tween ρ and r, as long as the stability condition (15.44) is satisfied. Or, to be more precise: the Blanchard model works well in the case $\rho < r$; in the opposite case, where $\rho > r$, the model works at least better than the Ramsey model, because it never implies that $C_t \to 0$ in the long run.

It should be admitted, however, that in the case of a very impatient country ($\rho > r$), even the OLG model implies a counterfactual prediction. What (15.47) tells us is that the impatient small open economy in a sense asymptotically mortgages all of its physical capital and part of its human capital. The OLG model predicts this will happen, *if* financial markets are perfect, and *if* the political sphere does not intervene. It certainly seems unlikely that an economic development, ending up with negative national wealth, is going to be observed in practice. There are two - complementary - explanations of this.

First, the international credit market is far from perfect. Because a full-scale supranational legal authority comparable with domestic courts is lacking, credit default risk in international lending is generally a more serious problem than in domestic lending. Physical capital can to some extent be used as a collateral on foreign loans, while human wealth is not suitable. Human wealth cannot be repossessed. This implies a constraint on the ability to borrow.[10] And lenders' risk perceptions depend on the level of debt.

Second, long before *all* the physical capital of an impatient country is mortgaged or have directly become owned by foreigners, the government presumably would intervene. In fear of losing national independence, it would use its political power to end the pawning of economic resources to foreigners.

This is a reminder, that we should not forget that the economic sphere of a society is just one side of the society. Politics as well as culture and religion are other sides. The economic outcome may be conditioned on these social factors, and the interaction of all these spheres determines the final outcome.

15.2 The housing market and residential construction

The housing market is from a macroeconomic point of view important for several reasons: a) housing makes up a substantial proportion of the consumption budget; b) housing wealth makes up a substantial part of private wealth of a major fraction

[10] We have been speaking as if domestic residents own the physical capital stock in the country, but have obtained part or all the financing of the stock by issuing bonds to foreigners. The results would not change if we allowed for foreign direct investment. Then foreigners would themselves own part of the physical capital rather than bonds. In such a context a similar constraint on foreign investment is likely to arise, since a foreigner can buy a factory or the shares issued by a firm, but it is difficult to buy someone else's stream of future labour income.

of the population; c) fluctuations in house prices and in construction activity are large and seem important for business cycles; and d) residential investment, whish typically is of magnitude 5 percent of GDP, and aggregate output are strongly positively correlated. The analysis will be based on a simple dynamic partial equilibrium model with rising marginal construction costs.

Let time be continuous. Let H_t denote the aggregate housing stock at time t and S_t the aggregate flow of housing services at time t. Ignoring heterogeneity, the housing *stock* can be measured in terms of m^2 floor area at a given point in time. For convenience we will talk about the stock as a certain number of houses of a standardized size. The supply of housing *services* at time t constitute a *flow*, thereby being measured *per time unit*, say per year: so and so many square meter-months are at the disposal for accommodation during the year. The two concepts are related through

$$S_t = \alpha H_t, \qquad \alpha > 0, \tag{15.49}$$

where we will treat α as a constant which depends only on the measurement unit for housing services. If these are measured in square meter-months, α equals the number of square meters of a "normal-sized" house times 12.

We ignore population growth and economy-wide technological progress.

15.2.1 The housing service market and the house market

There are two goods, houses and housing services, and therefore also two markets and two prices:

$$\begin{aligned} p_t &= \text{ the (real) price of a "normal-sized'" house at time } t, \\ R_t &= \text{ the rental rate } \equiv \text{ the (real) price of housing services at time } t. \end{aligned}$$

The price R_t of housing services is known as the *rental rate* at the housing market. Buying a housing service means *renting* the apartment or the house for a certain period. Or, if we consider an owner-occupied house (or apartment), R_t is the imputed rental rate, that is, the owner's opportunity cost of occupying the house. The prices R_t and p_t are measured in *real* terms, or more precisely, they are deflated by the consumer price index. We assume perfect competition in both markets.

The market for housing services

In the short run the housing stock is historically given. Construction is time-consuming and houses cannot be imported. Owing to the long life of houses, investment in new houses per year tends to be a small proportion of the available housing stock (in advanced economies about 3 percent, say). So also the supply, S_t, of housing services is given in the short run.

Suppose the aggregate demand for housing services at time t is

$$S_t^d = D(R_t, A, PV(wL)), \qquad D_1 < 0, D_2 > 0, D_3 > 0, \tag{15.50}$$

where A is aggregate financial wealth and $PV(wL)$ is human wealth, i.e., the present discounted value of expected future labor income after tax for those alive. That demand depends negatively on the rental rate reflects that both the substitution effect and the income effect of a higher rental rate are negative. The wealth effect on housing demand of a higher rental rate is likely to be positive for owners and negative for tenants.[11]

The market for housing services is depicted in Fig. 15.6. We get a characterization of the equilibrium rental rate in the following way. In equilibrium at time t, $S_t^d = S_t$, that is,

$$D(R_t, A, PV(wL)) = \alpha H_t. \tag{15.51}$$

This equation determines R_t as an implicit function, $R_t = \tilde{R}(H_t,\ A,\ PV(wL))$, of H_t, A, and $PV(wL)$. By implicit differentiation in (15.51) we find the partial derivatives of this function, $\tilde{R}_H = \alpha/D_R < 0$, $\tilde{R}_A = -D_A/D_S > 0$, and $\tilde{R}_{PW} = -D_{PV}/D_R > 0$.

The supply of housing services is inelastic in the short run and the market clearing rental rate immediately moves up and down as the demand curve shifts rightward or leftward. But in our partial equilibrium framework, we will consider A and $PV(wL)$ as exogenous and constant. Hence we suppress these two variables as arguments in the functions and define $R(H_t) \equiv \tilde{R}(H_t, A, PV(wL))$, whereby

$$R_t = R(H_t), \qquad R' = \alpha/D_R < 0. \tag{15.52}$$

From now on our time unit will be one year and we define one unit of housing service per year to mean disposal of a house of standard size one year. By this, α in (15.49) equals 1.

