Chapter 13

General equilibrium analysis of public and foreign debt

This chapter reviews long-run dynamics of public and foreign debt in the light of the continuous time OLG model of the previous chapter. Section 13.1 reconsiders the *Ricardian equivalence* issue. In Section 13.2 we extend the enquiry to a *general equilibrium analysis of budget deficits and debt dynamics* in a closed economy. Section 13.3 addresses general equilibrium aspects of public and foreign debt in a small open economy. Issues of *twin deficits* and the current account of a growing economy are considered. In Section 13.4 the assumption of lump-sum taxes is replaced by income taxation in order to examine the relationship between *debt and distortionary taxation.* The theme of *optimal debt* is addressed in Section 13.5, and the concluding Section 13.6 addresses the *time-inconsistency problem* faced by economic policy when outcomes depend on private sector expectations.

13.1 Reconsidering the issue of Ricardian equivalence

Recall that Ricardian equivalence is the claim that, given the (expected) future path of government spending, it does not matter for aggregate private consumption and saving whether the government finances its current spending by lump-sum taxes or borrowing. Whether this claim is an acceptable approximation is still a subject of debate among macroeconomists.

As we know from earlier chapters, the representative agent approach and the life-cycle-OLG approach lead to opposite conclusions regarding the issue. In models with a representative household with infinite horizon (the Barro and Ramsey dynasty models) a change in the timing of lump-sum taxes does not change the present value of the infinite stream of taxes imposed on the individual dynasty. A cut in current taxes is offset by the expected higher future taxes. Private saving goes up just as much as current taxes are reduced. This is exactly what is needed for paying the higher taxes in the future and maintain the preferred time path of consumption. Current consumption is thus not affected. And aggregate saving in society as a whole stays the same (the higher government dissaving being matched by higher private saving).

It is different in the life-cycle-OLG models (without a Barro-style bequest motive). For instance the Diamond OLG model with a public sector reveals how taxes levied at different times are levied on different sets of agents. In the future some of the currently alive will be gone and there will be newcomers to bear part of the higher tax burden. A current tax cut thus makes current tax payers feel wealthier and this leads to an increase in their current consumption. So current private consumption in the economy ends up higher. The present generations consequently benefit and future generations bear the cost in the form of smaller national wealth than otherwise.

Because of the more refined notion of time in the Blanchard OLG model from Chapter 12 and its capability of treating wealth effects more aptly, let us see what this model precisely says about the issue. A simple book-keeping exercise will show that the size of the public debt *does* matter. By affecting private wealth, it affects private consumption.

To keep things simple, we ignore retirement ($\lambda = 0$). To avoid notational confusion of the birth rate with the debt-income ratio, the former will in this chapter be denoted β while we still denote the latter by b. As in the previous chapters, B_t will denote net government debt, G_t government spending on goods and services, and T_t net tax revenue, $\tilde{T}_t - X_t$, where $\tilde{T}_t$ is gross tax revenue while X_t is transfers, all in real terms. We assume that the interest rate is in the long run higher than the output growth rate. Hence, to remain solvent the government has to satisfy its intertemporal budget constraint. Ignoring seigniorage and presupposing the government does not plan to procure more tax revenue than needed to satisfy its intertemporal budget constraint, as seen from time 0 (interpreted as "now"), we have the condition

$$\int_0^\infty T_t e^{-\int_0^t r_s ds} dt = \int_0^\infty G_t e^{-\int_0^t r_s ds} dt + B_0, \tag{GIBC}$$

where the expected future time paths of G_t and r_t are considered given and B_0 is historically given. In brief, (GIBC) says that the present value of future net tax revenues must equal the sum of the present value of future spending on goods and services and the current level of debt. A temporary cut in taxes in an early time interval after time 0 must be offset in a later time interval by a rise in taxes of the same present value.

Given aggregate private financial wealth, A_0, and aggregate human wealth, H_0, aggregate private consumption is

$$C_0 = (\rho + m)(A_0 + H_0). \tag{13.1}$$

Because of the logarithmic specification of instantaneous utility, the propensity to consume out of wealth is a constant equal to the sum of the pure rate of time preference, ρ, and the mortality rate, m. Human wealth is the present value of expected future net-of-tax labor earnings of those currently alive:

$$H_0 = N_0 \int_0^\infty (w_t - \tau_t) e^{-\int_0^t (r_s + m) ds} dt. \tag{13.2}$$

Here, τ_t is the per capita lump-sum net taxation at time t, i.e., $\tau_t \equiv T_t/N_t \equiv (\tilde{T}_t - X_t)/N_t$, where N_t is the size of the population (here equal to the labor force, which in turn equals employment). The discount rate is the sum of the risk-free interest rate, r_t, and the actuarial compensation which is identical to the mortality rate, m.

