Written re-exam for the M. Sc. in Economics, Winter 2012/2013

Advanced Macroeconomics

Master's Course

February 20, 2013

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is:

Problem 1: 50 %, Problem 2: 35 %, Problem 3: 15 %.[1]

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[1]The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1   We consider a Blanchard model of a full-employment closed economy with a public sector, public debt, and lump-sum taxation. The dynamics of the economy are described by the differential equations:

$$\dot{C}_t = (F_K(K_t, N) - \delta - \rho)C_t - m(\rho + m)(K_t + B_t), \qquad (*)$$

$$\dot{K}_t = F(K_t, N) - \delta K_t - C_t - G, \qquad K_0 > 0 \text{ given}, \qquad (**)$$

$$\dot{B}_t = [F_K(K_t, N) - \delta]B_t + G - T_t, \qquad B_0 \text{ given}, \qquad (***)$$

the condition

$$\lim_{t \to \infty} B_t e^{-\int_0^t [F_K(K_s, N) - \delta]ds} \leq 0, \qquad (****)$$

and a requirement that households satisfy their transversality conditions. Here, C_t is aggregate private consumption, K_t is physical capital, N is population = labor supply, B_t is public debt, G is public consumption (government spending on goods and services), T_t is net tax revenue (= gross tax revenue−transfer payments), and F is an aggregate neoclassical production function with constant returns to scale and satisfying the Inada conditions. The other symbols stand for parameters and all these are positive; N and G are positive constants. A dot over a variable denotes the derivative w.r.t. time, t.

a) Briefly interpret (*), (**), (***), and (****), including the parameters.

b) Assuming $B_0 > 0$ and a balanced budget for all $t \geq 0$, construct a phase diagram. It is understood that G and B_0 are "modest" relative to the production possibilities of the economy, given an arbitrary initial $K_0 > 0$. Indicate by arrows the path the economy follows over time for $t \geq 0$. Give your argument as to why the economy takes that path and not another one.

  Assume the economy is in steady state.

c) Find the real interest rate and denote it r^*. Compare its size with that of the pure rate of time preference. Comment.

d) Find the tax revenue in the steady state and denote it $\bar{T}$.

  From now on suppose instead that after the economy has been in its steady state until time $t_0 > 0$, fiscal policy unexpectedly changes.

e) Let the unexpected fiscal event at time $t_0 > 0$ be that public consumption is raised to a higher constant level G' (still of "modest" size, though). At the same time, taxes are raised so as to maintain a balanced budget. We assume that everybody rightly expect the new policy to continue forever. Illustrate by the same or a new phase diagram the evolution of the economy for $t \geq t_0$. Sign the short- and long-run effects on private consumption of the new fiscal policy. Explain the intuition.

f) Suppose instead that at time t_0, although consumption is raised to the level G', taxes are left unchanged at $\bar{T}$. Now phase diagram analysis is not adequate for an explanation of the dynamics. Why?

g) What is meant by fiscal policy being sustainable. Is the fiscal policy $(G', \bar{T})$ sustainable? Why or why not? *Hint:* there are different possible approaches; one compares various present values of primary surpluses and applies, among other things, the mathematical rule that if a and b are constants, $b > 0$, then $\int_{t_0}^{\infty} a e^{-b(t-t_0)} dt = a/b$.

h) Suppose that G' is maintained for all $t \geq t_0$ while the tax revenue is at time $t_1 > t_0$ raised so as to maintain a balanced budget for all $t \geq t_1$. Illustrate by a new phase diagram the evolution of the economy for $t \geq t_1$.

i) Suppose the shift in tax policy at t_1 was already announced at time t_0 and that people believed the government. Does it make any difference for private consumption in this model whether the time interval (t_0, t_1) is short or long? Why or why not?

j) Can you imagine another type of model where the answer to i) would be different? Explain.