[11] A simple microeconomic "rationale" behind the aggregate demand function (15.50) is obtained by assuming an instantaneous utility function $u(h_t, c_t) = \ln(h_t^\gamma c_t^{1-\gamma})$, where $0 < \gamma < 1$, and h_t is consumption of housing services at time t, whereas c_t is non-housing consumption. Then the share of housing expenditures in the total instantaneous consumption budget is a constant, γ. This is broadly in line with empirical evidence for the US (Davis and Heathcote, 2005). In turn, according to standard neoclassical theory, the total consumption budget will be an increasing function of total wealth of the household, cf. Chapter 9. Separation between the two components of wealth, A and $PV(wl)$, is relevant when credit markets are imperfect.

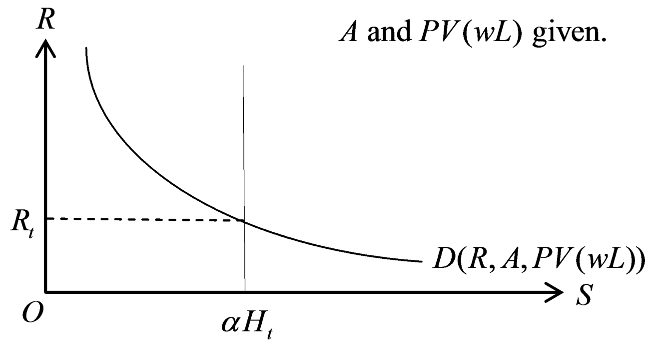

Figure 15.6: Supply and demand in the market for housing services at time t.

The market for existing houses

Because a house is a durable good with market value, it is an *asset*. This asset typically constitutes a substantial share of the wealth of a large fraction of the population, the house-owners. At the same time the supply of the asset can change only slowly.

Assume there is an exogenous and constant risk-free real interest rate $r > 0$. This is a standard assumption in partial equilibrium analysis. If the economy is a small open economy with perfect capital mobility, the exogeneity of r (if not constancy) is warranted even in general equilibrium analysis.

Considering the asset motive associated with housing, a series of aspects are central. We let houses depreciate physically at a constant rate $\delta > 0$. Suppose there is a constant tax rate $\tau_R \in [0, 1)$ applied to rental income (possibly imputed) after allowance for depreciation. In case of an owner-occupied house the owner must pay the tax $\tau_R(R_t - \delta p_t)$ out of the imputed income $(R_t - \delta p_t)$ per house per year. Assume further there is a constant property tax (real estate tax) $\tau_p \geq 0$ applied to the market value of houses. Finally, suppose that a constant tax rate $\tau_r \in [0, 1)$ applies to interest income. There is symmetry in the sense that if you are a debtor and have negative interest income, then the tax acts as a rebate. We assume capital gains are not taxed and we ignore all complications arising from the fact that most countries have tax systems based on nominal income rather than real income. In a low-inflation world this limitation may not be serious.[12]

Suppose there are no credit market imperfections, no transaction costs, and no uncertainty. Assume further that the user of housing services value these services

[12] Note, however, that if all capital income should be taxed at the same rate, capital gains should also be taxed at the rate τ_r, and τ_R should equal τ_r. In Denmark, in the early 2000s, the government replaced the rental value tax, τ_R, on owner-occupied houses by a lift in the property tax, τ_p. Since then, due to a *nominal* "tax freeze", τ_p has been gradually decreasing in real terms.

independently of whether he/she owns or rent. Under these circumstances the price of houses, p_t, will adjust so that the expected after-tax rate of return on owning a house equals the after-tax rate of return on a safe bond. We thus have the no-arbitrage condition

$$\frac{(1-\tau_R)(R(H_t)-\delta p_t)-\tau_p p_t+\dot{p}_t^e}{p_t}=(1-\tau_r)r, \qquad (15.53)$$

where $\dot{p}_t^e$ denotes the expected capital gain per time unit (so far $\dot{p}_t^e$ is just a commonly held subjective expectation).

For given $\dot{p}_t^e$ we find the equilibrium price

$$p_t=\frac{(1-\tau_R)R(H_t)+\dot{p}_t^e}{(1-\tau_r)r+(1-\tau_R)\delta+\tau_p}.$$

Thus p_t depends on H_t, $\dot{p}_t^e$, r, and tax rates in the following way:

$$\begin{aligned}
\frac{\partial p_t}{\partial H_t} &= \frac{(1-\tau_R)R'(H_t)}{(1-\tau_r)r+(1-\tau_R)\delta+\tau_p}<0,\\
\frac{\partial p_t}{\partial \dot{p}_t^e} &= \frac{1}{(1-\tau_r)r+(1-\tau_R)\delta+\tau_p}>0,\\
\frac{\partial p_t}{\partial \tau_R} &= \frac{-\left[(1-\tau_r)r+\tau_p\right]R(H_t)+\delta\dot{p}_t^e}{\left[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p\right]^2}\lesseqqgtr 0 \text{ for } \dot{p}_t^e \lesseqqgtr \frac{\left[(1-\tau_r)r+\tau_p\right]R(H_t)}{\delta},\\
\frac{\partial p_t}{\partial \tau_p} &= -\frac{(1-\tau_R)R(H_t)+\dot{p}_t^e}{\left[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p\right]^2}<0,\\
\frac{\partial p_t}{\partial \tau_r} &= \frac{\left[(1-\tau_R)R(H_t)+\dot{p}_t^e\right]r}{\left[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p\right]^2}>0,\\
\frac{\partial p_t}{\partial r} &= -\frac{\left[(1-\tau_R)R(H_t)+\dot{p}_t^e\right](1-\tau_r)}{\left[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p\right]^2}<0,
\end{aligned}$$

where the sign of the last three derivatives are conditional on $\dot{p}_t^e$ being nonnegative or at least not “too negative”.

Note that a higher expected increase in p_t, $\dot{p}_t^e$, implies a higher house price p_t. Over time this feeds back and may confirm and sustain the expectation, thus generating a further rise in p_t. Like other assets, a house is thus a good with the property that the expectation of price increases make buying more attractive and may become self-fulfilling if the expectation is generally held.

15.2.2 Residential construction

It takes time for the stock H_t to change. While manufacturing typically involves mass production of similar items, construction is generally done on location for a known client and within intricate legal requirements. It is time-consuming to design, contract, and execute the sequential steps involved in residential construction. Careful guidance and monitoring is needed. These features give rise to fixed costs (to management, architects etc.) and thereby rising marginal costs in the short run. Congestion and bottlenecks may easily arise.