To fix ideas, consider a closed economy. In view of the presence of government debt, aggregate private financial wealth in the closed economy is $A_0 = K_0 + B_0$, where K_0 is aggregate (private) physical capital and B_0 is assumed positive. Thus, (13.1) can be written

$$C_0 = (\rho + m)(K_0 + B_0 + H_0), \tag{13.3}$$

where ρ is the pure rate of time preference and m is the mortality rate. We ask whether B_0 is net wealth , for a given K_0, the sum $B_0 + H_0$ depends on the size of B_0, given the expected future path of G_t in (GIBC). We will see that the answer is yes. This is because, contrary to the Ricardian equivalence hypothesis, a higher B_0 is *not* offset by an equally reduced H_0 brought about by the higher future lump-sum taxes. Such a fully offsetting reduction of H_0 will not occur. Therefore C_0 *is* increased. Aggregate consumption depends positively on B_0.

The argument is the following. Rewrite (13.2) as

$$\begin{aligned}
H_0 &= N_0 \int_0^\infty \frac{w_t N_t - T_t}{N_t} e^{-\int_0^t (r_s + m) ds} dt \qquad \text{(from } \tau_t = T_t/N_t\text{)} \\
&= \int_0^\infty (w_t N_t - T_t) e^{-nt} e^{-\int_0^t (r_s + m) ds} dt \qquad \text{(since } N_0 = N_t e^{-nt}\text{)} \\
&= \int_0^\infty (w_t N_t - T_t) e^{-\int_0^t (r_s + n + m) ds} dt = \int_0^\infty (w_t N_t - T_t) e^{-\int_0^t (r_s + \beta) ds} dt,
\end{aligned}$$

using that the population growth rate, n, equals $\beta - m$. Therefore,

$$\begin{aligned} H_0 + B_0 &= \int_0^\infty (w_t N_t - T_t) e^{-\int_0^t (r_s+\beta)ds} dt + B_0 = \int_0^\infty (w_t N_t - G_t) e^{-\int_0^t (r_s+\beta)ds} dt \\ & \quad - \int_0^\infty (T_t - G_t) e^{-\int_0^t (r_s+\beta)ds} dt + B_0. \qquad (13.4) \end{aligned}$$

Note that the first integral on the right-hand side of (13.4) is given (independent of a changed time profile of τ_t).

Reordering (GIBC), we have

$$B_0 = \int_0^\infty (T_t - G_t) e^{-\int_0^t r_s ds} dt. \qquad (13.5)$$

Hence, the last line of (13.4) can be written

$$\begin{aligned} & - \int_0^\infty (T_t - G_t) e^{-\int_0^t (r_s+\beta)ds} dt + \int_0^\infty (T_t - G_t) e^{-\int_0^t r_s ds} dt \\ = & \int_0^\infty \left((T_t - G_t) e^{-\int_0^t r_s ds} - (T_t - G_t) e^{-\int_0^t r_s ds} e^{-\int_0^t \beta ds} \right) dt \\ = & \int_0^\infty (T_t - G_t) e^{-\int_0^t r_s ds} \left(1 - e^{-\int_0^t \beta ds} \right) dt. \qquad (13.6) \end{aligned}$$

As $B_0 > 0$, in view of (13.5), the primary surplus, $T_t - G_t$, is positive "most of the time". Then from (13.6) follows

$$H_0 + B_0 = \int_0^\infty (w_t N_t - G_t) e^{-\int_0^t (r_s+\beta)ds} dt + \int_0^\infty (T_t - G_t) e^{-\int_0^t r_s ds} \left(1 - e^{-\int_0^t \beta ds} \right) dt. \qquad (13.7)$$

There are two cases regarding the birth rate β to consider: $\beta = 0$ and $\beta > 0$. The first case turns the Blanchard model into a representative agent model. Now, if $\beta = 0$, the second term on the right-hand side of (13.7) vanishes. Then the remaining term indicates that $H_0 + B_0$ is independent of the time profile of taxes. Only the given time path of G_t matters. A higher B_0 does not affect the $w_t N_t - G_t$ flow, and so the *sum* $H_0 + B_0$ is unaffected. That is, the only effect of a higher B_0 is to make H_0 equally much lower so as to leave $H_0 + B_0$ unchanged. The case $\beta = 0$ thus implies Ricardian equivalence.

When $\beta > 0$ (positive birth rate), both terms on the right-hand side of (13.7) becomes decisive (generally). When $B_0 > 0$, the primary surplus, $T_t - G_t$, is positive "most of the time", in view of (13.5). The right-hand side of (13.7) will thus generally depend on the time profile of taxes and so be affected by a temporary tax cut. Moreover, a higher B_0 will tend to make the second term in

(13.7) larger (more or larger primary surpluses will be needed). This is exactly what does *not* happen if $\beta = 0$, because in that case the second term is and remains nil.

We conclude:

$$\begin{cases} H_0 + B_0 \text{ is independent of } B_0, \text{ if } \beta = 0, \text{ while} \\ H_0 + B_0 \text{ depends positively on } B_0, \text{ if } \beta > 0. \end{cases} \tag{13.8}$$

The intuition is that when the birth rate is positive, the tax burden in the future falls partly on new generations. Larger holdings of government bonds thus make the current generations feel wealthier in spite of future taxes being raised.

