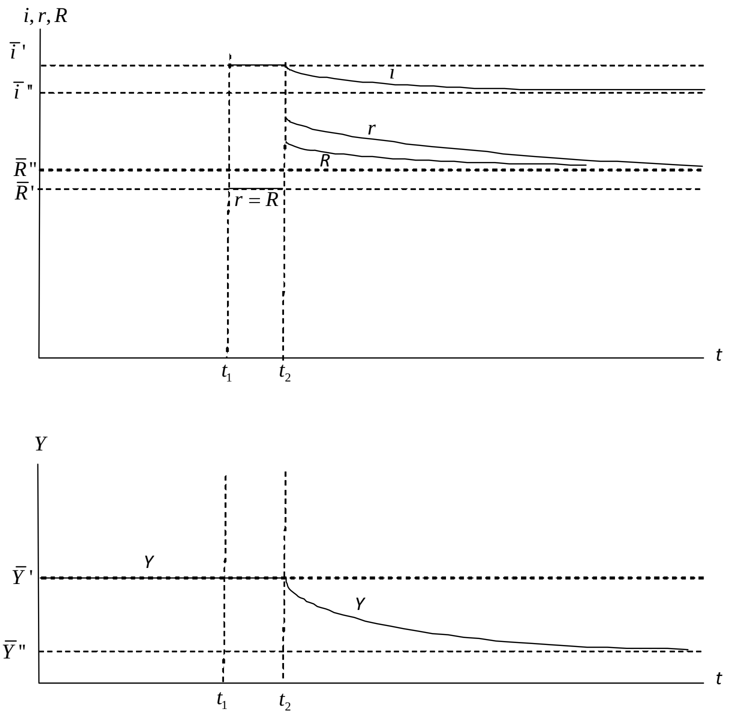

Figure 1.5:

the no-arbitrage condition. Indeed, (1.7) implies

$$R = r + \dot{R}/R \gtreqless r \qquad \text{for} \qquad \dot{R} \gtreqless 0, \qquad \text{respectively.}$$

For $t > t_2$, we have $\dot{R} < 0$ along the saddle path. This reflects that $\dot{Q} > 0$, that is, capital gains on the consol are expected all along the adjustment process. To compensate for this, the *level* of R (which always equals $1/Q$) must be lower than r such that the no-arbitrage condition is still satisfied.

g) The new monetary policy rule implies that for $t > t_3$, equation (1.2) is replaced by

$$M_t = P_t L(Y_t, \bar{i}''),$$

where $P_t = P_{t_3} e^{\bar{\pi}'(t-t_3)}$ and $\bar{i}'' \approx 0$.

h) Yes, the coordinated fiscal and monetary policy is more potent than a conventional expansionary *fiscal* policy in isolation. This is because, if the real money supply was just kept constant, the rise in G would raise the nominal short-term interest rate. With inflation remaining unaffected, the real short-term rate would rise and so would the real long-term rate. Hence, there would be a negative feedback on aggregate demand, partly counteracting the expansionary fiscal policy; this is the phenomenon called financial crowding out which is avoided by the simultaneous expansionary monetary policy (making the $\dot{R} = 0$ curve a horizontal line in the phase diagram).

The coordinated fiscal and monetary policy is also more potent than a conventional expansionary *monetary* policy in isolation. This is because a conventional expansionary monetary policy is a policy which by raising the money supply (i.e., buying bonds) aims at lowering the nominal short-term interest rate. But the latter is already at the zero floor and cannot become lower (the liquidity trap).

i) To the extent that the stimulation of aggregate demand succeeds in raising economic activity, it is likely that (outside the model) also inflation begins to rise towards a more "normal level". This feature is likely to *support* the effectiveness of the coordinated fiscal and monetary policy because inflation makes the *real* interest lower than otherwise.

Here it is important that we consider a closed economy. In a small open economy with a fixed exchange rate, the *competitiveness* of the economy would be adversely affected by higher inflation. This would give a boost to the imports leak of the demand stimulus and weaken exports, thereby partly counteracting the expansionary policy.

