Written exam for the M. Sc. in Economics, Winter 2013-14

Advanced Macroeconomics

Master's Course

January 15, 2014

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 5 pages in total.

The weighting of the problems is:

Problem 1: 35 %, Problem 2: 45 %, Problem 3: 20 %.[1]

[1]The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a firm facing the following decision problem: choose a plan $(L_t, I_t)_{t=0}^{\infty}$ to maximize

$$V_0 = \int_0^{\infty} (F(K_t, L_t) - G(I_t, K_t) - w_t L_t - (1-\sigma)I_t)\, e^{-rt} dt \quad \text{s.t.} \tag{1}$$

$$L_t \geq 0, I_t \text{ free (i.e., no restriction on } I_t), \tag{2}$$

$$\dot{K}_t = I_t - \delta K_t, \qquad K_0 > 0 \text{ given,} \tag{3}$$

$$K_t \geq 0 \text{ for all } t. \tag{4}$$

Here F is a CRS neoclassical production function satisfying the Inada conditions. Inputs of capital and labor are denoted K_t and L_t, respectively, G is a capital installation cost function, I_t is gross investment, w_t is a given market real wage, $r > 0$ is a given constant real interest rate, and $\delta > 0$ is a given constant capital depreciation rate. There is a constant investment subsidy $\sigma \in (0, 1)$. The installation cost function G satisfies, for all (I, K),

$$G(0, K) = 0,\ G_I(0, K) = 0,\ G_{II}(I, K) > 0, \quad \text{and} \quad G_K(I, K) \leq 0.$$

a) Set up the current-value Hamiltonian. Let the adjoint variable, also called the co-state variable, be denoted q_t. Derive the first-order conditions and state the necessary transversality condition. *Hint:* in this problem the necessary transversality condition is of standard form, given an infinite horizon optimal control problem with discounting.

b) Interpret q_t. Show that optimal investment at time t is a function of q_t, K_t, and σ.

From now, let $G(I, K) = \beta \frac{I^2}{2K}$, where $\beta > 0$.

c) Express the optimal I_t/K_t as a function of q_t and σ.

Suppose the firm considered is the representative firm in a small open economy, SOE, with perfect mobility of financial capital and no mobility of labor. The labor force of the SOE is a constant, $\bar{L}$.

d) Show that the first-order conditions of the firm combined with full employment result in two coupled differential equations in K_t and q_t.

e) Construct the corresponding phase diagram and indicate the evolution over time of (K_t, q_t) for a given $K_0 > 0$. Comment.

f) Suppose that until time $t_0 > 0$ the system has been in steady state with $(K, q) = (K^*, q^*)$. Then, unexpectedly the government raises the investment subsidy to $\sigma' > \sigma$. Assume the investment subsidy is rightly expected to remain at the new level forever. Illustrate by the same or a new phase diagram the path (K_t, q_t) follows for $t > t_0$. Comment. *Hint:* to find out whether the new steady-state value of K, $K^{*\prime}$, is larger or smaller than the old, K^*, one approach is to combine your $\dot{q} = 0$ equation with the fact that $q^{*\prime} + \sigma' - 1 = q^* + \sigma - 1 = \delta$.

Problem 2 Consider the following model in continuous time for a closed economy:

$$\dot{Y}_t = \lambda(D(Y_t, R_t, \tau) + G - Y_t), \quad \lambda > 0, 0 < D_Y < 1, D_R < 0, -1 < D_\tau < 0, \tag{1}$$
$$\frac{M_t}{P_t} = L(Y_t, i_t), \qquad L_Y > 0, \ L_i < 0. \tag{2}$$
$$R_t = \frac{1}{q_t}, \tag{3}$$
$$\frac{1 + \dot{q}_t^e}{q_t} = r_t^e, \tag{4}$$
$$r_t^e \equiv i_t - \pi_t^e, \tag{5}$$
$$\pi_t = \pi, \tag{6}$$

where the superscript e denotes subjective expectation. Further, Y_t = output, q_t = real price of a consol paying one unit of output per time unit forever, G = government spending on goods and services, M_t = money supply, P_t = output price, R_t = real long-term interest rate, i_t = nominal short-term interest rate, r_t = real short-term interest rate, and $\pi_t \equiv \dot{P}_t/P_t$ = rate of inflation. The variables λ, τ, G, and π are exogenous constants. The initial values Y_0 and P_0 are historically given.

In questions a) - f) we assume that the central bank maintains the real money supply, $m_t \equiv M_t/P_t$, at a given constant level, m, by letting the (nominal) money supply grow at a rate equal to the rate of inflation.

a) Briefly interpret the model including the parameters.

Suppose expectations are rational and that speculative bubbles never arise.

b) To characterize the movement over time of the economy, derive from the model a dynamic system in Y and R. Draw the corresponding phase diagram and illustrate the path that the economy follows. Comment.

