The balanced budget after time t_2 implies

$$T_t = (F_K(K_t, N) - \delta)B_{t_2} + \bar{G}. \tag{13.27}$$

The dynamics are therefore again governed by a two-dimensional system,

$$\dot{K}_t = F(K_t, N) - \delta K_t - C_t - \bar{G}. \tag{13.28}$$

$$\dot{C}_t = [F_K(K_t, N_t) - \delta - \rho]C_t - m(\rho + m)(K_t + B_{t_2}), \tag{13.29}$$

Consequently phase diagram analysis can again be used.

The phase diagram for $t \geq t_2$ is depicted in Fig. 13.4. The new initial K is K_{t_2}, which is smaller than the previous steady-state value K^* because of the negative net investment in the time interval $[t_1, t_2)$. Relative to Fig. 13.2, the $\dot{K} = 0$ locus is unchanged (since $\bar{G}$ is unchanged). But in view of the new constant debt level B_{t_2} being higher than B_0, the $\dot{C} = 0$ locus has turned counter-clockwise. For any given $K \in (0, \bar{K})$, the value of C required for $\dot{C} = 0$ is higher than before, cf. (13.20). The intuition is that for every given K, private financial wealth is higher than before in view of the possession of government bonds being higher. For every given K, therefore, the generation replacement effect on the change in aggregate consumption is greater and so is then the level of aggregate consumption that via the operation of the Keynes-Ramsey rule is required to offset the generation replacement effect and ensure $\dot{C} = 0$ (cf. Section 12.2 of the previous chapter).

The new saddle-point stable steady state is denoted E' in Fig. 13.4 and it has capital stock $K^{*\prime} < K^*$ and consumption level $C^{*\prime} < C^*$. As the figure is drawn, K_{t_2} is larger than $K^{*\prime}$. This case seems likely to arise if the tax cut at t_1 is "large" but does not last long. The level of consumption immediately after t_2, where the fiscal tightening sets in, is found where the vertical line $K = K_{t_2}$ crosses the new saddle path, i.e., the point P in Fig. 13.4. Immediately before t_2, consumption was at a higher level. The movement of the economy after t_2 implies gradual lowering of the capital stock and consumption until the new steady state, E', is reached. (More details below in the section on time profiles.)

Alternatively, it seems possible that K_{t_2} can be smaller than $K^{*\prime}$ so that the new initial point, A, is to the left of the new steady state E'. This case is illustrated in Fig. 13.5 and seems likely to arise if the tax cut at t_1 is "small" but lasts a long time. The low amount of capital at t_2 implies a high interest rate and the fiscal tightening must now be tough. This induces a low consumption level − so low that net investment becomes positive. Then the capital stock and output increase gradually during the adjustment to the steady state E'.[13]

[13] A precise determination of conditions under which the case $K^{*\prime} < K_{t_2}$ or the case $K^{*\prime} > K_{t_2}$ occurs is complicated. The difficulty arises from the three-dimensional dynamics and

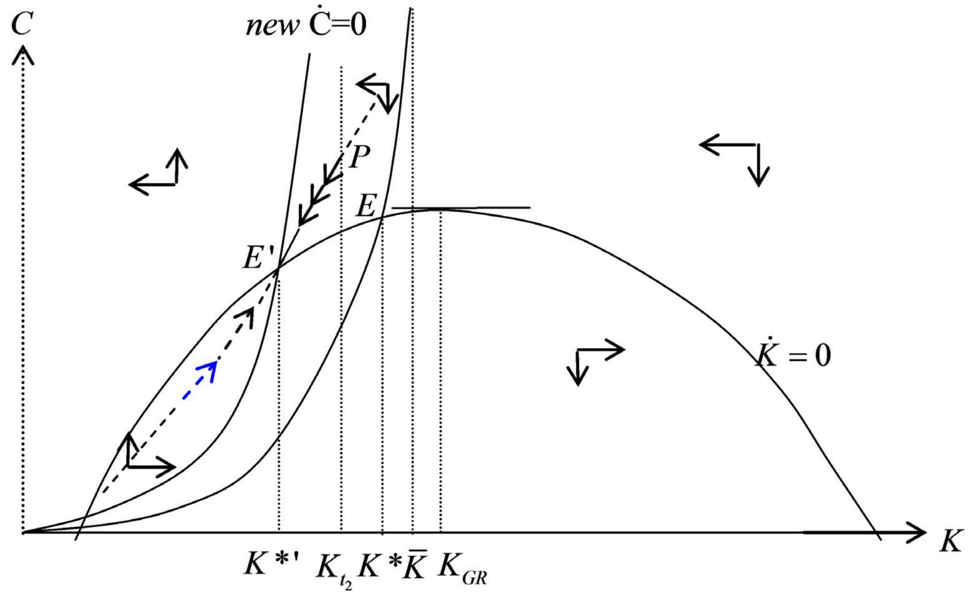

Figure 13.4: The adjustment after fiscal tightening at time t_2, presupposing $K^{*\prime} < K_{t_2}$.