The construction process

Assume the construction industry is competitive. At time t the *representative construction firm* produces B_t units of housing per time unit (B for "building"), thereby increasing the aggregate housing stock according to

$$\dot{H}_t = B_t - \delta H_t, \delta > 0. \tag{15.54}$$

The construction technology is described by a production function $\tilde{F}$:

$$B_t = \tilde{F}(K_t, L_t, \bar{M}; E_t) \equiv \bar{F}(F(K_t, L_t), \bar{M}; E_t) = \bar{F}(I_t, \bar{M}; E_t) \equiv T(I_t; E_t).$$

The last argument of $\tilde{F}$, E_t, is not a production factor but stands for construction experience acquired through accumulated learning in the construction industry. It determines the efficiency of the current technology. The three other arguments of $\tilde{F}$ represent input of capital, K_t, blue-collar labor, L_t, and "management labor", $\bar{M}$, which includes working hours of specialists like architects and lawyers. There are constant returns to scale with respect to these three production factors. We treat $\bar{M}$ as a fixed production factor even in the medium run. Hence the associated fixed cost (salaries) is, in real terms, constant for quite some time. We denote this fixed cost $\bar{f}$.

The remaining two production inputs, capital and blue-collar labor, produces components for residential construction − intermediate goods − in the amount $I_t = F(K_t, L_t)$ per time unit. The production function, F, is "nested" in the "global" production function, $\bar{F}$. Thus construction is modeled as if it makes up a two-stage process. First, capital and blue-collar labor produce intermediate goods for construction. Next, management accomplishes quality checks and "assembling" of these intermediate goods into new houses or at least final new components built into existing houses. The final output is measured in units corresponding to a standard house. This does not rule out that a large part of the output is really in the form of renovations, additions of a room etc.

We treat both blue-collar labor and capital as variable production factors in the short run and assume F has constant returns to scale. The intermediate goods are produced on a routine basis at minimum costs (convex *capital* adjustment costs, as in Chapter 14, are for simplicity ignored). Let the real cost per unit of I_t be denoted c. In our short-to-medium run perspective we treat c as a constant.

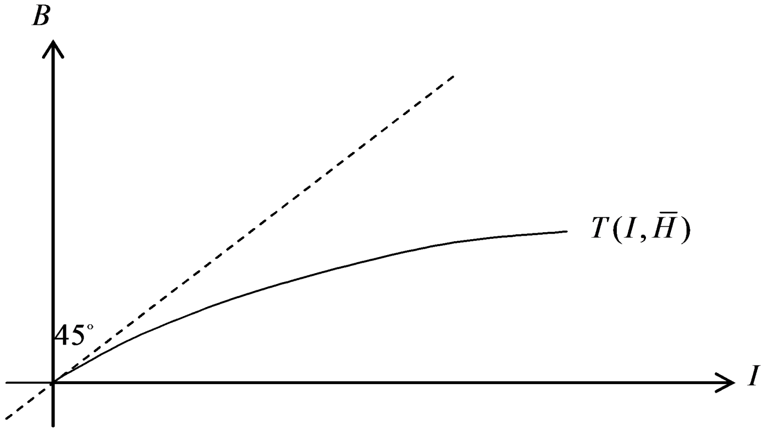

Figure 15.7: The number of new houses as a function of residential investment (for given $E = \bar{H}$).

The marginal productivity of I_t is decreasing in I_t. That is, keeping $\bar{M}$ fixed, the final output, B_t, has *diminishing* returns with respect to the level of *construction activity* per time unit as measured by the flow variable I_t. In the short run thus rising marginal costs obtains, "haste is waste".

To save notation, from now on, with the purpose of suppressing the constant argument $\bar{M}$, we introduce the production function T. Moreover, we suppress the explicit dating of the variables unless needed for clarity. To help intuition, we shall speak of the function T as a *transformation function.* This function is assumed to be strictly concave in I : the larger is I, the smaller is the rate at which a unit increase in I is transformed into new houses.

To summarize: the amount of new houses built per time unit is

$$\begin{aligned} B &= T(I,E), \quad \text{where} \\ T(0,E) &= 0, \quad T_I(0,E) = 1, \quad T_I > 0, \quad T_{II} < 0, \quad T_E \geq 0. \end{aligned} \tag{15.55}$$

A higher level of construction activity per time unit means that a larger fraction of I is "wasted" because of control, coordination, and communication difficulties. Hence $T_{II}(I,E) < 0$, i.e.,.T is strictly concave in I

The second argument in the transformation function is the construction experience, E. More experience means that the intermediate goods can be designed in a better way thus implying higher productivity of a given I than otherwise, hence $T_E \geq 0$.[13] As an indicator of cumulative experience it would be natural to use

[13] In a long-run perspective, the increasing scarcity of available land may hamper the productivity of the intermediate goods, for given I and E. This is ignored in our medium-run perspective. All the same, in the real world construction technology improves over time and the limited availability of land can to some extent be dealt with by building taller structures.

cumulative gross residential investment, $\int_{-\infty}^{t} B_s ds$, reflecting cumulative learning by doing. It is simpler, however, to use cumulative *net* residential investment, H_t. We thus assume that E_t is (approximately) proportional to H_t.[14] Normalizing the factor of proportionality to one, we have

$$E_t = H_t.$$

For fixed $E = \bar{H}$, Fig. 15.7 shows the graph of $T(I, \bar{H})$ in the $(I,\ B)$ plane. The assumptions $T_I(0, \bar{H}) = 1$ and $T_{II} < 0$ imply $T_I(I, \bar{H}) < 1$ for $I > 0$, as visualized in the figure. An example satisfying all the conditions in (15.55) is a CES function,[15]

$$T(I, H) = A(aI^\beta + (1-a)H^\beta)^{1/\beta}, \quad \text{with } 0 < A < 1, 0 < a < 1, \text{ and } \beta < 0.$$

From the perspective of Tobin's q-theory of investment, we may let the "waste" be represented by a kind of adjustment cost function $G(I, H)$ akin to that considered in Chapter 14. Then $T(I, H) \equiv I - G(I, H)$. In Chapter 14 convex adjustment costs were associated with the installation of firms' fixed capital and acted as a reduction in the firms' output available for sale. In construction we may speak of analogue costs acting as a reduction in the productivity of the intermediate goods in the construction process. It is easily seen that, on the one hand, all the properties of G *required* in Chapter 14 when $I \geq 0$ are maintained. On the other hand, not all properties required of T in (15.55) need be satisfied in Tobin's q-theory (see Appendix B).

