Christian Groth

A suggested solution to the problem set at the re-exam in Advanced Macroeconomics February 15, 2017

(3-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates (a) accurate and thorough understanding of the concepts, methods, and models in the course, (b) knowledge of the major empirical regularities for aggregate economic variables, and (c) ability to use these theoretical tools and this empirical knowledge to answer macroeconomic questions.

1. Solution to Problem 1

The given equations from the Blanchard OLG model are:

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \frac{\lambda + b}{b}\tilde{c}_t - (\delta + g + b - m)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given}, \tag{1.1}$$

$$\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho + \lambda - g\right]\tilde{c}_t - b(\rho + m)\tilde{k}_t, \tag{1.2}$$

together with the condition that for any fixed pair (t_0, v_0), where $t_0 \geq 0$ and $v \leq t_0$,

$$\lim_{t\to\infty} a_{t,v} e^{-\int_{t_0}^{t}(f'(\tilde{k}(s))-\delta+m)ds} = 0. \tag{1.3}$$

Notation: $\tilde{k}_t \equiv K_t/(T_t L_t)$ and $\tilde{c}_t \equiv C_t/(T_t N_t) \equiv c_t/T_t$, where K_t and C_t are aggregate capital and aggregate consumption, respectively; N_t is population, L_t is labor supply, and T_t is the technology level, all at time t; f is a production function in intensive form, satisfying $f(0) = 0$, $f' > 0$, $f'' < 0$, and the Inada conditions. Finally, $a_{t,v}$ is financial wealth at time t of an individual born at time v. The remaining symbols stand for parameters and it is assumed all these are strictly positive. Furthermore, $\rho \geq b-m \geq 0$ and $\lambda < \delta + \rho + g$.

[1] The solution below contains more details and more precision than can be expected at a three hours exam.

a) Parameters: λ = retirement rate, b = birth rate, δ = capital depreciation rate, g = growth rate of technology, m = mortality rate, ρ = pure rate of time preference (utility discount rate, a measure of impatience). The model relies on the simplifying assumption that for a given individual the probability of having a remaining lifetime, X, longer than some arbitrary number x is $P(X > x) = e^{-mx}$, the same for all (i.e., independent of age). Individual labor supply is assumed to decline exponentially at rate λ with increasing age. Both population and labor force grow at the rate $n \equiv b - m$.

The equation (1.1) is essentially just national income accounting for a closed economy. Isolating $f(\tilde{k}_t)$ on one side we have aggregate gross income per unit of effective labor on this side and consumption plus gross investment per unit of effective labor on the other side. Because the technology-corrected capital-labor ratio, $\tilde{k}_t$, has employment, L_t, in the denominator while the technology-corrected per-capita consumption, $\tilde{c}_t$, has population, N_t, in the denominator, $\tilde{c}_t$ is multiplied by the inverse of the participation rate, $L_t/N_t = b/(\lambda + b)$.

As to the first term on the right-hand side of (1.2), notice that instantaneous utility in the Blanchard OLG model is logarithmic, so that the individual Keynes-Ramsey rule at time t for a person born at time v is simply

$$\frac{\partial c_{t,v}}{\partial t} = [r_t + m - (\rho + m)]\, c_{t,v} = (r_t - \rho)c_{t,v}. \tag{1.4}$$

In general equilibrium with perfect competition, $r_t = f'(\tilde{k}_t) - \delta$. The corresponding Keynes-Ramsey rule for growth-corrected per-capita consumption is therefore

$\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho - g\right] \tilde{c}_t$. But due to the gradual replacement of dying elder individuals with low labor supply by younger individuals supplying more labor, the first term on the right-hand side of (1.2) also includes $+\lambda\tilde{c}_t$.

The second term on the right-hand side of (1.2) represents another aspect of generation replacement. The arrival of newborns is Nb per time unit. The fact that they have *more human* wealth than those who they replace has already been taken into account by the mentioned $+\lambda\tilde{c}_t$. But the newborns enter the economy with *less financial* wealth than the "average citizen". This lowers aggregate consumption by $b(\rho+m)A_t$ per time unit, where A_t is aggregate private financial wealth. In general equilibrium in the closed economy (without government debt) we have $A_t = K_t$. Correcting for population and technology growth, we end up with a lowering of $\dot{\tilde{c}}_t$ equal to $b(\rho + m)\tilde{k}_t$. This explains the second term in (1.2).