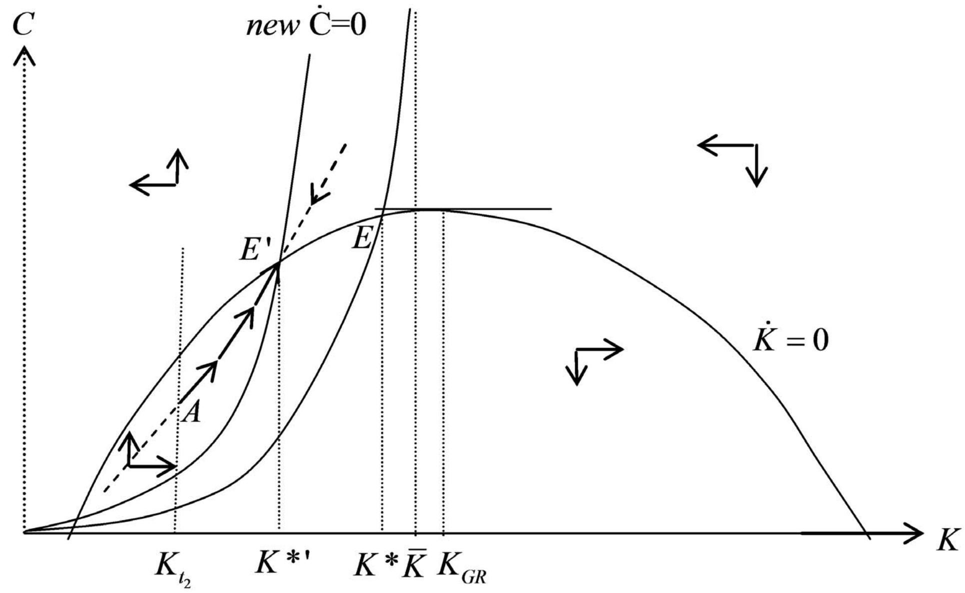

Figure 13.5: The adjustment after fiscal tightening at time t_2, presupposing $K^{*\prime} > K_{t_2}$.

Thus, in both cases the long-run effect of the transitory budget deficit is qualitatively the same, namely that the larger supply of government bonds crowds out physical capital in the private sector. Intuitively, a certain feasible time profile for financial wealth, $A = K + B$, is desired and the higher is B, the lower is the needed K. To this "stock" interpretation we may add a "flow" interpretation saying that the budget deficit offers households a saving outlet which is an alternative to capital investment. All the results of course hinge on the assumption of permanent full capacity utilization in the economy (no idle capital, no idle labor).

To be able to quantify the long-run effects of a change in the debt level on K and C we need the long-run multipliers. By equalizing the right-hand sides of (13.19) and (13.20), with B_0 replaced by $\bar{B}$, and using implicit differentiation w.r.t. $\bar{B}$, we get

$$\frac{\partial K^*}{\partial \bar{B}} = \frac{m\,(\rho + m)}{\mathcal{D}} < 0, \tag{13.30}$$

where $\mathcal{D} \equiv C^* F_{KK}^* - (r^* + m)(\rho + m - r^*) < 0$.[14] Next, by using the chain rule on $C^* = F(K^*, N) - \delta K^* - \bar{G}$ from (13.19), we get

$$\frac{\partial C^*}{\partial \bar{B}} = \frac{\partial C^*}{\partial K^*}\frac{\partial K^*}{\partial \bar{B}} = (F_K(K^*, N) - \delta)\frac{m\,(\rho + m)}{\mathcal{D}} = r^* \frac{m\,(\rho + m)}{\mathcal{D}} < 0.$$

The multiplier $\partial K^*/\partial \bar{B}$ tells us the approximate size of the long-run effect on the capital stock, when a temporary tax cut causes a unit increase in public debt. The resulting change in long-run output is approximately $\partial Y^*/\partial \bar{B} = (\partial Y^*/\partial K^*)(\partial K^*/\partial \bar{B}) = (r^* + \delta)m\,(\rho + m)\,/\mathcal{D} < 0$. The elasticity of long-run output with respect to public debt is $(\bar{B}/Y^*)\partial Y^*/\partial \bar{B} = (\bar{B}/Y^*)(r^* + \delta)m\,(\rho + m)\,/\mathcal{D} < 0$.