Profit maximization

The representative construction firm takes the current economy-wide experience $E = H$ as given. The gross revenue of the firm is pB and costs are cI. Given the market price p, the firm maximizes profit:

$$\begin{aligned} \max_I \Pi &= pB - cI \quad \text{s.t.} \quad B = T(I, H) \text{ and} \\ I &\geq 0. \end{aligned}$$

Inserting $B = T(I, H)$, we find that an interior solution satisfies

$$\frac{d\Pi}{dI} = pT_I(I, H) - c = 0, \text{ i.e., } \frac{p}{c}T_I(I, H) = 1. \tag{15.56}$$

[14] At least in an economic growth context, where H would almost never be decreasing, this approximation of the learning effect would not seem too coarse.

[15] As shown in the appendix to Chapter 4, by defining $T(I, H) = 0$ when $I = 0$ or $H = 0$, the domain of the CES function can be extended to include all $(I, H) \in \mathbb{R}_+^2$ also when $\beta < 0$, while maintaining continuity.

In view of $T_I(I,H) < 1$ for $I > 0$, the latter equation has a solution $I > 0$ only if $p > c$. For $p \leq c$, we get the corner solution $I = 0$. Naturally, when the current market price of houses is below marginal construction cost (which equals $c/(T_I(I,H) \geq c)$, no new houses will be built. This is a desired property of the model. On the other hand, when $p > c$, the construction firm will supply new houses up to the point where the rising marginal cost equals the current house price, p.[16]

A precise determination of optimal I is obtained the following way. For $p > c$, the first-order condition (15.56) defines construction activity, I, as an implicit function of p/c and H :

$$I = M\left(\frac{p}{c}, H\right), \quad \text{where } M(1,H) = 0. \tag{15.57}$$

By implicit differentiation with respect to p/c in (15.56), we find

$$M_{p/c} = \frac{\partial I}{\partial (p/c)} = \frac{-1}{(p/c)^2 T_{II}(I,H)} > 0,$$

where the argument I can be written as in (15.57).

Special case

From now on we assume the transformation function T is homogeneous of degree one. Thus, $B = T(I/H,1)H$. Then, by Euler's theorem, $T_I(I,H)$ is homogeneous of degree 0. So, with explicit timing of the time-dependent variables, (15.56) can be written

$$\frac{p}{c} T_I\left(\frac{I}{H}, 1\right) = 1.$$

This first-order condition defines I_t/H_t as an implicit function of p_t/c :

$$\frac{I}{H} = m\left(\frac{p}{c}\right), \quad \text{where } m(1) = 0. \tag{15.58}$$

By implicit differentiation with respect to p/c in the first-order condition we find

$$m' = \frac{-1}{(p/c)^2 T_{II}\left(I/H, 1\right)} > 0,$$

where $I/H = m(p/c)$ can be inserted. A construction activity function m with this property is shown in Fig. 15.8, where $c = 1$.

[16] How to come from the transformation function $T(I,H)$ to the marginal cost schedule is detailed in Appendix C.

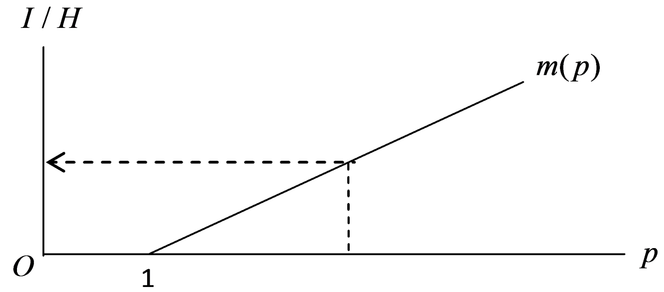

Figure 15.8: Construction activity (relative to the housing stock) as a function of the market price of houses ($c = 1$).

With explicit timing of the time-dependent variables, and letting b_t denote the flow of new houses relative to the stock of houses, we now have

$$b_t \equiv \frac{B_t}{H_t} = \frac{T(I_t, H_t)}{H_t} = T\left(\frac{I_t}{H_t}, 1\right) = T\left(m\left(\frac{p_t}{c}\right), 1\right) \equiv b\left(\frac{p_t}{c}\right), \qquad (15.59)$$

where $b(1) = T(m(1), 1) = T(0, 1) = 0$, $b' = T_I m' > 0$.

Remark. Like Tobin's q, the house price p is the market value of a produced asset whose supply changes only slowly. As is the case for firms' fixed capital there are strictly convex stock adjustment costs, represented by the rising marginal construction costs. As a result the stock of houses does not change instantaneously if for instance p changes. But as shown by the above analysis, the flow variable, residential construction, responds to p in a way similar to the way firm's fixed-capital investment responds to Tobin's q according to the q theory. Recall that Tobin's q is defined as the economy-wide ratio $V/(p_I K)$, where V is the market value of the firms, p_I is a price index for investment goods, and K is the stock of physical capital. The analogue ratio in the housing sector is $V^{(H)}/(p_I \cdot H) \equiv p \cdot H/(p_I \cdot H) = p/c$, in view of $p_I = c$. A higher p/c results in more construction activity. □

15.2.3 Equilibrium dynamics under perfect foresight

To determine the evolution over time in H and p, we derive two coupled differential equations in these two variables. When the transformation function T is homogeneous of degree one, we can in view of 15.59) write (15.54) as

$$\dot{H}_t = \left(b\left(\frac{p_t}{c}\right) - \delta\right) H_t, \qquad (15.60)$$

where $b(1) = 0$ and $b' = T_I m' > 0$.