2. Solution to Problem 2

a) We have

$$GBD_t = rB_t + G_t - T_t, \tag{GBD}$$

and

$$B_{t+1} = (1+r)B_t + G_t - T_t. \tag{DGBC}$$

As an implication,

$$B_{t+1} = B_t + GBD_t. \tag{2.1}$$

b) The debt-income ratio $b_t \equiv B_t/Y_t$ changes over time according to

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} = \frac{(1+r)B_t + G_t - T_t}{(1+g)Y_t} = \frac{1+r}{1+g}b_t + \frac{\gamma - \tau_t}{1+g}. \tag{2.2}$$

As the question is stated, we should look for a *constant* net tax burden. Hence, we replace τ_t by τ in this equation. Then we can use the hint, from which follows that the solution to this linear difference equation is

$$b_t = (b_0 - b^*)\left(\frac{1+r}{1+g}\right)^t + b^*, \quad \text{where} \quad b^* = \frac{\tau - \gamma}{r - g}, \tag{2.3}$$

and $b_0 > 0$ is historically given.

Only if the current fiscal policy, (γ, τ), implies a non-exploding debt-income ratio, is the policy sustainable. Since $r > g$, this requires that τ and γ are such that $b^* \geq b_0$. That is, the requirement is that

$$\tau - \gamma \geq (r - g)b_0.$$

Hence, the minimum sustained net tax burden consistent with fiscal sustainability is

$$\bar{\tau} = \gamma + (r - g)b_0. \tag{2.4}$$

c) From (2.4) follows:

$$\begin{aligned}
\frac{\partial \bar{\tau}}{\partial r} &= b_0 > 0, \\
\frac{\partial \bar{\tau}}{\partial g} &= -b_0 < 0, \\
\frac{\partial \bar{\tau}}{\partial \gamma} &= 1, \\
\frac{\partial \bar{\tau}}{\partial b_0} &= r - g > 0.
\end{aligned}$$

d) We have $\partial B_0 / \partial G_0 = 0$, since B_0 is predetermined at the beginning of period 0. Hence, with $t = 0$, from (GBD) follows

$$\frac{\partial GBD_0}{\partial G_0} = 1 - T'(Y_0)\frac{\partial Y_0}{\partial G_0} = 1 - 0.5 \cdot 1.5 = 1 - 0.75 = 0.25,$$

and from (2.1)

$$\frac{\partial B_1}{\partial G_0} = \frac{\partial GBD_0}{\partial G_0} = 0.25.$$

The effects of the rise in G_0 are then given by

$$\Delta GBD_0 \approx \frac{\partial GBD_0}{\partial G_0}\Delta G = 0.25\Delta G,$$

and

$$\Delta B_1 \approx \frac{\partial B_1}{\partial G_0}\Delta G = \frac{\partial GBD_0}{\partial G_0}\Delta G = 0.25\Delta G.$$

e) At $t = t_1$, we have, from (DGBC),

$$B_2 = (1 + r_1)B_1 + G_1 - T(Y_1).$$

The expansionary fiscal policy affects B_2 both directly via a higher G_1 (partly offset by the associated extra tax revenue) and indirectly via the higher G_0 which generated a higher B_1. The total effect is

$$\begin{aligned}
\Delta B_2 &\approx (1 + r_1)\Delta B_1 + \Delta G_1 - T'(Y_1)\Delta Y_1 \approx (1 + r_1)0.25\Delta G + (1 - T'(Y_1)\frac{\partial Y_1}{\partial G_0})\Delta G \\
&= (1 + r_1)0.25\Delta G + 0.25\Delta G = (2 + r_1)0.25\Delta G.
\end{aligned}$$

f) The extra interest payments on the debt incurred by the expansionary fiscal policy is $r_2\Delta B_2$ per period. On the other hand, we are told that laissez-faire during the slump would have resulted in more people experiencing *long-term* unemployment, hence more people becoming de-qualified and in effect driven out of the effective labor force. The implied loss in "full employment" output from period 2 and onwards is assumed equal to a positive constant, ΔY, per period.

Hence, for the expansionary fiscal policy to "pay for itself" in period 2 and onwards it is required that $r_2\Delta B_2 \leq \tau\Delta Y$, that is,

$$\Delta Y \geq \frac{r_2\Delta B_2}{\tau}.$$

g) We get

$$\begin{aligned}\Delta Y &\geq \frac{r_2\Delta B_2}{\tau} = \frac{r_2(2+r_1)0.25\Delta G}{\tau} = \frac{0.03(2+0.02)0.25\Delta G}{0.3} \\ &= \frac{2.02}{4\cdot 10}\Delta G = 0.0505\Delta G.\end{aligned}$$

So if the loss in "full employment" output from period 2 and onwards, incurred under laissez faire, is not smaller than about 5% of the fiscal stimulus, ΔG, the expansionary fiscal policy "pays for itself" in the sense of not implying a worsening of the long-run fiscal stance.