Time profiles It is also useful to consider the time profiles of the variables.

Case 1: $K^{*\prime} < K_{t_2}$. Fig. 13.6 shows the time profile of T and B, respectively. The upper panel visualizes that the increase in taxation at time t_2 is larger than the decrease at time t_1. As (13.27) shows, this is due to public expenses being larger after t_2 because both the government debt B_t and the interest rate,

uncertainty during the time interval (t_1, t_2) as well as from the fact that the longer the time interval (t_1, t_2), the larger is not only the fall in K but also the rise in B. While there is a lower bound, $-\delta$, on the proportionate rate of change of the capital stock, there is no comparable upper bound on how fast the government debt can increase. Hence, if the tax cut is substantial and the time interval (t_1, t_2) "small", it seems likely that the fall in K is "dominated" by the rise in B as in Fig. 13.4. In the polar case, however, an outcome as in Fig. 13.5 might occur. Numerical simulation and sensitivity analysis should be able to settle the matter but is not pursued here.

[14] For details, see Appendix B.

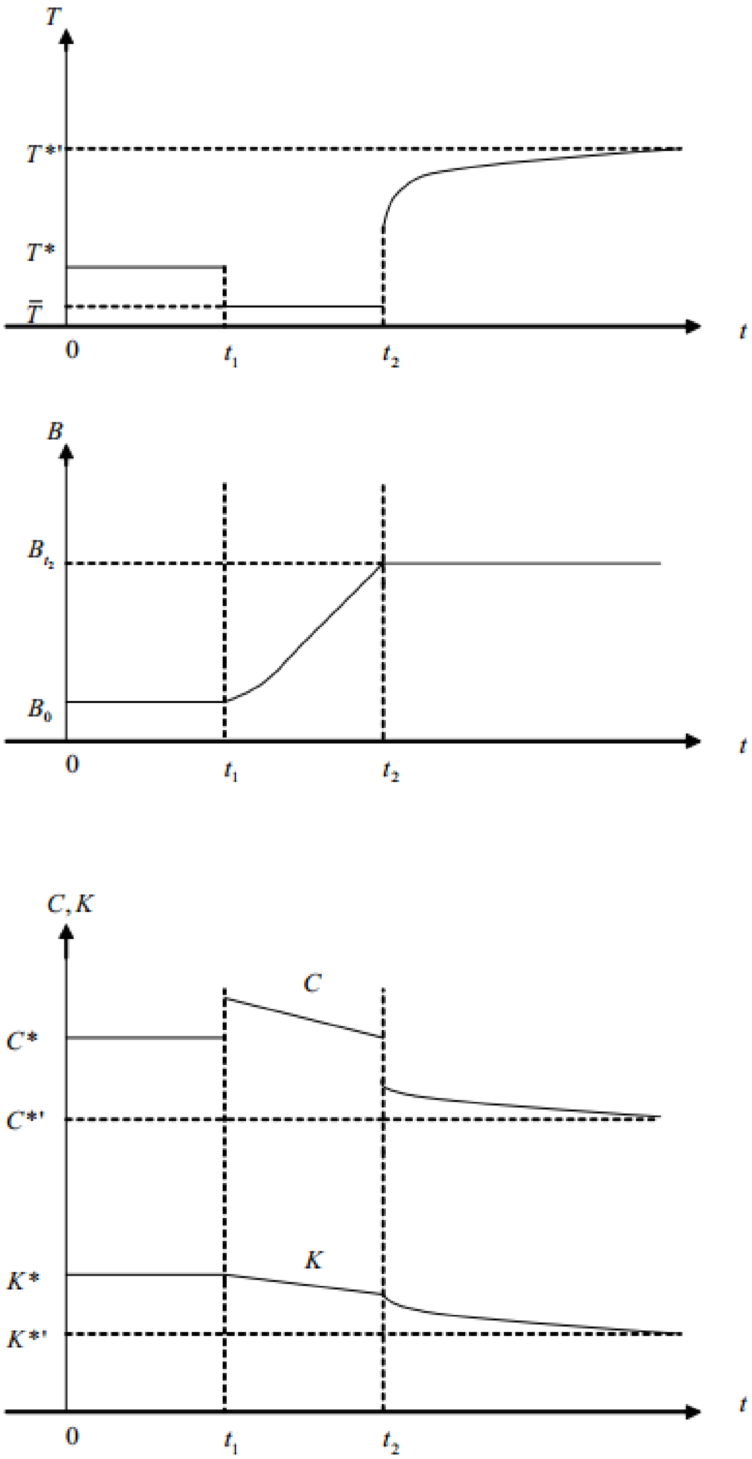

Figure 13.6: Case 1: $K^{*\prime} < K_{t_2}$. Regarding time path of C in the time interval (t_1, t_2) only one possibility shown.

