Written exam for the M. Sc. in Economics, Winter 2011/2012

Advanced Macroeconomics 2

Master's Course

January 16, 2012

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is:

Problem 1: 25 %, Problem 2: 30 %, Problem 3: 30 %Problem 4: 15 %.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a *non-standard* budget deficit rule saying that $\lambda \cdot 100$ percent of the interest expenses on public nominal debt, D, plus the primary budget deficit must not be above $\alpha \cdot 100$ percent of nominal GDP, PY, where Y is real GDP, growing at a constant rate, $g_Y > 0$, and P is the GDP deflator. So the rule requires that

$$\lambda i D_t + P_t(G_t - T_t) \leq \alpha P_t Y_t, \tag{*}$$

where $\lambda > 0$, $\alpha > 0$, and

$$\begin{aligned}
G_t &= \text{real government spending on goods and services in period } t, \\
T_t &= \text{real net tax revenue in period } t, \\
i &= (1+r)(1+\pi) - 1, \text{ where } r \text{ is the real interest rate, a given non-negative constant,} \\
\pi &= \frac{P_t - P_{t-1}}{P_{t-1}} = \text{the inflation rate, a given non-negative constant.}
\end{aligned}$$

Assume that money financing of the government deficit never occurs. The motivation for the policy parameter λ might be to open up for giving different weight to the primary budget deficit and the interest expenses.

a) Write down two equations, one for the nominal budget deficit, GBD_t, and one indicating the accumulation of nominal government debt.

b) Is the deficit rule of the Stability and Growth Pact in the EMU a special case of (*)? Why or why not?

c) Let $b_t \equiv D_t/(P_{t-1}Y_t)$. Derive the law of motion (difference equation) for b_t, assuming the "ceiling" (*) is always binding.

d) Suppose λ is such that $0 < 1+(1-\lambda)i < (1+\pi)(1+g_Y)$. For an arbitrary $b_0 > 0$, find the time path of b. Briefly comment. *Hint:* the difference equation $x_{t+1} = ax_t + c$, where a and c are constants, $a \neq 1$, has the solution $x_t = (x_0 - x^*)a^t + x^*$, where $x^* = c/(1-a)$.

e) How does a rise in λ affect the long-run debt-income ratio? Comment.

Problem 2 In this problem we consider a model of a closed economy in the "very short run". Time is continuous. The assumptions are:

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\begin{aligned}
\dot{Y}_t &= \lambda(D(Y_t, R_t, \rho) + G - Y_t), \qquad \lambda > 0,\ 0 < D_Y < 1,\ D_R < 0,\ D_\rho > 0, && (1)\\
\frac{M_t}{P_t} &= L(Y_t, i_t), \qquad L_Y > 0,\ L_i < 0, && (2)\\
R_t &= \frac{1}{q_t}, && (3)\\
\frac{1 + \dot{q}_t^e}{q_t} &= r_t^e, && (4)\\
r_t^e &\equiv i_t - \pi_t^e, && (5)\\
\pi_t &= \pi, && (6)\\
i_t &= \alpha_0 + \alpha_1 Y_t, \qquad \alpha_1 > 0. && (7)
\end{aligned}
$$

where a dot over a variable denotes the derivative w.r.t. time t, and the superscript e indicates expected value (until further notice a subjective expectation). Further, Y_t = output, R_t = real long-term interest rate, q_t = real price of a consol paying one unit of output per time unit forever, i_t = nominal short-term interest rate, ρ = level of "confidence", G = government spending on goods and services, M_t = money supply, P_t = output price, and $\pi_t \equiv \dot{P}_t/P_t$ = rate of inflation. The tax revenue function is implicit in the demand function $D(\cdot)$. The variables ρ, G, and π are exogenous positive constants. The initial values Y_0 and P_0 are historically given. In questions a) - e) it is assumed that exogenous variables and initial conditions are such that $i_t > 0$ for all $t \geq 0$.

a) Briefly interpret the model. Make sure you explain why the real long-term interest rate can be written as in (3).

From now on we assume perfect foresight and that speculative bubbles do not arise.

b) To characterize the movement of the economy over time, derive from the model a dynamic system in Y_t and R_t. Comment on what the role of equation (2) is in the model.

c) Draw the corresponding phase diagram, assuming that parameters are such that there exists a steady state with a nominal short-term interest rate $\bar{i} > \pi$. Illustrate the path that the economy follows for an arbitrary $Y_0 > 0$. Comment.

Now suppose that the economy is already in its steady state ("short-run equilibrium"). Let the steady state values of Y and R be denoted $\bar{Y}$ and $\bar{R}$, respectively.

d) Find an analytical expression for the effect on $\bar{Y}$ of a unit increase in G (the "spending multiplier").