Finally, (1.3) is a transversality condition as seen from time t_0 for a person born at time v. The condition says that the No-Ponzi-Game condition is not over-satisfied (a necessary condition for individual optimality).

b) The equation describing the $\dot{\tilde{k}} = 0$ locus is

$$\tilde{c} = \frac{b}{\lambda + b}\left[f(\tilde{k}) - (\delta + g + b - m)\tilde{k}\right]. \tag{1.5}$$

The equation describing the $\dot{\tilde{c}} = 0$ locus is

$$\tilde{c} = \frac{b\,(\rho + m)\,\tilde{k}}{f'(\tilde{k}) - \delta - \rho + \lambda - g}. \tag{1.6}$$

Let $\bar{\tilde{k}}$ be defined by

$$f'(\bar{\tilde{k}}) - \delta = \rho - \lambda + g. \tag{1.7}$$

That is, $\bar{\tilde{k}}$ is defined as the value of $\tilde{k}$ at which the denominator of (1.6) vanishes. Such a value exists since, in addition to the Inada conditions, the inequality

$$\lambda < \delta + \rho + g$$

is assumed to hold. Another benchmark value of $\tilde{k}$ is the golden-rule value, $\tilde{k}_{GR}$, determined by the requirement

$$f'\left(\tilde{k}_{GR}\right) - \delta = n + g, \qquad \text{where } n = b - m.$$

The phase diagram and the $\dot{\tilde{k}} = 0$ and $\dot{\tilde{c}} = 0$ loci are shown in Fig. 1.1. The $\dot{\tilde{c}} = 0$ locus is everywhere to the left of the line $\tilde{k} = \bar{\tilde{k}}$ and is asymptotic to this line for $\tilde{k} \to \bar{\tilde{k}}$. The figure also displays the steady-state point, E, where the $\dot{\tilde{c}} = 0$ locus intersects the $\dot{\tilde{k}} = 0$ locus. The corresponding capital intensity is $\tilde{k}^*$, to which corresponds the (growth-corrected) per-capita consumption level $\tilde{c}^*$. Fig. 1.1 depicts a case where $\bar{\tilde{k}} \leq \tilde{k}_{GR}$ so that $\tilde{k}^* < \tilde{k}_{GR}$, that is, the economy is dynamically efficient. Yet, since we may have $b - m \leq \rho < \lambda + b - m$, so that $\bar{\tilde{k}} > \tilde{k}_{GR}$, dynamic inefficiency cannot be ruled out theoretically (a typical feature of an OLG model).

Concerning the directions of movement in the different regions of the phase diagram: From (1.1) follows

$$\dot{\tilde{k}} \gtreqless 0 \text{ for } \tilde{c} \lesseqgtr \frac{b}{\lambda + b}\left[f(\tilde{k}) - (\delta + g + b - m)\tilde{k}\right],$$

respectively. From (1.2) follows

$$\text{when } \tilde{k} < \bar{\tilde{k}},\ \dot{\tilde{c}} \gtreqless 0 \text{ for } \tilde{c} \lesseqgtr \frac{b\,(\rho + m)\,\tilde{k}}{f'(\tilde{k}) - \delta - \rho + \lambda - g} \text{ respectively; when } \tilde{k} \geq \bar{\tilde{k}},\ \dot{\tilde{c}} < 0.$$

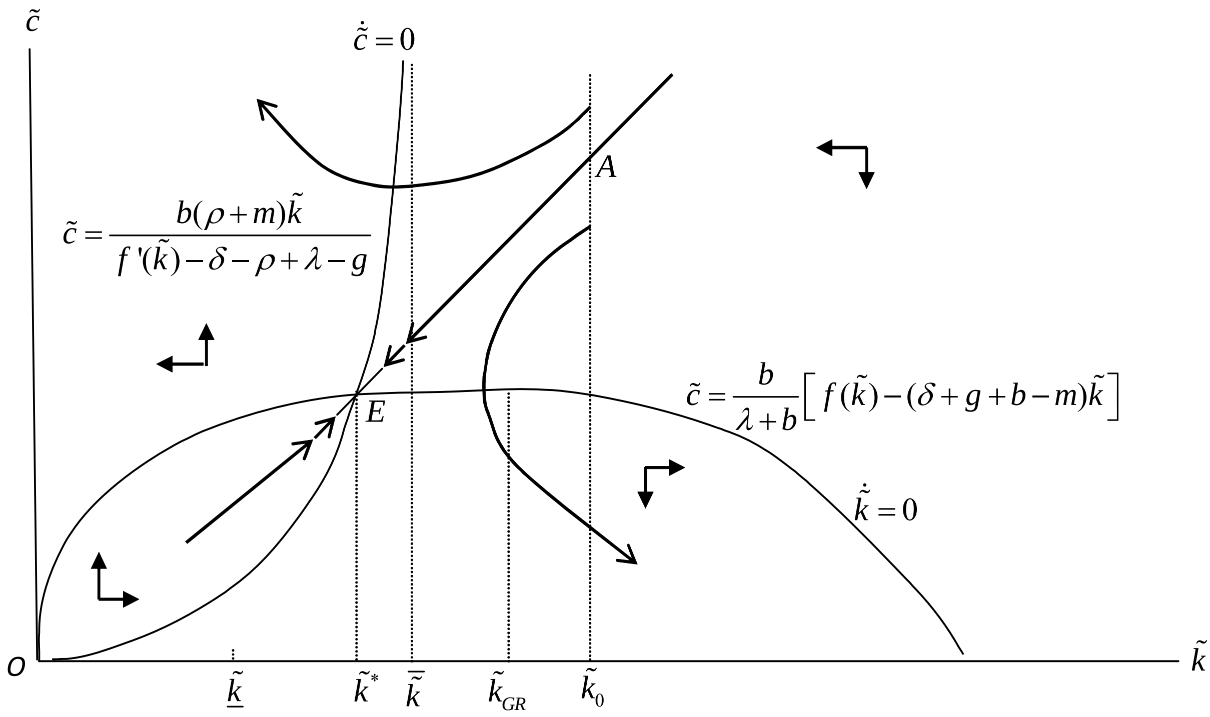

Figure 1.1:

The directions of movement are shown by arrows in Fig. 1.1. The arrows taken together indicate that the steady state, E, is a saddle point. Moreover, the following holds: there is one predetermined variable, $\tilde{k}$, and one jump variable, $\tilde{c}$, the saddle path is not parallel to the jump-variable axis, and the diverging paths can be ruled out as equilibrium paths (see below). Hence the steady state is saddle-point stable.

The saddle path is the only path that satisfies *all* the conditions of general equilibrium (individual utility maximization for given expectations, profit maximization by firms, continuous market clearing, and perfect foresight). The other paths in the diagram are diverging and violate either the transversality conditions of the individuals (paths that in the long run point South-East) or the NPG conditions of the individuals[2] (paths that in the long run point North-West).

This explains why initial consumption in equilibrium is determined as the ordinate, $\tilde{c}_0$, to the point where the vertical line $\tilde{k} = \tilde{k}_0$ intersects the saddle path. Over time the economy moves from this point, along the saddle path, towards the steady state. As the figure is drawn, $\tilde{k}_0$ happens to exceed both $\tilde{k}^*$ and $\bar{\bar{\tilde{k}}}$. We could alternatively have $0 < \tilde{k}_0 < \tilde{k}^*$.

[2] And therefore also the transversality conditions.

c) In view of stability we have for $t \to \infty$,

$$r_t = f'(\tilde{k}_t) - \delta \to f'(\tilde{k}^*) - \delta \equiv r^*, \tag{1.8}$$

where r^* is the long-run rate of return. From the definition of $\bar{\tilde{k}}$ and the fact that $\bar{\tilde{k}} > \tilde{k}^*$, follows

$$f'(\bar{\tilde{k}}) - \delta = \rho + g - \lambda < f'(\tilde{k}^*) - \delta = r^*,$$

in view of $f'' < 0$. So the lower end point of the interval to which r^* belongs is $\rho + g - \lambda$.

d) 1) In a representative agent model *Ricardian Equivalence* holds, but in the Blanchard OLG model it does not. By definition, Ricardian Equivalence holds in a model if, for a given time path of future government spending, aggregate private consumption is unaffected by a temporary (lump-sum) tax cut according to the model. Owing to the *generation turnover* in OLG models, the current generations fully benefit from a tax cut while they do not fully bear the burden of the higher future taxes as they do in a representative agent model, for instance the Ramsey model.

2) In a representative agent model the households are alike and are described as being infinitely lived, thereby having infinite horizon. Hence, in such a model general equilibrium cannot have the long-run (real) interest rate, r^*, lower than the long-run GNP growth rate, $g + n$; if it had, the present value of the household's expected future labor income, wL, would be infinite, which is inconsistent with general equilibrium. This means that *dynamic inefficiency* cannot occur within the model.

In the Blanchard OLG model, however, $r^* < g + n$ is not inconsistent with general equilibrium. This is because households have finite (but uncertain) lifetime and horizon. Whether $r^* \geq g + n$ or $r^* < g + n$ depends on the parameter values.

3) A third difference is that in a representative agent model the long-run interest rate is determined in a very simple way, and for instance independently of the production function and demographic parameters, namely by the formula: $r^* = \rho + \theta g$. This is contrary to the Blanchard OLG model.

e) To answer how a shift in b affects r^* we have to find out, how it affects $\tilde{k}^*$. An unambiguous conclusion can be obtained in the following way. In steady state we have $\dot{\tilde{k}} = 0$ and $\dot{\tilde{c}} = 0$ at the same time, implying that the right-hand sides of (1.5) and (1.6) must equal each other. By rearranging we get

$$\left(\frac{f(\tilde{k})}{\tilde{k}} - (\delta + g + b - m) \right) \left[f'(\tilde{k}) - \delta - \rho + \lambda - g \right] = (\lambda + b)(\rho + m), \tag{1.9}$$