$F_K(K_t, N_t) - \delta$, are higher. The further gradual rise in T_t towards its new steady-state level is due to the rising interest service along with a rising interest rate, caused by the falling K.

The middle panel of Fig. 13.6 is self-explanatory.

As visualized by the lower panel of Fig. 13.6, the tax cut at time t_1 results in an upward jump in consumption. This implies negative net investment, so that K begins to fall. The size of the upward jump in consumption at time t_1 and the subsequent time path of consumption in the time interval $[t_1, t_2)$ can not be precisely pinned down. We can not even be sure that C will be gradually falling. Therefore the downward-sloping time path of C in the lower panel of Fig. 13.6 in this time interval illustrates just one of the possibilities.

The ambiguity arises for the following reason. Though the current generations will immediately feel wealthier and increase their consumption as a result of the tax cut, they have rational expectations and are thereby aware that sooner or later fiscal policy will have to be changed again. As the households may have uncertain and different beliefs about *when* and *how* the fiscal sustainability problem will be remedied, we can not theoretically assign a specific value to the new after-tax human wealth, even less a constant value. What we can tell is that H_{t_1}, and therefore C_{t_1}, will be "somewhat" larger than immediately before time t_1. Also private saving will rise, however. This is because the rise in consumption at time t_1 will be less than the fall in taxes. To see this, imagine first that the households expect a constant level, T, to last for a long time during which also the real interest rate and the real wage remain approximately unchanged. Perceived human wealth would then be $H \approx (w^*N - T)/(r^* + m)$, from (13.15). By $C_t = (\rho + m)(A_t + H)$, we would have

$$\Delta C_t \approx dC_t = \frac{\partial C_t}{\partial T} dT = (\rho + m)\frac{\partial H}{\partial T} dT = -\frac{\rho + m}{r^* + m} dT < -dT, \qquad (13.31)$$

in view of $dT = \bar{T} - T^* < 0$ and $r^* > \rho$. To the extent that the households expect the new tax level $\bar{T}$ to last a *shorter* time, the boost to H, and therefore also to C, will be *less* than indicated by this equation. The boost to H and C is further dampened by the (correct) anticipation that the ongoing negative net investment will imply a falling K and thereby a falling real wage (due to the falling marginal productivity of labor) and a rising interest rate (due to the rising net marginal productivity of capital). So, at least for a while, there *will* be *positive private saving*, hence rising private financial wealth A. Meanwhile H *will be falling* after t_1 due to the falling real wage, the rising interest rate, and the fact that the date of likely fiscal tightening is approaching, although uncertain.

So the two components of total wealth, A and H, move in opposite directions. Depending of which of these opposite movements is dominating, consumption will