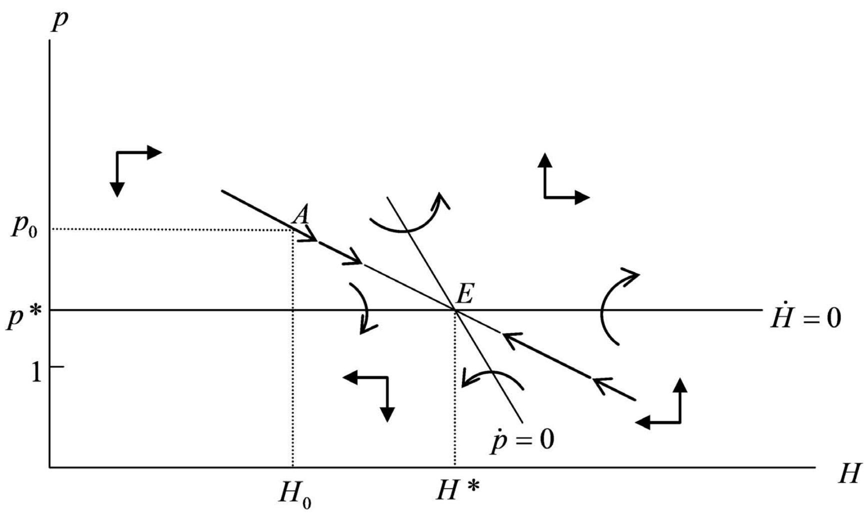

Figure 15.9: Phase diagram of aggregate construction activity ($c = 1$).

Assuming *perfect foresight*, we have $\dot{p}_t^e = \dot{p}_t$ for all t. Then we can write (15.53) on the standard form for a first-order differential equation:

$$\dot{p}_t = [(1 - \tau_r)r + (1 - \tau_R)\delta + \tau_p]\, p_t - (1 - \tau_R)R(H_t), \tag{15.61}$$

where $R' < 0$. We have hereby obtained a dynamic system in H and p, the coupled differential equations (15.60) and (15.61). The corresponding phase diagram is shown in Fig. 15.9.

We have $\dot{H} = 0$ for $b(p/c) = \delta > 0$. The unique p satisfying this equation is the steady state value p^*. The $\dot{H} = 0$ locus is thus represented by the horizontal line segment $p = p^*$. The direction of movement for H is positive if $p > p^*$ and negative if $p < p^*$. Since $b(1) = 0$ and $b' > 0$, we have $p^* > c$.

We have $\dot{p} = 0$ for $p = (1 - \tau_R)R(H)/\left[(1 - \tau_r)r + (1 - \tau_R)\delta + \tau_p\right]$. Since $R'(H) < 0$, the $\dot{p} = 0$ locus has negative slope. The unique steady state value of H is denoted H^*. To the right of the $\dot{p} = 0$ locus, p is rising, and to the left p is falling. The directions of movement of H and p in the different regions of the phase plane are shown in Fig. 15.9.

The unique steady state is seen to be a saddle point with housing stock H^* and housing price p^*. The initial housing stock, H_0, is predetermined. Hence, at time $t = 0$, the economic system must be somewhere on the vertical line $H = H_0$. The question is whether there can be asset price bubbles in the system. An *asset price bubble* is present if the market value of the asset for some time systematically exceeds its *fundamental value* (the present value of the expected future services or dividends from the asset). Agents might be willing to buy at a price above the fundamental value if they expect the price will rise further in the future. The divergent paths ultimately moving North-East in the phase diagram are actually, by construction, consistent with the no-arbitrage condition and are thus candidates for asset price bubbles generated by self-fulfilling expectations. The fact that houses have clearly defined reproduction costs, however, implies a ceiling on the ultimate level of p since potential buyers of already existing houses have the alternative of initiating construction at "normal pace" of a new house at the cost p^*. Then, by backward induction, these explosive price paths will not, under rational expectations, get started in the first place. Given rational expectations, these paths - and therefore "rational bubbles" - can thus be ruled out.[17] This leaves us with the converging path as the unique solution to the model. At time 0 the residential construction sector will be at the point A in the diagram and then it will move along the saddle path and after some time the housing stock and the house price settle down at the steady state, E.

In this model (without economic growth) the steady-state price level, p^*, of houses equal the marginal building costs when building activity exactly matches the physical wearing down of houses so that the stock of houses is constant. Owing to the specific form of the positive relationship between building productivity and H, implied by the transformation function T being homogeneous of degree one in I and H, the marginal building costs are unchanged in the medium run even if r or one of the taxes change. The steady-state level of H is at the level required for the rental rate $R(H)$ to yield an after-tax rate of return on owning a house equal to $(1-\tau_r)r$. This level of H is H^*. The corresponding level of R is R^* $= R(H^*)$, which is that level at which the demand for housing services equals the steady-state supply, i.e., $D(R^*, A, PV(wl)) = S^* = H^*$.

Effect of a fall in the property tax

Suppose the residential construction sector has been in the steady state E in Fig. 15.10 until time t_1. Then there is an unanticipated downward shift in the property tax τ_p to a new constant level τ_p' rightly expected to last forever in the future. The resulting evolution of the system is shown in the figure. The new steady state is called E'. The new medium-run level of H is $H^{*\prime} > H^*$, because $R'(H) < 0$. On impact, p jumps up to the point where the vertical line $H = H^*$ crosses the new (downward-sloping) saddle path. The intuition is that the after-tax return on owning a house has been increased. Hence, by arbitrage the market price p rises to a level such that the after-tax rate of return on houses is as before, namely equal to $(1-\tau_r)r$. After t_1, owing to the high p relative to the unchanged building cost schedule, H increases gradually and p falls gradually (due to R

[17] In the last section we briefly return to the issue whether *other* kinds of housing price bubbles might arise.

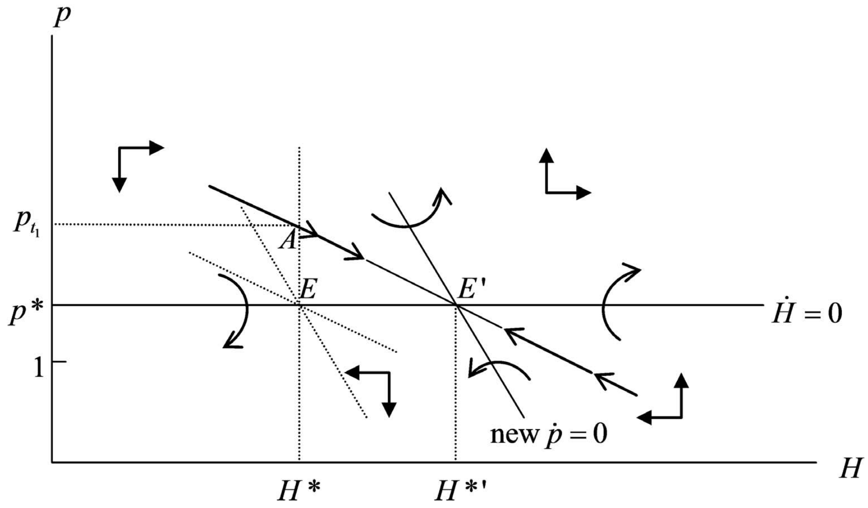

Figure 15.10: Response to a fall in the property tax ($c = 1$).

falling with the rising H). This continues until the new steady state is reached with unchanged p^*, but higher H.