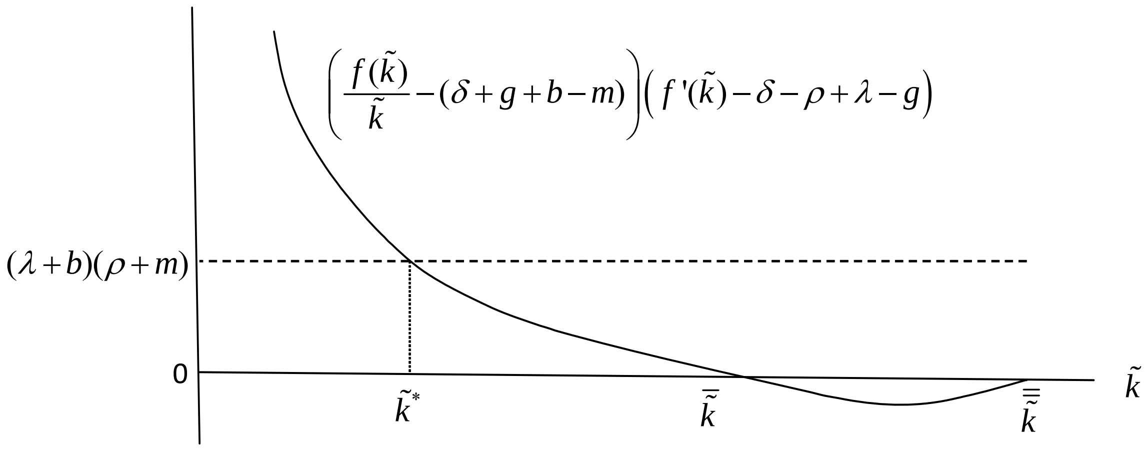

Figure 1.2:

where on the left-hand side the first factor is positive for all $\tilde{k} < \bar{\bar{\tilde{k}}}$, while the second factor shifts from positive, when $\tilde{k} < \bar{\tilde{k}}$, to negative, when $\bar{\tilde{k}} > \bar{\tilde{k}}$. Fig. 1.2 illustrates.

A lower b makes the left-hand side of (1.9) larger for fixed $\tilde{k}$ and the right-hand side smaller. Thereby $\tilde{k}^*$ is raised and so $r^* = f'(\tilde{k}^*) - \delta$ becomes lower (since $f''(\tilde{k}^*) < 0$).

f) Likewise, when a lower m *along with* the lower b makes $n = b - m$ *smaller*, again the left-hand side of (1.9) becomes larger (although less so than at e)) for fixed $\tilde{k}$, whereas the right-hand side as before becomes smaller (although more so than at e)). So again $\tilde{k}^*$ rises and r^* falls.[3]

g) A lower g makes the left-hand side of (1.9) larger and does not affect the right-hand side. This implies a further rise in $\tilde{k}^*$ and thus a further reduction in r^*.

h) In the new interpretation output and employment are primarily demand-determined. A downward shift in aggregate demand may for instance be triggered by a bursting housing bubble leading to a financial crisis and defaults. A credit crunch arises in the banking sector, thus causing a drop in spending on especially durable consumption and investment by households and firms in need of credit. A fall in output and employment sets in, which may prompt precautionary saving and thus, in “the second round”, cause a further decline in aggregate demand and employment (a vicious spiral).

An upward shift in aggregate demand may for instance be triggered by a wave of optimism due to invention of new consumer goods whereby households’ spending may be profoundly stimulated (historical examples: automobiles, the radio, the ICT revolution).

[3] A fall in λ, however, has an ambiguous effect on $\tilde{k}^*$.

Similarly, news about new technology may suddenly stimulate firms' investment spending (think of the economic expansion leading to the dot.com bubble). For an open economy a rise in exports of the kind of goods the economy produces may spark a virtual upward demand spiral in the economy.

Shifts in monetary or fiscal policy are also examples of circumstances that may suddenly change aggregate demand.

It may be added (although the notion of demand *shock* is not appropriate here) that some economists fear that a gradual reduction in trend aggregate demand may be a consequence of reduced population growth leading to less demand for residential construction and less capital investment to furnish new workers with production equipment. In turn, this may mean less learning by investing and less new technology than otherwise.

i) Let us consider the nominal interest rate on short-term government bonds to be the primary "instrument" used by the CB to control aggregate demand (the aim may be to affect inflation, employment, competitiveness, and exchange rate). Let this interest rate, the policy rate, be denoted i_t. Let the inflation rate, $\dot{P}_t/P_t$, be denoted π_t. Then the real interest rate of relevance for households' consumption and firms' investment is $r_t = i_t + \omega - \pi_t$, where $\omega \geq 0$ is the spread between the "official" nominal interest rate i_t and the nominal interest rate faced by the non-bank general public. For simplicity, let us consider ω as given, at least for some time. Aggregate demand depends negatively on the expected real interest rate $r_t^e = i_t + \omega - \pi_t^e$. To ease the discussion, let $\pi_t^e = \pi_t$, so that we do not have to distinguish between r_t^e and r_t. We further assume that π_t is relatively sticky within the time horizon considered.