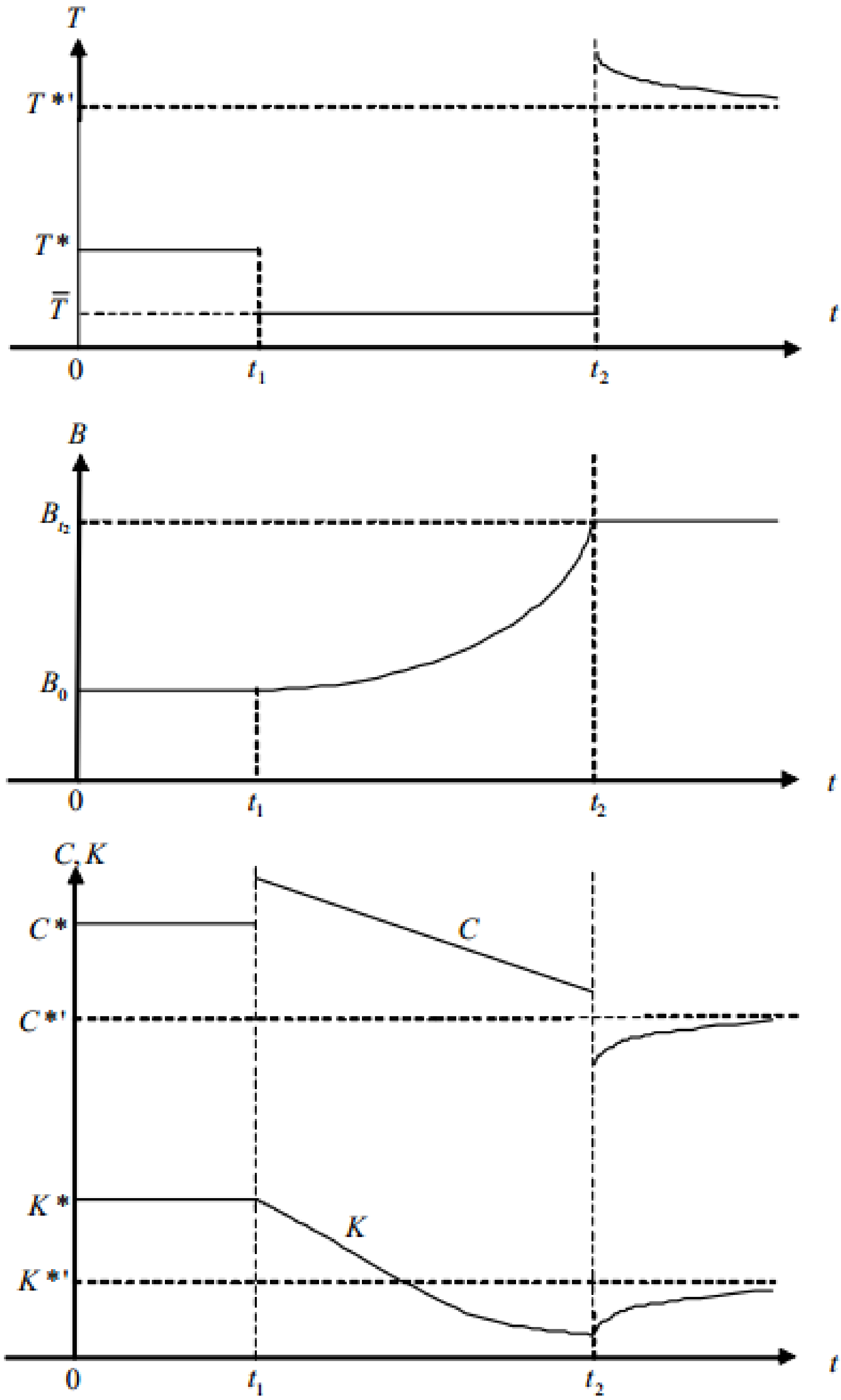

Figure 13.7: Case 2: $K^{*\prime} > K_{t_2}$. Regarding time path of C in the time interval (t_1, t_2) only one possibility shown.

be rising or falling for a while after t_1 (Fig. 13.6 depicts the latter case). Anyway, because the exact time and form of the fiscal tightening is not anticipated, a sharp decrease in the present discounted value of after-tax labor income occurs at time t_2, which induces a downward jump in consumption as indicated in the lower panel of Fig. 13.6. Although the fall in consumption makes room for increased net investment, by definition of $t_2 - t_1$ *being* "small", net investment remains negative so that the fall in K continues after t_2. Therefore, also the real wage continues to fall, implying continued fall in H, cf. (13.14) and (13.15), hence further fall in C, until the new steady-state level is reached.

If the time of the fiscal tightening were anticipated, consumption would not jump at time t_2. But the long-run result would be qualitatively the same.

Case 2: $K^{*\prime} > K_{t_2}$. In this case the tax revenue after t_2 has to exceed what is required in the new steady state. During the subsequent adjustment the taxation level will be gradually falling which reflects the gradual fall in the interest rate generated by the rising K, cf. Fig. 13.5. Private consumption will at time t_2 jump to a level *below* the new (in itself lower) steady state level, $C^{*\prime}$.

The above analysis is in a sense "biased" against budget deficits because it ignores economic growth. Thereby persistent budget deficits necessarily become incompatible with fiscal sustainability. With economic growth, persistent budget deficits are compatible with fiscal sustainability as long as the resulting government debt does not persistently grow faster than GDP. A further limitation of the analysis is its abstraction from the role of Keynesian aggregate demand factors in the process.

13.3 Public and foreign debt: a small open economy

Let the country considered be a small open economy (SOE). Suppose there is perfect substitutability and mobility of goods and financial capital across borders, but no mobility of labor. The main difference compared with the above analysis is then that the interest rate will not be affected by the public debt of the country (as long as its fiscal policy seems sound). Besides making the analysis simpler, this entails a *stronger* crowding out effect of public debt than in the closed economy. The lack of an offsetting increase in the interest rate means absence of the feedback which in a closed economy limits the fall in aggregate saving. In the open economy national wealth equals the stock of physical capital plus net foreign assets. And it is national wealth rather than the capital stock which is crowded out.