Now we will consider effects of shifts in policy. Suppose that the economy has been in its steady state until time $t_0 > 0$.

c) Then, at time t_0, an unanticipated downward shift in G occurs. But after this shift everybody rightly expects G to remain unchanged forever. Graphically illustrate by means of a phase diagram. Comment. *Hint:* the following formula may be helpful for intuition:

$$R_t = \frac{1}{q_t} = \frac{1}{\int_t^\infty e^{-\int_t^s r_u du} ds}.$$

d) Suggest a "free" interpretation of G and the downward shift in G such that the result under c) can be seen as a "rough" picture of events in the wake of the financial crisis 2008-2009.

e) What is the sign of the slope of the yield curve immediately after the shock to G? Comment.

We now consider a different monetary policy. Suppose that the central bank applies the short-term nominal interest rate as the monetary policy instrument and does so in accordance with the following rule:

$$i_t = \max(0, \alpha + \beta Y_t), \tag{*}$$

where α and β are constants, $\beta > -L_Y/L_i$.

f) Maintaining the interpretation from d), briefly answer question c) under these circumstances.

g) Compare the output stabilization capability of this policy rule to that of the original monetary policy above. *Hint:* compare the slope of the $\dot{R} = 0$ locus under the two alternative policies.

Suppose that the economy has ended up close to a steady state with nominal interest rate $\bar{i}$ close to zero and that the government finds the level of economic activity (output and employment) unsatisfactorily low.

h) In this situation the government decides to raise the level of spending to $G' > G$. So, at time $t_1 > t_0$ the government announces a shift to G' to be implemented at time $t_2 > t_1$ and maintained forever. After this announcement everybody rightly expect this fiscal policy to be carried out as announced. Graphically illustrate by means of a phase diagram what happens to R_t, r_t, Y_t, and M_t for $t \geq t_0$. Comment.

i) Given the intension of the government: 1) Is it a good or bad idea to let the time interval (t_1, t_2) be short? Why? 2) Is it a good or bad idea to have the monetary policy (*) replaced by an interest rate targeting policy, $i_t = \bar{i}$, for a while? Why?

Problem 3 *Short questions*

In a theoretical debate about expansionary fiscal policy in a given economy the debaters agree to argue within a framework where the economy is described this way:

1. The economy is closed, the production factors are capital and labor, the production function is neoclassical with CRS, and aggregate output demand is $C_t + I_t + G_t$, where C_t is private consumption, I_t is private investment, and G_t is public consumption. Time is discrete.

2. The household sector consists of Barro-style dynasties with infinite horizon, strictly concave period utility function, and perfect foresight.

3. Labor supply is inelastic and constant. There is no technological progress.

4. The government exactly satisfies its intertemporal budget constraint and there is no reason to fear government default.

5. Taxes are lump sum and credit markets are perfect.
6. The private agents rightly believe in the announcements made by the government.

Two alternative fiscal policies, A and B, are considered. Policy A is a "business-as-usual" policy and Policy B is intended as a "fiscal stimulus policy" implemented as of period t_0 without previous announcement. With G_t^A and G_t^B denoting public consumption implied by Policy A and Policy B, respectively, and $\bar{G}$ and $\Delta\bar{G}$ denoting positive constants of moderate size, it holds that:

$$G_t^A = \bar{G} \text{ for all } t,$$
$$G_t^B = \begin{cases} \bar{G} + \Delta\bar{G} \text{ for } t = t_0, t_0 + 1, \ldots, t_0 + n, \\ \bar{G} \text{ for all other } t, \end{cases}$$

where $1 \leq n \leq \infty$, n being credibly announced already in period t_0.

Taxes are not immediately adjusted and so Policy B allows a budget deficit to arise (or be larger than otherwise).

In the debate Policy B is compared with Policy A and the following claims are made:

a) "Suppose the economy has perfect competition and for $t < t_0$ is in steady state with constant C_t. Then, if the increased level of public consumption is permanent (i.e., $n = \infty$), the representative household will reduce its consumption in period t_0 and onwards by the amount $\Delta\bar{G}$ per period and so the sum $C_t + G_t$ will not be raised by Policy B." Evaluate this claim. *Hint:* thinking the situation through, a clear evaluation of the claim is possible.

b) "Suppose the economy has perfect competition and for $t < t_0$ is in steady state with constant C_t. Then, if the increased level of public consumption is only temporary (i.e., $n < \infty$), the representative household will reduce its consumption in period t_0 and onwards by less than the amount $\Delta\bar{G}$ per period and so Policy B will temporarily increase the sum $C_t + G_t$." Evaluate this claim.

c) "Suppose the economy has imperfect competition and that due to deficient aggregate demand, the economy has involuntary unemployment and abundant production capacity in the periods at least up to period t_0. Suppose further that the increased level of public consumption implied by Policy B is only temporary (i.e., $n < \infty$). Then the outcome of Policy B may be no reduction in private consumption at all, possibly a rise." Evaluate this claim.

d) State what the proposition of Ricardian Equivalence exactly asserts. In what sense is the above debate not directly a debate about this proposition?

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