EXAMPLE Let $B_0 > 0$. Suppose T_0 is proportional to G_0 for all $t \geq 0$ with the factor of proportionality $1 + \xi$. Then, inserting $T_0 = (1 + \xi)G_0$ into (13.7) gives

$$H_0 + B_0 = \int_0^\infty (w_t N_t - G_t)e^{-\int_0^t (r_s+\beta)ds}dt + \xi \int_0^\infty G_t e^{-\int_0^t r_s ds}\left(1 - e^{-\int_0^t \beta ds}\right) dt,$$

which for $\beta > 0$ is an increasing function of ξ. In turn, ξ is an increasing function of B_0 because inserting $T_0 = (1 + \xi)G_0$ into (13.5) and solving for ξ gives $\xi = B_0 / \int_0^\infty G_t e^{-\int_0^t r_s ds}dt > 0$. So, for $\beta > 0$, $H_0 + B_0$ depends positively on B_0. $\square$

The result may be seen in the light of the different discount rates involved. The discount rate relevant for the government when discounting future tax receipts and future spending is just the market interest rate, r. But the discount rate relevant for the households currently alive is $r + \beta$. This is because the present generations are, over time, a decreasing fraction of the tax payers, the rate of decrease being larger the larger is the birth rate. In the Barro and Ramsey models the "birth rate" is effectively zero in the sense that no *new* tax payers are born. When the bequest motive (in Barro's form) is operative, those alive today will take the tax burden of their descendents fully into account.

This takes us to the distinction between *new individuals* and *new decision makers*, a distinction related to the fundamental difference between representative agent models and overlapping generations models.

It is neither finite lives nor population growth

It is sometimes claimed that finite lives or the presence of population growth are basic theoretical reasons for the absence of Ricardian equivalence. This is a misunderstanding, however. The distinguishing feature is whether new decision makers continue to enter the economy or not.

To sort this out, let $\bar{\beta}$ be a constant birth rate of *decision makers.* That is, if the population of decision makers is of size N, then $N\bar{\beta}$ is the inflow of new decision makers per time unit.[1] Given the assumption of a perfect credit market, we claim:

$$\text{there is Ricardian equivalence if and only if } \bar{\beta} = 0. \tag{13.9}$$

Indeed, with (13.8) in mind, when $\bar{\beta} = 0$, future taxes have to be paid by those current tax payers who are still alive in the future. In the absence of credit market imperfections the current tax payers will thus respond to deficit finance (deferment of taxation) by increasing current saving out of the currently higher after-tax income. This increase in saving matches the expected extra taxes in the future. So current private consumption is unaffected by the deficit finance.

If $\bar{\beta} > 0$, however, deficit finance means shifting part of the tax burden from current tax payers to new tax payers in the future whom current tax payers do not care about. Even though representative agent models like the Ramsey and Barro models may include population growth in a demographic sense, they have a *fixed* number of dynastic families (decision makers) and whether the *size* of these dynastic families rises (population growth) or not is of no consequence for the question of Ricardian equivalence.

Another implication of (13.9) is that it is not the *finite lifetime* that is decisive for absence of Ricardian equivalence in OLG models. Indeed, even if we imagine the agents in a Blanchard-style model have a zero death rate, there will still be a *positive* birth rate. New decision makers continue to enter the economy through time. When deficit finance occurs, part of the tax burden is shifted to these newcomers.

To be specific, let $\bar{m}$ be a constant and age-independent death rate of existing decision makers. Then $\bar{n} \equiv \bar{\beta} - \bar{m}$ is the growth rate of the number of decision makers. With β, m, and n denoting the birth rate, death rate, and population growth rate, respectively, in the usual *demographic* sense, we have in Blanchard's model $\bar{\beta} = \beta$, $\bar{m} = m$, and $\bar{n} = n$. In the Ramsey model, however, $\bar{\beta} = \bar{m} = \bar{n} = 0 \leq n = \beta - m$. With this interpretation, both the Blanchard and the Ramsey model fit into (13.9). In the Blanchard model every new generation consists of new decision makers, i.e., $\bar{\beta} = \beta > 0$. In that setting, whether or not the population grows, the generations now alive know that the higher taxes in the future implied by deficit finance today will in part fall on the new generations. We therefore have $n \geq 0$, $\bar{\beta} = \bar{n} + \bar{m} \geq \bar{m} > 0$, and in accordance with (13.9) there is not Ricardian equivalence. In the Ramsey model where, in principle, the new generations are not new decision makers since their utility were already taken care of through bequests by their forerunners, there is Ricardian equivalence. This is in accordance with (13.9), since $\bar{\beta} = 0$, whereas $n \geq 0$.

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[1] In view of the law of large numbers, we do not distinguish between expected and actual inflow.