The dichotomy between the short and medium run

There is a dichotomy between the price and quantity adjustment in the short and medium run:

1. In the *short* run, H, hence also the supply of housing services, is given. The rental rate R as well as the house price p immediately shifts up (down) if the demand for housing services shifts up (down).

2. In the medium run (i.e., without new disturbances), it is H that adjusts and does so gradually. The adjustment of H is in a direction indicated by the sign of the initial price difference, $p - p^*$, which in turn reflects the initial position of the demand curve in Fig. 15.6. On the other hand, the house price, p, converges towards the cost-determined level, p^*. This price level is constant as long as technical progress in the production of intermediate goods for construction follows the general trend in the economy.

15.2.4 Discussion

In many countries a part of the housing market is under some kind of rent control. Then there is, of course, rationing on the demand side of the housing market. It may still be possible to use the model in a modified version since the part of the housing market, which is *not* under regulation and therefore has a market determined price, p, usually includes the new building activity.

We have carried out partial equilibrium analysis in a simplified framework. Possible refinements of the analysis include considering household optimization with an explicit distinction between durable consumption (housing demand) and non-durable consumption and allowing uncertainty and credit market imperfections. Moreover, a general equilibrium approach would take into account the possible feedbacks on the financial wealth, A, from changes in H and possibly also p.[18] Allowing economic growth with rising wages in the model would also be preferable, so that a steady state with a *growing* housing stock can be considered (a growing housing stock at least in terms of quality-adjusted housing units). A more complete analysis would also include land prices and ground rent.

The issue of housing bubbles After a decade of sharply rising house prices, the US experienced between 2006 and 2009 a fall in house prices of about 30% (Shiller,), in Denmark about 20% (Economic Council, Fall 2011). We argued briefly that in the present model with rational expectations, housing bubbles can be ruled out. Let us here go a little more into detail about the concepts involved.

The question is whether the large volatility in house prices should be seen as reflecting the rise and burst of housing bubbles or just volatility of fundamentals. A *house price bubble* is present if the market price, p_t, of houses for some time systematically exceeds the *fundamental value,* that is, if $p_t > \hat{p}_t$, where $\hat{p}_t$ is the fundamental value (the present value of the expected future services or dividends from the asset). The latter can be found as the solution to the differential equation (15.61), assuming absence of housing price bubbles (see Appendix D).

Our model assumes rational expectations which in the absence of stochastic elements in the model amounts to perfect foresight. What we ruled out by referring to the well-defined reproduction costs of houses was that a *rational* deterministic asset price bubble could occur in the system. A *rational* asset price bubble is an asset price bubble that is consistent with the relevant no-arbitrage condition, here (15.53), when agents have model-consistent expectations. If stochastic elements are added to the model, a rational housing bubble (which would in this case be

[18] Feedbacks from changes in p are more intricate than one might imagine at first glance. In a representative agent model everybody is an average citizen and owns the house she lives in. Nobody is better off by a rise in house prices. In a model with heterogeneous agents, those who own more houses than they use themselves gain by a rise in house prices. And those in the opposite situation lose. Whether and how aggregate consumption is affected depends on differences in the marginal propensity to consume and on institutional circumstances concerning collaterals in credit markets. In two papers by Case, Quigley, and Shiller (2005, 2011) empirical evidence of a positive relationship between consumption and housing wealth in the US is furnished.

stochastic) can still be ruled out (the argument is similar to the one given for the deterministic case).

Including land and unique building sites with specific amenity values into the model will, however, make the argument against rational bubbles less compelling (see, e.g., Kocherlakota, 2011). Moreover, there are reasons to believe that in the real world, expectations are far from always rational. The behavioral finance literature has suggested alternative theories of speculative bubbles where market psychology (herding, fads, etc.) plays a key role. We postpone a more detailed discussion of asset price bubbles to Part VI.

15.3 Literature notes

(incomplete)

Poterba (1984).

Attanasio et al., 2009.

Buiter, Housing wealth isn't wealth, WP, London School of Economics, 20-07-2008.

The question of systematic bias in homebuyer's expectations in four U.S. metropolitan areas over the period 2003-2012 is studied in Case, Shiller, and Thompson (2012), based on questionnaire surveys. See also Cheng, Raina, and Xiong (2012).

Campbell and Cocco, 2007.

Mayer (2011) surveys theory and empirics about the cyclical movement of house prices.

The phenomenon that fast expansion may reduce efficiency when managerial capability is a fixed production factor is known as a *Penrose effect*, so named after a book from 1959 on management by the American economist Edith Penrose (1914-1996). Uzawa (1969) explores Penrose's ideas in different economic contexts. The construction process is sensitive to managerial capability which is a scarce resource in a construction boom.

15.4 Appendix

A. Complementary inputs

In Section 15.1.2 we claimed, without proof, certain properties of the oil demand function and the marginal productivities of capital and labor, respectively, in general equilibrium, given firms' profit maximization subject to a three-factor production function with inputs that exhibit direct complementarity. Here, we use the attributes of the production function F, including (15.2), and the first-order conditions of the representative firm, to derive the claimed signs of the partial derivatives of the functions $M(K, p_M)$, $w(K, p_M)$, and $MPK(K, p_M)$.

First, taking the total derivative w.r.t. K and M in (15.13) gives

$$F_{MK}dK + F_{MM}dM = dp_M.$$

Hence, $\partial M/\partial K = -F_{MK}/F_{MM} > 0$, and $\partial M/\partial p_M = 1/F_{MM} < 0$.