Whether this is likely to hold in practice or not will depend on a lot of circumstances, including the severity of the economic downturn.

3. Solution to Problem 3

a) Consider an economy with lump-sum taxation. The economy has the *Ricardian equivalence* property if, for a given path of government spending the timing of the taxes does not affect aggregate saving in the economy. So, if Ricardian equivalence is present, it does not matter for aggregate saving and thereby resource allocation whether the government finances its current spending by taxes or borrowing.

Robert Barro's claim is that this form of "debt neutrality" is not a bad characterization of actual market economies.

The theoretical basis for Barro's claim is his dynasty model, i.e., his model of overlapping generations linked through altruistic bequests. Ricardian equivalence holds in this model if the intergenerational discount rate is sufficiently low. Then a temporary tax cut will *not* motivate the "dynasties" to raise their current consumption. The reason is that the dynasty will not feel more wealthy. Indeed, the present value of the infinite stream of taxes imposed on the infinitely-lived dynasty will not be affected by the tax cut. The parents know that to cover the government's higher future interest payment, the present value of future taxes will have to rise exactly as much as current taxes have been decreased. To compensate the descendants for these higher taxes in the future, the parents will use the rise in current after-tax income to save more and leave higher bequests to their descendants.

It is otherwise in OLG models without Barro's altruistic bequest motive. Blanchard's OLG model is an example. Here taxes levied at different times are levied on different sets of agents. In the future there are newcomers and they will bear part of the higher tax burden and some of those people alive today will be gone. Therefore a current tax cut make current tax payers feel wealthier and this leads to an increase in their consumption. So Ricardian equivalence is not present.

In a "full-employment economy" the result of the temporary tax cut is that aggregate current saving in the economy becomes lower. The present generations benefit, but future generations bear the cost in the form of smaller national wealth than otherwise.

According to Keynesian theory, if the economy is in a depression, there need not be a cost to future generations. Indeed, a temporary budget deficit, whether caused by a tax cut or a spending rise, *may* be beneficial to both current and future generations. The last part of Problem 2 above exemplified this "spend now, save later" philosophy.

b) True. In the Blanchard OLG model the consumption function for the individual is

$$c_t = (\rho + m)(a_t + h_t), \tag{3.1}$$

where ρ and m are the pure rate of time preference and the mortality rate, respectively (both constant), and a_t denotes financial wealth, whereas h_t is present discounted value of future labor income. The aggregate consumption function has the same simple form (which stems from the assumption of log utility).

To understand the different Slutsky effects, the intertemporal budget constraint of the individual is helpful:

$$\int_t^{\infty} c_\tau e^{-\int_t^\tau (r_s+m)ds} d\tau = a_t + h_t. \tag{IBC}$$

An increase in the interest rate level, makes future consumption cheaper as seen from today. Hence there is a negative *substitution effect* on current consumption c_t. At the same time, the present discounted value of a given consumption plan becomes smaller. Hence, for a given total wealth (i.e., a given $a_t + h_t$), a rise in r makes it possible to consume more at any time. This amounts to a positive *income effect* on current consumption.

When discussing substitution and income effects, we do not take the effect of the price change, here a rise in the interest rate level, on the total *wealth* of the individual into account. The total wealth appears on the right-hand side of both (IBC) and (3.1). If the net effect on current consumption of the substitution and income effects should depend on the interest rate, r should affect the coefficient to total wealth on the right-hand side of (3.1). This coefficient is $\rho + m$, which obviously is independent of r. So (3.1) shows that these two effects exactly cancel each other.

But there is generally a third Slutsky effect, namely a *wealth effect.* This effect indicates the effect on current consumption through the change in total wealth generated by the price change, here the rise in the interest rate level. Whereas in the Blanchard model, financial wealth, a_t, is predetermined, human wealth, h_t, obviously depends on how heavily future labor income is discounted. An increase in r affects h_t *negatively.*

The wealth effect of a rise in the interest rate is thus negative. And since

$$\begin{aligned} \text{total effect} &= \text{substitution effect} + \text{income effect} + \text{wealth effect} \\ &= \text{wealth effect}, \end{aligned}$$

in this model, the total effect is negative as well.

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