Unexpectedly an adverse demand shock occurs at time $t_1 > 0$ (that is, a shift of confidence level from ρ to $\rho' < \rho$). Until further notice, we assume that after this shift everybody rightly expect the new level of confidence to be maintained for a long time.

e) Illustrate by a phase diagram what happens to Y_t and R_t over time, presupposing that a steady state with $R > 0$ still exists. Illustrate in another figure the time profiles of i_t, r_t, R_t, and Y_t for $t \geq t_1$. Explain the intuition in words. *Hint:* the following formula may be helpful:

$$R_t = \frac{1}{Q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds}.$$

f) Briefly evaluate the model.

Problem 3 In this problem we consider a model of a closed economy in the "short-to-medium run". Time is continuous. The inflation rate, π_t, is sticky (can not jump) but adjusts gradually over time according to

$$\dot{\pi}_t = \delta(Y_t - Y^*), \qquad \delta > 0, Y^* > 0 \qquad \pi_0 \text{ given}, \tag{1}$$

where Y_t is aggregate output at time t. As we do not want the dynamic system to be too complicated, we assume that aggregate private demand is a function of the expected real *short-term* interest rate, r_t^e, and we ignore any time lags between aggregate demand and actual output. Moreover, we approximate aggregate demand by a linear function so that we can write output on reduced form this way:

$$Y_t = \gamma - \beta r_t^e, \qquad \gamma > 0, \beta > 0, \tag{2}$$

where γ is a shift parameter (subsuming effects of changes in confidence and fiscal policy). We have

$$r_t^e \equiv i_t - \pi_t^e, \tag{3}$$

where i_t is the nominal short-term interest rate, π_t is the rate of inflation, and the superscript e indicates expected value. The central bank is assumed to know the constant Y^*. Monetary policy is described by

$$i_t = \max\left[0, \hat{r} + \hat{\pi} + \alpha_1(Y_t - Y^*) + \alpha_2(\pi_t - \hat{\pi})\right], \qquad \hat{r} \equiv \frac{\bar{\gamma} - Y^*}{\beta} > 0, \hat{\pi} > 0, \alpha_1 > 0, \alpha_2 > 1, \tag{4}$$

where $\bar{\gamma}$ is the "normal" value of γ. We assume that $\bar{\gamma} > Y^*$.

The model consists of (1), (2), (3), and (4) together with an assumption of perfect foresight.

a) Briefly interpret (1) and (4), including the constants $Y^*, \hat{r}$, and $\hat{\pi}$.

b) Presupposing that the lower bound in (4) is *not* binding, show that Y_t can be written as a linear function of the inflation rate, $Y_t = \theta_0(\gamma) - \theta_1 \pi_t$, where

$$\begin{aligned} \theta_0(\gamma) &= \frac{\gamma - \beta\left(\hat{r} + \hat{\pi} - \alpha_1 Y^* - \alpha_2 \hat{\pi}\right)}{1 + \beta\alpha_1}, \\ \theta_1 &= \frac{\beta(\alpha_2 - 1)}{1 + \beta\alpha_1} > 0. \end{aligned}$$

c) Consider the relationship $Y_t = \theta_0(\bar{\gamma}) - \theta_1 \pi_t$ as the AD curve associated with the model when $\gamma = \bar{\gamma}$. Graphically illustrate this AD curve as well as the steady state of the economy either in the (Y, π) plane or, after substitution of the AD curve into (1), in the $(\pi, \dot{\pi})$ plane. Comment.

Suppose that up until time t_1, $\gamma = \bar{\gamma}$ and the economy is in steady state with $\pi = \hat{\pi}$. Then unexpectedly an adverse demand shock occurs such that the new γ is $\gamma' < \bar{\gamma}$ at least for a while. Until further notice, we still assume that the lower bound in (4) is *not* binding.

d) Illustrate the resulting dynamics in a diagram similar to the one you have drawn in connection with question c). Explain by words.

e) Suppose that after some time, confidence is restored. As a crude representation of this, we imagine that a complete restoration takes place in a jump at time $t_2 > t_1$. So, for $t \geq t_2$, (2) with $\gamma = \bar{\gamma}$ is again valid. Illustrate the resulting dynamics in a new diagram.

f) Now, presuppose instead that the lower bound in (4) is *binding* immediately after the adverse demand shock. Briefly characterize the resulting incipient dynamics. *Hint:* As the linear approximation, (2), will after some time no longer be valid, you are only asked to comment on the resulting *incipient* dynamics, i.e., the dynamics that take place during a bounded time interval immediately after the shock. A verbal answer to the question will be enough.

Problem 4 *Short questions*

a) "A given fiscal policy is sustainable if and only if it maintains compliance with the No-Ponzi-Game condition of the government." True or false? Briefly explain.

b) Suppose there is a market for actuarially fair life annuity contracts. What property defines actuarial fairness? If life annuity contracts have this property, why would anybody bother to buy them?

c) "In the q-theory of investment, the steady state value of q must equal one". True or false? Why?

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