Second, taking the total derivative w.r.t. K and p_M in (15.12) gives

$$dw = F_{LK}dK + F_{LM}(M_K dK + M_{p_M} dp_M).$$

Hence, $\partial w/\partial K = F_{LK} + F_{LM}M_K > 0$, and $\partial w/\partial p_M = F_{LM}M_{p_M} < 0$.

Third, $\partial MPK/\partial p_M = F_{KM}M_{p_M} < 0$, since $F_{KM} > 0$ and $M_{p_M} < 0$. As to the sign of $\partial MPK/\partial K$, observe that

$$\begin{aligned}\partial MPK/\partial K &= F_{KK} + F_{KM}M_K = F_{KK} + F_{KM}(-F_{MK}/F_{MM}) \\ &= \frac{1}{F_{MM}}(F_{KK}F_{MM} - F_{KM}{}^2) < 0,\end{aligned}$$

where the inequality follows from $F_{MM} < 0$, if $F_{KK}F_{MM} - F_{KM}{}^2 > 0$. And the latter inequality does indeed hold. This follows from (15.62) in the lemma below.

Lemma. Let $f(x_1, x_2, x_3)$ be some arbitrary concave $\mathbb{C}^2$-function defined on $\mathbb{R}^3_+$. Assume $f_{ii} < 0$ for $i = 1, 2, 3$, and $f_{ij} > 0, i \neq j$. Then, concavity of f implies that

$$f_{ii}f_{jj} - f_{ij}{}^2 > 0 \quad \text{for } i \neq j. \tag{15.62}$$

Proof. By the general theorem on concave $\mathbb{C}^2$-functions (see Math Tools), f satisfies

$$f_{11} \leq 0, \qquad f_{11}f_{22} - f_{12}{}^2 \geq 0 \text{ and}$$

$$f_{11}(f_{22}f_{33} - f_{23}{}^2) - f_{12}(f_{21}f_{33} - f_{23}f_{31}) + f_{13}(f_{21}f_{32} - f_{22}f_{31}) \leq 0 \tag{15.63}$$

in the interior of $\mathbb{R}^3_+$. Combined with the stated assumptions on f, (15.63) implies (15.62) with $i = 2, j = 3$. In view of symmetry, the numbering of the arguments of f is arbitrary. So (15.62) also holds with $i = 1, j = 3$ as well as $i = 1, j = 2$. □

The lemma applies because F satisfies all the conditions imposed on f in the lemma. First, the direct complementarity condition $f_{ij} > 0, i \neq j$, is directly assumed in (15.2). Second, the condition $f_{ii} < 0$ for $i = 1, 2, 3$ is satisfied by F since, in view of F being neoclassical, the marginal productivities of F are diminishing. Finally, as F in addition to being neoclassical has non-increasing returns to scale, F is concave.

B. The transformation function and the adjustment cost function in Tobin's q-theory

As mentioned in Section 15.2.2 we may formulate the strictly concave transformation function $T(I, H)$ as being equal to $I - G(I, H)$, where the "waste" is represented by an adjustment cost function $G(I, H)$ familiar from Chapter 14. Then, on the one hand, all the properties of G *required* in Chapter 14.1 when $I \geq 0$ are maintained. On the other hand, not all properties required of T in (15.55) need be satisfied in Tobin's q-theory.

As to the first claim, note that when the function $T(I, H) \equiv I - G(I, H)$ has all the properties stated in (15.55), then the function G must, for $(I, H) \in \mathbb{R}^2_+$, satisfy:

$$\begin{aligned}
G(I,H) &= I - T(I,H), \\
G(0,H) &= 0 - T(0,H) = 0, \\
G_I(I,H) &= 1 - T_I(I,H) \geq 0, \text{ with } G_I \geq 0 \text{ for } I \geq 0, \text{ respectively}, \\
G_{II}(I,H) &= -T_{II}(I,H) > 0 \text{ for all } I \geq 0, \\
G_H(I,H) &= -T_H(I,H) \leq 0,
\end{aligned}$$

where the second line is implied by $T_I(0, H) = 1$ and $T_{II} < 0$. These conditions on G for $(I, H) \in \mathbb{R}^2_+$ are exactly those required in Chapter 14.1.

As to second claim, a requirement on the function T in (15.55) is that $T_I(0, H) = 1$ and $T_I(I, H) > 0$ for all $I \geq 0$ at the same time as $T_{II} < 0$. This requires that $0 < T_I(I, H) < 1$ for all $I > 0$. For $G(I, H) = I - T(I, H)$ to be consistent with this, we need that $0 < G_I < 1$ for all $I > 0$. So the G function should not be "too convex" in I. We would have to impose the condition that $\lim_{I\to\infty} G_{II} = 0$ holds with "sufficient" speed of convergence. Whereas for instance

$$G(I,H) = I - A(aI^\beta + (1-a)H^\beta)^{1/\beta}, \quad \text{with } 0 < A < 1, 0 < a < 1, \text{ and } \beta < 0,$$

will do, a function like $G(I/H) = (\alpha/2)I^2/H$, $\alpha > 0$, will *not* do for large I. Nevertheless, the latter function satisfies all conditions required in Tobin's q-theory.

If for some reason one would like to use such a quadratic function to represent waste in construction, one could relax the in (15.55) required condition $T_I(I, H) > 0$ to hold only for I below some upper bound.

Finally, we observe that when $T(I, H) \equiv I - G(I, H)$, then, if the function G is homogeneous of degree k, so is the function T, and vice versa.

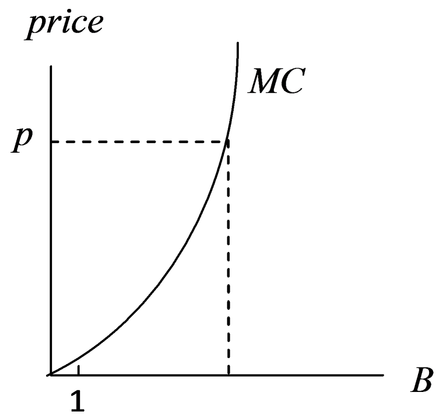

Figure 15.11: Marginal costs in house construction (housing stock given).