The term "zero lower bound", ZLB, refers to the fact that the nominal interest rate even on short-term bonds can (essentially) not go below zero.[4] This is because agents would prefer holding cash at zero interest rather than bonds at negative interest. So,

$$i_t = \max(0, i_t^p),$$

where i_t^p is the level of the interest rate *desired* by the CB, the "policy rate". The ZLB becomes a binding constraint when, in a recession, the interest rate needed to obtain the desired stimulus of aggregate demand is negative. In this situation conventional monetary policy can thus only bring i_t down to zero, which is not sufficient for recovery of the economy. The economy is in a "liquidity trap".

[4] We ignore that strictly speaking the lower bound is slightly below zero because the alternative to holding bonds is holding cash which gives zero interest but involves costs of storage, insurance, and transport.

So, the situation calls for additional policies. See answer to k) below.

j) To answer this, let us consider the "normal case" where i_t^p is given by a Taylor rule which in the present context could have the form:

$$i_t^p = \hat{\imath} + \alpha_1 \frac{Y_t - Y_t^*}{Y_t^*} + \alpha_2 \frac{\pi_t - \hat{\pi}}{\hat{\pi}}, \qquad \alpha_1 > 0, \alpha_2 > 1,$$

where $\hat{\imath} \equiv r^* - \omega + \hat{\pi}$, Y_t is actual output, Y_t^* is NAIRU output, and $\hat{\pi}$ the inflation target.

For a given $\hat{\pi}$, a reduced r^* implies a lower i_t^p everything else equal. So the distance to ZLB becomes smaller and therefore the likelihood that the ZLB becomes binding is raised.

k) Conceivable alternatives or supplements to conventional monetary policy that may lessen a possible tendency to "secular stagnation" due a binding ZLB include:

i) *Raising the inflation target* is a possible non-conventional monetary policy to deal with the problem. A difficulty is that the announcement of a higher inflation target may not be taken as credible.

ii) *Quantitative easing (QE).* This can take several forms. The central bank may offer credit to financial intermediaries (banks, mutual funds, mortgage credit companies, insurance firms, etc.) on more gentle conditions than usually. And it may try directly to reduce the spread, ω, by buying long-term government bonds and other assets in the market. This stimulates aggregate demand.

Another form of QE is "helicopter money" as Milton Friedman called it. This is fiscal policy in the form of income transfers to the private sector directly financed by money issue. This relaxes the intertemporal budget constraint of the government by effectively financing a budget deficit by money instead of new government bonds. Here QE is a kind of coordinated fiscal and monetary policy.

iii) *Expansionary fiscal policy* as such is also an option. Most macroeconomists agree that when the economy is in a liquidity trap, fiscal policy multipliers tend to be considerably larger than otherwise. This is so for several reasons. One is that there will be no financial crowding out as long as the aim of the CB is to maintain i_t as low as possible. Another reason is that the economic situation which has triggered the liquidity trap is also a situation where involuntary unemployment tends to be considerable.

Moreover, times with low interest rates are the right time for public investment of which a part is normally financed by borrowing.

More structural policies are also conceivable: subsidize fertility by child benefits, invest more in public health, support pharmaceutical research to improve life expectancy, and support R&D with the aim of raising g (easier said than done).

2. Solution to Problem 2

Given the time horizon $T \geq 2$, the optimization problem is:

$$\begin{aligned}
\max E_0 U_0 &= E_0[\sum_{t=0}^{T-1} u(c_t)(1+\rho)^{-t}] \quad \text{s.t.} && (2.1)\\
c_t &\geq 0, && (2.2)\\
a_{t+1} &= (1+r_t)a_t + w_t \ell_t - c_t, \qquad a_0 \text{ given}, && (2.3)\\
a_T &\geq 0, && (2.4)
\end{aligned}$$

where $u' > 0$ and $u'' < 0$. We think of "period t" as the time interval $[t, t+1)$; the last period within the planning horizon T is thus period $T-1$. There is uncertainty about future values of r_t, w_t, and ℓ_t, but the household knows the stochastic processes that these variables follow.

a) In (1) we see the objective function which is to maximize expected discounted utility; ρ is the discount rate (a measure of impatience). E_0 is the expectation operator, conditional on the information available in period 0. (2) indicates the control region. (3) indicates the dynamic budget identity, saying that a_{t+1}, financial wealth at the beginning of the next period, equals financial wealth at the beginning of the current period, a_t, plus saving in the current period, which equals income, $r_t a_t + w_t \ell_t$, minus consumption, c_t. Initial financial wealth, a_0, is historically given. (4) is a solvency condition, saying that the household is not allowed to leave the terminal period, $T-1$, with positive debt.

b) We consider the generic expression for expected discounted utility as seen from an arbitrary period $t \in \{0, 1, 2, ..., T-2\}$:

$$E_t U_t = u(c_t) + (1+\rho)^{-1} E_t[u(c_{t+1}) + u(c_{t+2})(1+\rho)^{-1} + ...] \tag{2.5}$$