Problem 2   We consider a small open economy (SOE) satisfying the following conditions:

1. Perfect mobility across borders of financial capital, but no mobility of labor.

2. Domestic and foreign bonds are perfect substitutes and command the same expected rate of return.

3. Domestic and foreign output goods are imperfect substitutes.

4. Nominal prices are sluggish and follow an exogenous constant inflation path.

Aggregate output demand is

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\begin{aligned}
Y_t^d \;=\; & C(Y_t^p, R_t) + I(Y_t, R_t) + N(Y_t, x_t) + G \equiv D(Y_t, R_t, x_t, \tau) + G, && (*) \\
& \text{where } Y_t^p \equiv Y_t - \mathbb{T} \text{ and } \mathbb{T} = \tau + T(Y), \; 0 < T' < 1, \\
0 \;<\; & C_{Y^p}(1 - T') + N_Y < C_{Y^p}(1 - T') < C_{Y^p}(1 - T') + I_Y + N_Y \equiv D_Y < 1, \\
0 \;<\; & D_Y < 1, C_R + I_R \equiv D_R < 0, D_x > 0, -1 < D_\tau = -C_{Y^p} < 0.
\end{aligned}
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Notation: Y_t^d is output demand, Y_t^p is after-tax income, Y_t is output, R_t is the long-term real interest rate, and x_t is the real exchange rate, XP_t^*/P_t, where X is a given and constant nominal exchange rate, and P_t is the domestic price level while P_t^* is the foreign price level; τ measures "fiscal tightness", G is government spending on goods and services, $\mathbb{T}$ is net tax revenue, and $T(Y)$ is a tax function.

We assume that the domestic (forward-looking) inflation rate, π, is constant and equals the foreign (forward-looking) inflation rate, π^*. Hence, P_t^*/P_t is a constant and so is the real exchange rate, from now denoted x.

The dynamics of the economy is described by the following equations:

$$\dot{Y}_t = \lambda(D(Y_t, R_t, x, \tau) + G - Y_t), \qquad \lambda > 0, \quad Y_0 > 0 \text{ given}, \tag{1}$$

$$\frac{M_t}{P_t} = L(Y_t, i^*), \qquad L_Y > 0, \; L_i < 0. \tag{2}$$

$$R_t = \frac{1}{q_t}, \tag{3}$$

$$\frac{1 + \dot{q}_t^e}{q_t} = r_t^e, \tag{4}$$

$$r_t^e \equiv i^* - \pi_t^e, \qquad \pi_t \equiv \frac{\dot{P}_t}{P_t}, \tag{5}$$

$$P_t = P_0 e^{\pi t}, \tag{6}$$

where i_t is the domestic short-term nominal interest rate, i^* the foreign short-term nominal interest rate, r_t the domestic short-term real interest rate, M_t the money supply, and q_t the real price of a long-term bond (a consol). The superscript e indicates an subjective expectation.

The variables x, τ, G, i^*, and π are exogenous and constant, $\pi < i^*$. The initial values, Y_0 and P_0, are given. Expectations are rational and there are never speculative bubbles. The parameters are such that the speed of adjustment towards steady state is high.

a) Briefly interpret (2), (3), and (4) of the model.

b) To characterize the movement over time of the economy, derive two differential equations in Y and R, respectively.

c) Construct the corresponding phase diagram and illustrate the path the economy follows for an arbitrary $Y_0 > 0$. Comment.

d) Let the steady-state values of the long-term real interest rate and output be denoted $\bar{R}$ and $\bar{Y}$, respectively. Find these values. Finally, derive the spending and tax multipliers, $\partial \bar{Y}/\partial G$ and $\partial \bar{Y}/\partial \tau$.

Suppose that the economy has been in its steady state until time t_0.

e) Suppose the government is dissatisfied with the level of employment and at time $t_0 > 0$ decides (unexpectedly) to increase G to a higher level, G'. Suppose further that, owing to the automatic budget reaction under high unemployment, people rightly expect this higher level of spending to be maintained for quite some time without a rise in τ. Under the simplifying assumption that the new spending level is permanent, illustrate by a new phase diagram what happens for $t \geq t_0$.

f) Assume instead that at time t_0, the government credibly announces an upward shift in the level of government spending from G to G' to take place at time $t_1 > t_0$. Illustrate by a phase diagram and by graphical time profiles what happens to R_t, r_t, Y_t, and $m_t \equiv M_t/P_t$ for $t \geq t_0$. Comment. *Hint:* the answer to this question may be easier than one might immediately think.

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Problem 3   *Short questions*

a) "The expectations theory of the term structure of interest rates predicts that the long-term interest rate tends to be higher than the short-term interest rate." True or false? Comment.

b) Among the "stylized facts" concerning business cycle fluctuations (based on time series data after de-trending) are the following:

(i) Employment (aggregate labor hours) is procyclical and fluctuates almost as much as GDP.

(ii) Aggregate consumption and employment are markedly positively correlated.

(iii) Real wages are weakly procyclical and do not fluctuate much.

Briefly discuss how these "stylized facts" relate to alternative business cycle theories.

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