C. Interpreting construction behavior in a marginal cost perspective (Section 15.2.2)

We may look at the construction activity of the representative construction firm from the point of view of increasing marginal costs in the short run. First, let $\mathcal{TC}$ denote the total costs per time unit of the representative construction firm. We have $\mathcal{TC} = \bar{f} + \mathcal{TVC}$, where $\bar{f}$ is the fixed cost to management and $\mathcal{TVC}$ is the total variable cost associated with the construction of B $(= T(I, H))$ new houses per time unit, given the economy-wide stock H. All these costs are measured in real terms. We have $\mathcal{TVC} = cI$. The input of intermediates, I, required for building B new houses per time unit is an increasing function of B. Indeed, the equation

$$B = T(I, H), \qquad (*)$$

where $T_I > 0$, defines I as an implicit function of B and H, say $I = \varphi(B, H)$. By implicit differentiation in (*) we find

$$\varphi_B = \partial I/\partial B = 1/T_I(\varphi(B, H), H) > 1, \quad \text{when } I > 0$$

So $\mathcal{TVC} = cI = c\varphi(B, H)$, and short-run marginal cost is

$$\mathcal{MC}(c, B, H) = \frac{\partial \mathcal{TVC}}{\partial B} = c\varphi_B = \frac{c}{T_I(\varphi(B, H), H)} > c, \quad \text{when } I > 0. \qquad (**)$$

CLAIM
(i) The short-run marginal cost, $\mathcal{MC}$, of the representative construction firm is increasing in B.
(ii) The construction sector produces new houses up the point where $\mathcal{MC} = p$.
(iii) The cost of building one new house per time unit is approximately c.

Proof. (i) By (**) and (*),

$$\frac{\partial \mathcal{MC}}{\partial B} = \frac{-cT_{II}(\varphi(B,H),H)\varphi_B}{T_I(\varphi(B,H),H)^2} = \frac{-cT_{II}(\varphi(B,H),H)}{T_I(\varphi(B,H),H)^3} > 0,$$

since $T_I > 0$ and $T_{II} < 0$. (ii) Follows from (**) and the first-order condition (15.56) found in the text. (iii) The cost of building ΔB, when $B = 0$, is $\mathcal{MC}(c, \Delta B, H) \approx [c/T_I(0,H)] \cdot \Delta B = c\Delta B = c$ when $\Delta B = 1$, where we have used (**). □

That it is profitable to produce new houses up the point where $\mathcal{MC} = p$ is illustrated in Fig. 15.11.

D. Solving the no-arbitrage equation for p_t in the absence of house price bubbles (Section 15.2.4)

By definition, if there are no housing bubbles, the market price of a house equals its *fundamental value*, i.e., the present value of expected (possibly imputed) after-tax rental income from owning the house. Denoting the fundamental value $\hat{p}_t$, we thus have

$$\begin{aligned} \hat{p}_t &= (1-\tau_R)\int_t^\infty R(H_s)e^{-(\tau_p+\delta)(s-t)}e^{\tau_R\delta(s-t)}e^{-(1-\tau_r)r(s-t)}ds, \qquad (15.64) \\ &= (1-\tau_R)\int_t^\infty R(H_s)e^{-[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p](s-t)}ds, \end{aligned}$$

where the three discount rates appearing in the first line are, first, $\tau_p + \delta$, which reflects the rate of "leakage" from the investment in the house due to the property tax and wear and tear, second, $\tau_R\delta$, which reflects the tax allowance due to wear and tear, and, finally, $(1-\tau_r)r$, which is the usual opportunity cost discount. In the second row we have done an addition of the three discount rates so as to have just one discount factor easily comparable to the discount factor appearing below.

In Section 15.2.4 we claimed that in the absence of housing bubbles, the linear differential equation, (15.61), implied by the no-arbitrage equation (15.53) under perfect foresight, has a solution p_t equal to the fundamental value of the house, i.e., $p_t = \hat{p}_t$. To prove this, we write (15.61) on the standard form for a linear differential equation,

$$\dot{p}_t + ap_t = -(1-\tau_R)R(H_t), \qquad (15.65)$$

where

$$a \equiv -\left[(1-\tau_r)r + (1-\tau_R)\delta + \tau_p\right] < 0. \qquad (15.66)$$

The general solution to (15.65) is

$$p_t = \left(p_{t_0} - (1-\tau_R)\int_{t_0}^{t} R(H_s)e^{a(s-t_0)}ds \right) e^{-a(t-t_0)}.$$

Multiplying through by $e^{a(t-t_0)}$ gives

$$p_t e^{a(t-t_0)} = p_{t_0} - (1-\tau_R)\int_{t_0}^{t} R(H_s)e^{a(s-t_0)}ds.$$

Rearranging and letting $t \to \infty$, we get

$$p_{t_0} = (1-\tau_R)\int_{t_0}^{\infty} R(H_s)e^{a(s-t_0)}ds + \lim_{t\to\infty} p_t e^{a(t-t_0)}.$$

Inserting (15.66), replacing t by T and t_0 by t, and comparing with (15.64), we see that

$$p_t = \hat{p}_t + \lim_{T\to\infty} p_T e^{-[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p](T-t)}.$$

The first term on the right-hand side is the fundamental value of the house at time t. The second term on the right-hand side thus amounts to the difference between the market price of the house and its fundamental value. By definition, this difference represents a bubble. In the absence of the bubble, the market price, p_t, therefore coincides with the fundamental value.

On the other hand, we see that a bubble being present requires that

$$\lim_{T\to\infty} p_T e^{-[(1-\tau_r)r+(1-\tau_R)\delta+\tau_p](T-t)} > 0.$$

In turn, this requires that the house price is explosive in the sense of ultimately growing at a rate not less than $(1-\tau_r)r + (1-\tau_R)\delta + \tau_p$. The candidate for a bubbly path ultimately moving North-East portrayed in Fig. 15.9 in fact has this property. Indeed, by (15.61), for such a path we have

$$\dot{p}_t/p_t = [(1-\tau_r)r + (1-\tau_R)\delta + \tau_p] - (1-\tau_R)R(H_t)/p_t \to (1-\tau_r)r + (1-\tau_R)\delta + \tau_p \text{ for } t \to \infty,$$

since $p_t \to \infty$ and $R'(H_t) < 0$.

15.5 Exercises

(15.61)