To solve the problem as seen from period t we will use the substitution method. From (2.3) we have

$$\begin{aligned}
c_t &= (1+r_t)a_t + w_t \ell_t - a_{t+1}, \qquad \text{and} && (2.6)\\
c_{t+1} &= (1+r_{t+1})a_{t+1} + w_{t+1}\ell_{t+1} - a_{t+2}.
\end{aligned}$$

Substituting this into (2.5), the problem is reduced to an essentially unconstrained maximization problem, namely one of maximizing the function E_tU_t w.r.t. $a_{t+1}, a_{t+2}, ..., a_T$ (thereby indirectly choosing $c_t, c_{t+1}, .., c_{T-1}$). Hence, we first take the partial derivative w.r.t. a_{t+1} in (2.5) and set it equal to 0:

$$\frac{\partial E_tU_t}{\partial a_{t+1}} = u'(c_t) \cdot (-1) + (1+\rho)^{-1}E_t[u'(c_{t+1})(1+r_{t+1})] = 0.$$

Reordering gives the stochastic Euler equation,

$$u'(c_t) = (1+\rho)^{-1}E_t[u'(c_{t+1})(1+r_{t+1})], \qquad t = 0, 1, 2, ..., T-2. \qquad (2.7)$$

This first-order condition describes the trade-off between consumption in period t and period $t+1$, as seen from period t. The optimal plan must satisfy that the current utility loss by decreasing consumption by one unit is equal to the discounted expected utility gain next period by having $1 + r_{t+1}$ extra units available for consumption, namely the gross return on saving one more unit. Considering $\partial E_tU_t/\partial a_{t+i}$ for $i = 2, 3, ..., T-t-2$, we get similar first-order conditions, in expected value, for each i.

c) In the final period, given the solvency condition $a_T \geq 0$, the decision must be to choose $a_T = 0$ (the transversality condition). Since $u' > 0$, the alternative, $a_T > 0$, could always be improved upon by increasing c_{T-1} without violating the solvency condition. So, the optimal c_{T-1} satisfies

$$c_{T-1} = (1 + r_{T-1})a_{T-1} + w_{T-1}\ell_{T-1}. \qquad (2.8)$$

d) With a CRRA utility function, $u(c) = (c^{1-\theta} - 1)/(1-\theta)$, we find

$$u' = c^{-\theta}, \quad u'' = -\theta c^{-\theta-1}, \quad u''' = -\theta(-\theta-1)c^{-\theta-2} > 0.$$

That $u''' > 0$ means that $(u')'' > 0$, i.e., for the CRRA utility function $u'(c)$ is a strictly convex function of c. The graph of $u'(c)$ is shown in Fig. 2.1.

We now assume that our $u(c)$ in (2.1) satisfies $u''' > 0$. We let the graph in Fig. 2.1 represent the strictly convex marginal utility of this $u(c)$. For reasons appearing below we let c refer to period 2. We are told to assume there is no uncertainty about the future value of r_t, only about future labor income because future employment and real wage are uncertain.

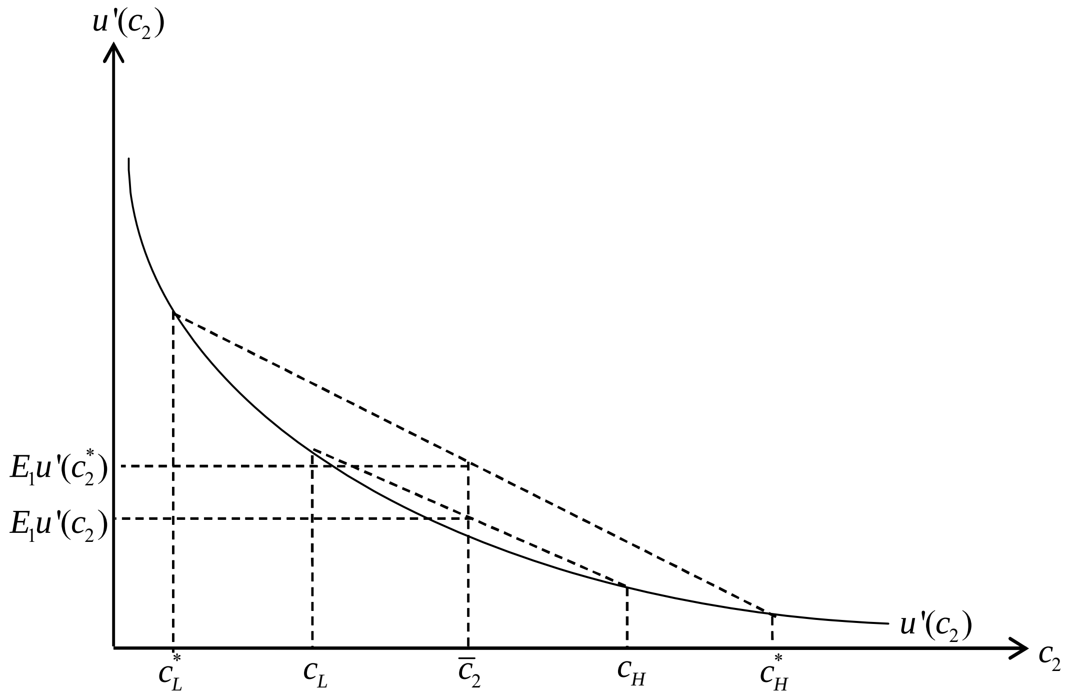

Figure 2.1:

e) We are asked to consider the problem as seen from period 1:

$$\begin{aligned} \max E_1 U_1 &= u(c_1) + (1+\rho)^{-1} E_1 u(c_2) \quad \text{s.t.} \\ a_2 &= (1+r_1)a_1 + w_1\ell_1 - c_1, \qquad a_1 \text{ given,} \\ a_3 &= (1+r_2)a_2 + w_2\ell_2 - c_2 \geq 0, \end{aligned}$$

where the only uncertainty is about the labor income $w_2\ell_2$. In line with (2.7) and no uncertainty regarding r_2, the relevant Euler equation becomes

$$u'(c_1) = \frac{1+r_2}{1+\rho} E_1 [u'(c_2)], \tag{2.9}$$

In view of $T = 3$ and the transversality condition $a_3 = 0$, (2.6) becomes

$$c_2 = (1+r_2)a_2 + w_2\ell_2.$$

It is known that there are only two possible outcomes for period 2's labor income, $w_2\ell_2$, namely y_L and y_H, each with probability $\frac{1}{2}$. Hence, given a_2, we have

$$c_2 = \begin{cases} c_L = (1+r_2)a_2 + y_L & \text{with probability} = \frac{1}{2}, \\ c_H = (1+r_2)a_2 + y_H & \text{with probability} = \frac{1}{2}. \end{cases} \tag{2.10}$$

Mean consumption will be $\bar{c}_2 = (1+r_2)a_2 + \bar{y}$, where $\bar{y} = \frac{1}{2}(y_L + y_H)$.

f) Suppose a_2 is fixed. Then Fig. 2.1 shows graphically how $E_1u'(c_2)$ is determined, given this a_2 together with (2.10).

g) *Precautionary saving* is defined as the increase in saving arising as a result of increased uncertainty.

To check whether or not precautionary saving can arise in the present setup we compare two situations, the one described so far and one with *more uncertainty* about labor income in period 2 as seen from period 1. We let the new situation be given by a *mean-preserving spread.* Variable values in the new situation are marked by *. So a *mean-preserving spread* is present if the new spread, $y_H^* - y_L^*$, is larger than the old, $y_H - y_L$, while the mean, $\bar{y}$, is unchanged.

So, if a_2 remains unchanged, now the two possible outcomes for c_2 are c_L^* and c_H^*, while the average equals $\bar{c}$ as before. The new situation is also illustrated in Fig. 2.1. Owing to the strict convexity of marginal utility, the expected marginal utility of consumption is now greater than before, as indicated by $E_1u'(c_2^*)$ in the figure. In order that (2.9) can still be satisfied, a lower value than before of c_1 must be chosen (since $u'' < 0$), hence, more saving occurs.

Yet, *this* lower value of c_1 is not the final outcome. Indeed, as soon as c_1 tends to be lowered, saving in period 1 tends to be raised. This means a higher a_2 than before so that the expected value of c_2 is in fact larger than $\bar{c}$ on the figure. This dampens, but does not eliminate, the effect of the mean-preserving spread on $E_1u'(c_2)$. This expected value ends up somewhere between the old $E_1u'(c_2)$ and the new $E_1u'(c_2^*)$ in the figure. The conclusion is still that the new c_1 has to be lower than the original c_1 in order for the first-order condition (2.9) to be satisfied in the new situation.

If instead the increased uncertainty pertains to period 1, the effect is again to decrease current consumption to provide for a buffer.

The conclusion is that precautionary saving does indeed arise. And the reason is the strict convexity of marginal utility. The intuition is that consumption is postponed in order to have a buffer-stock. The household wants to be prepared for meeting the bad outcome, because it wants to avoid the risk of having to end up “starving” (“save for the rainy day”).

3. Solution to Problem 3

a) Since the 1970s two quite different approaches to the explanation of business cycle fluctuations in industrialized market economies have been pursued. We may broadly classify them as either of a new-Classical or a Keynesian orientation. Since the mid 1980s the new-Classical approach settled for what became known as the Real Business Cycle (RBC) theory where output and employment fluctuations are seen as movements in productivity and labor supply. The Keynesian, including new-Keynesian, approach attempts to explain the fluctuations as movements in aggregate demand and the degree of capacity utilization.

Let us call the two approaches *RBC theory* versus *K-theory*.

The RBC theory considers perfect competition and price flexibility in all markets as an acceptable approximation. This implies general market clearing. So employment fluctuations *must* be fluctuations in labor supply. According to this theory, the driving force behind the fluctuations is technology shocks.

Let TFP be denoted A_t. Suppose a positive technology shock occurs. This raises the marginal productivity of labor, which in turn positively affects the equilibrium real wage. According to the theory, labor supply and employment go up, with further repercussions in the economy.

In standard notation:

$$A_t \quad \uparrow \quad \Rightarrow \begin{cases} Y_t \uparrow \\ \frac{\partial Y_t}{\partial N_t} \uparrow \Rightarrow w_t \uparrow \Rightarrow N_t^s \uparrow \Rightarrow N_t \uparrow \Rightarrow Y_t \uparrow \end{cases}$$

$$\Rightarrow \begin{cases} C_t \uparrow \\ S_t \uparrow \Rightarrow K_{t+1} \uparrow \Rightarrow \begin{cases} Y_{t+1} \uparrow \\ \frac{\partial Y_{t+1}}{\partial N_{t+1}} \uparrow \Rightarrow w_{t+1} \uparrow \Rightarrow N_{t+1}^s \uparrow \Rightarrow N_{t+1} \uparrow \Rightarrow Y_{t+1} \uparrow \end{cases} \end{cases} \Rightarrow \text{etc.}$$

In this way persistence in the effects of temporary shocks arises, both through the *capital accumulation mechanism* (the "$S_t \uparrow \Rightarrow K_{t+1} \uparrow \Rightarrow$" part) and through the *intertemporal substitution in labor supply mechanism* (the "$\frac{\partial Y_{t+1}}{\partial N_{t+1}} \uparrow \Rightarrow w_{t+1} \uparrow \Rightarrow N_{t+1}^s \uparrow$").

A problem faced by the RBC theory is that these two mechanisms are not capable at generating the *large* fluctuations in output and employment that we observe. Both mechanisms imply little amplification of the shocks. Most critically, the intertemporal substitution in labor supply mechanism is not able to generate much amplification. This is because changes in real wages tend to be permanent rather than purely transitory. Permanent wage increases tend to have little or no effect on labor supply because the wealth effect will tend to offset the substitution and income effects. Given the very minor temporary movements in the real wage that occur at the empirical level, a high intertemporal elasticity of substitution in labor supply is required to generate the large fluctuations in employment observed in the data. But the micro-econometric evidence suggests that this requirement is not met.

Proponents of the RBC theory have made several attempts to overcome the absence of noteworthy internal propagation and amplification. One is the involvement of *recurrent shocks*. As already Slutzky showed, if A_t follows a stationary AR(1) process with a positive root, the geometrically declining sums of past shocks can endow the fluctuations of A_t with considerably larger amplitudes and more duration than the shocks themselves display.

In contrast, the K-theory rejects perfect competition and price flexibility. Hence, the theory does not connect ups and downs in employment to ups and downs in labor supply. And the K-theory does not depend on presence of recurrent shocks.

This latter feature is due to aggregate demand mechanisms implying strong *internal* propagation and amplification. The K-theory emphasizes that the endogenous variables enter into *cumulative causation* patterns. These patterns are in our syllabus referred to as sometimes *virtuous circles*, generating a boom, and sometimes *vicious circles*, generating a recession, which may be triggered by just one large specific event. In syllabus the following examples of such circles are mentioned:

(1) The spending multiplier. (2) Destabilizing price flexibility. (3) The balance sheet channel. (4) The bank lending channel. (5) Multiple equilibria, self-fulfilling expectations, and coordination failure. (6) Hysteresis.

b) (i) Movements in the labor market. Workers' quits are pro-cyclical. In a boom quits go up and in a recession they go down. This is contrary to the prediction from the RBC theory that quits increase in the downturn since variation in employment is voluntary. But the pro-cyclicality of quits fits well with K-theory since recessions are times where firms generally need fewer workers to satisfy the slack demand, and where workers are hesitant to quit because they are aware that vacant jobs are scarce.

Other empirical regularities are:

(ii) Employment (aggregate labor hours) is *pro-cyclical*, i.e., varies in the same direction as GDP, and fluctuates almost as much as GDP. As mentioned under a), the RBC theory has difficulties to make employment fluctuate as much as output. The K-theory has no such difficulties.

(iii) Real wages are *weakly* pro-cyclical and do not fluctuate much. The RBC theory rather predicts real wages are *strongly* pro-cyclical and fluctuate almost as much as output. In Keynes' theory as presented in his *General Theory* from 1936 it is only nominal wages that are sticky. Nominal prices are assumed flexible. The resulting prediction that real wages should be counter-cyclical was soon empirically falsified. Keynes admitted in 1939 that he had relied too much on perfect competition and price flexibility in the output market. Modern K-theory fits well with the observation that real wages are weakly pro-cyclical. The relative strength of the parties in wage negotiations in the labor market matters.

(iv) Aggregate consumption and employment are markedly positively correlated. This fits well with K-theory. But, given the fact that real wages do not fluctuate much, the RBC-theory rather predicts that *leisure* is positively correlated with aggregate consumption.

—